Reflection Models I: Ideal Materials

Today

- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Next lecture (Thursday)

- Glossy and specular reflection models
- Rough surfaces and microfacets

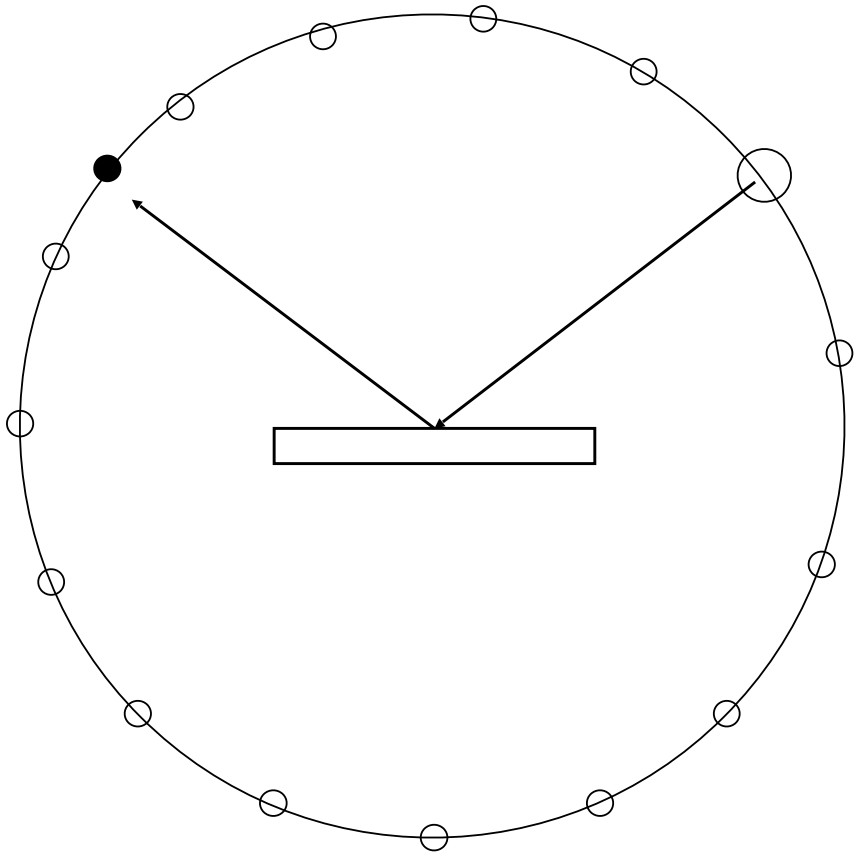
Reflection Models

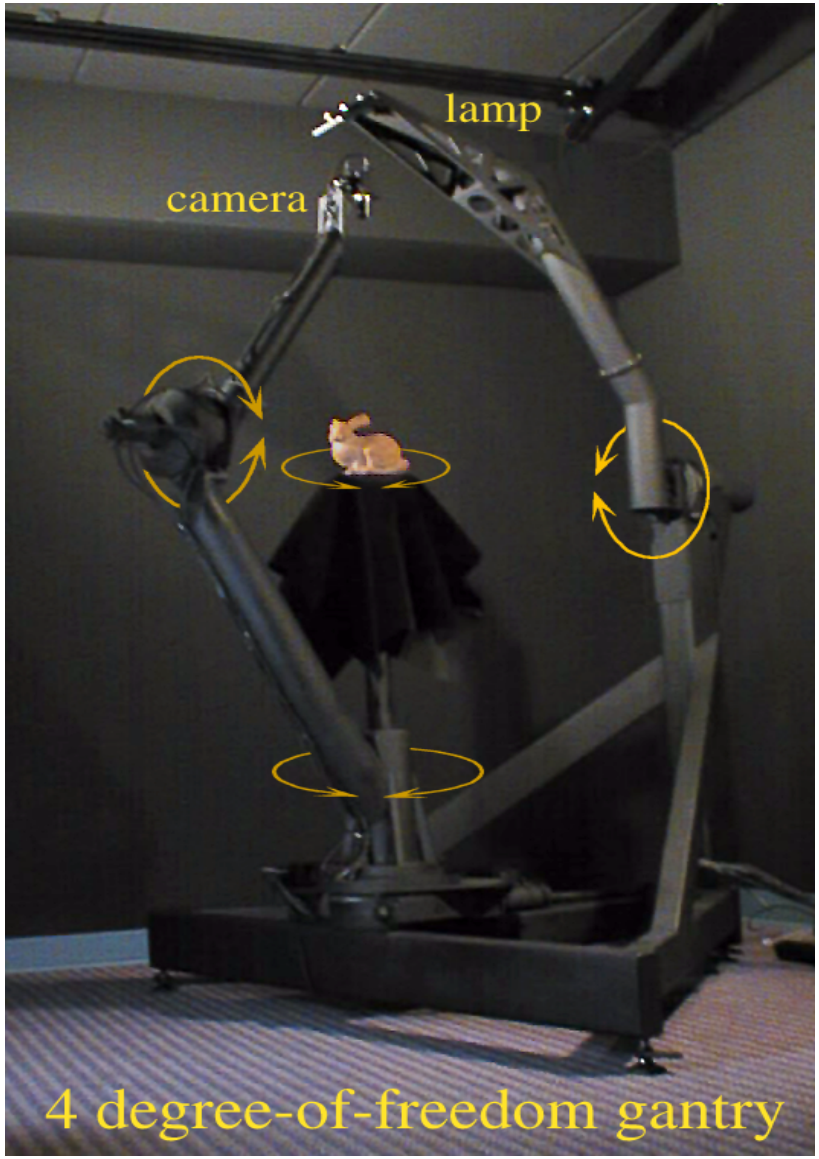
Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

