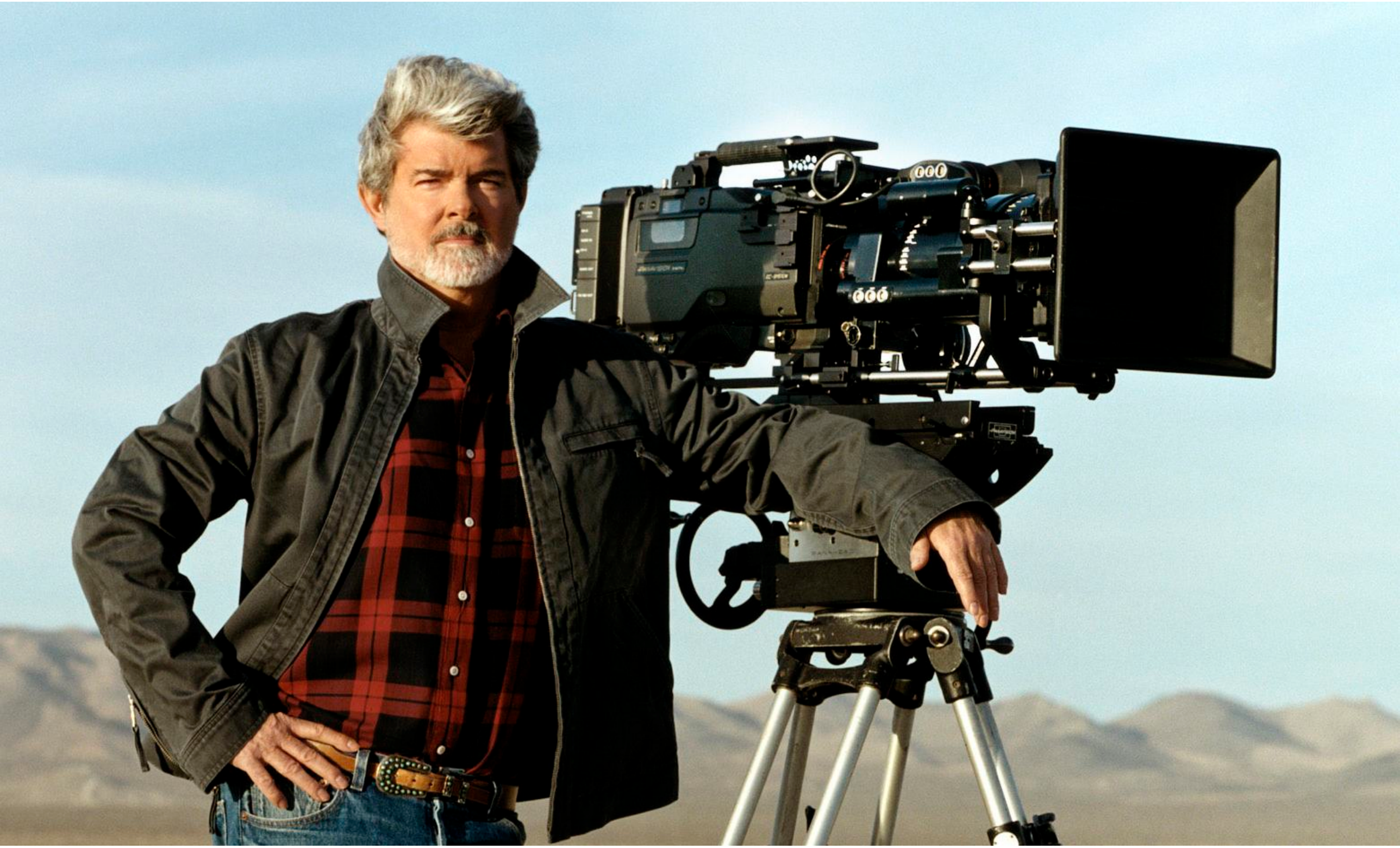
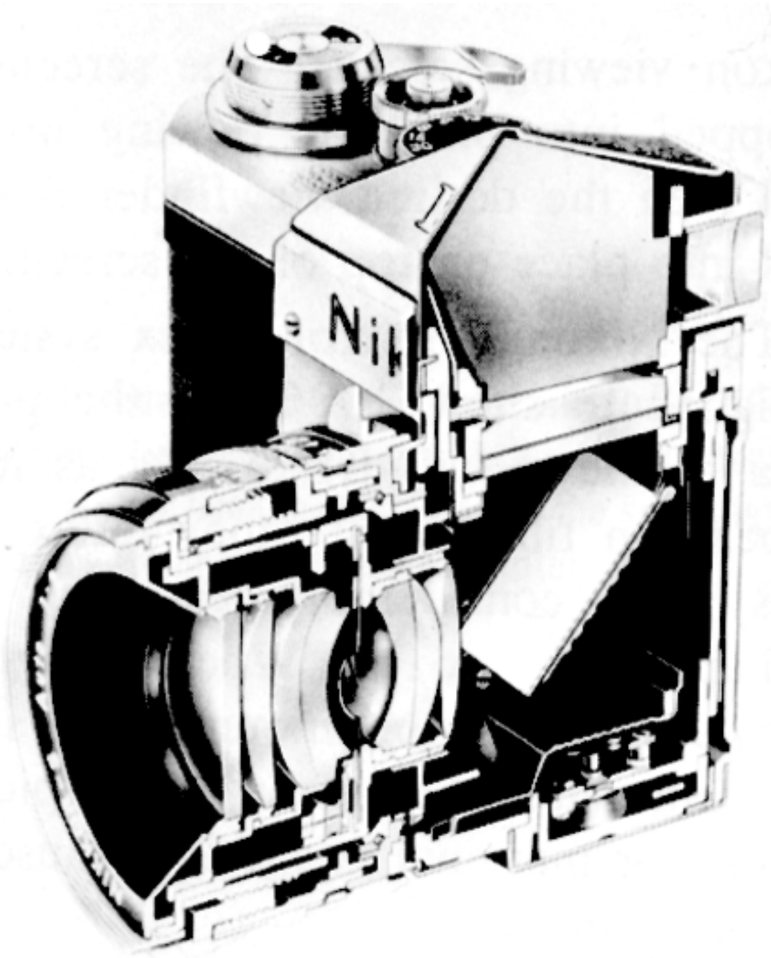


Camera Simulation



Effect	Cause
Field of view	Film size, focal length
Depth of field	Aperture, focal length
Exposure	Film speed, aperture, shutter
Motion blur	Shutter

References

Photography, B. London and J. Upton

Optics in Photography, R. Kingslake

The Camera, *The Negative*, *The Print*, A. Adams

Cameras (5D integral)

$$R = \int_T \iiint_\Omega \iint_A L(x,\omega,t)\ P(x)\ S(t)\ \cos\theta\ dA\ d\omega\ dt$$

Motion Blur

Cook, Porter, Carpenter, 1984

Depth of Field

Mitchell, 1991

Topics

Lenses

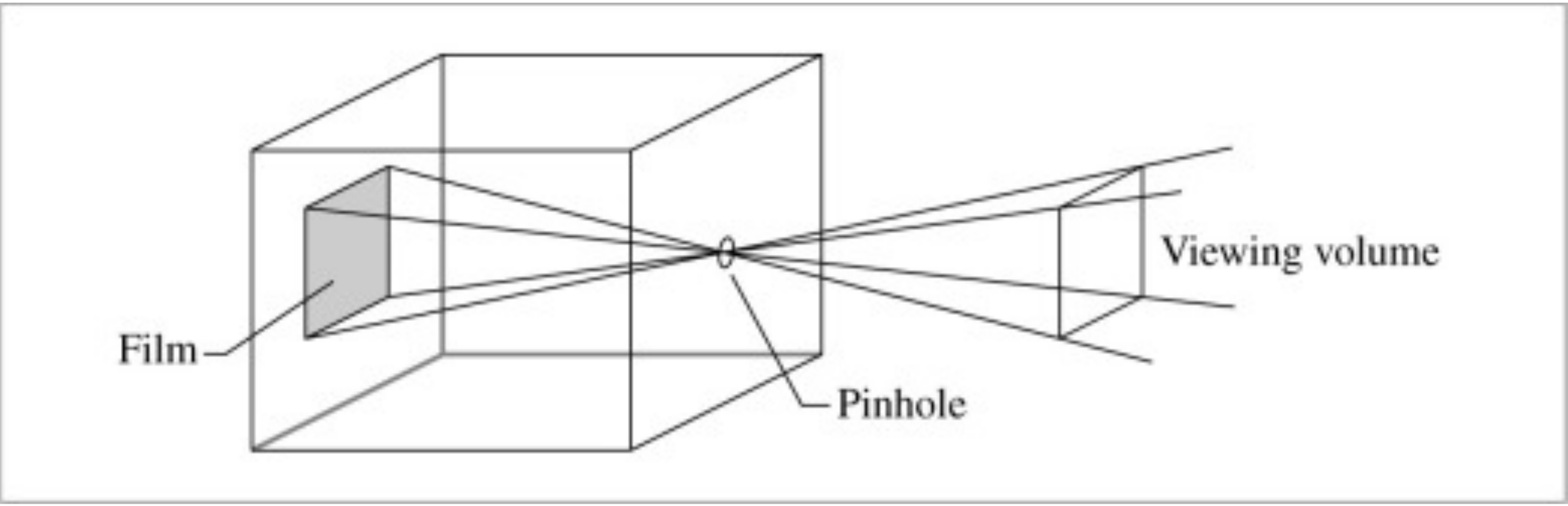
Depth of field

Exposure

Lightfield Cameras

Lenses

Pinhole Camera



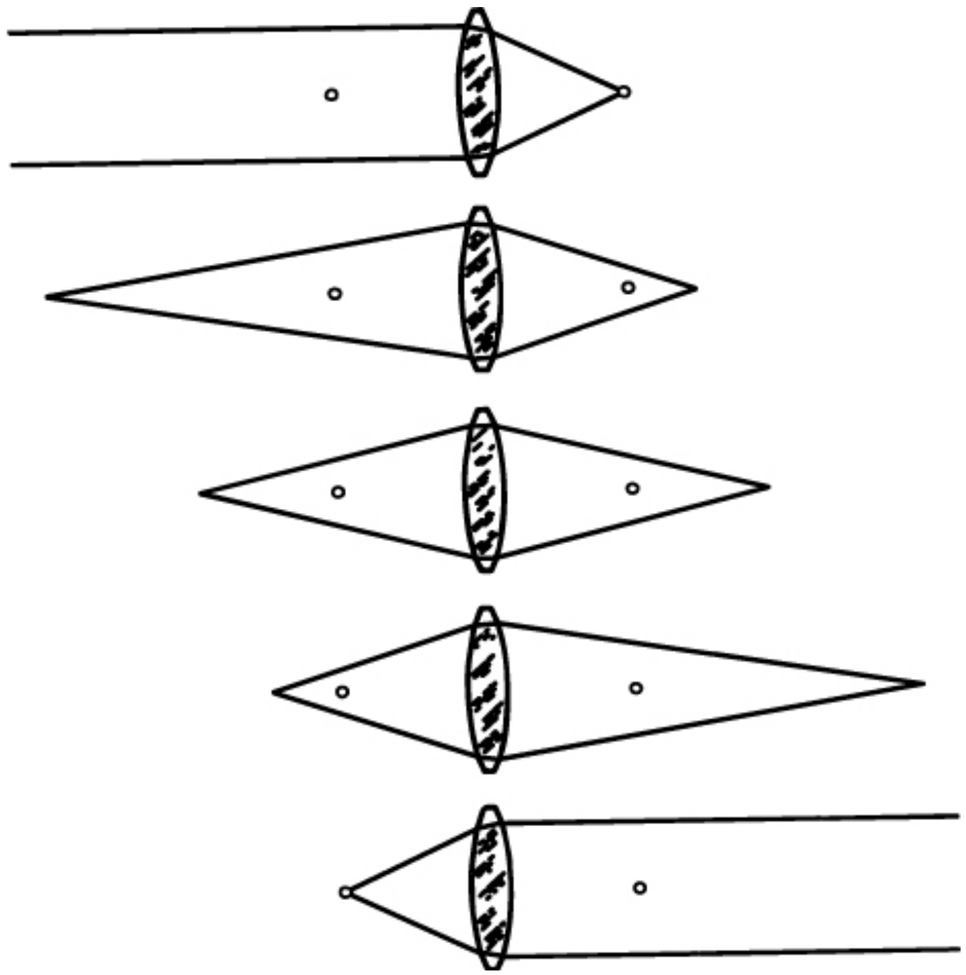
From Pharr et al. Physically Based Rendering (3rd Edition)

Thin Lens Demonstration

http://graphics.stanford.edu/courses/cs178-10/applets/gaussian.html

Lenses Focus!

