Camera Simulation

<table>
<thead>
<tr>
<th>Effect</th>
<th>Cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field of view</td>
<td>Film size, stops and pupils</td>
</tr>
<tr>
<td>Depth of field</td>
<td>Aperture, focal length</td>
</tr>
<tr>
<td>Exposure</td>
<td>Film speed, aperture, shutter</td>
</tr>
<tr>
<td>Motion blur</td>
<td>Shutter</td>
</tr>
</tbody>
</table>

References
Photography, B. London and J. Upton
Optics in Photography, R. Kingslake
The Camera, The Negative, The Print, A. Adams

Topics

Lenses
Focus
Depth of focus / depth of field
Exposure
Lenses

Refraction at a Spherical Lens

\[ n : \text{index of refraction} \]
Angle of Incidence

\[ I = U + \phi \]

The sum of the interior angles is equal to the exterior angle.

Angle of Refraction

\[ I' = U' + \phi \]
Snell’s Law

\[ n' \sin I' = n \sin I \]

Snell’s Law

\[ n' \sin(U' + \phi) = n \sin(U + \phi) \]

\[ I = U + \phi \]

\[ I' = U' + \phi \]
Refraction

\[ n' \sin(U' + \phi) = n \sin(U + \phi) \]

Rays deviate only slightly from the axis

Paraxial Approximation: Small Angles

\[ n'(u' + \phi) = n(u + \phi) \]

\[ \sin A = a = \tan A \]
### Angles to Slopes

\[ u = \frac{h}{z} \quad u' = \frac{h}{z'} \quad \phi = -\frac{h}{R} \]

\[ \sin \theta = a = \tan \theta \]

### Gauss' Formula

**Paraxial approximation to Snell's Law**

\[ n'(u' + \phi) = n(u + \phi) \]

**Ray coordinates**

\[ u = \frac{h}{z} \quad u' = \frac{h}{z'} \quad \phi = -\frac{h}{R} \]

\[ n'(\frac{h}{z'} - \frac{h}{R}) = n(\frac{h}{z} - \frac{h}{R}) \]

\[ \frac{n'}{z'} = \frac{n}{z} + \frac{(n' - n)}{R} \]

\[ \leftarrow \text{Holds for any height, any ray!} \]
Thin Lens Demonstration

http://www.physics.metu.edu.tr/~bucurgat/ntnujava/Lens/lens_e.html

Gauss’ Ray Tracing Construction

Parallel Ray
Focal Ray
Chief Ray
Object
Image
Conjugate Points

To focus: move lens relative to backplane
Horizontal rays converge on focal point in the focal plane

\[ \frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \]

Perspective Transformation

Thin lens equation – how z transforms?

\[ \frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \implies z' = \frac{fz}{z + f} \]

\[ \Rightarrow x' = \frac{fx}{z + f} \]

\[ \Rightarrow y' = \frac{fy}{z + f} \]

Represent lens transformation as a 4x4 matrix
Ray Tracing: Finite Aperture

1. Pick a point on image plane \( x' \)
2. Pick a point on the lens \( u \)
3. Transform \( x' \) to \( x \); form ray \((u,x-u)\)

Real Lens

Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, *Optics in Photography*
## Double Gauss

Data from W. Smith, Modern Lens Design, p 312

<table>
<thead>
<tr>
<th>Radius (mm)</th>
<th>Thick (mm)</th>
<th>$n_d$</th>
<th>V-no</th>
<th>aperture</th>
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<td>40.0</td>
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</tbody>
</table>

## Ray Tracing Through Lenses

From Kolb, Mitchell and Hanrahan (1995)

- 200 mm telephoto
- 35 mm wide-angle
- 50 mm double-gauss
- 16 mm fisheye
Depth of Field

From London and Upton

CS348B Lecture 7

Pat Hanrahan, 2009
Circle of Confusion

Circle of confusion proportional to the size of the aperture

\[
\frac{c}{a} = \frac{d'}{z'} = \frac{s' - z'}{z'}
\]

Resolving Power

- **Diffraction limit**
  \[
  c = 1.22 \frac{f}{\lambda} \quad [= 1.22 \times 64 \times .500\,\mu m = 0.040\,\text{mm}]
  \]