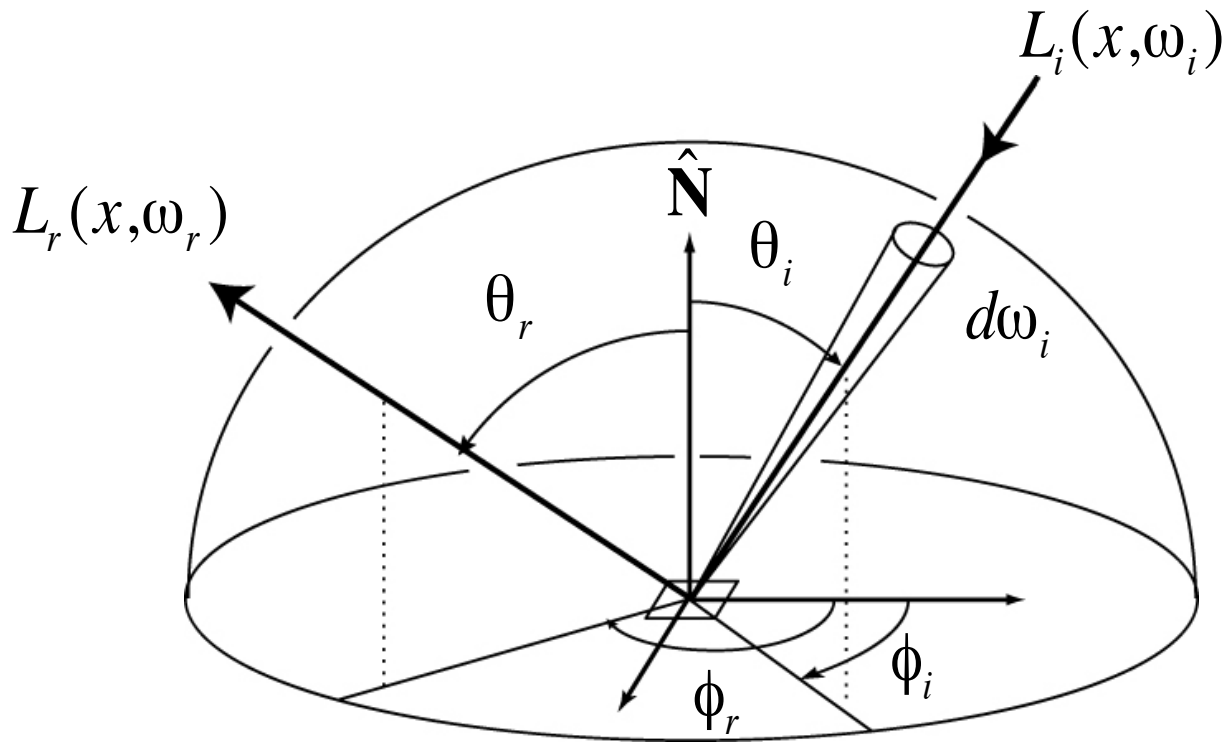
- **Spectra and Color**
- **Polarization**
- **Directional distribution**