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$



Rays from a point in object space intersect at a point in image space

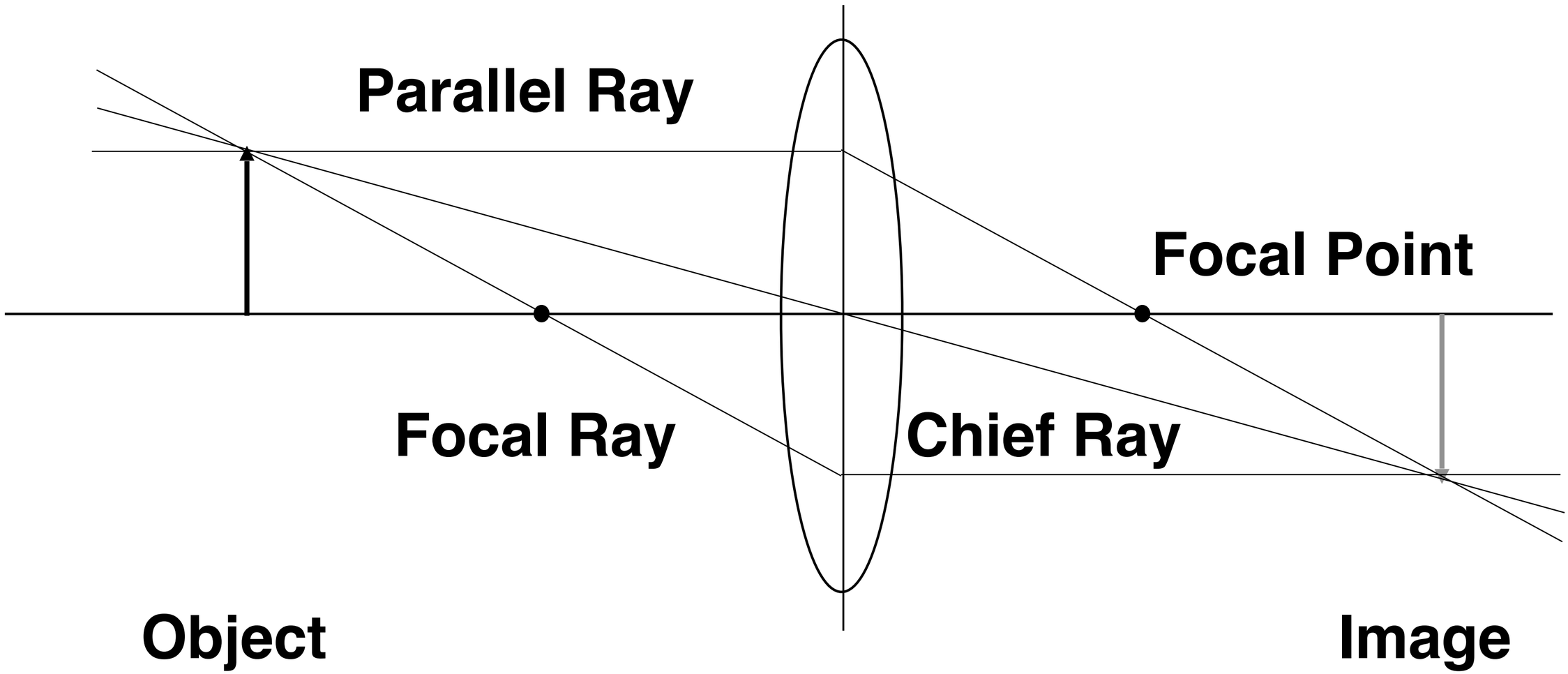
These are conjugate points

Points at infinity converge at the focal point or focus

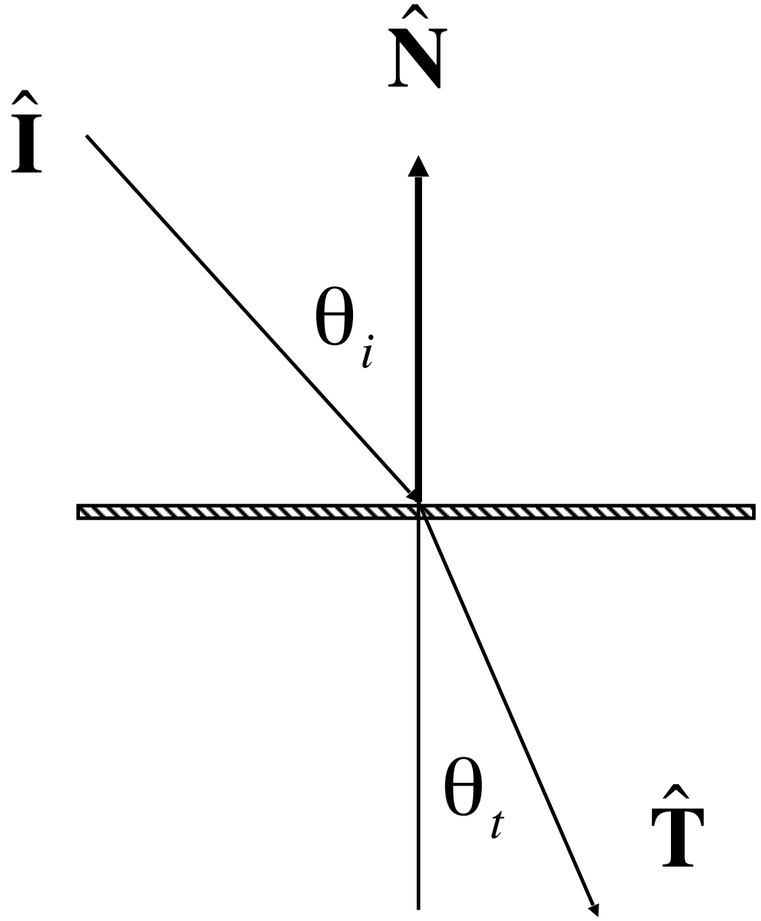
Points on different object planes focus on different image planes

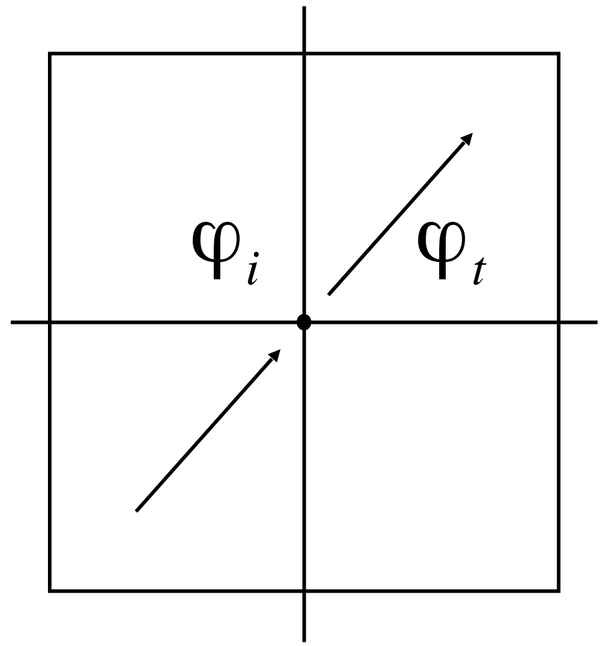
The position of the point of focus can be changed by moving the lens

Gauss' Ray Tracing Construction



Snell's Law



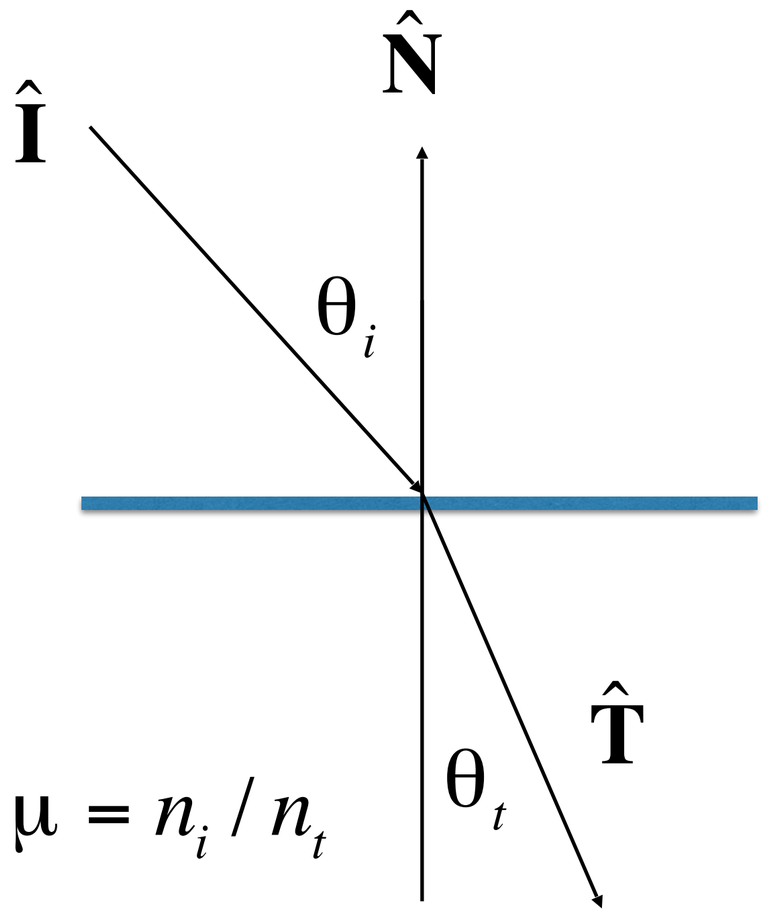


$$\varphi_t = \varphi_i \pm \pi$$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_i \hat{\mathbf{N}} \times \hat{\mathbf{I}} = n_t \hat{\mathbf{N}} \times \hat{\mathbf{T}}$$

Law of Refraction



$$\mu = n_i / n_t$$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$\sin\theta_t = \mu \sin\theta_i$$

$$\hat{\mathbf{T}} = \mu \ \hat{\mathbf{I}} + \gamma \ \hat{\mathbf{N}}$$

$$\hat{\mathbf{T}}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \ \hat{\mathbf{I}} \bullet \hat{\mathbf{N}}$$

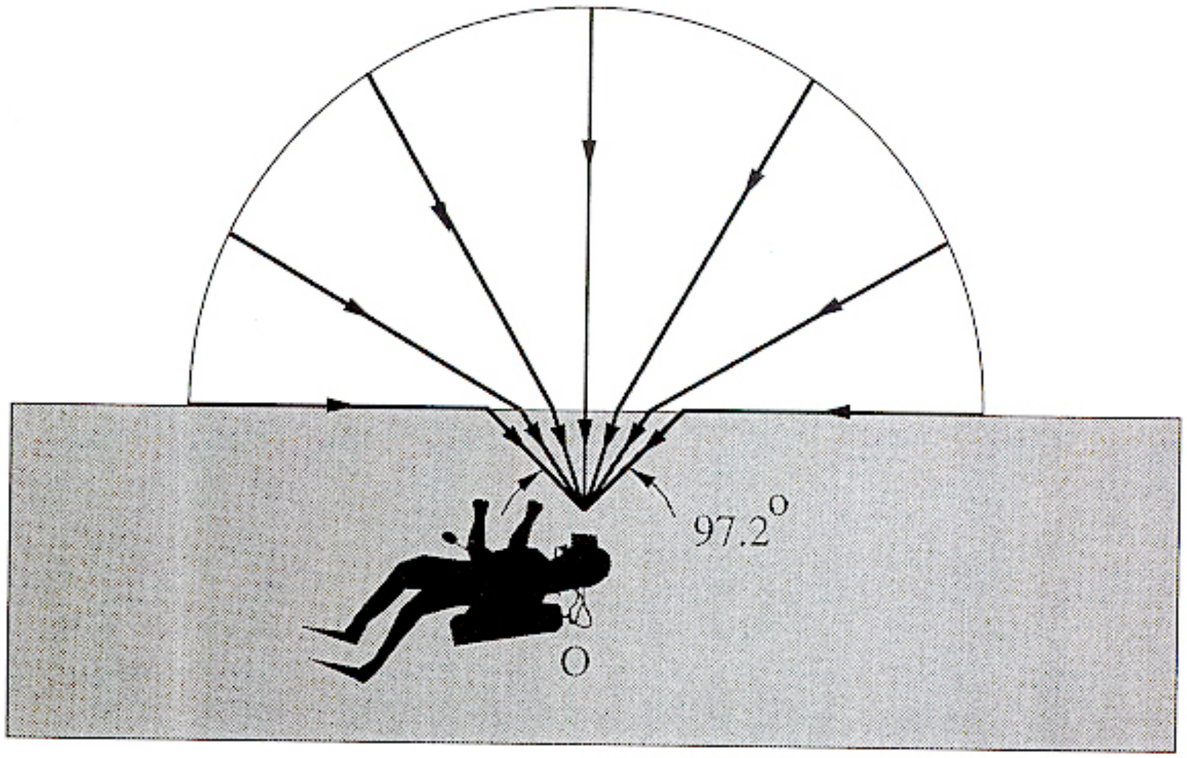
$$\begin{aligned}
\gamma &= -\mu\hat{\mathbf{I}} \cdot \hat{\mathbf{N}} \pm \sqrt{1 - \mu^2 \left(1 - (\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})^2\right)} \\
&= \mu \cos\theta_i \pm \sqrt{1 - \mu^2 \sin^2\theta_i} \\
&= \mu \cos\theta_i \pm \cos\theta_t \\
&= \mu \cos\theta_i - \cos\theta_t
\end{aligned}$$

Total internal reflection:

$$1 - \mu^2(1 - \left(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}}\right)^2) < 0$$

