Light

Visible electromagnetic radiation

Power spectrum

Polarization

Photon (quantum effects)

Wave (interference, diffraction)

Radiometry and Photometry

Measuring spatial properties of light

- Radiant power
- Radiant intensity
- Irradiance
  - Inverse square law and cosine law
- Radiance
- Radiant exitance (radiosity)

*Goal is to perform lighting calculations in the physically correct way*
Radiometry and Photometry

Radiant Energy and Power

Power: Watts (radiometry) vs. Lumens (photometry)
- Spectral efficacy
- Energy efficiency

Energy: Joules vs. Talbot
- Exposure
  - Film response
  - Skin - sunburn

Photometric luminance

\[ Y = \int V(\lambda) L(\lambda) d\lambda \]
Radiant Intensity

**Definition:** The radiant (luminous) intensity is the power per unit solid angle emanating from a point source.

\[ I(\omega) \equiv \frac{d\Phi}{d\omega} \]

\[
\left[ \frac{W}{sr} \right] \left[ \frac{lm}{sr} = cd = candela \right]
\]
Angles and Solid Angles

■ Angle \( \theta = \frac{l}{r} \)
  \( \Rightarrow \) circle has \( 2\pi \) radians

■ Solid angle \( \Omega = \frac{A}{R^2} \)
  \( \Rightarrow \) sphere has \( 4\pi \) steradians

Differential Solid Angles

\( dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \)
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \sin \theta \, d\theta \, d\phi \]

\[ \Omega = \int_{s^2} d\omega = \int_{0}^{\frac{\pi}{2}} \int_{0}^{2\pi} \sin \theta \, d\theta \, d\phi = \int_{-1}^{1} \int_{0}^{2\pi} d \cos \theta \, d\phi = 4\pi \]
**Isotropic Point Source**

\[ \Phi = \int_{S^2} I \, d\omega \]
\[ = 4\pi I \]
\[ I = \frac{\Phi}{4\pi} \]

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**Warn’s Spotlight**

\[ I(\omega) = \cos^s \theta = (\hat{\omega} \cdot \hat{A})^s \]
Warn’s Spotlight

\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s \]

\[ \Phi = \int_0^{2\pi} \int_0^1 I(\omega) d\cos \theta d\varphi \]

\[ \Phi = 2\pi \int_0^1 \cos^s \theta d\cos \theta = \frac{2\pi}{s+1} \]
Warn’s Spotlight

\[ I(\omega) = \cos^s \theta = (\hat{\omega} \cdot \hat{A})^{s} \]

\[ \Phi = \int_{0}^{2\pi} \int_{0}^{1} I(\omega) \, d\theta \, d\phi = 2\pi \int_{0}^{1} \cos^s \theta \, d\theta \cos = \frac{2\pi}{s + 1} \]

\[ I(\omega) = \Phi \frac{s + 1}{2\pi} \cos^s \theta \]

Light Source Goniometric Diagrams

1. Porcelain-enamed, ventilated standard dome with incandescent lamp
2. Concentric ring unit with incandescent silvered-bowl lamp
3. Pendant diffusing sphere with incandescent lamp
4. R-40 flood with specular anodized reflector slant; 45° cutoff
Irradiance

**Definition:** The *irradiance* (illuminance) is the power per unit area incident on a surface.

\[ E(x) \equiv \frac{d\Phi_i}{dA} \]

\[ \left[ \frac{W}{m^2} \right] \left[ \frac{lm}{m^2} = \text{lux} \right] \]

Sometimes referred to as the radiant (luminous) incidence.
## Typical Values of Illuminance [lm/m²]

<table>
<thead>
<tr>
<th>Source</th>
<th>Illuminance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunlight plus skylight</td>
<td>100,000 lux</td>
</tr>
<tr>
<td>Sunlight plus skylight (overcast)</td>
<td>10,000</td>
</tr>
<tr>
<td>Interior near window (daylight)</td>
<td>1,000</td>
</tr>
<tr>
<td>Artificial light (minimum)</td>
<td>100</td>
</tr>
<tr>
<td>Moonlight (full)</td>
<td>0.02</td>
</tr>
<tr>
<td>Starlight</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

## Beam Power in Terms of Irradiance

\[
\Phi = E A \\
E = \frac{\Phi}{A}
\]
Beam Power Falling on the Surface

\[ \Phi' = E' A' \]

\[ E' = \frac{\Phi'}{A'} \]

Projected Area

\[ A = A' \cos \theta \]
Conservation of Energy

\[ A = A' \cos \theta \]

\[ \Phi = \Phi' \]

Lambert’s Cosine Law

\[ A = A' \cos \theta \]

\[ \Phi = \Phi' \]

\[ E' = \frac{\Phi'}{A'} = \frac{\Phi}{A} \cos \theta = E \cos \theta \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ d\Phi = I \, d\omega \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ d\omega = \frac{\cos \theta}{r^2} dA \]

\[ I \, d\omega = \frac{\Phi}{4\pi} \frac{\cos \theta}{r^2} dA \]
**Irradiance: Isotropic Point Source**

\[
I = \frac{\Phi}{4\pi}
\]

\[
I \, d\omega = \frac{\Phi \cos \theta}{4\pi \, r^2} \, dA = E \, dA
\]

\[
E = \frac{\Phi \cos \theta}{4\pi \, r^2}
\]

