The Light Field

Concepts
- Light field = radiance function on rays
- Conservation of radiance
- Throughput and counting rays
- Measurement equation
- Irradiance calculations

From London and Upton

Light Field = Radiance(Ray)
Field Radiance

Definition: The field radiance (luminance) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction

\[ dA \quad r(x, \omega) \quad d\omega \quad L(x, \omega) \]

Radiance is the quantity associated with a ray

Gazing Ball ⇒ Environment Maps

Miller and Hoffman, 1984

- Photograph of mirror ball
- Reflection direction indexed by normal
- Image is the radiance in the reflected dir.
Environment Maps

Interface, Chou and Williams (ca. 1985)

The Sky Radiance Distribution

From Greenler, Rainbows, halos and glories
Spherical Gantry $\Rightarrow$ 4D Light Field

Capture all the light leaving an object - like a hologram

Multi-Camera Array $\Rightarrow$ Light Field
Two-Plane Light Field

\[ L(u, v, s, t) \]

Properties of Radiance
Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.
   - Radiance is a function on rays
   - All other field quantities are derived from it
2. Radiance invariant along a ray.
   - 5D ray space reduces to 4D
3. Response of a sensor proportional to radiance.

1st Law: Conservation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates

\[ d^2\Phi_1 = d^2\Phi_2 \]
\[ d^2\Phi_1 = L_1d\omega_1dA_1 \]
\[ d^2\Phi_2 = L_2d\omega_2dA_2 \]
\[ d\omega_1dA_1 = \frac{dA_1dA_2}{r^2} = d\omega_2dA_2 \]
\[ \therefore L_1 = L_2 \]
Quiz

Does radiance increase under a magnifying glass?

No!!

Measuring Rays = Throughput
Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) \quad \text{and} \quad dA_2(u_2, v_2) \]

The differential throughput measures size of the beam:

\[ d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2} \]

Parameterizing Rays

Parameterize rays with respect to receiver \( r(u_2, v_2, \theta_2, \phi_2) \)

\[ d\omega_2(\theta_2, \phi_2) \quad \text{and} \quad dA_2(u_2, v_2) \]

\[ d^2T = \frac{dA_1}{|x_1 - x_2|^2} dA_2 = d\omega_2 dA_2 \]
Parameterizing Rays

Parameterize rays wrt to source $r(u_1, v_1, \theta_1, \phi_1)$

$$dA_1(u_1, v_1) \rightarrow \int \ldots \int \ldots d\omega_1(\theta_1, \phi_1)$$

$$d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1$$

Parameterizing Rays

Tilting the surfaces reparameterizes the rays!

$$dA_1(u_1, v_1) \rightarrow \int \ldots \int \ldots dA_2(u_2, v_2)$$

$$d^2T = \frac{\cos \theta_1 \cos \theta_2}{|x_1 - x_2|^2} dA_1 dA_2$$
Parameterizing Rays: $S^2 \times R^2$

Parameterize rays by $r(x, y, \theta, \phi)$

Projected area $\tilde{A}(\tilde{\omega})$

Measuring the number or rays that hit a shape

$T = \int_{S^2} d\omega(\theta, \phi)\, dA(x, y) = \int_{S^2} d\omega(\theta, \phi) \int_{r^2} dA(x, y)$

$= \int_{S^2} \tilde{A}(\theta, \phi)\, d\omega(\theta, \phi)$

Sphere:

$T = 4\pi A = 4\pi^2 R^2$

Parameterizing Rays: $M^2 \times S^2$

Parameterize rays by $r(u, v, \theta, \phi)$

Sphere: $T = \pi S = 4\pi^2 R^2$

Crofton’s Theorem: $4\pi A = \pi S \Rightarrow A = \frac{S}{4}$
The Measurement Equation

Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.

\[ R = \iiint_A \omega dA = L \bar{T} \]

\[ T = \iiint_A dA \bar{T} \]

L is what should be computed and displayed.

T quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered.
Quiz

Does the brightness that a wall appears to the sensor depend on the distance?

No!!

Irradiance
Directional Power Arriving at a Surface

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta \, dA \, d\omega \]

Irradiance from the Environment

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta \, dA \, d\omega \]
\[ dE(x, \omega) = L_i(x, \omega) \cos \theta \, d\omega \]

\[ E(x) = \int_{4\pi} L_i(x, \omega) \cos \theta \, d\omega \]
Irradiance Environment Maps

\[ L(\theta, \varphi) \quad R \quad E(\theta, \varphi) \quad N \]

Radiance Environment Map

Irradiance Environment Map

Irradiance Map or Light Map

Isolux contours
Uniform Area Source

\[
E(x) = \int_{H^2} L \cos \theta d\omega \\
= L \int_{\Omega} \cos \theta d\omega \\
= L \tilde{\Omega}
\]

Uniform Disk Source

**Geometric Derivation**

\[
\tilde{\Omega} = \pi \sin^2 \alpha
\]

**Algebraic Derivation**

\[
\tilde{\Omega} = \int_1^\alpha \int_0^{2\pi} \cos \theta \, d\phi \, d\cos \theta \\
= 2\pi \left. \frac{\cos^2 \theta}{2} \right|_{\cos \alpha}^{\cos \alpha} \\
= \pi \sin^2 \alpha \\
= \pi \frac{r^2}{r^2 + h^2}
\]
### Spherical Source

<table>
<thead>
<tr>
<th>Geometric Derivation</th>
<th>Algebraic Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Geometric Diagram" /></td>
<td>( \Omega = \int \cos \theta , d\omega )</td>
</tr>
</tbody>
</table>

\[
\Omega = \pi \sin^2 \alpha
\]

### The Sun

**Solar constant (normal incidence at zenith)**

- **Irradiance** 1353 W/m²
- **Illuminance** 127,500 lm/m² = 127.5 kilolux

**Solar angle**

\[
\alpha = .25 \text{ degrees} = .004 \text{ radians (half angle)}
\]

\[
\Omega = \pi \sin^2 \alpha \approx \pi \alpha^2 = 6 \times 10^{-5} \text{ steradians}
\]

**Solar radiance**

\[
L = \frac{E}{\Omega} = \frac{1.353 \times 10^3 \text{ W/m}^2}{6 \times 10^{-5} \text{ sr}} = 2.25 \times 10^7 \frac{\text{W}}{\text{m}^2 \cdot \text{sr}}
\]
Polygonal Source

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![Diagram of a polygonal source](image)

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Polygonal Source

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![Diagram of a polygonal source](image)
Consider 1 Edge

\[ A = \gamma \cos \theta = \gamma \hat{N}_E \cdot \hat{N} \]
Lambert’s Formula

\[ \sum_{i=1}^{3} A_i = A_1 - A_2 - A_3 \]

\[ \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{N}_i \cdot \vec{N} \]

Penumbras and Umbras

emitter

occluder

receiver

umbra

tenumbra