Optical Manhole

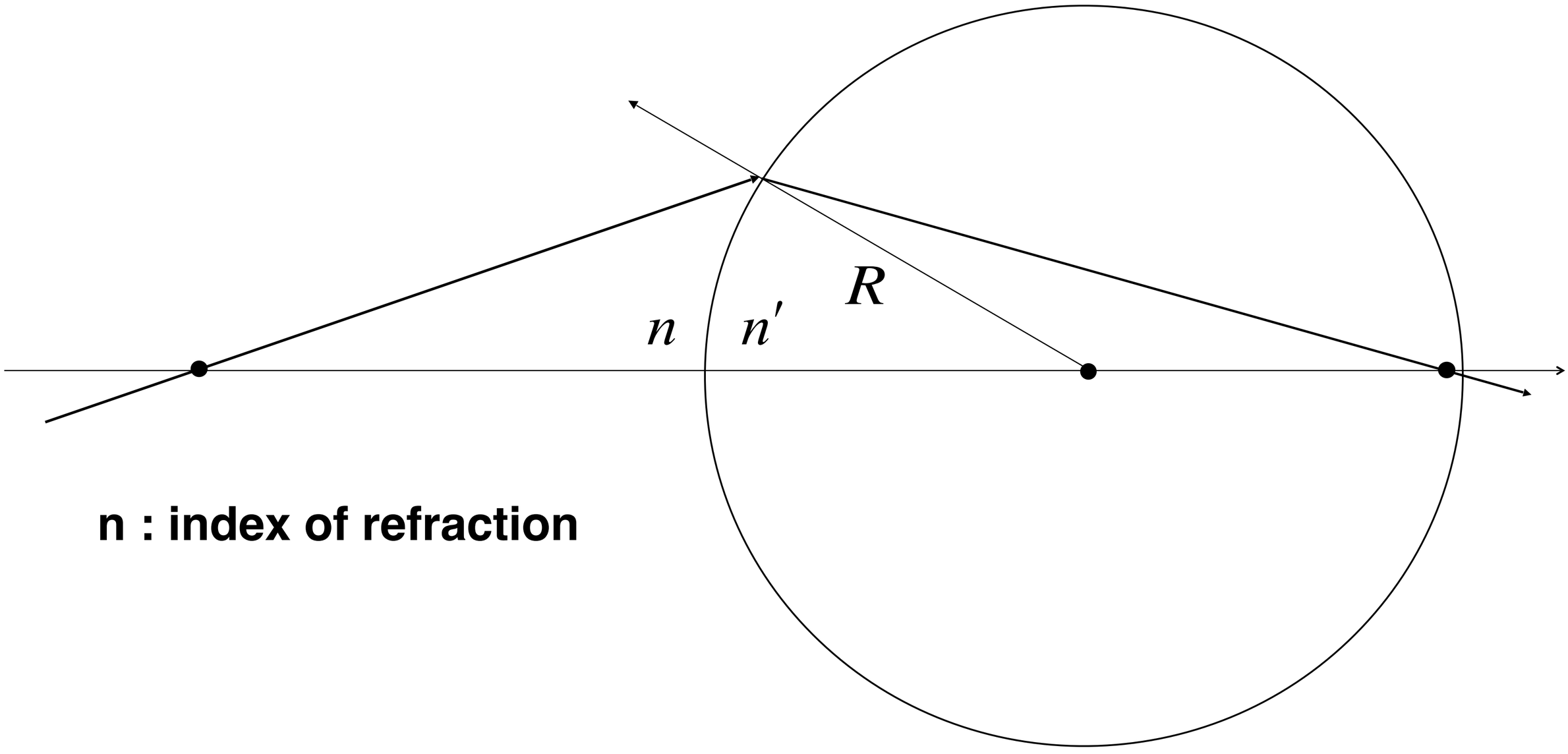
Total internal reflection



$$n_w = \frac{4}{3}$$

From Livingston and Lynch

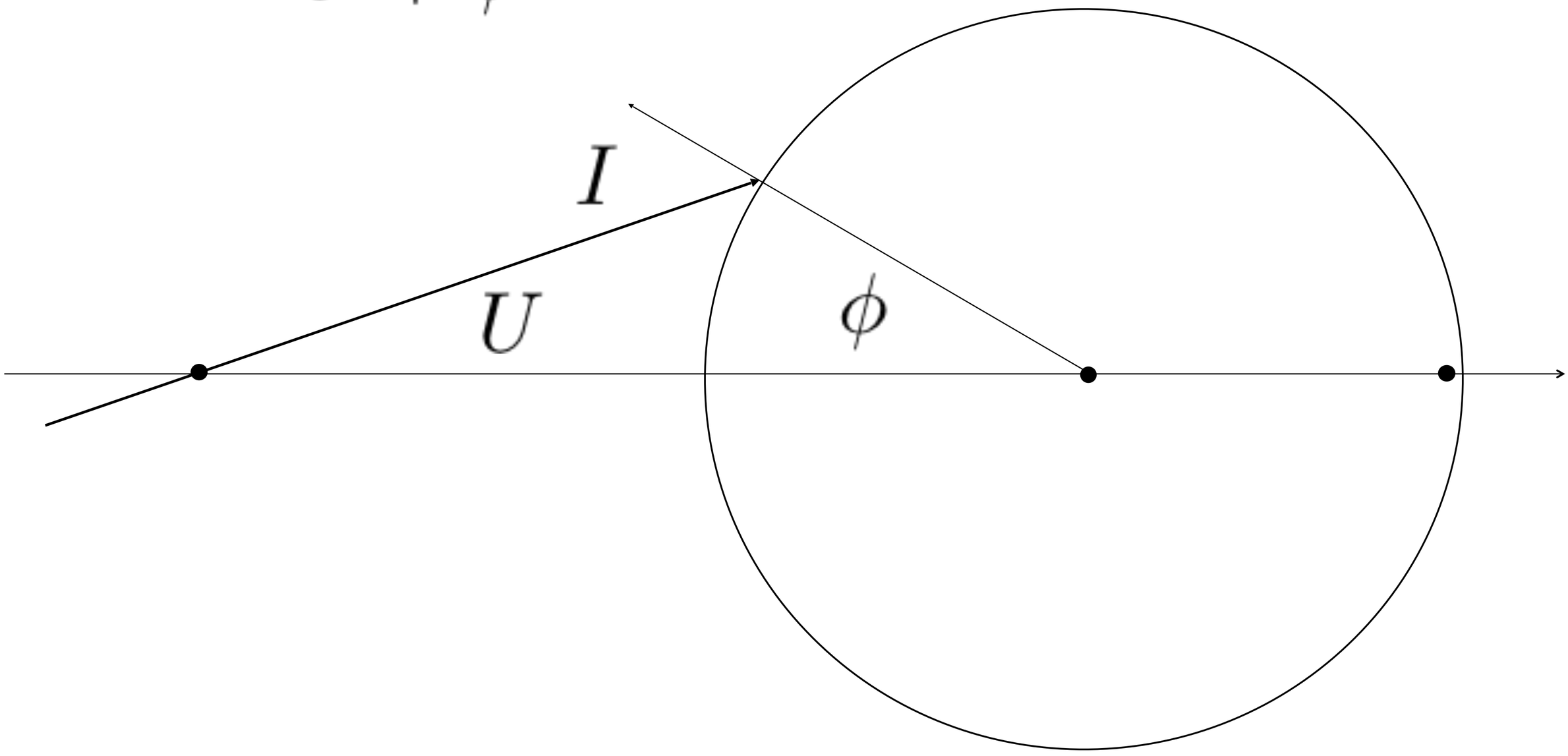
Refraction at a Spherical Lens



n : index of refraction

Angle of Incidence

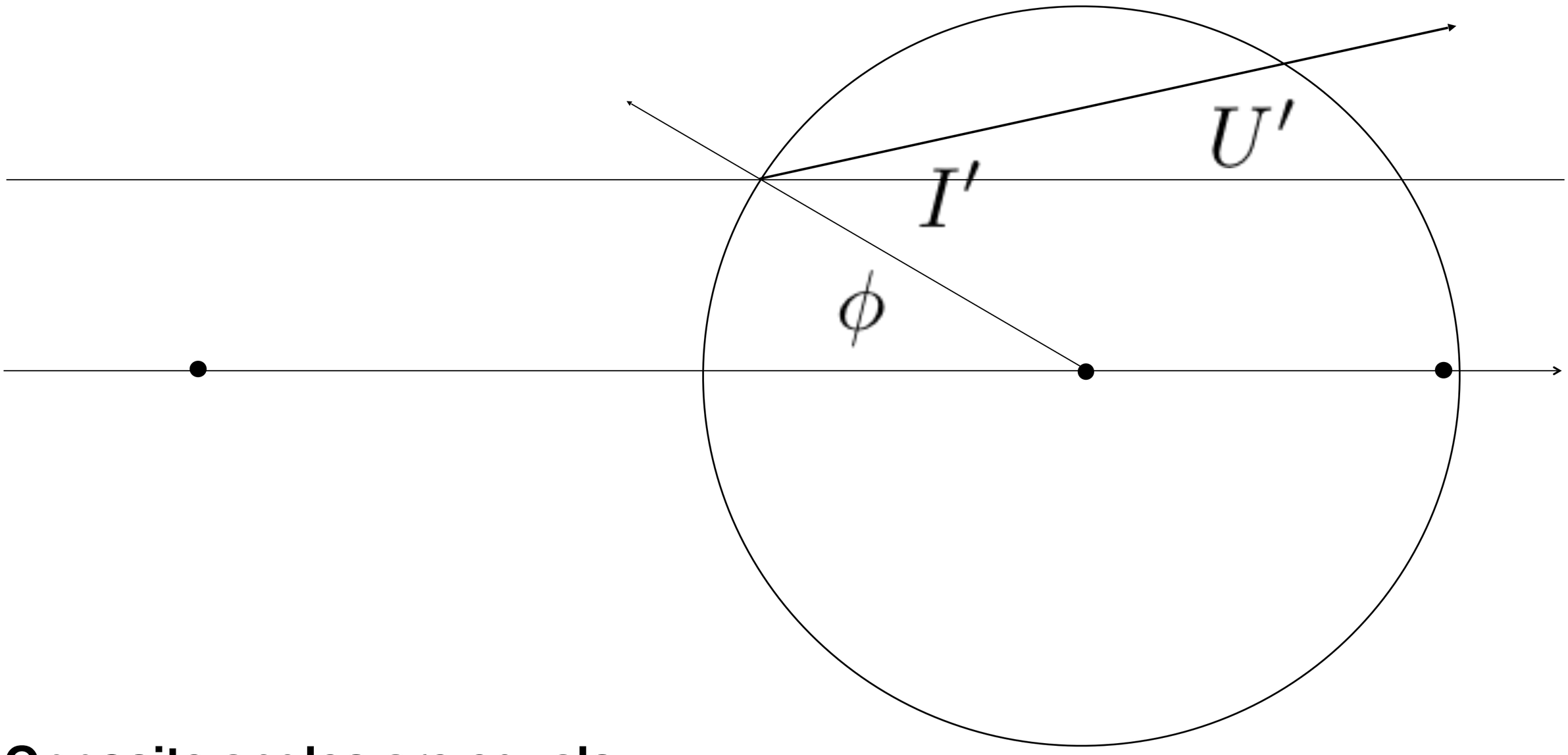
$$I = U + \phi$$



The sum of the interior angles is equal to the exterior angle.

