Reflection Models I

Today

- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Next lecture

- Glossy and specular reflection models
- Rough surfaces and microfacets

Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

- Spectra and Color
- Polarization
- Directional distribution
Types of Reflection Functions

Ideal Specular
- Reflection Law
- Mirror

Ideal Diffuse
- Lambert’s Law
- Matte

Specular
- Glossy
- Directional diffuse

Materials

From Apodaca and Gritz, Advanced RenderMan
The BRDF

Bidirectional Reflectance-Distribution Function

\[ dL_r(x, \omega_i) \]

\[ L_r(x, \omega_i) \]

\[ \hat{N} \]

\[ \theta_r \]

\[ \theta_i \]

\[ d\omega_i \]

\[ f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \left[ \frac{1}{sr} \right] \]

The Reflection Equation

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i \rightarrow \omega_r)L_i(x, \omega_i)\cos\theta_i \, d\omega_i \]
Gonioreflectometer

Properties of BRDF’s

1. Linearity

From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle

\[ f_r(\omega_i \rightarrow \omega_r) = f_r(\omega_r \rightarrow \omega_i) \]
Properties of BRDF’s

3. Isotropic vs. anisotropic

\[ f_r(\theta, \varphi, \theta_r, \varphi_r) = f_r(\theta, \varphi, \theta_r, \varphi_r - \phi_r) \]

Reciprocity and isotropy

\[ f_r(\theta, \varphi, \theta_r, \varphi - \phi_r) = f_r(\theta, \varphi, \theta_r, \varphi_r - \phi_r) = f_r(\theta, \varphi, \varphi_r, \varphi_r - \phi_r) \]

4. Energy conservation

Energy Conservation

\[
\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i \ d\omega_i}{\Omega_i \int_{\Omega_i} L_i(\omega_i) \cos \theta_i \ d\omega_i} = \int_{\Omega_i} \int_{\Omega_i} f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i \ d\omega_i \cos \theta_r \ d\omega_r \leq 1
\]
Law of Reflection

\[ \mathbf{R} + (-\hat{\mathbf{I}}) = 2 \cos \theta \hat{\mathbf{N}} = -2(\hat{\mathbf{I}} \cdot \hat{\mathbf{N}}) \hat{\mathbf{N}} \]
\[ \mathbf{R} = \hat{\mathbf{I}} - 2(\hat{\mathbf{I}} \cdot \hat{\mathbf{N}}) \hat{\mathbf{N}} \]

Ideal Reflection (Mirror)

\[ L_i(\theta_i, \varphi_i) \quad L_r(\theta_r, \varphi_r) \]
\[ \begin{align*}
L_{r,m}(\theta_r, \varphi_r) &= L_i(\theta_r, \varphi_r, \pm \pi) \\
L_{r,m}(\theta_r, \varphi_r) &= \int f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) L_i(\theta_i, \varphi_i) \cos \theta_i \, d \cos \theta_i \, d \varphi_i \\
&= \int \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r, \pm \pi) L_i(\theta_i, \varphi_i) \cos \theta_i \, d \cos \theta_i \, d \varphi_i \\
&= L_i(\theta_r, \varphi_r, \pm \pi)
\end{align*} \]
**Snell’s Law**

\[ n_i \sin \theta_i = n_r \sin \theta_r \]

\[ n_i \hat{N} \times \hat{I} = n_r \hat{N} \times \hat{T} \]

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**Law of Refraction**

\[ \hat{T} = \mu \hat{I} + \gamma \hat{N} \]

\[ \hat{T}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \hat{I} \cdot \hat{N} \]

\[ \gamma = -\mu \hat{I} \cdot \hat{N} \pm \left\{ -\mu^2 \left(1 - (\hat{I} \cdot \hat{N})^2\right) \right\}^{\frac{1}{2}} \]

Total internal reflection:

\[ 1 - \mu^2 (1 - (\hat{I} \cdot \hat{N})^2) < 0 \]
Optical Manhole

Total internal reflection

\[ n_w = \frac{4}{3} \]

From Livingston and Lynch

Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97
Fresnel Reflectance

Dielectric (N=1.5)

Glass $n=1.5$ $F(0)=0.04$
Diamond $n=2.4$ $F(0)=0.15$

Schlick Approximation

$$F(\theta) = F(0) + (1 - F(0))(1 - \cos \theta)^5$$

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Fresnel Reflectance

Metal (Aluminum)

Gold $F(0)=0.82$
Silver $F(0)=0.95$
**Cook-Torrance Model for Metals**

Reflectance of Copper as a function of wavelength and angle of incidence

Light spectra

\[
\rho \\
\pi/2 \\
\theta \\
\lambda
\]

Copper spectra

**Measured Reflectance**

**Approximated Reflectance**

Cook-Torrance approximation

\[
R = R(0) + R(\pi/2) \left[ \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)} \right]
\]

**Copper spectra**
Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction

\[ L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos \theta_i \, d\omega_i \]
\[ = f_{r,d} \int L_i(\omega_i) \cos \theta_i \, d\omega_i \]
\[ = f_{r,d} E \]

\[ f_{r,d} = \frac{M}{L_r} = \frac{\pi L_r}{\int \cos \theta_r \, d\omega_r} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \Rightarrow f_{r,d} = \frac{\rho_d}{\pi} \]

Lambert’s Cosine Law

\[ M = \rho_d E = \rho_d E \cos \theta_s \]

“Diffuse” Reflection

Theoretical

- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

Experimental

- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

Paint manufacturers attempt to create ideal diffuse