The Light Field

Concepts
- Light field = radiance function on rays
- Conservation of radiance
- Throughput and counting rays
- Measurement equation
- Irradiance calculations

Light Field = Radiance(Ray)

From London and Upton
**Field Radiance**

**Definition:** The field *radiance (luminance)* at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction.

\[ r(x, \omega) \]

\[ dA \]

\[ d\omega \]

\[ L(x, \omega) \]

Radiance is the quantity associated with a ray.

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**Gazing Ball Environment Maps**

Miller and Hoffman, 1984

- Photograph of mirror ball
- Reflection direction indexed by normal
- Image is the radiance in the reflected dir.
The Sky Radiance Distribution

From Greenler, Rainbows, halos and glories

Spherical Gantry $\Rightarrow$ 4D Light Field

$L(x, y, \theta, \varphi)$

Capture all the light leaving an object - like a hologram
Multi-Camera Array $\Rightarrow$ Light Field

Two-Plane Light Field

$L(u,v,s,t)$
Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.
   - Radiance is a function on rays
   - All other field quantities are derived from it

2. Radiance invariant along a ray.
   - 5D ray space reduces to 4D

3. Response of a sensor proportional to radiance.
1st Law: Conservation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates.

\[ d^2 \Phi_1 = d^2 \Phi_2 \]

\[ d^2 \Phi_1 = L_1 d\omega_1 dA_1 \]

\[ d^2 \Phi_2 = L_2 d\omega_2 dA_2 \]

\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \]

\[ \therefore L_1 = L_2 \]

Quiz

Does radiance increase under a magnifying glass?

No!!
Measuring Rays = Throughput

Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements:

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) dA_2(u_2, v_2) \]

The differential throughput measures size of the beam:

\[ d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2} \]
Parameterizing Rays

Parameterize rays wrt to receiver \( r(u_2, v_2, \theta_2, \phi_2) \)

\[
d\omega_2(\theta_2, \phi_2) \quad \bigcirc \quad \bigcirc \quad dA_2(u_2, v_2)
\]

\[
d^2T = \frac{dA_1}{|x_1 - x_2|^2} dA_2 = d\omega_2 dA_2
\]

Parameterizing Rays

Parameterize rays wrt to source \( r(u_1, v_1, \theta_1, \phi_1) \)

\[
dA_1(u_1, v_1) \quad \bigcirc \quad \bigcirc \quad d\omega_1(\theta_1, \phi_1)
\]

\[
d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1
\]
Parameterizing Rays

Tilting the surfaces reparameterizes the rays!

\[ dA_1(u_1, v_1) \rightarrow dA_2(u_2, v_2) \]

\[ d^2T = \frac{\cos\theta_1 \cos\theta_2}{|x_1 - x_2|^2} dA_1 dA_2 \]

Parameterizing Rays: \( S^2 \times \mathbb{R}^2 \)

Parameterize rays by \( r(x, y, \theta, \phi) \)

Projected area 

\[ \tilde{A}(\tilde{\omega}) \]

Measuring the number or rays that hit a shape

\[ T = \int_{S^2} d\omega(\theta, \varphi) dA(x, y) = \int_{S^2} d\omega(\theta, \varphi) \int_{\mathbb{R}^2} dA(x, y) \]

\[ = \int_{S^2} \tilde{A}(\theta, \varphi) d\omega(\theta, \varphi) \quad \text{Sphere:} \]

\[ = 4\pi \tilde{A} \]

\[ T = 4\pi \tilde{A} = 4\pi^2 R^2 \]
Parameterizing Rays: $M^2 \times S^2$

Parameterize rays by $r(u, v, \theta, \phi)$

\[
T = \frac{1}{\sin \theta} \int_{\mathcal{H}^2(\mathbf{N})} \cos \theta \, d\omega(\theta, \varphi) \int_{M^2} dA(u, v)
\]

Sphere: $T = \pi S = 4\pi^2 R^2$

Crofton's Theorem: $4\pi \bar{A} = \pi S \Rightarrow \bar{A} = \frac{S}{4}$

The Measurement Equation
Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.

\[ R = \int \int_{A \Omega} Ld\omega dA = \overline{L}T \quad T = \int \int_{A \Omega} d\omega dA \]

\(L\) is what should be computed and displayed.

\(T\) quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered.

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Quiz

Does the brightness that a wall appears to the sensor depend on the distance?

\[ \text{No!!} \]
Irradiance

**Directional Power Arriving at a Surface**

\[
d^2 \Phi_i (x, \omega) = L_i(x, \omega) \cos \theta \, dA \, d\omega
\]
Irradiance from the Environment

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta \ dA \ d\omega \]

\[ dE(x, \omega) = L_i(x, \omega) \cos \theta \ d\omega \]

\[ E(x) = \int_{H^2} L_i(x, \omega) \cos \theta \ d\omega \]

Irradiance Map or Light Map

Isolux contours
**Irradiance Environment Maps**

\[ L(\theta, \varphi) \quad \text{Radiance Environment Map} \]

\[ E(\theta, \varphi) \quad \text{Irradiance Environment Map} \]

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**Uniform Area Source**

\[
E(x) = \int_{H^2} L \cos \theta \, d\omega \\
= L \int_{\Omega} \cos \theta \, d\omega \\
= L \tilde{\Omega}
\]
### Uniform Disk Source

**Geometric Derivation**

\[ \tilde{\Omega} = \pi \sin^2 \alpha \]

**Algebraic Derivation**

\[
\tilde{\Omega} = \int_{0}^{\cos \alpha} \int_{1}^{2 \pi} \cos \theta \, d\phi \, d \cos \theta \\
= 2\pi \cos^2 \theta \left[ \cos \alpha \right]_{1}^{\cos \alpha} \\
= \pi \sin^2 \alpha \\
= \pi \frac{r^2}{r^2 + h^2}
\]

### Spherical Source

**Geometric Derivation**

\[ \tilde{\Omega} = \pi \sin^2 \alpha \]

**Algebraic Derivation**

\[
\tilde{\Omega} = \int \cos \theta \, d\omega \\
= \pi \sin^2 \alpha \\
= \pi \frac{r^2}{R^2}
\]
The Sun

Solar constant (normal incidence at zenith)

Irradiance 1353 W/m²
Illuminance 127,500 lm/m² = 127.5 kilolux

Solar angle

$\alpha = .25$ degrees = .004 radians (half angle)

$\tilde{\Omega} = \pi \sin^2 \alpha \approx \pi \theta^2 \times 10^{-5}$ steradians

Solar radiance

$L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \, \text{W/m}^2}{6 \times 10^{-5} \, \text{sr}} = 2.25 \times 10^7 \frac{\text{W}}{\text{m}^2 \cdot \text{sr}}$

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Polygonal Source

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Consider 1 Edge

\[ \gamma_i \text{ Area of sector} \]

\[ \gamma \vec{N}_E \cdot \theta \]

\[ A = \gamma \cos \theta = \gamma \vec{N}_E \cdot \vec{N} \]

Lambert’s Formula

\[ \sum_{i=1}^{3} A_i = A_1 - A_2 - A_3 \]

\[ \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{N}_i \cdot \vec{N} \]
Penumbras and Umbras