Angle of Refraction

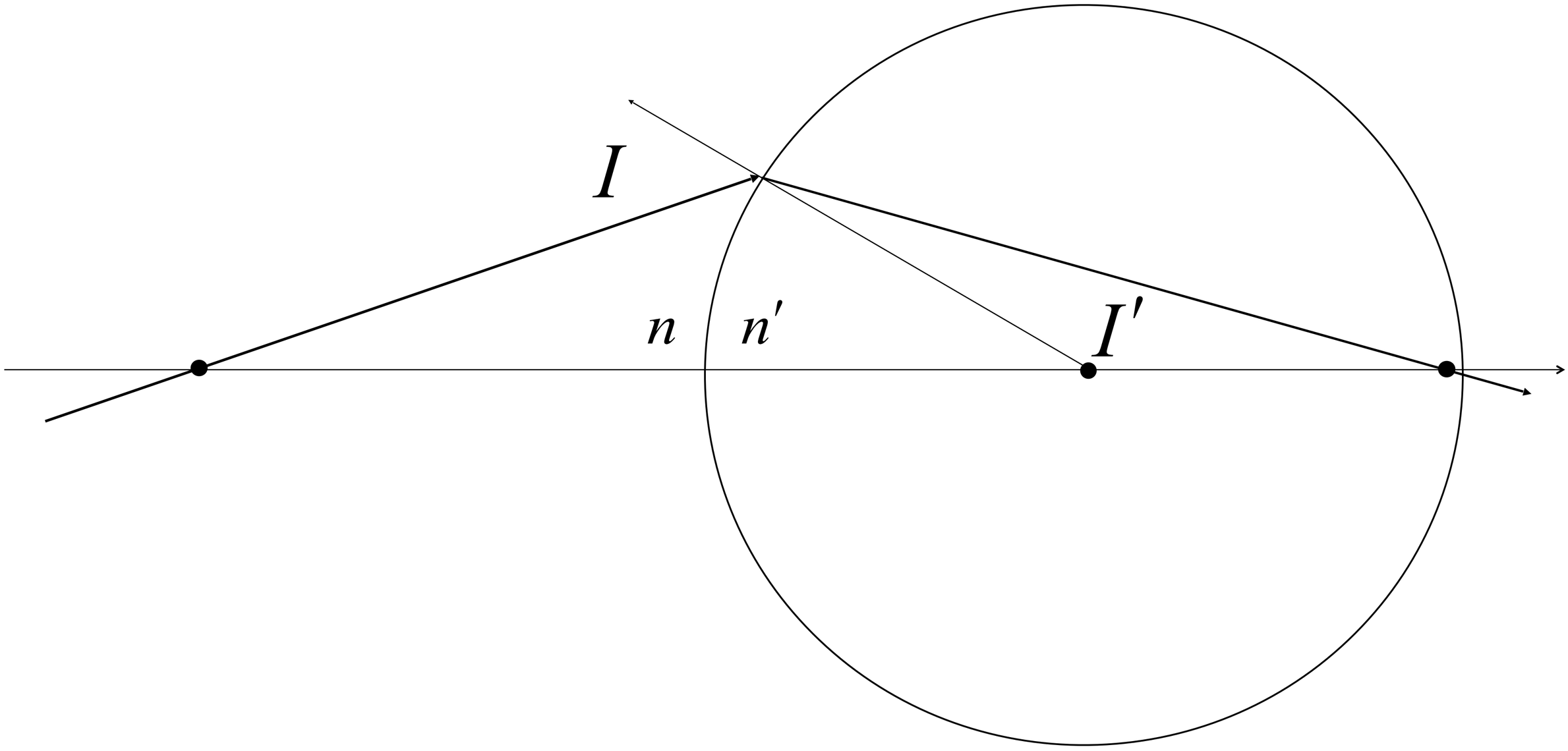
$$I' = U' + \phi$$



Opposite angles are equals

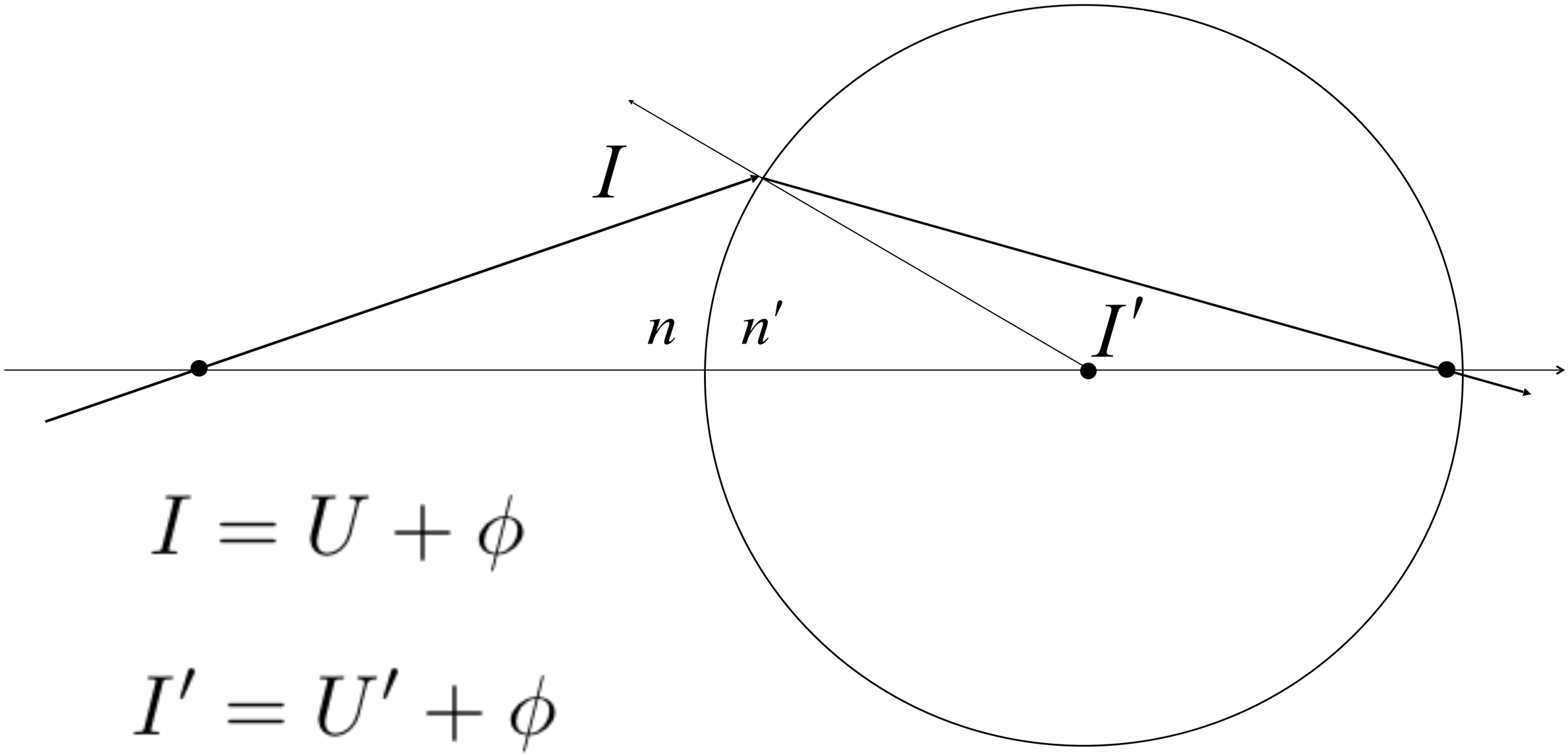
Snell's Law

$$n' \sin I' = n \sin I$$



Snell's Law

$$n' \sin(U' + \phi) = n \sin(U + \phi)$$

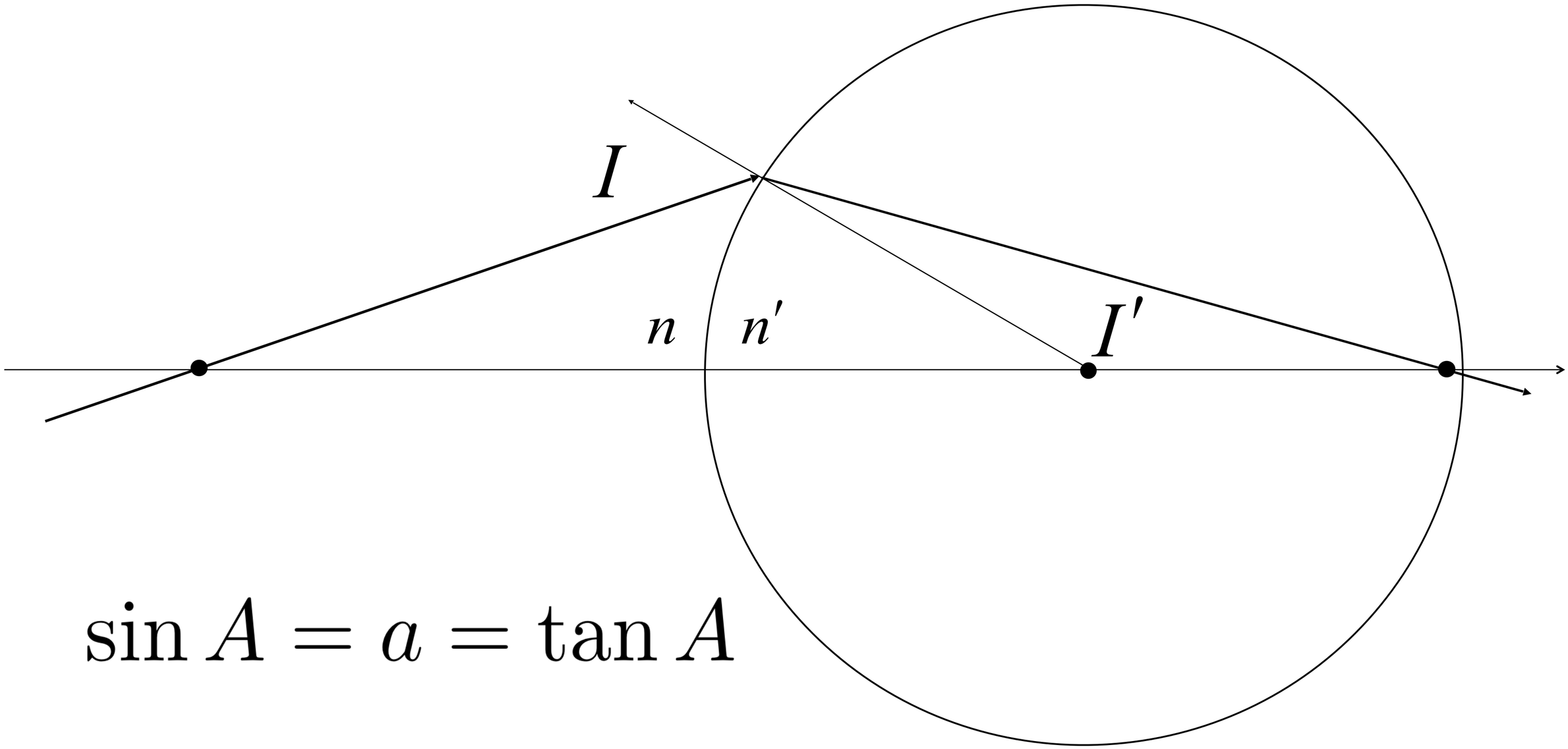


$$I = U + \phi$$

$$I' = U' + \phi$$

Paraxial Approximation: Small Angles

$$n' \sin(U' + \phi) = n \sin(U + \phi)$$

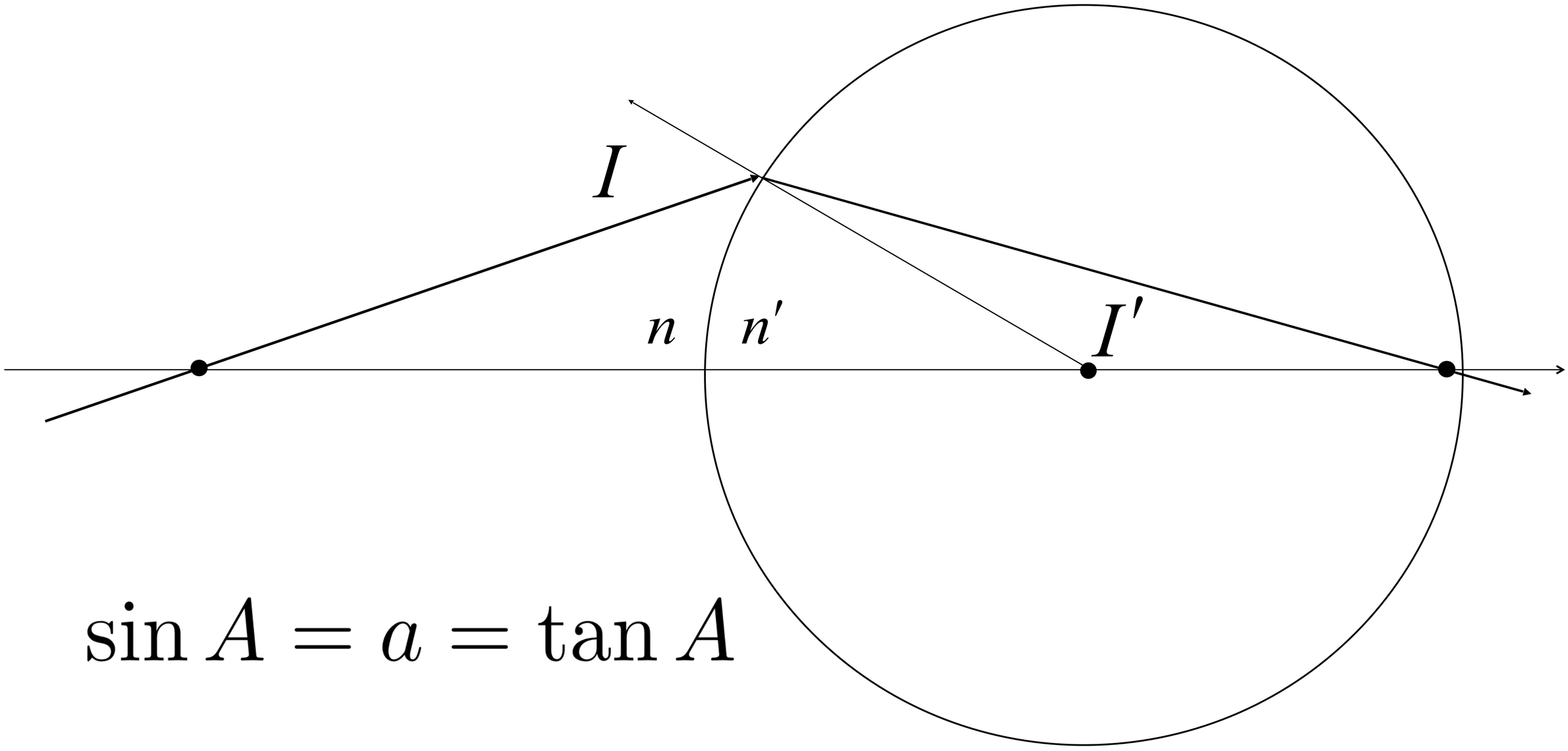


$$\sin A = a = \tan A$$

Rays deviate only slightly from the axis

Paraxial Approximation: Small Angles

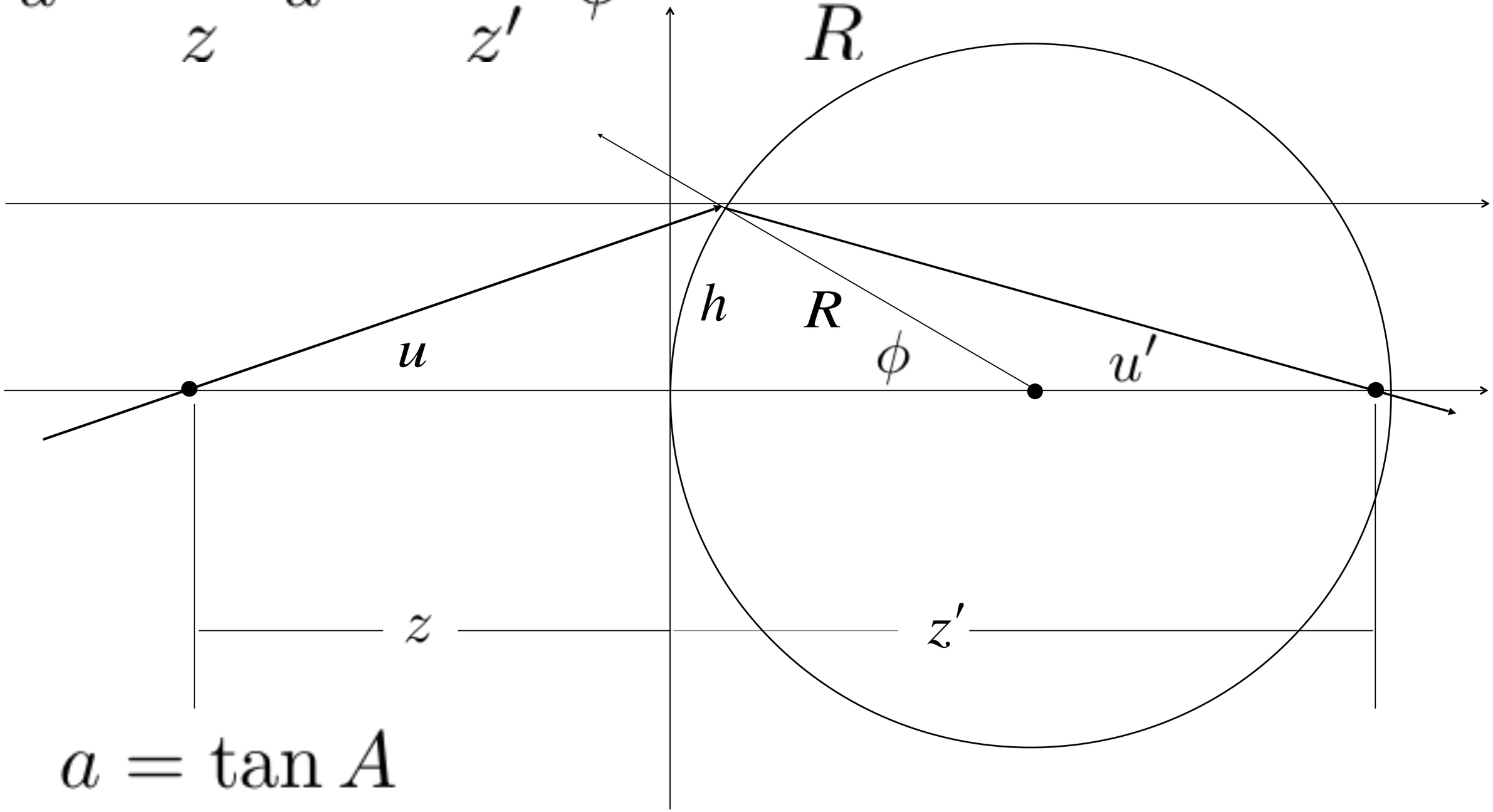
$$n'(u' + \phi) = n(u + \phi)$$



$$\sin A = a = \tan A$$

Angles to Slopes

$$u = \frac{h}{z} \quad u' = \frac{h}{z'} \quad \phi = -\frac{h}{R}$$