Gonio-reflectometer





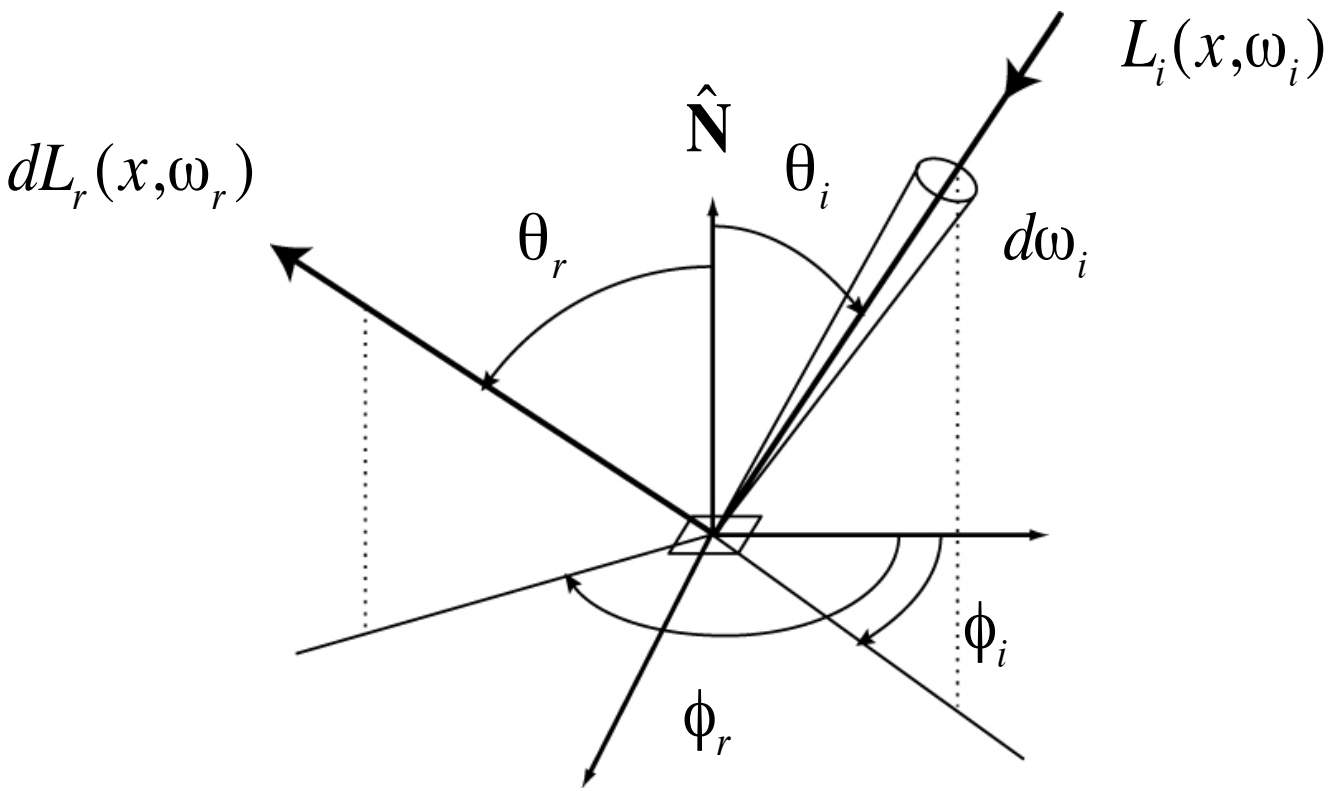
The Reflection Equation



$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \rightarrow \omega_r) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

The BRDF

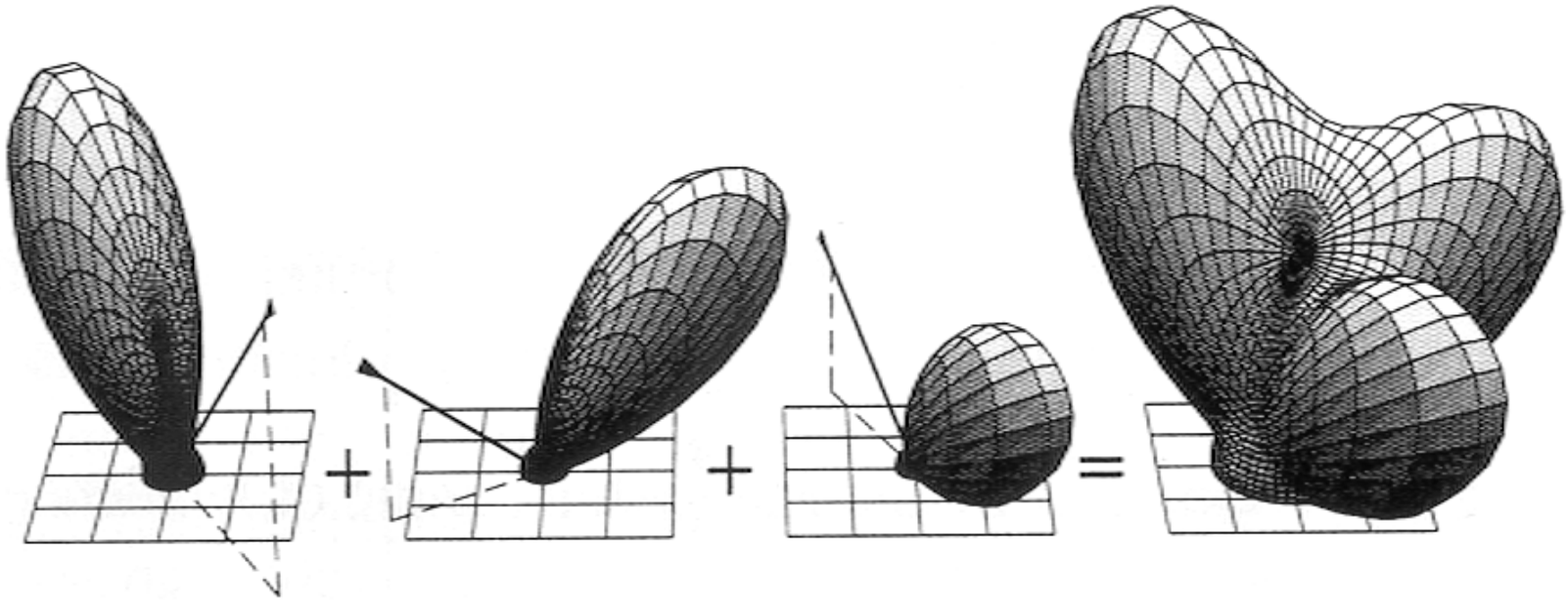
Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

