Light

Visible electromagnetic radiation

Power spectrum

Polarization

Photon (quantum effects)

Wave (interference, diffraction)

Topics

Radiometry and photometry

Measuring spatial properties of light

- Radiant power
- Radiant intensity
- Irradiance
  - Inverse square law and cosine law
- Radiance
- Radiant exitance (radiosity)
Radiometry and Photometry

Radiant Energy and Power

- Power: Watts (radiometry) vs. Lumens (photometry)
  - Energy efficiency
  - Spectral efficacy

Energy: Joules vs. Talbot
- Exposure
  - Film response
  - Skin - sunburn

Luminance
\[ Y = \int YV(\lambda)\Phi L(\lambda)d\lambda \]
Radiant Intensity

**Definition**: The radiant (luminous) intensity is the power per unit solid angle emanating from a point source.

\[ I(\omega) \equiv \frac{d\Phi}{d\omega} \]

\[ \frac{W}{sr} \equiv \frac{lm}{sr} = \text{candela} \]
## Angles and Solid Angles

**Angle** \( \theta = \frac{l}{r} \)

\( \Rightarrow \) circle has \( 2\pi \) radians

**Solid angle** \( \Omega = \frac{A}{R^2} \)

\( \Rightarrow \) sphere has \( 4\pi \) steradians

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## Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]
Differential Solid Angles

\[ d\omega = \sin \theta \, d\theta \, d\phi \]

\[ \Omega = \int_{S^2} d\omega \]
\[ = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \sin \theta \, d\theta \, d\phi \]
\[ = \int_{\theta=-\pi}^{\pi} \int_{\phi=-\pi}^{\pi} d\cos \theta \, d\phi \]
\[ = 4\pi \]

Isotropic Point Source

\[ \Phi = \int_{S^2} I \, d\omega \]
\[ = 4\pi I \]
\[ I = \frac{\Phi}{4\pi} \]
Warn’s Spotlight

\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{\mathbf{A}})^s \]

\[ \Phi = \int_{0}^{2\pi} \int_{0}^{1} I(\omega) \cos \theta \, d\varphi \]

Warn’s Spotlight

\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{\mathbf{A}})^s \]

\[ \Phi = \int_{0}^{2\pi} \int_{0}^{1} I(\omega) d \cos \theta \, d\varphi = 2\pi \int_{0}^{1} \cos^s \theta \, d\cos \theta = \frac{2\pi}{s+1} \]
Warn’s Spotlight

\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \vec{A})^s \]

\[ \Phi = \int_0^{2\pi} \int_0^1 I(\omega) \, d\cos\theta \, d\phi = 2\pi \int_0^1 \cos^s \theta \, d\cos\theta = \frac{2\pi}{s+1} \]

\[ I(\omega) = \Phi \frac{s+1}{2\pi} \cos^s \theta \]

Light Source Goniometric Diagrams

1. Porcelain-enameded ventilated standard dome with incandescent lamp
2. Concentric ring unit with incandescent silvered-bowl lamp
3. Pendant diffusing sphere with incandescent lamp
4. R-40 flood with specular anodized reflector skirt; 45° cutoff
PIXAR Point Light Source

UberLight( )
{
  Clip to near/far planes
  Clip to shape boundary
  foreach superelliptical blocker
    atten *= …
  foreach cookie texture
    atten *= …
  foreach slide texture
    color *= …
  foreach noise texture
    atten, color *= …
  foreach shadow map
    atten, color *= …
  Calculate intensity fall-off
  Calculate beam distribution
}

Irradiance
Irradiance

**Definition:** The *irradiance* (illuminance) is the power per unit area incident on a surface.

\[ E(x) \equiv \frac{d\Phi_i}{dA} \]

\[
\left[ \frac{W}{m^2} \right] = \left[ \frac{lm}{m^2} = lux \right]
\]

Sometimes referred to as the radiant (luminous) incidence.

Lambert’s Cosine Law

\[ \Phi = EA \]

\[ E = \frac{\Phi}{A} \]
Lambert’s Cosine Law

\[ E = \frac{\Phi}{A \cos \theta} = \frac{\Phi}{A} \cos \theta \]

Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ d\Phi = I \, d\omega \]

\[ d\omega = \frac{\cos \theta}{r^2} \, dA \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ I \, d\omega = \frac{\Phi}{4\pi} \frac{\cos \theta}{r^2} \, dA \]

\[ E = \frac{\Phi}{4\pi} \frac{\cos \theta}{r^2} \]

Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ I \, d\omega = \frac{\Phi}{4\pi} \frac{\cos \theta}{r^2} \, dA = E \, dA \]
**Irradiance: Isotropic Point Source**

\[ I = \frac{\Phi}{4\pi} \]

\[ h = r \cos \theta \]

\[ E = \frac{\Phi \cos \theta}{4\pi r^2} = \frac{\Phi \cos^3 \theta}{4\pi h^2} \]

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**The Invention of Photometry**

- **Bouguer’s classic experiment**
  - Compare a light source and a candle
  - Move until they both appear equally bright
  - Intensity is proportional to ratio of distances squared

