The Light Field

Concepts
- Light field = radiance function on rays
- Conservation of radiance
- Measurement equation
- Throughput and counting rays
- Irradiance calculations

Lines and Light

From London and Upton

Field Radiance or Light Field

**Definition:** The *field radiance* (*luminance*) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction.

\[
dA \quad r(x, \omega) \quad d\omega \quad L(x, \omega)
\]

The radiance is a function on rays, this function defines the *light field*
Spherical Gantry ⇒ 4D Light Field

$\ L(x,y,\theta,\varphi) \quad (\theta,\varphi) \$

Capture all the light leaving an object - like a hologram

Two-Plane Light Field

$\ L(u,v,s,t) \$

2D Array of Cameras 2D Array of Images
Multi-Camera Array $\Rightarrow$ 4D Light Field

Properties of Radiance
Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment
   - Radiance is the quantity associated with a ray
   - Rendering is all about computing the radiance

2. Radiance is invariant along a ray
   - Reduces parameters from 6D to 4D

3. Response of a sensor proportional to the radiance
   - Cameras measure radiance

1st Law: Conservation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates

\[ d^2 \Phi_1 = d^2 \Phi_2 \]

\[ d^2 \Phi_1 = L_1 d\omega_1 dA_1 \]

\[ d^2 \Phi_2 = L_2 d\omega_2 dA_2 \]

\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \]

\[ \therefore L_1 = L_2 \]
Alternative Definition of Radiance

Two physical laws
1. Energy is conserved
2. The size of a beam does not change as rays propagate

Radiance is the ratio of 2 conserved quantities, therefore, the radiance is also conserved

\[ L(r) = \lim_{\Delta T \to r} \frac{\Delta \Phi}{\Delta T} \]

Quiz

Does radiance increase under a magnifying glass?

No!!
The Measurement Equation

Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.

\[ R = \int_\Omega \int_A L d\omega dA = \bar{L}T \]

\[ T = \int_\Omega \int_A d\omega dA \]

L is what should be computed and displayed.

T (throughput) quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered.
Quiz

Does the brightness that a wall appears to the sensor depend on the distance?

No!!

Throughput

Measuring the Number of Rays
Beam of Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) \quad \rightarrow \quad dA_2(u_2, v_2) \]

How many rays are in this beam?

Number of Rays in a Beam

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

\[ r(u_1, v_1, u_2, v_2) \]

\[ dA_1(u_1, v_1) \quad \rightarrow \quad dA_2(u_2, v_2) \]

This many:

\[ d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2} \]

The number of rays in a beam is the throughput
Why is this the Right Measure?

\[ dA_1(u_1, v_1) \rightarrow dA_2(u_2, v_2) \]

\[ d^2T = \frac{dA_1 dA_2}{|x_1 - x_2|^2} \]

Suppose we double the area \( dA_1 \) (or \( dA_2 \)) AND also double the distance \( |x_1 - x_2|^2 \) between the two areas. This transformation doesn’t change the set of rays comprising the beam (the number of rays) and it also doesn’t change the throughput.

Changing Ray Coordinates

Parameterize rays wrt to receiver \( r(u_2, v_2, \theta_2, \phi_2) \)

\[ d\omega_2(\theta_2, \phi_2) \rightarrow dA_2(u_2, v_2) \]

\[ d^2T = \frac{dA_1}{|x_1 - x_2|^2} dA_2 = d\omega_2 dA_2 \]

Same value for the throughput, different formula
Changing Ray Coordinates

Parameterize rays wrt to source \( r(u_1, v_1, \theta_1, \phi_1) \)

\[
dA_1(u_1, v_1) \quad \frac{dA_2}{\left| x_1 - x_2 \right|^2} = dA_1 d\omega_1
\]

Changing Ray Coordinates Again

Tilting the surfaces reparameterizes the rays!

\[
dA_1(u_1, v_1) \quad \frac{r(u_1, v_1, u_2, v_2)}{dA_2(u_2, v_2)}
\]

\[
d^2T = \frac{\cos \theta_1 \cos \theta_2}{\left| x_1 - x_2 \right|^2} dA_1 dA_2
\]
Number of Rays Hitting a Shape

Parameterize rays by \( r(x, y, \theta, \phi) \)

Projected area \( \tilde{A}(\hat{\omega}) \)