Properties of BRDF's

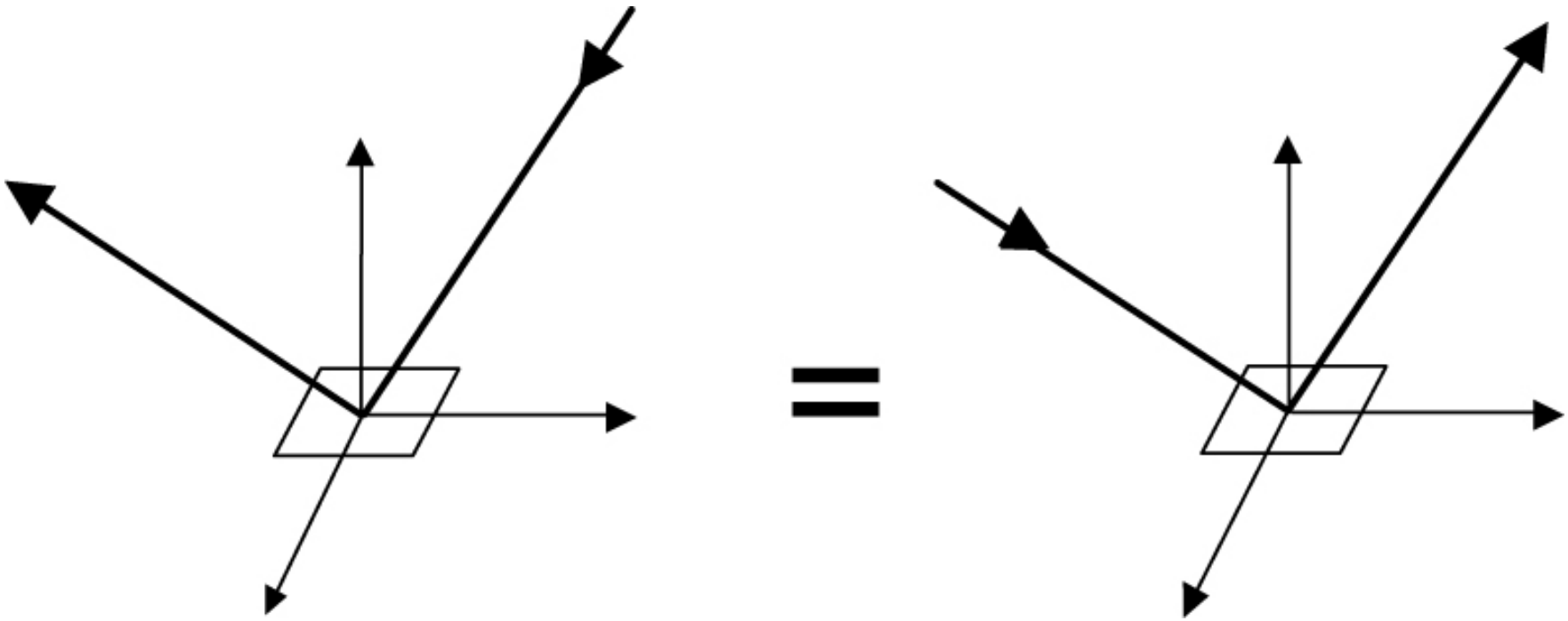
1. Linearity



From Sillion, Arvo, Westin, Greenberg

Properties of BRDF's

2. Reciprocity principle

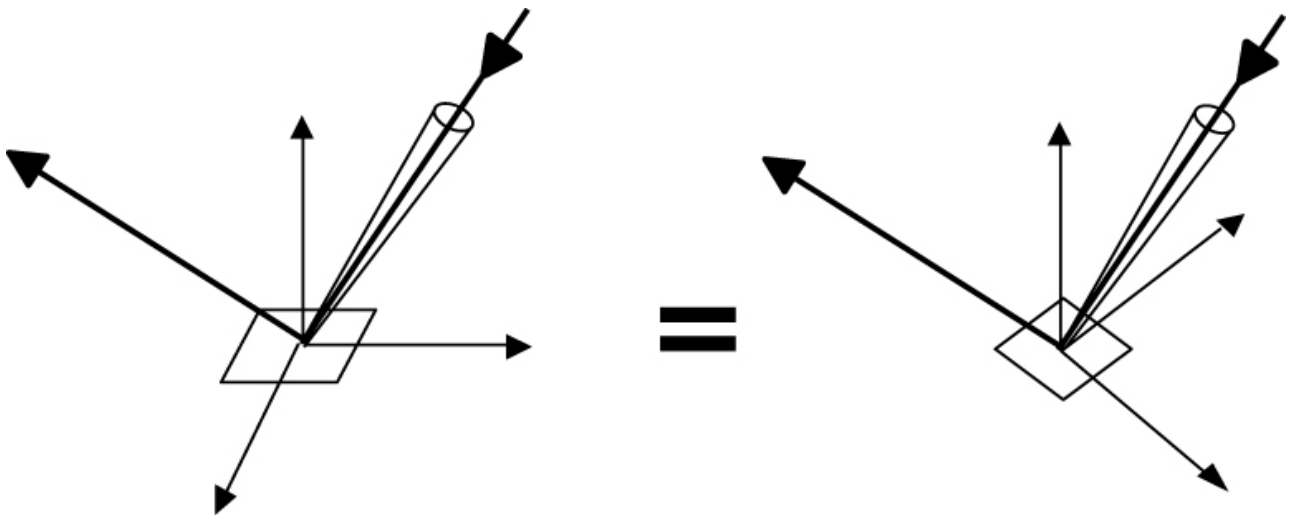


$$f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r)$$

Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i,\varphi_i;\theta_r,\varphi_r) = f_r(\theta_i,\theta_r,\varphi_r - \varphi_i)$$

