Ray Tracing

Ray Tracing 1
- Basic algorithm
- Overview of pbrt
- Ray-surface intersection (triangles, ...)

Ray Tracing 2
- Problem: brute force = |Image| x |Objects|
- Acceleration data structures

Primitives

pbrt primitive base class
- Shape
- Material (reflection and emission)

Subclasses
- Primitive instance
  - Transformation and pointer to a primitive
- Aggregate (collection)
  - Treat collections just like single primitives
  - Incorporate acceleration structures into collections
  - May nest accelerators of different types
  - Types: grid.cpp and kdtree.cpp
Uniform Grids

Preprocess scene
1. Find bounding box

Uniform Grids

Preprocess scene
1. Find bounding box
2. Determine resolution

\[ n_v = n_x n_y n_z \propto n_o \]
\[ \max(n_x, n_y, n_z) = d \sqrt[3]{n_o} \]
Uniform Grids

Preprocess scene
1. Find bounding box
2. Determine resolution
   \[ \max(n_x, n_y, n_z) = d \sqrt[3]{n_0} \]
3. Place object in cell, if object overlaps cell
4. Check that object's surface intersects cell
Uniform Grids

Preprocess scene
Traverse grid
3D line – 3D-DDA
6-connected line

Section 4.3

Caveat: Overlap

Problem: Don’t output first intersection found!
Caveat: Overlap

Problem: Don’t output first intersection found!

Problem: Redundant intersection tests

Solution: Mailboxes
- Assign each ray an increasing number
- Primitive intersection cache (mailbox)
  - Store last ray number tested in mailbox
  - Only intersect if ray number is greater
Spatial Hierarchies

Letters correspond to planes (A)

Spatial Hierarchies

Letters correspond to planes (A, B)
Point Location by recursive search
Spatial Hierarchies

Letters correspond to planes (A, B, C, D)

Variations

kd-tree  oct-tree  bsp-tree
Ray Traversal Algorithms

Recursive inorder traversal
[Kaplan, Arvo, Jansen]

\[ t^* = \frac{S - O(a)}{D[a]} \]

\[ \text{Intersect}(L, t_{\text{min}}, t_{\text{max}}) \]
\[ \text{Intersect}(R, t_{\text{min}}, t_{\text{max}}) \]
\[ \text{Intersect}(L, t_{\text{min}}, t^*) \]
\[ \text{Intersect}(R, t^*, t_{\text{max}}) \]

How to Build the Hierarchy?
Build Hierarchy Top-Down

Methods to choose axis and splitting plane

- Midpoint
- Median cut (balanced)
- Surface area heuristic

Cost

What is the cost of tracing a ray through a node?

\[
\text{Cost}(\text{node}) = \text{C}_{\text{trav}} + \text{Prob(hit L)} \times \text{Cost}(\text{L}) + \text{Prob(hit R)} \times \text{Cost}(\text{R})
\]

- \text{C}_{\text{trav}} = \text{cost of traversing a cell}
- \text{Cost}(\text{L}) = \text{cost of traversing left child}
- \text{Cost}(\text{R}) = \text{cost of traversing right child}
Splitting with Cost in Mind

From Gordon Stoll

Split in the Middle = Bad!

From Gordon Stoll

Makes the L & R probabilities equal
Cost of R greater than cost of L
Split at the Median = Bad!

Makes the L & R costs equal
Probability of hitting L greater than R

Cost-Optimized Split = Good!

Cost(cell) = C_trav + Prob(hit L) * Cost(L) + Prob(hit R) * Cost(R)
Cost

Need the probabilities

- Turns out to be proportional to surface area

Need the child cell costs

- Triangle count is a good approximation

Cost(cell) = C_{trav} + SA(L) * TriCount(L) + SA(R) * TriCount(R)

C_{trav} is the ratio of the cost to traverse to the cost to intersect

- C_{trav} = 1:80 in pbrt
- C_{trav} = 1:1.5 in a highly optimized version

Projected Area and Ray Intersection

Number of rays in a given direction that hit an object is proportional to its projected area

\[ A \]
Projected Area and Surface Area

Number of rays in a given direction that hit an object is proportional to its projected area

\[ A \]

