Ray Tracing

Ray Tracing 1
- Basic algorithm
- Ray-surface intersection (triangles, ...)

Ray Tracing 2
- Problem: Brute force = $|\text{Image}| \times |\text{Objects}|$
- Solution: Acceleration $\approx |\text{Image}| \times \log |\text{Objects}|$

Later
- How to choose the best rays to trace?
- How to minimize the number of rays traced?
300k tris
Time: Accel 27.4%, Tri 10.2%

5.3M tris
Time: Accel 42.7%, Tri 13.3%

3.1B tris
Time: Accel 74.7%, Tri 13.3%
Primitives

pbtr Primitive base class

- Shape
- Material (emission, reflection and transmission)

```cpp
class Primitive {
    public:
        virtual Bounds3f WorldBound() const = 0;
        virtual bool Intersect(const Ray &r,
            SurfaceInteraction *) const = 0;
        virtual bool IntersectP(const Ray &r) const = 0;
        virtual const AreaLight *GetAreaLight() const = 0;
        virtual const Material *GetMaterial() const = 0;
        virtual void ComputeScatteringFunctions(...) const = 0;
};
```
Primitives

Collections

- TransformedPrimitive: Transformation + primitive
- Aggregate
  - Treat acceleration data structures as primitives
- Two types of accelerators: kdtree.cpp, and bvh.cpp
- May nest accelerators of different types

```cpp
class Scene {
    // ...
    bool Intersect(const Ray &ray,
                    SurfaceInteraction *isect) const {
        return aggregate->Intersect(ray, isect);
    }
    std::shared_ptr<Primitive> aggregate;
};
```
Uniform Grids
Preprocess Scene

1. Find bounding box
Preprocess Scene

1. Find bounding box
2. Determine resolution
Preprocess Scene

1. Find bounding box
2. Determine resolution
3. Place objects in cells if bbox overlaps
Preprocess Scene

1. Find bounding box
2. Determine resolution
3. Place objects in cells
   if bbox overlaps
   if surface intersects
Ray-Intersection

For each ray, step through grid
3D line – 3D-DDA
Output all grid cells the ray pierces
In 3D, 6-connected line
Resolution?

1 cell

No speedup
Resolution?

Large number of cells
Too many empty cells
Extra cost!
Resolution?

Heuristic

\[ n_v = n_x n_y n_z \propto n_o \]

\[ \max(n_x, n_y, n_z) = d^{3/2} n_o \]

\[ d \approx 3 \]
Objects Overlapping Multiple Cells

Mistake: Output first intersection found
Objects Overlapping Multiple Cells

Solution: Check whether intersection is inside cell
Objects Overlapping Multiple Cells

Solution: Check whether intersection is inside cell
Mailboxes

Solution: Check whether intersection is inside cell

Problem: Objects tested for intersection multiple times
Mailboxes

Solution: Check whether intersection is inside cell

Problem: Objects tested for intersection multiple times

Solution: Mailboxes

- Assign each ray an increasing number
- Primitive intersection cache (mailbox)
  - Give each ray a number \( n \)
  - Store intersection point and ray \( n \) with each primitive
- Only re-intersect if ray \( n \) is greater than last ray \( n \)
Uniform grids work well for large collections of objects that are uniform in size and distribution.

http://www.kevinboulanger.net/grass.html
Uniform Grids: When They Work Poorly

Teapot in the Stadium Problem

“For example, imagine you have a football stadium made of, say, 5K primitives. Sitting on a goal line is a shiny polygonalized teapot of 5K quadrilaterals (note that the teapot is teapot sized compared to the stadium).”

Eric Haines

http://tog.acm.org/resources/RTNews/html/rtnews1b.html#art4

Problem: non-uniform size distribution
Nested Grids
Nested Grids

Refine grid cells with many primitives to have second-level grids in those cells.
Nested Grids

Refine grid cells with many primitives to have second-level grids in those cells.
Spatial Hierarchies
Spatial Hierarchies

Letters correspond to planes (A)
Letters correspond to planes (A, B)
Point Location by recursive search
Letters correspond to planes (A, B, C, D)
Boxes at leaves correspond to regions
Variations

kd-tree  oct-tree  bsp-tree
Review: Ray-Plane Intersection

**Ray:** $r(t) = o + t \overrightarrow{d}$

**Plane:** $(p - p') \cdot \overrightarrow{n} = 0$

$t = -\frac{\overrightarrow{o - p} \cdot \overrightarrow{n}}{\overrightarrow{d} \cdot \overrightarrow{n}}$

**General case**

$t = -\frac{o_x - p_x}{d_x}$

**Axis-aligned**
Recursive Inorder Traversal

[Kaplan, Arvo, Jansen]

\[
\begin{align*}
\text{Intersect}(L, t_{\min}, t_{\max}) & \quad \text{Intersect}(L, t_{\min}, t^*) \\
\text{Intersect}(R, t^*, t_{\max}) & \quad \text{Intersect}(R, t^*, t_{\min})
\end{align*}
\]
Building the Hierarchy

Midpoint?  Median?
Which Hierarchy is Fastest?