- **Definition of a candela**
  - Originally a “standard” candle
  - Currently 550 nm laser w/ 1/683 W/sr
  - 1 of 6 fundamental SI units
<table>
<thead>
<tr>
<th>Typical Values of Illuminance [lm/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunlight plus skylight               100,000 lux</td>
</tr>
<tr>
<td>Sunlight plus skylight (overcast)    10,000</td>
</tr>
<tr>
<td>Interior near window (daylight)      1,000</td>
</tr>
<tr>
<td>Artificial light (minimum)           100</td>
</tr>
<tr>
<td>Moonlight (full)                     0.02</td>
</tr>
<tr>
<td>Starlight                            0.0003</td>
</tr>
</tbody>
</table>

Radiance
Area Lights – Surface Radiance

**Definition:** The surface *radiance* (*luminance*) is the intensity per unit area leaving a surface.

\[
L(x, \omega) = \frac{dI(x, \omega)}{dA} = \frac{d^2 \Phi(x, \omega)}{d\omega dA}
\]

\[
L(x, \omega) = \frac{W}{sr \ m^2} = \frac{cd}{m^2} = \frac{lm}{sr \ m^2} = \text{nit}
\]

Power Leaving a Surface

\[
d^2 \Phi(x, \omega) = L(x, \omega) d\omega dA
\]
## Typical Values of Luminance [cd/m²]

<table>
<thead>
<tr>
<th>Surface of the sun</th>
<th>2,000,000,000 nit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunlight clouds</td>
<td>30,000</td>
</tr>
<tr>
<td>Clear day</td>
<td>3,000</td>
</tr>
<tr>
<td>Overcast day</td>
<td>300</td>
</tr>
<tr>
<td>Moon</td>
<td>0.03</td>
</tr>
</tbody>
</table>
**Radiant Exitance**

Definition: The radiant (luminous) exitance is the energy per unit area leaving a surface.

\[
M(x) \equiv \frac{d\Phi_o}{dA}
\]

\[
\left[ \frac{W}{m^2} \right] \left[ \frac{lm}{m^2} = \text{lux} \right]
\]

In computer graphics, this quantity is often referred to as the *radiosity* (B).

---

**Power Leaving a Surface**

\[
L(x, \omega) \text{d}\omega \text{d}A
\]

\[
d^2\Phi(x, \omega) = L(x, \omega) \text{d}\omega \text{d}A
\]
**Directional Power Leaving a Surface**

\[ d^2 \Phi_o(x, \omega) = L_o(x, \omega) \cos \theta dA d\omega \]

**Area Light Source**

\[ d^2 \Phi_o(x, \omega) = L_o(x, \omega) \cos \theta dA d\omega \]

*Note cosine*

Same \( dA \) for all directions
Area Light Source

\[ dM(x, \omega) = \frac{d^2 \Phi_o(x, \omega)}{dA} = L_o(x, \omega) \cos \theta \, d\omega \]

Same \( dA \) for all directions

Area Light Source

\[ dM(x, \omega) = L_o(x, \omega) \cos \theta \, d\omega \]

Same \( dA \) for all directions
Area Light Source

\[
M = \int_{H^2} dM(x, \omega) = \int_{H^2} L_o(x, \omega) \cos \theta \, d\omega
\]

\(H^2\) Hemisphere

Uniform Diffuse Emitter

\[
M = \int_{H^2} L_o \cos \theta \, d\omega = L_o \int_{H^2} \cos \theta \, d\omega
\]

\(L_o(x, \omega) = L_o\)
Projected Solid Angle

\[ \tilde{\Omega} \equiv \int_{\Omega} \cos \theta \, d\omega \]

\[ \tilde{\Omega} = \int_{\Omega} \cos \theta \, d\omega = \pi \]
Uniform Diffuse Emitter

\[ M = \int_{H^2} L_o \cos \theta \, d\omega \]

\[ = L_o \int_{H^2} \cos \theta \, d\omega \]

\[ = \pi L_o \]

\[ L_o = \frac{M}{\pi} \]

Radiometry and Photometry

Summary
# Radiometric and Photometric Terms

<table>
<thead>
<tr>
<th>Physics</th>
<th>Radiometry</th>
<th>Photometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>Radiant Energy</td>
<td>Luminous Energy</td>
</tr>
<tr>
<td>Flux (Power)</td>
<td>Radiant Power</td>
<td>Luminous Power</td>
</tr>
<tr>
<td>Flux Density</td>
<td>Irradiance</td>
<td>Illuminance</td>
</tr>
<tr>
<td></td>
<td>Radiosity</td>
<td>Luminosity</td>
</tr>
<tr>
<td>Angular Flux Density</td>
<td>Radiance</td>
<td>Luminance</td>
</tr>
<tr>
<td>Intensity</td>
<td>Radiant Intensity</td>
<td>Luminous Intensity</td>
</tr>
</tbody>
</table>

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# Photometric Units

<table>
<thead>
<tr>
<th>Photometry</th>
<th>Units</th>
<th>MKS</th>
<th>CGS</th>
<th>British</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminous Energy</td>
<td>Talbot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Luminous Power</td>
<td>Lumen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Luminance</td>
<td>Lux</td>
<td></td>
<td>Phot</td>
<td>Footcandle</td>
</tr>
<tr>
<td>Luminosity</td>
<td>Nit, Apostilb, Blondel</td>
<td></td>
<td>Stilb</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Apostilb, Blondel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lambert</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>Candela (Candle, Candlepower, Carcel, Hefner)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?”, James Kajiya