Biased Monte Carlo Ray Tracing
CS348B

Alexis Haraux
(with Pat Hanrahan and Henrik Wann Jensen's slides)

Biased Monte Carlo Ray Tracing

Last time
- Path tracing
- Bidirectional ray tracing

Today
- Irradiance caching
- Photon mapping
Classic Ray Tracing

Forward (from eye): \( E S^* (D|G) L \)

Photon Paths

Radiosity

Caustics

From Heckbert
Irradiance Caching

Caching Techniques

Irradiance caching

- Compute irradiance at selected points and then interpolate

Photon mapping

- Trace “photons” from the lights and store them in a “photon map.” Gather light from photon map during eye ray tracing
Direct Lighting

Direct + Indirect Illumination
Indirect Illumination (Irradiance)

Irradiance Caching: Idea

“A ray tracing solution for diffuse interreflection.”,
G. Ward, F. Rubinstein, R. Clear, SIGGRAPH 1988

Idea: Irradiance changes slowly → Interpolate

Can use any irradiance calculation technique
Irradiance Calculation

\[ E(x) = \int_{\mathbb{S}^2} L_t(x, \omega) \cos \theta \, d\omega \]

Change in Irradiance

\[ \epsilon(x) \leq \left| \frac{\partial E}{\partial x}(x - x_0) + \frac{\partial E}{\partial \theta}(\theta - \theta_0) \right| \]

Position Rotation

\[ \leq E_0 \left( \frac{4}{\pi} \frac{||x - x_0||}{x_{avg}} + \sqrt{2 - 2\vec{N}(x) \cdot \vec{N}(x_0)} \right) \]

Derivation in Ward paper
Irradiance Interpolation

\[ E(x) = \frac{\sum_i w(x_i) E_i(x_i)}{\sum_i w(x_i)} \]

\[ w(x) = \frac{1}{\epsilon(x)} \approx \frac{1}{\frac{||x-x_0||}{x_{avg}} + \sqrt{1 - \vec{N}(x) \cdot \vec{N}(x_0)}} \]

Irradiance Caching Algorithm

Find all samples with \( w(x) > q \)

if (samples found)
    \interpolate
else
    \compute new irradiance

N.B. Subsample the image first and then fill in
Irradiance Caching

100 sample rays; \( w > 10 \)

Irradiance Cache Positions

100 sample rays; \( w > 10 \)
Irradiance Caching

100 sample rays; w > 20

Irradiance Cache Positions

100 sample rays; w > 20
Unbiased and Consistent

Unbiased

\[ E[X] = \int \ldots \]

Consistent

\[ \lim_{N \to \infty} E[X] = \int \ldots \]

Unbiased and Consistent

Unbiased

\[ \frac{1}{N} \sum_{i=1}^{N} f(x_i) \]

Consistent

\[ \frac{1}{N + 1} \sum_{i=1}^{N} f(x_i) \]
Unbiased and Consistent

Graphics interpretation:

• Consistent: Image approaches correct solution as some parameter is increased

• Unbiased: Produces correct image on average

Biased can be Good

A biased estimate might have much lower variance
A biased estimate may look better (less noise)
Caustics using Path Tracing

Path tracing: 1000 paths/pixels

Noisy
A Simple Test Scene

Photon Tracing
Photons

The photons are stored in a kd-tree

```c
struct photon = {
    float position[3];
    rgbe power;
    char phi, theta; // low-res direction
};
```
Radiance Estimate

Density Estimation

• Take a small spherical neighborhood containing k photons
• Count the power of the photons inside
• Compute enclosed surface area
• power / area = density

\[ L(x, \omega) = \int f_r(x, \omega', \omega) L(x, \omega') \cos \theta' \, d\omega' \]

\[ = \int f_r(x, \omega', \omega) \frac{d\Phi^2(x, \omega')}{d\omega \cos \theta' \, dA} \cos \theta' \, d\omega' \]

\[ = \int f_r(x, \omega', \omega) \frac{d\Phi^2(x, \omega')}{dA} \]

\[ \approx \sum_{p=1}^{n} f_r(x, \omega_p', \omega) \frac{\Delta \Phi_p(x, \omega_p')}{\pi r^2} \]
Direct visualization of radiance estimate

100000 photons, 50 photons in radiance estimate

500000 photons, 500 photons in radiance estimate

Direct visualization of radiance estimate

500000 photons, 500 photons in radiance estimate
**Fast Estimate of Indirect**

![Image](image1.png)

200 photons, 50 photons in radiance estimate

**Cornell Box: Photons**

![Image](image2.png)

200,000 global photons
Photon Map

Two photon maps:

- Global photon map (low frequency variation)
- Caustics photon map (high frequency detail)

Different Types of Paths

1. Direct lighting
2. Whitted ray tracing
3. Caustics (caustics photon map)
4. Diffuse interreflection (global photon map)
5. Glossy reflection (global photon map)

Need the right method for each type!
Direct Illumination

Path:
Specular Reflection

Path:

Caustics

Photon Path:
Eye Path:
Indirect Illumination (Final Gather)

Photon Path: \( L(S|D) \times D \)

Eye Path: \( ES \times D \)

What about convergence?

Photon Mapping is
• Biased
• Consistent
Cornell Box

200000 global photons, 50000 caustic photons

Fractal Cornell Box

200000 global photons, 50000 caustic photons
Caustic from a Glass Sphere

Photon mapping: 10000 photons, 50 photons in estimate

Path tracing: 1000 paths/pixels
Sphereflake Caustic

Reflection Inside a Metal Ring

50000 photons, 50 photons in radiance estimate
Caustics from Glossy Surfaces

Lambertian

340000 photons, 100 photons in radiance estimate

HDR Caustic from the Environment
Caustic from a Cognac Glass