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**The Invention of Photometry**

**Bouguer's classic experiment**
- Compare a light source and a candle
- Move until they both appear equally bright
- Intensity is proportional to ratio of distances squared

**Definition of a candela**
- Originally a “standard” candle
- Currently 550 nm laser with 1/683 W/sr
- 1 of 6 fundamental SI units
Radiance

Area Lights – Surface Radiance

Definition: The surface radiance (luminance) is the intensity per unit area leaving a surface

\[ L(x, \omega) = \frac{dI(x, \omega)}{dA} = \frac{d^2 \Phi(x, \omega)}{d\omega dA} \]

\[
\frac{W}{sr\ m^2} \quad \left[ \frac{cd}{m^2} = \frac{lm}{sr\ m^2} = \text{nit} \right]
\]
Typical Values of Luminance [cd/m²]

<table>
<thead>
<tr>
<th>Surface</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface of the sun</td>
<td>2,000,000,000 nit</td>
</tr>
<tr>
<td>Sunlight clouds</td>
<td>30,000</td>
</tr>
<tr>
<td>Clear sky</td>
<td>3,000</td>
</tr>
<tr>
<td>Overcast sky</td>
<td>300</td>
</tr>
<tr>
<td>Moon</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Directional Power Leaving a Surface

\[ d^2 \Phi_o(x, \omega) = L_o(x, \omega) \cos \theta dA d\omega \]

Same \(dA\) for all directions
Radiant Exitance

(Radiosity)

Definition: The radiant (luminous) exitance is the energy per unit area leaving a surface.

\[ M(x) \equiv \frac{d\Phi_o}{dA} \]

\[
\left[ \frac{W}{m^2} \right] \left[ \frac{lm}{m^2} = lux \right]
\]

In computer graphics, this quantity is usually referred to as radiosity (B)
Area Light Source

\[ d^2 \Phi_o(x, \omega) = L_o(x, \omega) \cos \theta \, dA \, d\omega \]

Area Light Source

\[ dM(x, \omega) = \frac{d^2 \Phi_o(x, \omega)}{dA} = L_o(x, \omega) \cos \theta \, d\omega \]
Area Light Source

\[ dM(x, \omega) = L_o(x, \omega) \cos \theta \, d\omega \]

Area Light Source

\[ M = \int_{H^2} dM(x, \omega) = \int_{H^2} L_o(x, \omega) \cos \theta \, d\omega \]
Uniform Diffuse Emitter

\[ M = \int_{H^2} L_o \cos \theta \, d\omega \]

\[ = L_o \int_{H^2} \cos \theta \, d\omega \]

Uniform means \( L_o \) is not a function of direction

Projected Solid Angle

\[ \tilde{\Omega} \equiv \int_{\Omega} \cos \theta \, d\omega \]
Projected Solid Angle

\[ \tilde{\Omega} = \int_{\Omega} \cos \theta \, d\omega \]

\[ \tilde{\Omega} = \int_{H^2} \cos \theta \, d\omega = \pi \]

Uniform Diffuse Emitter

\[ M = \int_{H^2} L_o \cos \theta \, d\omega \]

\[ = L_o \int_{H^2} \cos \theta \, d\omega \]

\[ = \pi L_o \]

\[ L_o = \frac{M}{\pi} \]
Radiometry and Photometry

Summary

### Radiometric and Photometric Terms

<table>
<thead>
<tr>
<th>Physics</th>
<th>Radiometry</th>
<th>Photometry</th>
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<tbody>
<tr>
<td>Energy</td>
<td>Radiant Energy</td>
<td>Luminous Energy</td>
</tr>
<tr>
<td>Flux (Power)</td>
<td>Radiant Power</td>
<td>Luminous Power</td>
</tr>
<tr>
<td>Flux Density</td>
<td>Irradiance</td>
<td>Illuminance</td>
</tr>
<tr>
<td></td>
<td>Radiosity</td>
<td>Luminosity</td>
</tr>
<tr>
<td>Angular Flux Density</td>
<td>Radiance</td>
<td>Luminance</td>
</tr>
<tr>
<td>Intensity</td>
<td>Radiant Intensity</td>
<td>Luminous Intensity</td>
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</table>
Photometric Units

<table>
<thead>
<tr>
<th>Photometry</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>MKS</td>
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<tr>
<td>Luminous Energy</td>
<td>Talbot</td>
</tr>
<tr>
<td>Luminous Power</td>
<td>Lumen</td>
</tr>
<tr>
<td>Illuminance</td>
<td>Lux</td>
</tr>
<tr>
<td>Luminosity</td>
<td>Nit</td>
</tr>
<tr>
<td>Luminance</td>
<td></td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>Candela (Candle, Candlepower, Carcel, Hefner)</td>
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