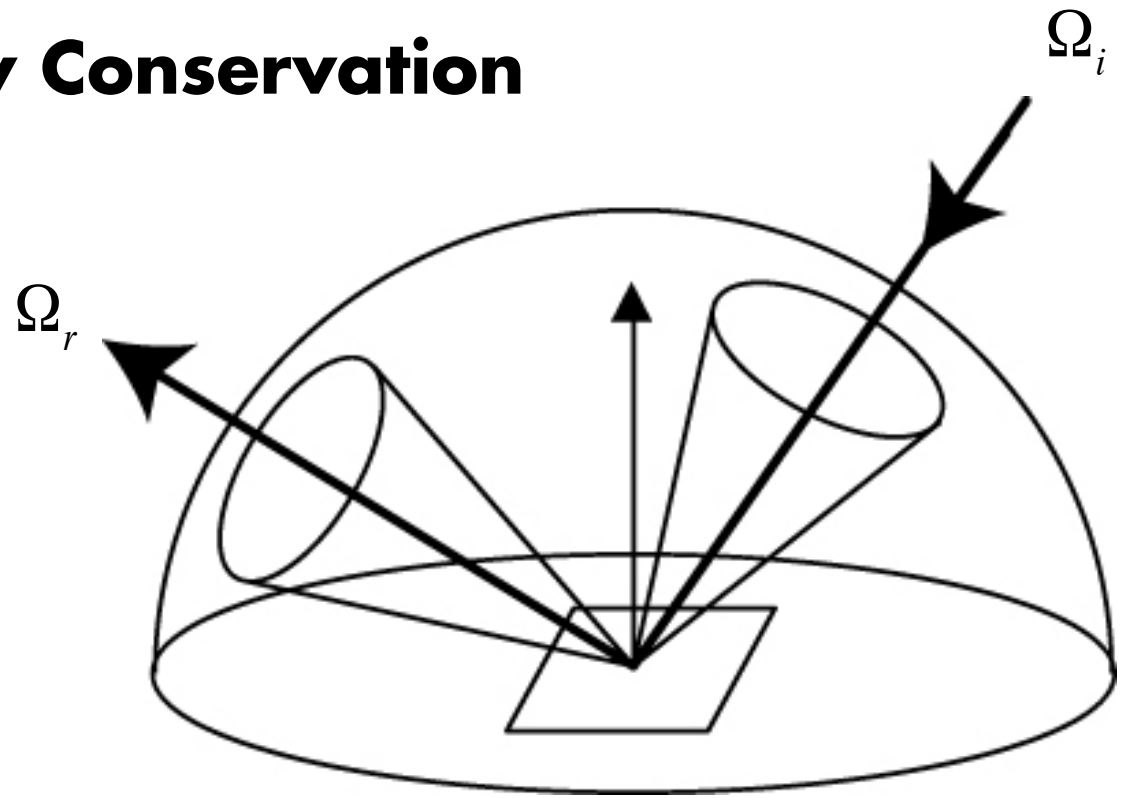


Reciprocity and isotropy

$$f_r(\theta_i,\theta_r,\varphi_r - \varphi_i) = f_r(\theta_r,\theta_i,\varphi_i - \varphi_r) = f_r(\theta_i,\theta_r,|\varphi_r - \varphi_i|)$$

Properties of BRDF's

4. Energy Conservation

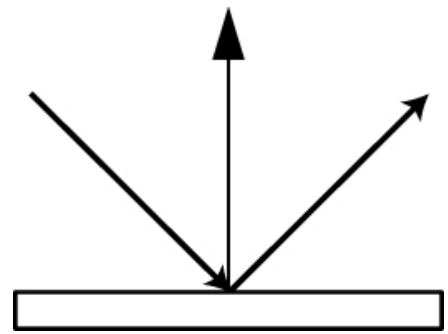


$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r)\cos\theta_r \, d\omega_r}{\int_{\Omega_i} L_r(\omega_o)\cos\theta_i \, d\omega_i} \leq 1$$