Measuring the number or rays that hit a shape

\[
T = \int_{S^2} d\omega(\theta, \varphi) dA(x, y) = \int_{S^2} d\omega(\theta, \varphi) \int_{R^2} dA(x, y) \\
= \int_{S^2} \tilde{A}(\theta, \varphi) d\omega(\theta, \varphi) \\
= 4\pi \tilde{A}
\]

Sphere:

\[
T = 4\pi \tilde{A} = 4\pi^2 R^2
\]

Calculated Another Way

Parameterize rays by \( r(u, v, \theta, \phi) \)

\[
T = \left[ \int_{H^2(N)} \int_{M^2} \cos \theta \ d\omega(\theta, \varphi) \right] \left[ \int_{\pi} dA(u, v) \right]
\]

Sphere: \( T = \pi S = 4\pi^2 R^2 \)

Crofton’s Theorem: \( 4\pi \tilde{A} = \pi S \Rightarrow \tilde{A} = \frac{S}{4} \)
Number of Rays Intersecting 2 Surfaces

The number of rays that intersect $dA_1$ and $dA_2$

$$d^2T = \frac{\cos \theta_1 \cos \theta_2}{|x_1 - x_2|^2} dA_1 dA_2$$

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Number of Rays Intersecting 2 Surfaces

The number of rays that intersect $A_1$ and $A_2$

$$T = \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{|x_1 - x_2|^2} dA_1 dA_2$$
Number of Rays Intersecting 2 Surfaces

The number of rays that intersect $A_1$ and $A_2$

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Form Factor

Form Factor = Probability of a ray hitting $A_2$ given that it hits $A_1$

$$F_{i,j} = \frac{1}{\pi A_1} \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{|x_1 - x_2|^2} dA_1 dA_2$$
Irradiance from the Environment

\[ d^2 \Phi_i(x, \omega) = L_i(x, \omega) \cos \theta \, dA \, d\omega \]

\[ dE(x, \omega) = L_i(x, \omega) \cos \theta \, d\omega \]

\[ E(x) = \int_{H^2} L_i(x, \omega) \cos \theta \, d\omega \]

Light meter

\[ L_i(x, \omega) \]
Irradiance from a Uniform Area Source

$$E(x) = \int_{\Omega} L \cos \theta \, d\omega$$

$$= L \int_{\Omega} \cos \theta \, d\omega$$

$$= L\tilde{\Omega}$$

The projected solid angle is the number of rays leaving $A$ that intersect $dA$.

It’s the probability of a light ray from $A$ arriving at $dA$.

Uniform Disk Source

Geometric Derivation

$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int_1^{\cos \alpha} \int_0^{2\pi} \cos \theta \, d\phi \, d\cos \theta$$

$$= 2\pi \frac{\cos^2 \theta}{2} \bigg|_1^{\cos \alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$
### Spherical Source

**Geometric Derivation**

\[ \tilde{\Omega} = \pi \sin^2 \alpha \]

**Algebraic Derivation**

\[ \tilde{\Omega} = \int \cos \theta \, d\omega = \pi \sin^2 \alpha \]

\[ = \pi r^2 \]

\[ = \pi \frac{r^2}{R^2} \]

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### The Sun

**Solar constant (normal incidence at zenith)**

- Irradiance 1353 W/m²
- Illuminance 127,500 lm/m² = 127.5 kilolux

**Solar angle**

\[ \alpha = .25 \text{ degrees} = .004 \text{ radians (half angle)} \]

\[ \tilde{\Omega} = \pi \sin^2 \alpha \approx \pi \alpha^2 \times 6 \times 10^{-5} \text{ steradians} \]

**Solar radianc**

\[ L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \text{ W/m}^2}{6 \times 10^{-5} \text{ sr}} = 2.25 \times 10^7 \frac{\text{W}}{\text{m}^2 \cdot \text{sr}} \]
Polygonal Source

Polygonal Source
Polygonal Source

Consider 1 Edge

\( A = \gamma \cos \theta = \gamma \mathbf{N}_E \cdot \mathbf{N} \)
Lambert’s Formula

\[ \sum_{i=1}^{3} A_i = A_1 - A_2 - A_3 \]

\[ \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{N}_i \cdot \vec{N} \]

Penumbra and Umbras

emitter

occluder

receiver

umbra

penumbra