- **35mm film (Leica standard)**
  \[
  c = 0.025\,\text{mm}
  \]

- **CCD/CMOS pixel aperture**
  \[
  c = 0.0116\,\text{mm} \, \text{(Nikon D1)}
  \]
Depth of Focus [Image Space]

Depth of focus $\equiv$

Equal circles of confusion

Extreme planes: near and far

$$\frac{c}{a} = \frac{d'_f}{z'_f} = \frac{s' - z'_f}{z'_f}$$

$$\frac{c}{a} = \frac{d'_n}{z'_n} = \frac{z'_n - s'}{z'_n}$$
Depth of Focus [Image Space]

Depth of focus $\equiv$

Equal circles of confusion

\[
\frac{1}{z'_f} = \frac{1}{s'} \left( 1 + \frac{c}{a} \right) \quad \frac{1}{z'_n} = \frac{1}{s'} \left( 1 - \frac{c}{a} \right)
\]

\[
\frac{1}{z'_f} + \frac{1}{z'_n} = 2 \frac{1}{s'}
\]

\[
\frac{1}{z'_f} - \frac{1}{z'_n} = \frac{2c}{a} \frac{1}{s'}
\]

Depth of Field [Object Space]

Depth of field $\equiv$

Equal circles of confusion

\[
\frac{1}{z_n} + \frac{1}{z_f} = 2 \frac{1}{s}
\]

\[
\frac{1}{z_n} - \frac{1}{z_f} = \frac{2c}{a} \left( \frac{1}{f} - \frac{1}{s} \right) = \frac{2c}{a} \frac{1}{f}
\]
Hyperfocal Distance

When

\[ \frac{1}{z_n} - \frac{1}{z_f} = \frac{2c}{a} \frac{1}{f} = 2 \frac{cN}{f^2} = \frac{2}{H} \]

\[ N = \frac{a}{f} \]

\[ \frac{1}{z_n} + \frac{1}{z_f} = 2 \frac{1}{s} \]

When

\[ s \rightarrow H \Rightarrow z_n = \frac{H}{2}, z_f = \infty \]

H is the hyperfocal distance

Factors Affecting DOF


\[ \frac{1}{H} = \frac{cN}{f^2} \]
Exposure

Image Irradiance

\[ E = \int_{\Omega} L \cos \theta \, d\omega = L \pi \sin^2 \theta = L \frac{\pi}{4} \left( \frac{a}{f} \right)^2 \]
**Uniform Area Source**

\[ E(x) = \int_{\Omega^2} L \cos \theta \, d\omega \]
\[ = L \int_{\Omega} \cos \theta \, d\omega \]
\[ = L \tilde{\Omega} \]

**Uniform Disk Source**

**Geometric Derivation**

\[ \tilde{\Omega} = \pi \sin^2 \alpha \]

**Algebraic Derivation**

\[ \tilde{\Omega} = \int_{1}^{\cos \alpha} \int_{0}^{2\pi} \cos \theta \, d\phi \, d\cos \theta \]
\[ = 2\pi \left[ \frac{\cos^2 \theta}{2} \right]_{1}^{\cos \alpha} \]
\[ = \pi \sin^2 \alpha \]
\[ = \pi \frac{r^2}{r^2 + h^2} \]
Relative Aperture or F-Stop

\[ a = \frac{f}{N} \]

F-Number and exposure:

\[ E = L \pi \frac{1}{4 N^2} \]

F-stops: 1.4 2 2.8 4.0 5.6 8 11 16 22 32 45 64
1 stop doubles exposure

Camera Exposure

Exposure \[ H = E \times T \]

Exposure overdetermined

Aperture: f-stop - 1 stop doubles \( H \)
Decreases depth of field

Shutter: Doubling the open time doubles \( H \)
Increases motion blur
Aperture vs Shutter

From London and Upton

Field of View
Field of View

From London and Upton
Field of View

Field of view

Redrawn from Kingslake, *Optics in Photography*

\[
\tan \left( \frac{\text{fov}}{2} \right) = -\frac{\text{filmsize}}{f}
\]

Types of lenses

- **Normal** 26°
  
  Film diagonal ~ focal length

- **Wide-angle** 75-90°

- **Narrow-angle** 10°