Types of Reflection Functions

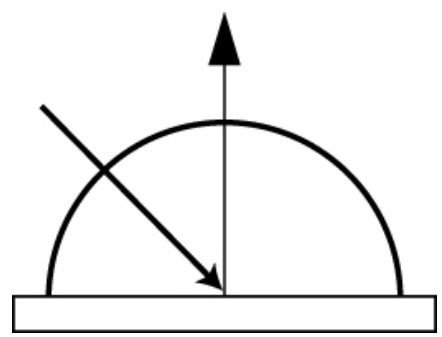
Ideal Specular

- **Reflection Law**
- **Mirror**



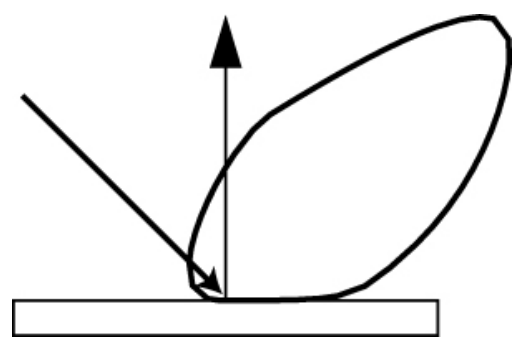
Ideal Diffuse

- **Lambert's Law**
- **Matte**

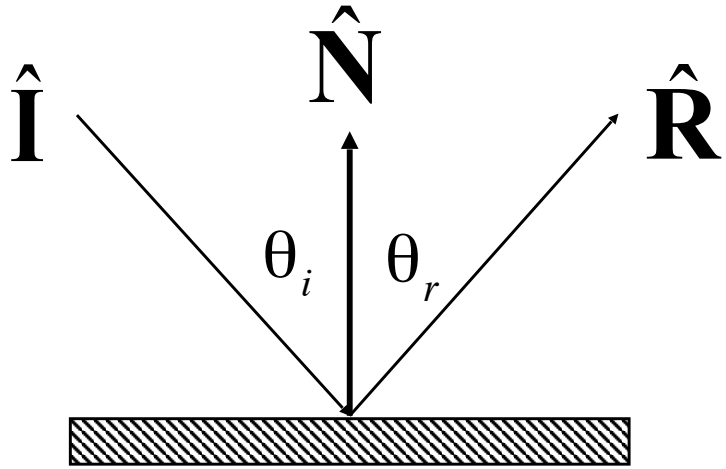


Specular

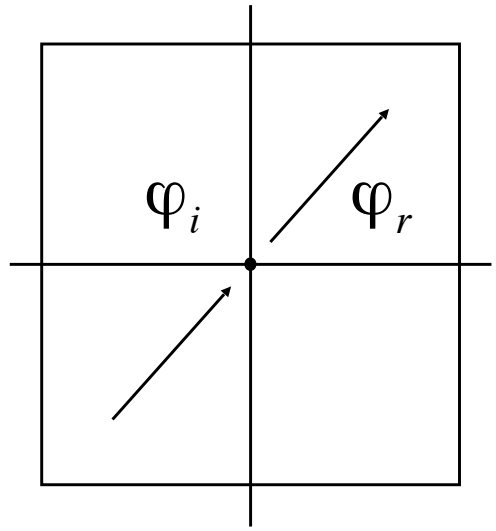
- **Glossy**
- **Directional diffuse**



Law of Reflection



$$\theta_r = \theta_i$$

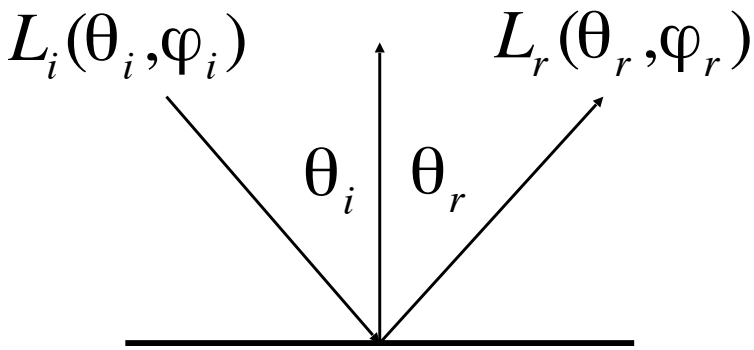


$$\varphi_r = (\varphi_i + \pi) \bmod 2\pi$$

$$\hat{\mathbf{R}} + (-\hat{\mathbf{I}}) = 2\cos\theta\,\hat{\mathbf{N}} = -2(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

$$\hat{\mathbf{R}} = \hat{\mathbf{I}} - 2(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

Ideal Reflection (Mirror)

