Biased Monte Carlo Ray Tracing

Last time
- Path tracing
- Bidirectional ray tracing

Today
- Filtering
- Irradiance caching
- Photon mapping

Based on a lecture by Henrik Wann Jensen
The Problem: Noise

10 paths / pixel

The Idea

The noise is high frequency

Remove high frequencies using filtering
Despeckle
Unfiltered

Median Filter
Energy Preserving Filters

Distribute noisy energy over several pixels
Similar in spirit to “error diffusion”

- Adaptive filter width [Rushmeier and Ward 940]
- Diffusion style filters [McCool 99]
- Splatting style filters [Suykens and Willems 00]

Problems with Filters

Everything is blurred

- Textures
- Highlights
- Caustics
- ...
Caching Techniques

Irradiance caching

- Compute irradiance at selected points and then interpolate

Photon mapping

- Trace “photons” from the lights and store them in a “photon map.” Gather light from photon map during eye ray tracing
Direct Lighting

10 samples / pixel

Direct + Indirect Illumination

10 paths / pixel
Indirect Illumination

Irradiance Caching: Idea

“A ray tracing solution for diffuse interreflection.”, G. Ward, F. Rubinstein, R. Clear, SIGGRAPH 1988

Idea: Irradiance changes slowly ➔ Interpolate
Irradiance Calculation

\[ E(x) = \int_{\mathbb{H}^2} L_t(x, \omega) \cos \theta \, d\omega \]

Change in Irradiance

\[ \epsilon(x) \leq \left| \frac{\partial E}{\partial x} (x - x_0) + \frac{\partial E}{\partial \theta} (\theta - \theta_0) \right| \]

Position \quad Rotation

\[ \leq E_0 \left( \frac{4}{\pi} \frac{||x - x_0||}{x_{avg}} + \sqrt{2 - 2 \vec{N}(x) \cdot \vec{N}(x_0)} \right) \]

Derivation in Ward paper
Irradiance Interpolation

\[ E(x) = \frac{\sum_i w(x_i) E_i(x_i)}{\sum_i w(x_i)} \]

\[ w(x) = \frac{1}{\epsilon(x)} \approx \frac{1}{\frac{|x-x_0|}{x_{avg}} + \sqrt{2 - 2\vec{N}(x) \cdot \vec{N}(x_0)}} \]

Irradiance Caching Algorithm

Find all samples with \( w(x) > q \)

if ( samples found )
    interpolate
else
    compute new irradiance

N.B. Subsample the image first and then fill in
Irradiance Caching

Irradiance Cache Positions
Irradiance Caching

Irradiance Cache Positions
Irradiance Caching

5000 sample rays; \( w > 10 \)

Irradiance Cache Positions

5000 sample rays; \( w > 10 \)
Unbiased and Consistent

Unbiased

\[ E[X] = \int ... \]

Consistent

\[ \lim_{N \to \infty} E[X] = \int ... \]

Unbiased and Consistent

Unbiased

\[ \frac{1}{N} \sum_{i=1}^{N} f(x_i) \]

Consistent

\[ \frac{1}{N + 1} \sum_{i=1}^{N} f(x_i) \]
Biased can be Good

A biased estimate might have much lower variance
A biased estimate may look better (less noise)

Photon Mapping
Caustics

Path tracing: 1000 paths/pixels

A simple test scene
Rendering

Photon Tracing
Radiance Estimate

\[ L(x, \omega) = \int f_r(x, \omega', \omega) L(x, \omega') \cos \theta' \, d\omega' \]

\[ = \int f_r(x, \omega', \omega) \frac{d\Phi^2(x, \omega')}{d\omega \cos \theta' \, dA} \cos \theta' \, d\omega' \]

\[ = \int f_r(x, \omega', \omega) \frac{d\Phi^2(x, \omega')}{dA} \]

\[ \approx \sum_{p=1}^{n} f_r(x, \omega'_p, \omega) \frac{\Delta \Phi_p(x, \omega'_p)}{\pi r^2} \]
Photon Map Data Structure

The photons are stored in a kd-tree

```c
struct photon = {
    float position[3];
    rgbe power;
    char phi, theta; // low-res direction
};
```
Rendering: Caustics

Caustic from a Glass Sphere

Photon mapping: 10000 photons, 50 photons in estimate
CS348B Lecture 15
Pat Hanrahan, Spring 2010
Caustic from a Glass Sphere

Sphereflake Caustic
Reflection Inside a Metal Ring

50000 photons, 50 photons in radiance estimate

Caustics from Glossy Surfaces

340000 photons, 100 photons in radiance estimate
HDR Caustic from the Environment

Caustic from a Cognac Glass
Direct and Indirect Illumination

100000 photons, 50 photons in radiance estimate
CS348B Lecture 15
Pat Hanrahan, Spring 2010

Direct and Indirect Illumination

500000 photons, 500 photons in radiance estimate
CS348B Lecture 15
Pat Hanrahan, Spring 2010

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Fast Estimate of Indirect

A Simple Test Scene

200 photons, 50 photons in radiance estimate

Rendering
Direct Illumination

Specular Reflection
Caustics

Indirect Illumination
Features

Photon map is
- Consistent
- The error is local

Illumination representation decoupled from geometry

Cornell Box

200000 global photons, 50000 caustic photons
Cornell Box: Photons

200000 global photons

Fractal Cornell Box

200000 global photons, 50000 caustic photons