$$a = \tan A$$

Gauss' Formula

Paraxial approximation to Snell's Law

$$n'(u' + \phi) = n(u + \phi)$$

Ray coordinates

$$u = \frac{h}{z} \quad u' = \frac{h}{z'} \qquad \phi = -\frac{h}{R}$$

$$n'(\frac{h}{z'} - \frac{h}{R}) = n(\frac{h}{z} - \frac{h}{R})$$

$$\frac{n'}{z'} = \frac{n}{z} + \frac{(n' - n)}{R} \quad \Leftarrow$$ **Holds for any height, any ray!**

Perspective Transformation

How does *z* transform?

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \Rightarrow z' = \frac{fz}{z+f}$$

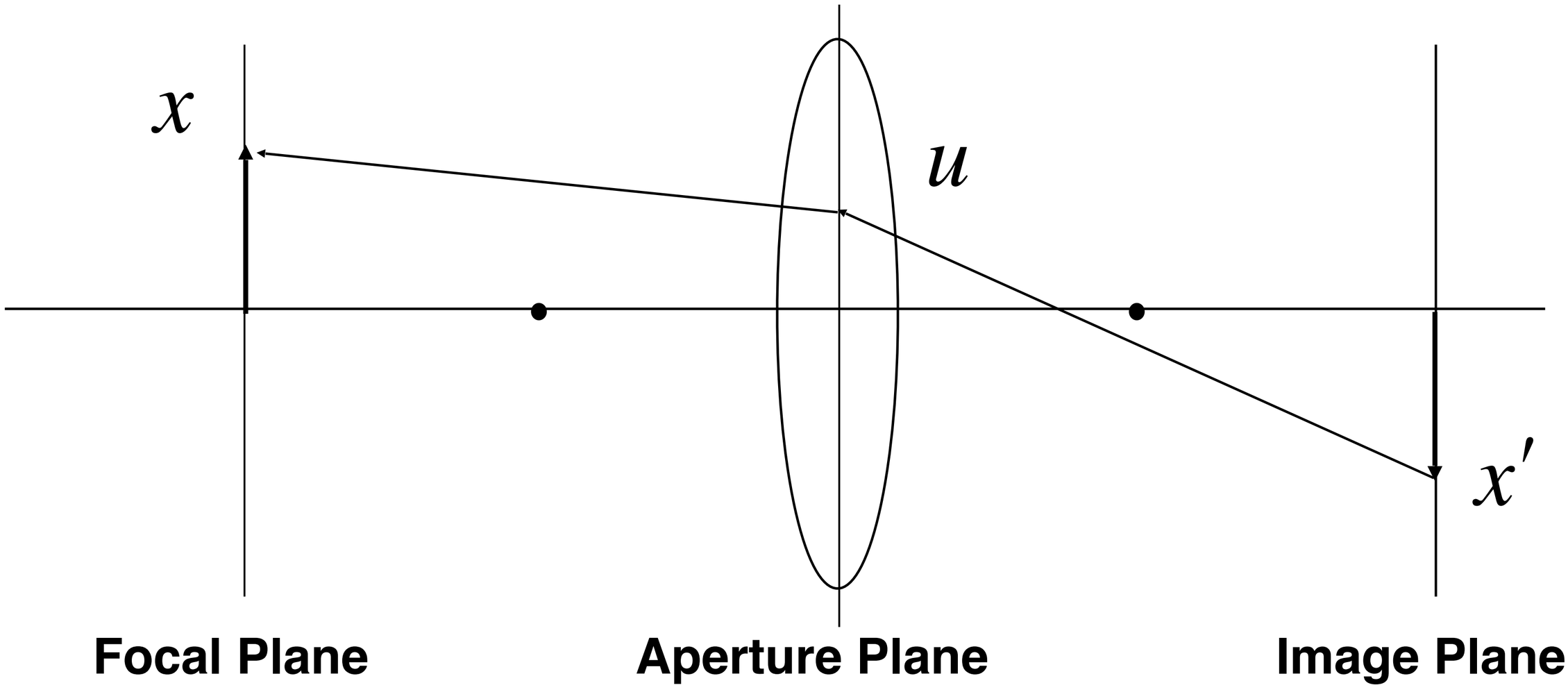
$$\Rightarrow x' = \frac{fx}{z+f}$$

$$\Rightarrow y' = \frac{fy}{z+f}$$

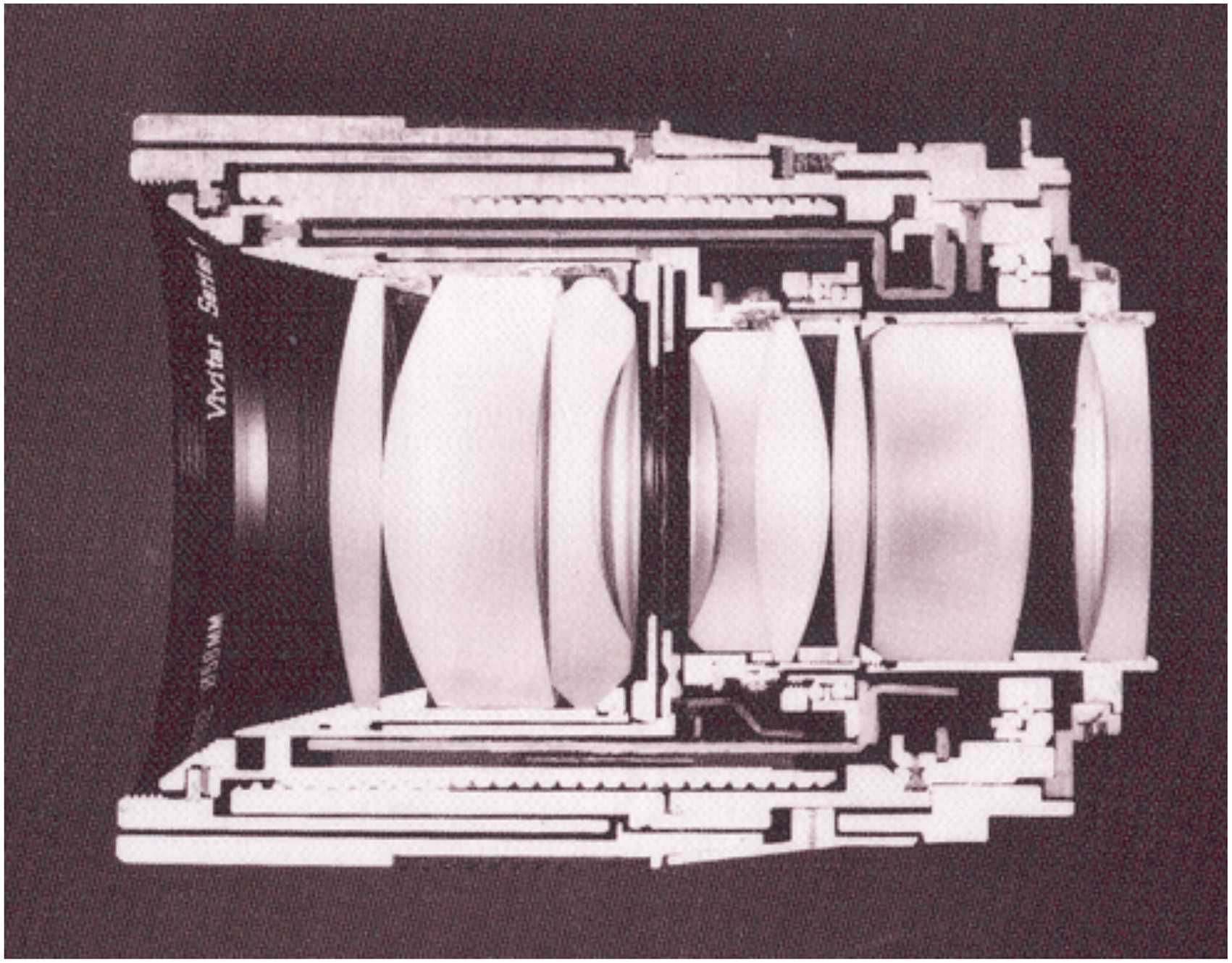
Represent lens transformation as a 4x4 matrix