What is the cost of tracing a ray?

\[
\text{Cost}(\text{node}) = C_{\text{trav}} + \text{Prob}(\text{hit L}) \times \text{Cost}(L) + \text{Prob}(\text{hit R}) \times \text{Cost}(R)
\]

\[C_{\text{trav}} = \text{cost of traversing a cell}\]

\[\text{Cost}(L) = \text{cost of traversing left child}\]

\[\text{Cost}(R) = \text{cost of traversing right child}\]
Splitting with Cost in Mind
Split in the Middle = Bad!

Midpoint: Makes the L & R probabilities equal
Cost of R greater than cost of L
Split at the Median = Bad!

Median: Makes the L & R costs equal
Probability of hitting L greater than R
Cost(cell) = \( C_{\text{trav}} + \text{Prob(hit L)} \times \text{Cost(L)} + \text{Prob(hit R)} \times \text{Cost(R)} \)
Cost

Need the probabilities

- Turns out to be proportional to surface area

Need the child cell costs

- Triangle count is a good approximation

\[
\text{Cost(cell)} = \text{C\_trav} + \text{SA(L)} \times \text{TriCount(L)} + \text{SA(R)} \times \text{TriCount(R)}
\]

C\_trav: the ratio of cost to traverse to cost to intersect

\[
\text{C\_trav} = 1:80 \text{ in pbrt}
\]

\[
\text{C\_trav} = 1:1.5 \text{ in a highly optimized version}
\]
Projected Area and Surface Area

The probability of ray in a given direction hitting an object with surface area $S$ is proportional to its projected area $A$ in the direction of the ray.
The probability of ray in a any direction hitting an object with surface area $S$ is proportional to its average projected area.

Average projected area: \[
\bar{A} = \frac{1}{4\pi} \int A \, d\omega
\]

Crofton’s theorem: \[
\bar{A} = \frac{S}{4} \quad \text{(Convex shapes only)}
\]
Ray Intersection Probability

The probability of a ray hitting a convex shape enclosed by another convex shape is

\[ \Pr[r \cap S_o \mid r \cap S_c] = \frac{S_o}{S_c} \]
Build Algorithm

Note: Cost minimum must be at extrema
Build Algorithm

\[ p_a = \frac{S_a}{S} \]
\[ N_a \]

\[ p_b = \frac{S_b}{S} \]
\[ N_b \]

2n candidate split locations
Sweep Build Algorithm (Triangles)

1. Pick an axis, or optimize across x, y, z
2. Build a set of “candidate” split locations
3. Sort the triangles into intervals
4. Sweep to incrementally track L/R counts, cost
5. Output position of minimum cost split

Running time: \[ T(N) = N \log N + 2T(N/2) \]
\[ T(N) = N \log^2 N \]
Termination Criteria

When should we stop splitting?

■ Bad: depth limit, number of triangles
■ Good: When split does not lower the cost

Threshold of cell size

■ Absolute probability $SA(\text{node})/SA(\text{scene})$ small
“Split Clipping”

Bounding box is conservative: may incorrectly report primitive overlap
# Results

<table>
<thead>
<tr>
<th># Triangles</th>
<th>600,127</th>
</tr>
</thead>
<tbody>
<tr>
<td>Render time</td>
<td>19328s</td>
</tr>
<tr>
<td>kd-tree Build</td>
<td>0.43s (~0%)</td>
</tr>
<tr>
<td>BVH Traverse</td>
<td>7775s (40.0%)</td>
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<td>Tri Intersect</td>
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<tr>
<td>Ray/Tri tests</td>
<td>107.4B</td>
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<td>Ray/Tri hits</td>
<td>4.4B (4.1%)</td>
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Primitive Hierarchies

Bounding Volume Hierarchies (BVH)
Bounding Volume Hierarchies

Spatial subdivision (grids, kd-trees): subdivide 3D space into non-overlapping cells

Primitive subdivision (e.g. bounding volume hierarchies): partition scene primitives into disjoint sets (note bounding volumes may overlap)
Building BVH

Can also apply sweep build

\[ \text{Cost} = N_A \times S_A + N_B \times S_B \]
Building BVH

Can also apply sweep build

\[
\text{Cost} = N_A \times S_A + N_B \times S_B
\]

Consider all three axes during partitioning,
Choose lowest cost

Caveat: “greedy” algorithm;
not guaranteed to give global optimum
BVH Traversal

```
stack<Node *> toVisit;
toVisit.push(root);
while (Node *cur = toVisit.pop()) {
    if (cur->bounds->Intersect(ray)) {
        if (cur->isLeaf) {
            /* intersect with primitives */
        } else {
            toVisit.push(cur->near);
            toVisit.push(cur->far);
        }
    }
}
```
# Results

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### Comparison

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What’s the difference?

Rendering time: 27m 38s
What’s the difference?

Rendering time: 1h 55m 45s
Axis-alignment and performance

Wall and its bounding box  Rotated wall and its bounding box
Axis-alignment and performance

Rotated wall and its bounding box  Work-around: refine bounding boxes
Theory

Computational geometry of ray shooting

1. Triangles (Pellegrini)
   - Time: \( O(\log n) \)
   - Space: \( O(n^{5+\varepsilon}) \)

2. Sphere (Guibas and Pellegrini)
   - Time: \( O(\log^2 n) \)
   - Space: \( O(n^{5+\varepsilon}) \)