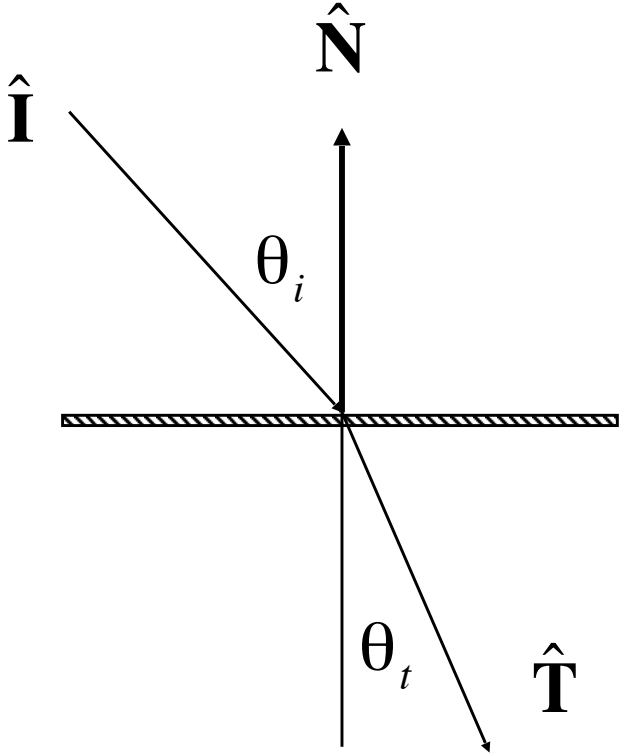


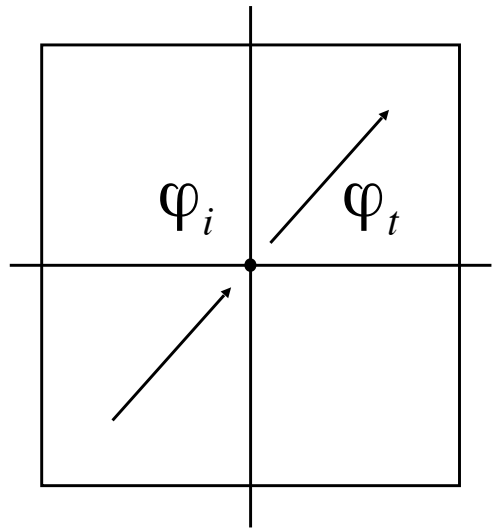
$$L_{r,m}(\theta_r, \varphi_r) = L_i(\theta_r, \varphi_r \pm \pi)$$

$$f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) = \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i} \delta(\varphi_i - \varphi_r \pm \pi)$$

$$\begin{aligned} L_{r,m}(\theta_r, \varphi_r) &= \int f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) L_i(\theta_i, \varphi_i) \cos\theta_i \, d\cos\theta_i \, d\varphi_i \\ &= \int \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i} \delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i) \cos\theta_i \, d\cos\theta_i \, d\varphi_i \\ &= L_i(\theta_r, \varphi_r \pm \pi) \end{aligned}$$