Ray Tracing: Finite Aperture

1. Pick a point on image plane *x'*
2. Pick a point on the lens *u*
3. Transform *x'* to *x*; form ray (*u*,*x*-*u*)



Real Lens

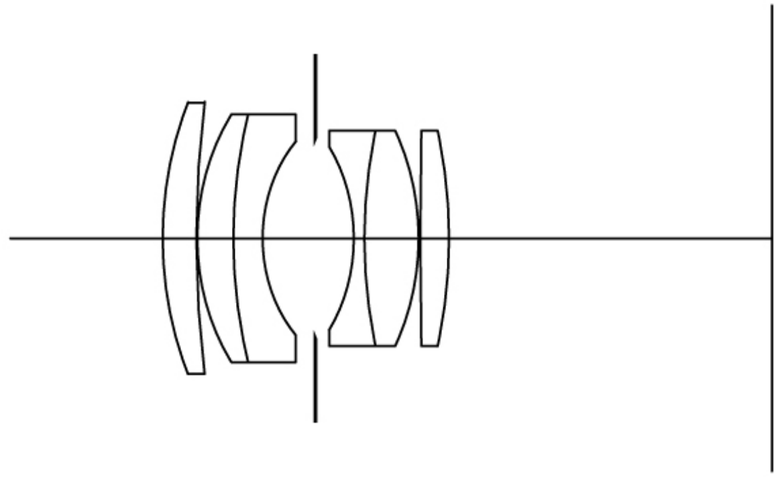


Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, Optics in Photography

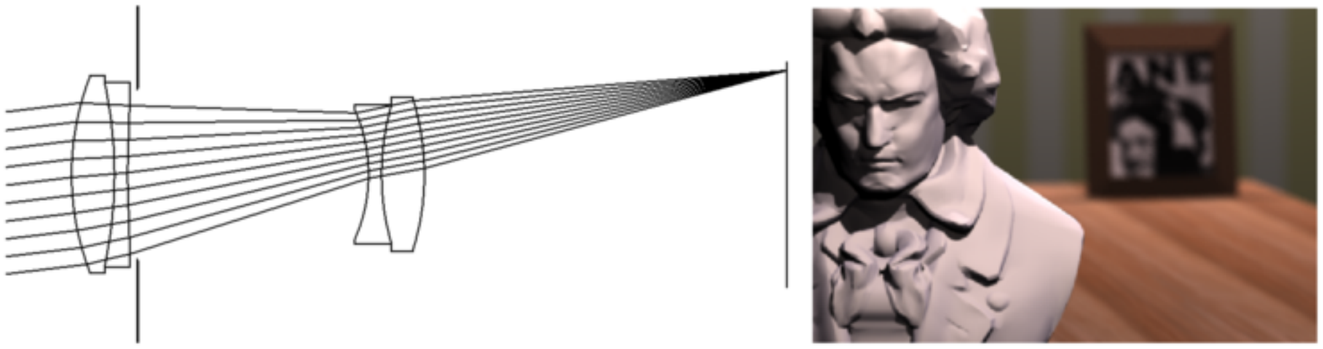
Double Gauss

Data from W. Smith,
Modern Lens Design, p 312

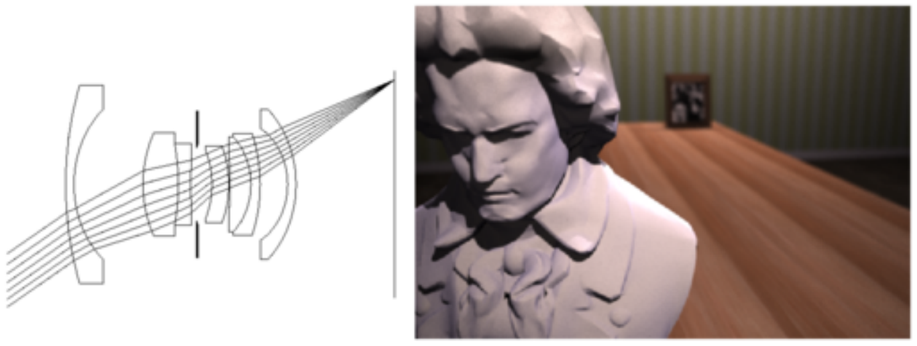
Radius	Thick	nd	V-no	aperture
58.950	7.520	1.670	47.1	50.4
169.660	0.240			50.4
38.550	8.050	1.670	47.1	46.0
81.540	6.550	1.699	30.1	46.0
25.500	11.410			36.0
	9.000			34.2
-28.990	2.360	1.603	38.0	34.0
81.540	12.130	1.658	57.3	40.0
-40.770	0.380			40.0
874.130	6.440	1.717	48.0	40.0
-79.460	72.228			40.0



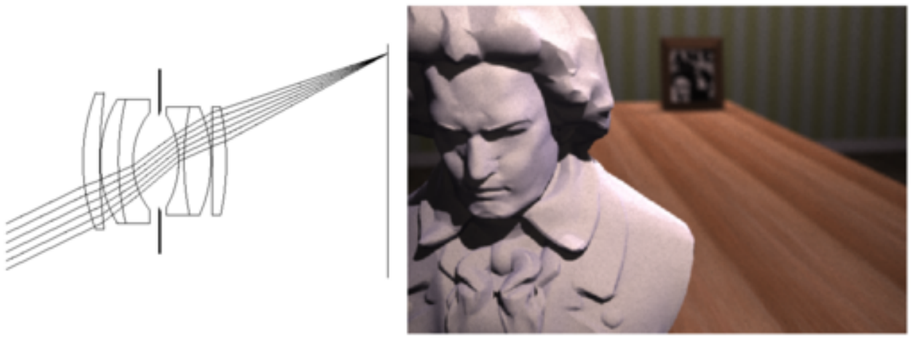
Ray Tracing Through Lenses



200 mm telephoto



35 mm wide-angle



50 mm double-gauss

16 mm fisheye

From Kolb, Mitchell and Hanrahan (1995)

Ray Tracing Through Lenses

R = Ray(point on film plane, point on rear-most element)
For each lens element E_i, from rear to front,
    p = intersection of R and E_i
    If p is outside clear aperture of E_i
        ray is blocked
    Else if medium on far side of $E_i \neq$ medium on near side
        compute new direction for R using Snell's law

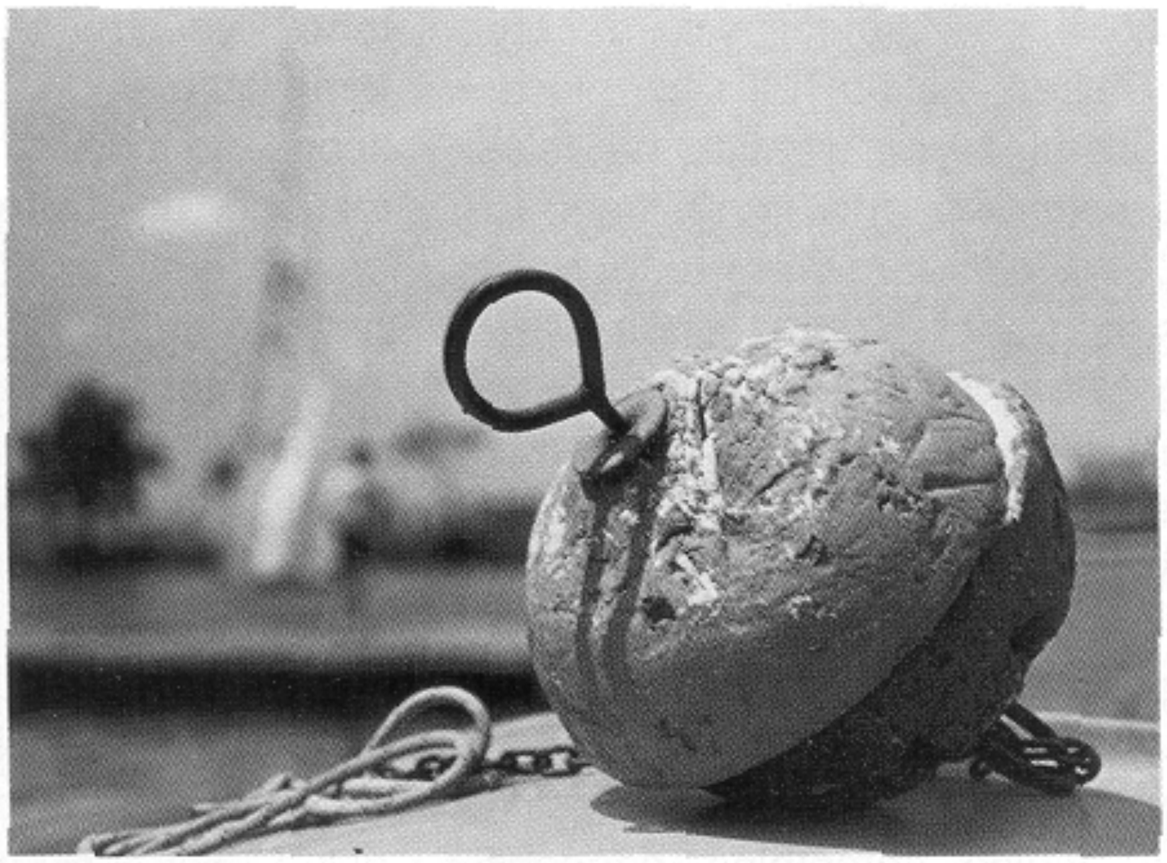
$$E(x') = \frac{1}{Z^2} \int_{x'' \in D} L(x'', x') \cos^4 \theta' \, dA''$$

From Kolb, Mitchell and Hanrahan (1995)

Depth of Field

Depth of Field

less depth of field



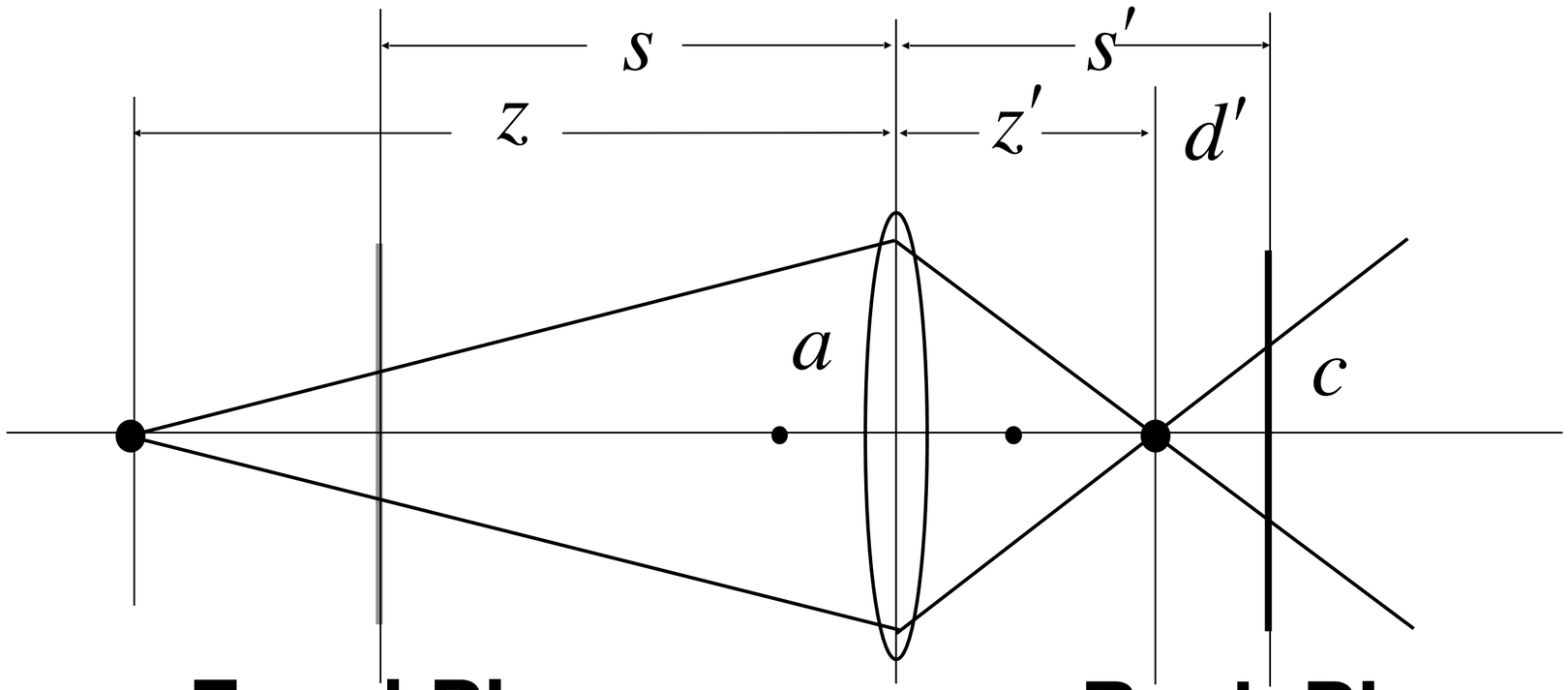
wider aperture

more depth of field

smaller aperture

From London and Upton

Circle of Confusion



Circle of confusion proportional to the size of the aperture

$$\frac{c}{a} = \frac{d'}{z'} = \frac{s' - z'}{z'}$$

Resolving Power

Diffraction limit

$$c = 1.22 \frac{f}{a} \lambda \quad \left[= 1.22 \times 64 \times .500 \mu\text{m} = 0.040\,\text{mm} \right]$$

35mm film (Leica standard)