The total number of rays hitting an object is \(4\pi \bar{A}\)

Crofton’s Theorem:

For a convex body \(\bar{A} = \frac{S}{4}\)

For a sphere \(S = 4\pi r^2\) and \(\bar{A} = A = \pi r^2\)

Surface Area and Ray Intersection

The probability of a ray hitting a convex shape enclosed by another convex shape is

\[ \Pr[r \cap S_o \mid r \cap S_c] = \frac{S_o}{S_c} \]
Sweep Build Algorithm

\[ 2n \text{ splits} \]

\[ \begin{align*}
    p_a &= \frac{S_a}{S} \\
    n_a &= \frac{S_a}{S} \\
    p_b &= \frac{S_b}{S} \\
    n_b &= \frac{S_b}{S}
\end{align*} \]

Basic Build Algorithm (Triangles)

1. Pick an axis, or optimize across x, y, z
2. Build a set of “candidate” split locations
   - Note: Cost extrema must be at bbox vertices
     - Vertices of triangle
     - Vertices of triangle clipped to node bbox
3. Sort the triangles into intervals
4. Sweep to incrementally track L/R counts, cost
5. Output position of minimum cost split

Running time:

\[ T(N) = N \log N + 2T(N/2) \]
\[ T(N) = N \log^2 N \]
Termination Criteria

When should we stop splitting?

- **Bad**: depth limit, number of triangles
- **Good**: When split does not lower the cost

Threshold of cost improvement

- Stretch over multiple levels
- For example, if cost doesn’t go down after three splits in a row, terminate

Threshold of cell size

- Absolute probability $\text{SA(node)}/\text{SA(scene)}$ small

---

Best Reported Timings

<table>
<thead>
<tr>
<th>Scene</th>
<th>Framerate (FPS) @ 1024x1024 resolution</th>
<th>OpenRT @ 2.5 GHz P4 1 thread</th>
<th>MLRTA @ 2.4 GHz P4 1 thread</th>
<th>MLRTA @ 3.2 GHz P4 with HT 2 threads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erw6 884</td>
<td>- shader</td>
<td>7.1</td>
<td>70.2</td>
<td>109.8</td>
</tr>
<tr>
<td></td>
<td>+ shader</td>
<td>2.3</td>
<td>37.8</td>
<td>50.7</td>
</tr>
<tr>
<td>Conference 274K</td>
<td>- shader</td>
<td>4.55</td>
<td>11.2</td>
<td>19.5</td>
</tr>
<tr>
<td></td>
<td>+ shader</td>
<td>1.93</td>
<td>9.5</td>
<td>15.6</td>
</tr>
<tr>
<td>Soda Hall 2195K</td>
<td>- shader</td>
<td>4.12</td>
<td>21.1</td>
<td>35.5</td>
</tr>
<tr>
<td></td>
<td>+ shader</td>
<td>1.8</td>
<td>15.3</td>
<td>24.1</td>
</tr>
</tbody>
</table>

Reshetov, Soupikov, Hurley, SIGGRAPH 2005
**Superoptimizations**

Lots of optimizations
- Carefully written inner loop (no recursion)
- Use vector instructions SSE2
- 64 bits per kd-tree node
  - 32 bit position
  - 32 bit pointer to pair of child nodes
  - 2 bits for split plane direction (x, y, or z)
- Trace packet of rays
  - 4 or more rays at a time
- Intersect beam at top of tree
- Encourage empty nodes
- Special case axis-aligned triangles
- …

---

**Theoretical Nugget 1**

**Computational geometry of ray shooting**

1. Triangles (Pellegrini)
   - **Time:** $O(\log n)$
   - **Space:** $O(n^{5+\varepsilon})$

2. Sphere (Guibas and Pellegrini)
   - **Time:** $O(\log^2 n)$
   - **Space:** $O(n^{5+\varepsilon})$
Theoretical Nugget 2

Optical computer = Turing machine
Reif, Tygar, Yoshida

Determining if a ray starting at \( y_0 \) arrives at \( y_n \) is undecidable

\[
y = y + 1
\]

\[
y = -2y
\]

\[
\text{if}( y > 0 )
\]