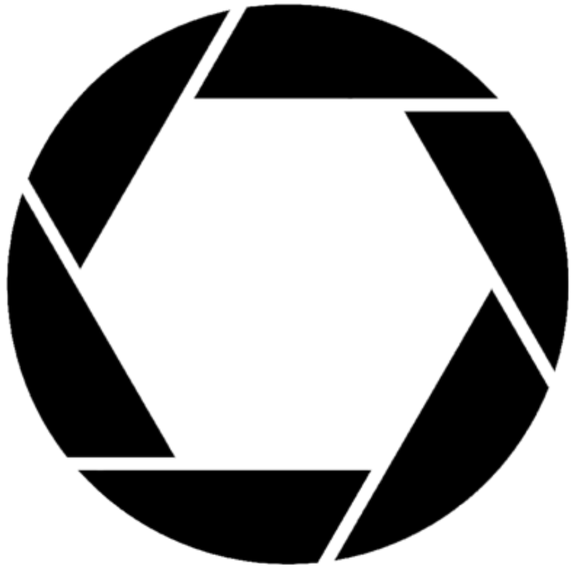
$$c = 0.025\,\text{mm}$$

CCD/CMOS pixel aperture

$$c = 0.0116\,\text{mm (Nikon D1)}$$

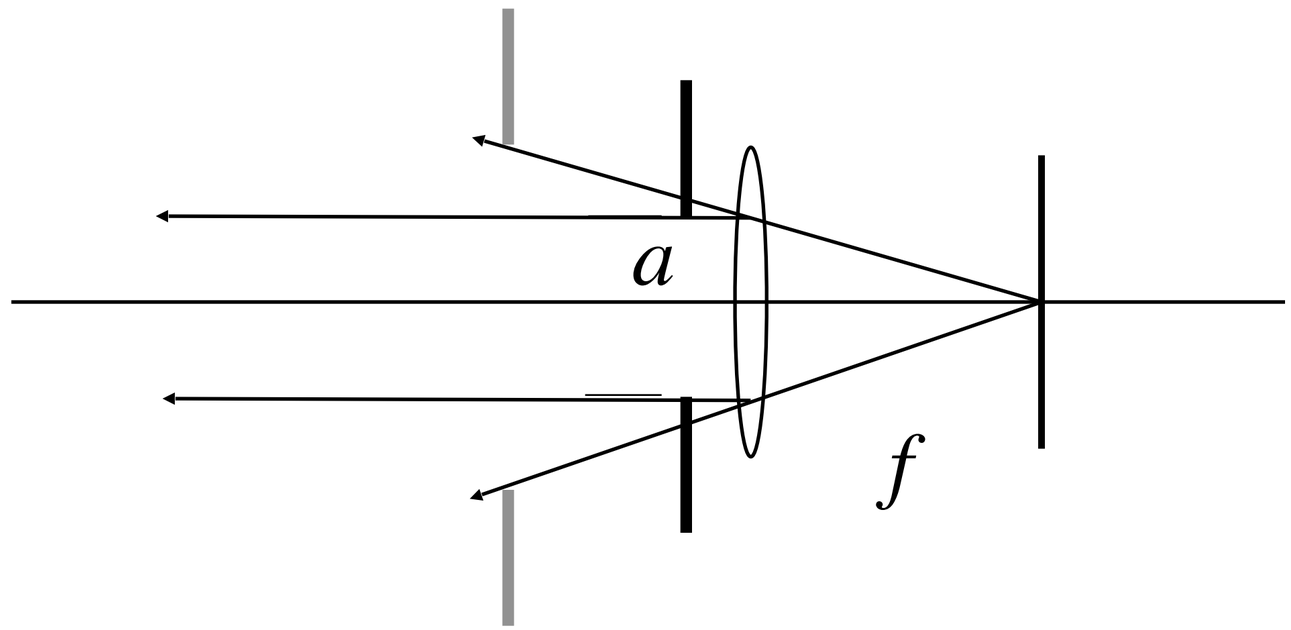
Bokeh

Photo credit: Wikimedia's The Photographer



Exposure

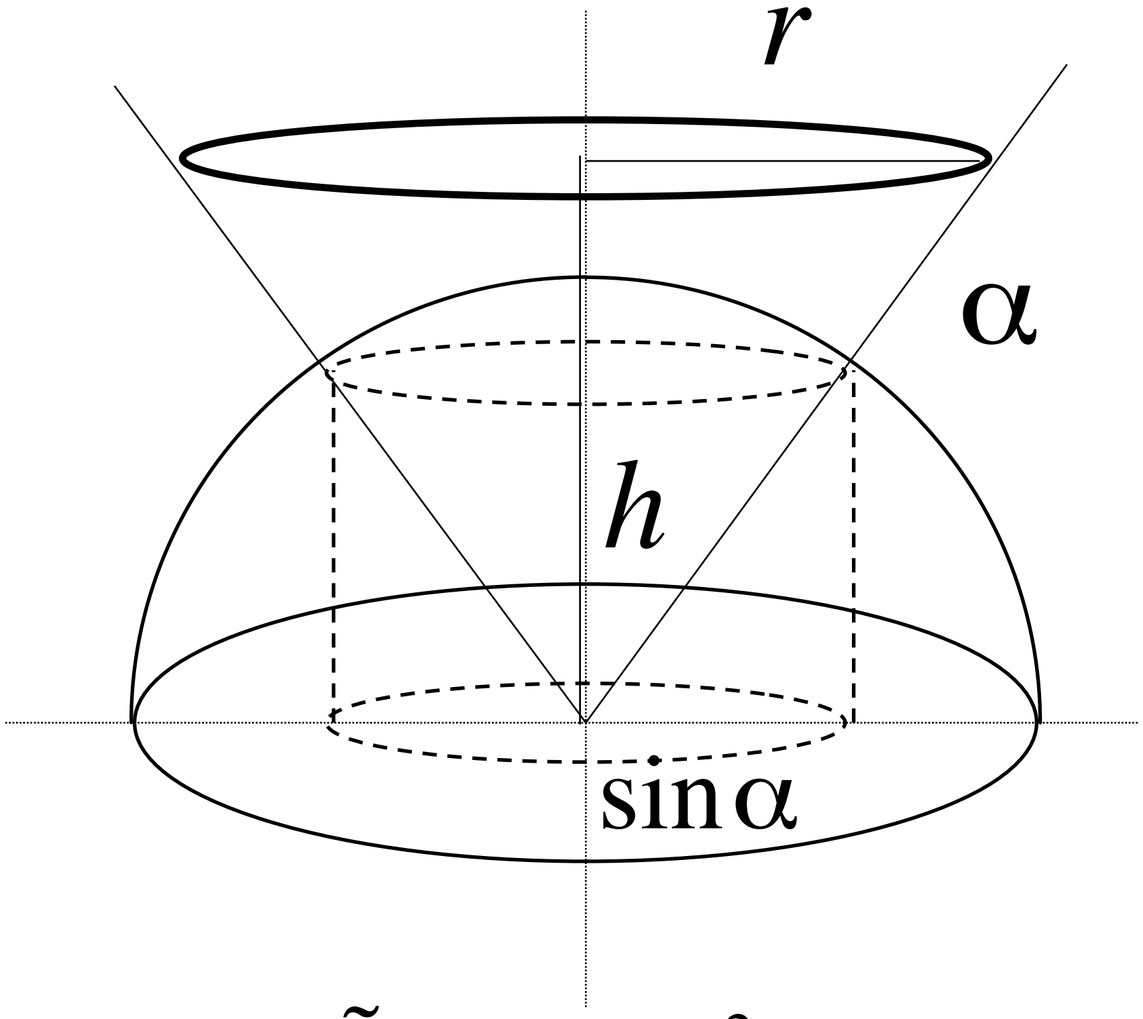
Image Irradiance



$$E = \int_{\Omega} L \cos\theta \, d\omega = L\pi \sin^2\theta = L\frac{\pi}{4}\left(\frac{a}{f}\right)^2$$

Uniform Disk Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

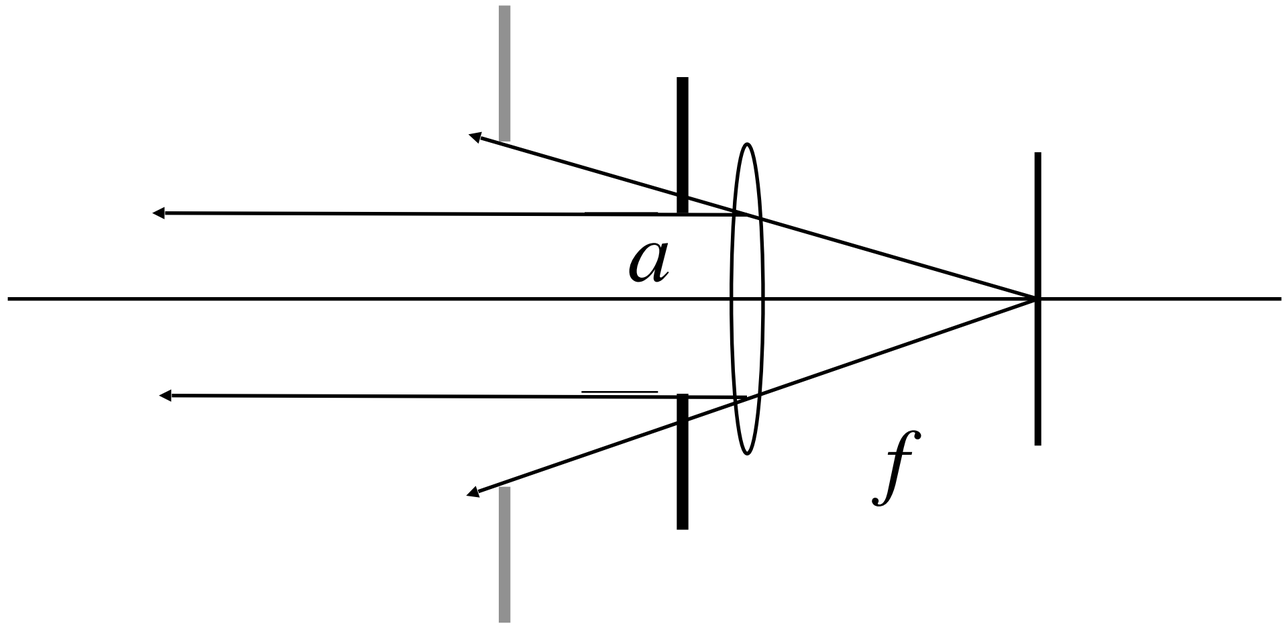
$$\tilde{\Omega} = \int_1^{\cos\alpha} \int_0^{2\pi} \cos\theta \, d\phi \, d\cos\theta$$

$$= 2\pi \left. \frac{\cos^2\theta}{2} \right|_1^{\cos\alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$

Relative Aperture or F-Stop



$$a = \frac{f}{N}$$

F-Number and exposure:

$$E = L \frac{\pi}{4} \frac{1}{N^2}$$

Fstops: 1.4 2 2.8 4.0 5.6 8 11 16 22 32 45 64

1 stop doubles exposure

Camera Exposure

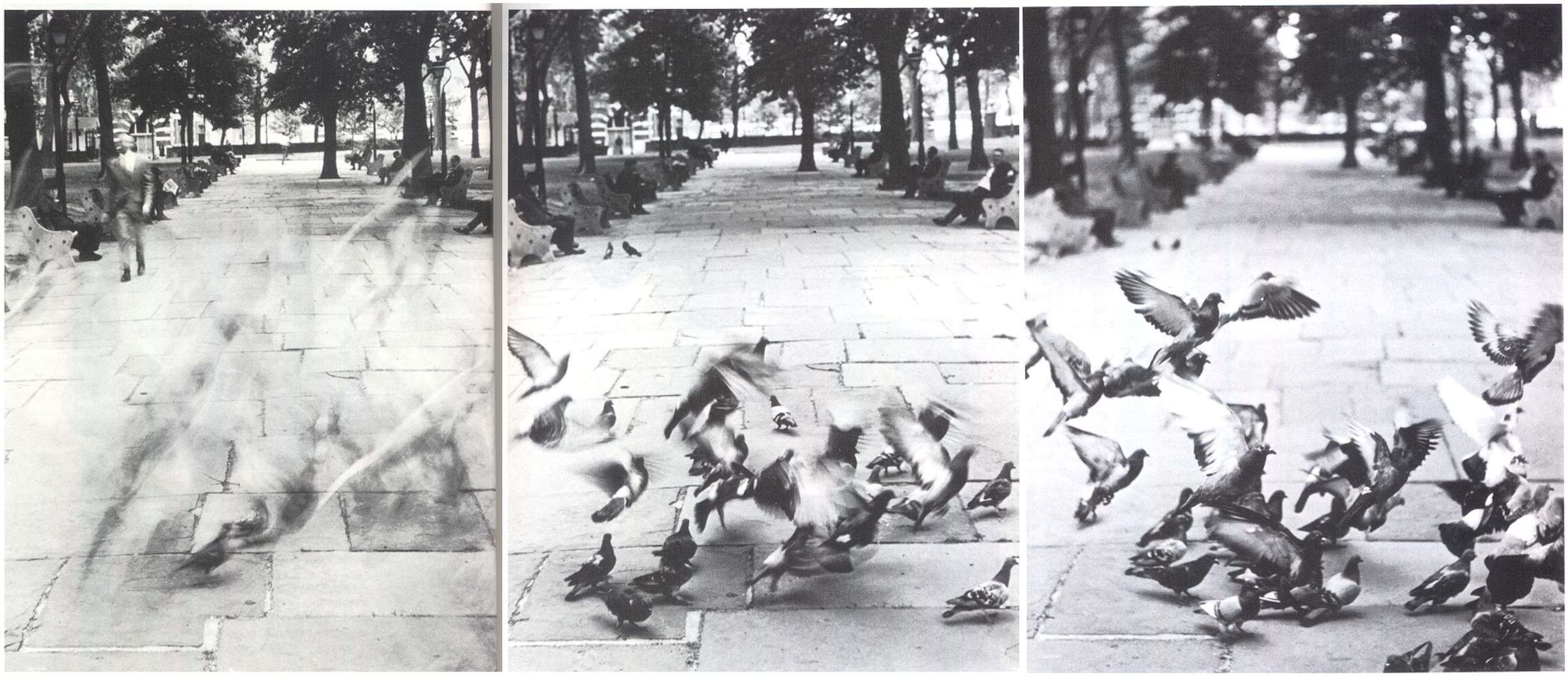
Exposure $H = E \times T$

Exposure overdetermined

- **Aperture: f-stop - 1 stop doubles *H***
  - **Decreases depth of field**
- **Shutter: Doubling the open time doubles *H***
  - **Increases motion blur**

Aperture vs Shutter

Constant Exposure



f/16
1/8s

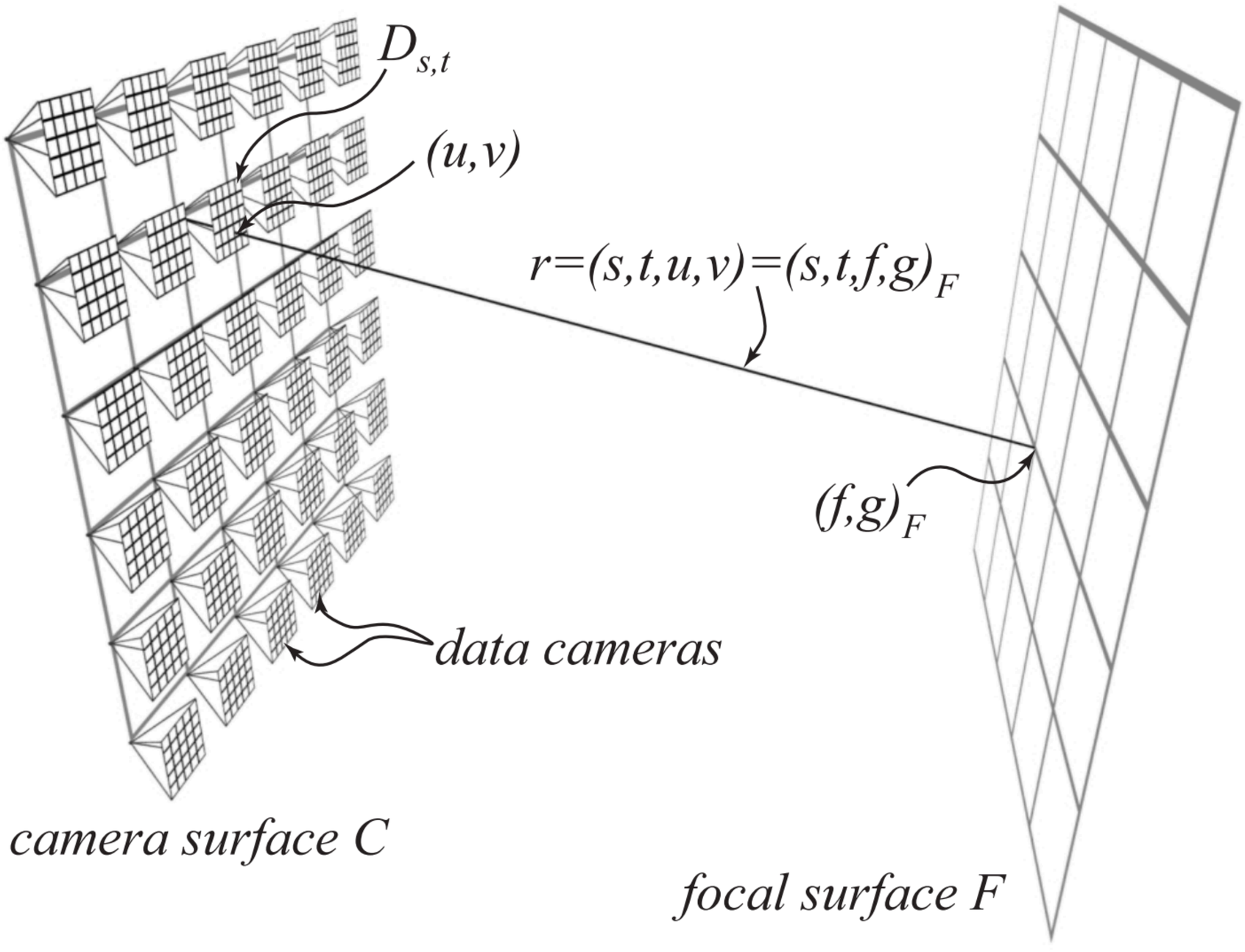
f/4
1/125s

f/2
1/500s

From London and Upton

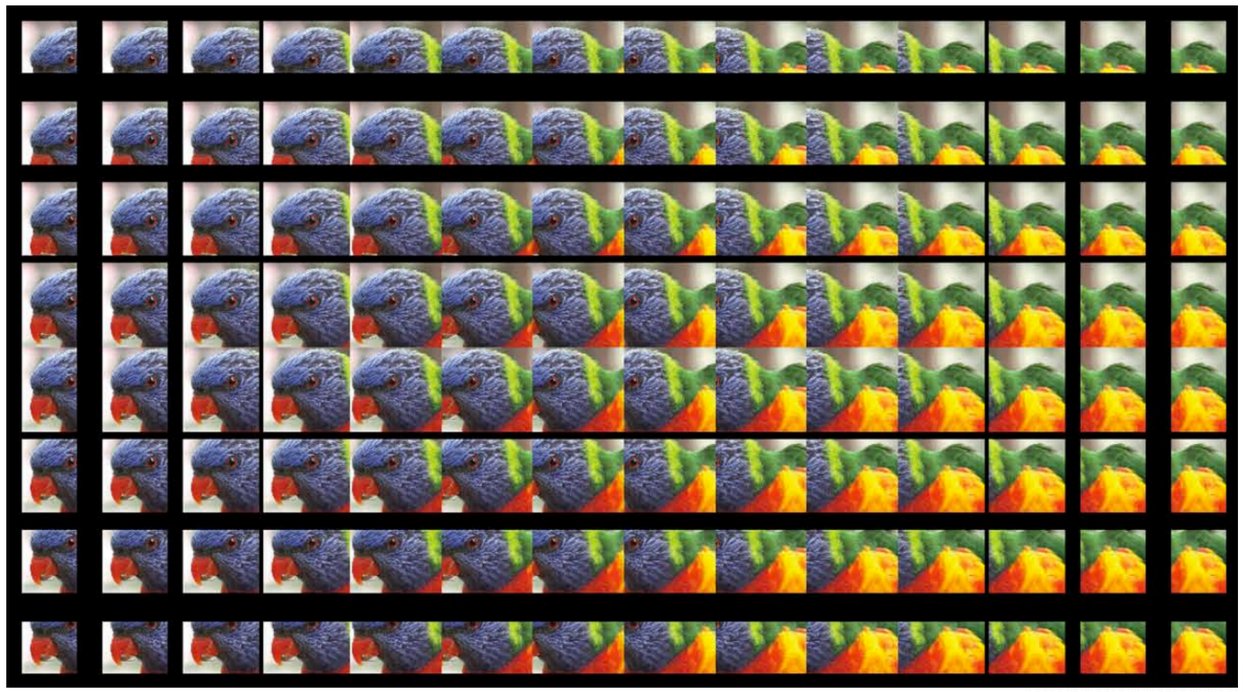
Lightfield Cameras

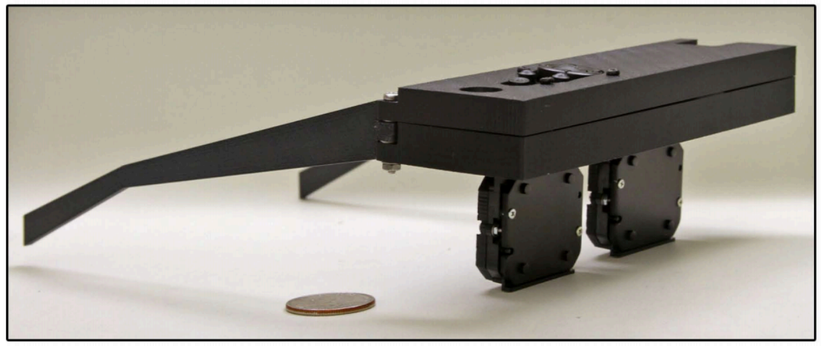
Lightfield Camera

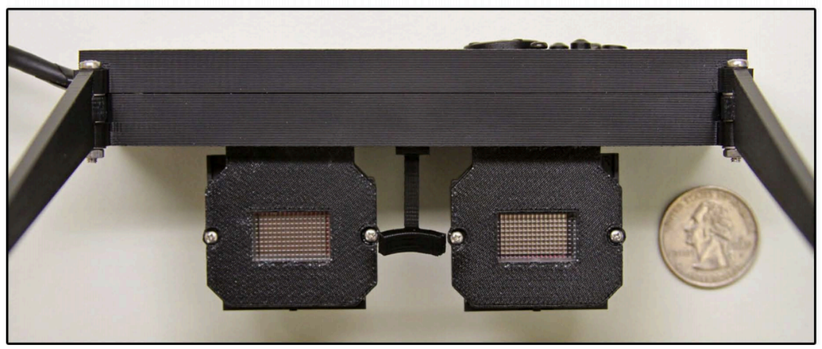


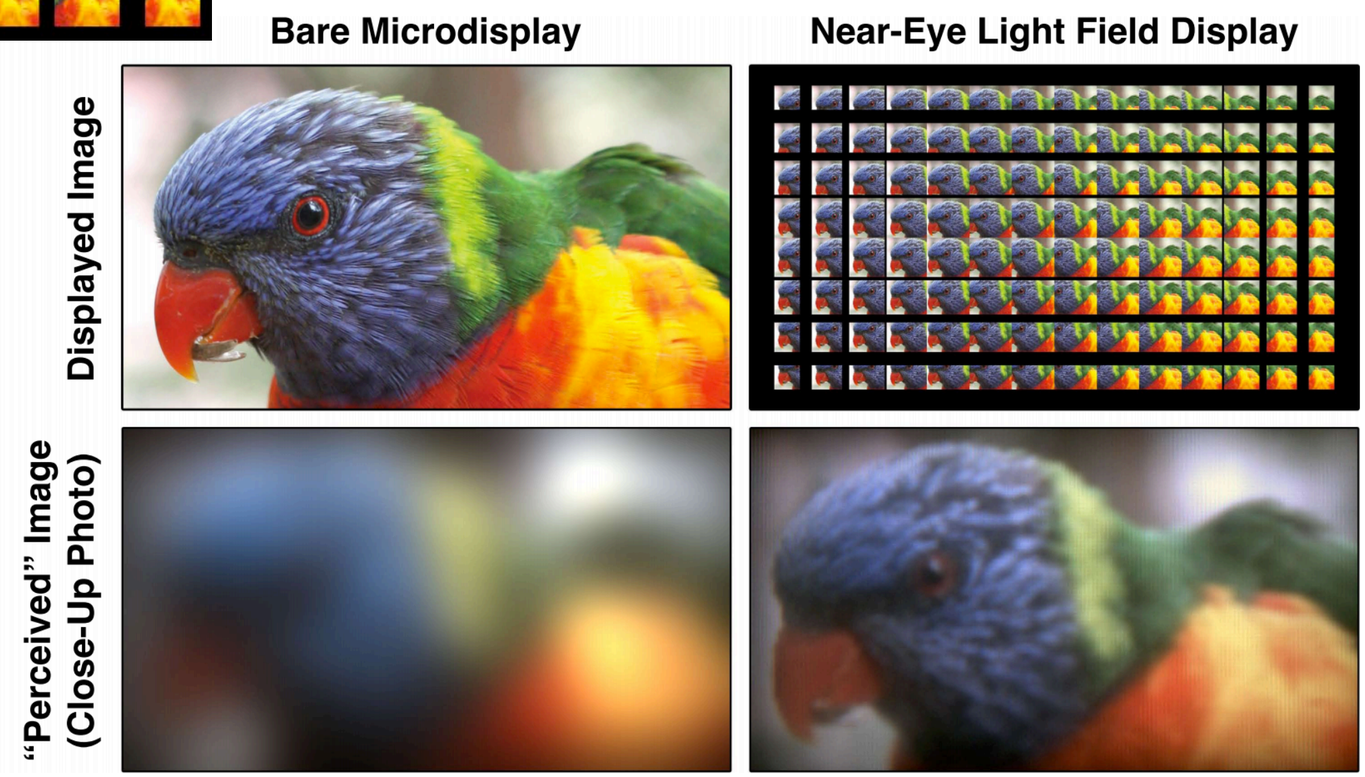
From Isaksen, McMillan, Gortler (2000)

Lightfield Displays









From Lanman and Luebke (2013)