Recall: Snell's Law



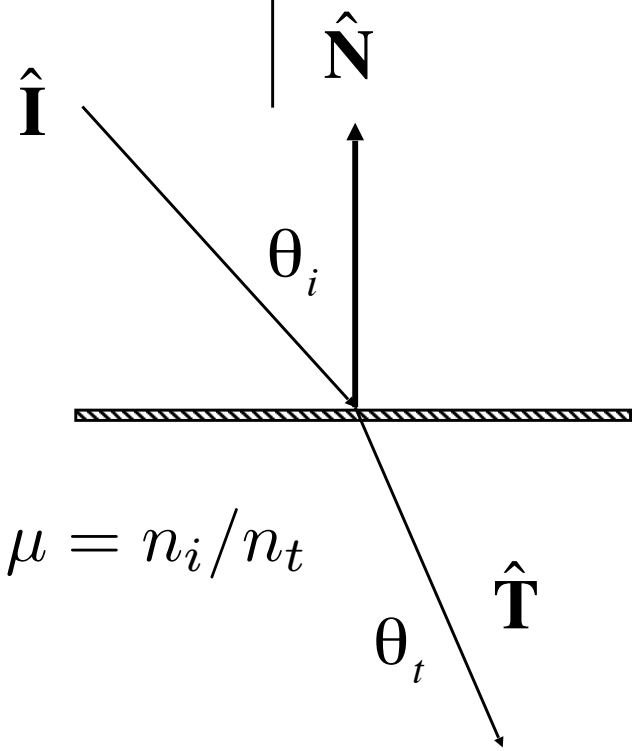


$$\varphi_t = \varphi_i \pm \pi$$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_i \, \hat{\mathbf{N}} \times \hat{\mathbf{I}} = n_t \, \hat{\mathbf{N}} \times \hat{\mathbf{T}}$$

Recall: Law of Refraction



$$\hat{\mathbf{T}} = \mu\,\hat{\mathbf{I}} + \gamma\,\hat{\mathbf{N}}$$

$$\hat{\mathbf{T}}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma\,\hat{\mathbf{I}}\cdot\hat{\mathbf{N}}$$

$$\mu = n_i/n_t$$

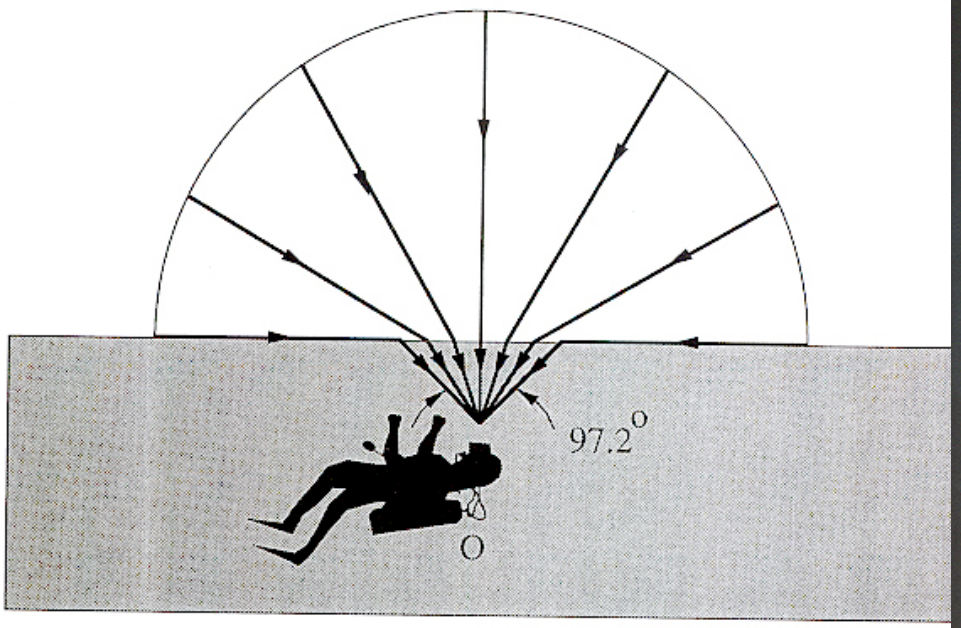
$$\begin{aligned}\gamma &= -\mu\,\hat{\mathbf{I}}\cdot\hat{\mathbf{N}} \pm \{-\mu^2(1-(\hat{\mathbf{I}}\cdot\hat{\mathbf{N}})^2)\}^{1/2} \\ &= \mu\cos\theta_i \pm (1-\mu^2\sin^2\theta_i)^{\frac{1}{2}} \\ &= \mu\cos\theta_i \pm \cos\theta_t \\ &= \mu\cos\theta_i - \cos\theta_t\end{aligned}$$

Total internal reflection:

$$1-\mu^2(1-(\hat{\mathbf{I}}\cdot\hat{\mathbf{N}})^2) < 0$$

Optical Manhole

Total internal reflection



$$n_w = \frac{4}{3}$$

From Livingston and Lynch

Experiment

Reflections from a shiny floor

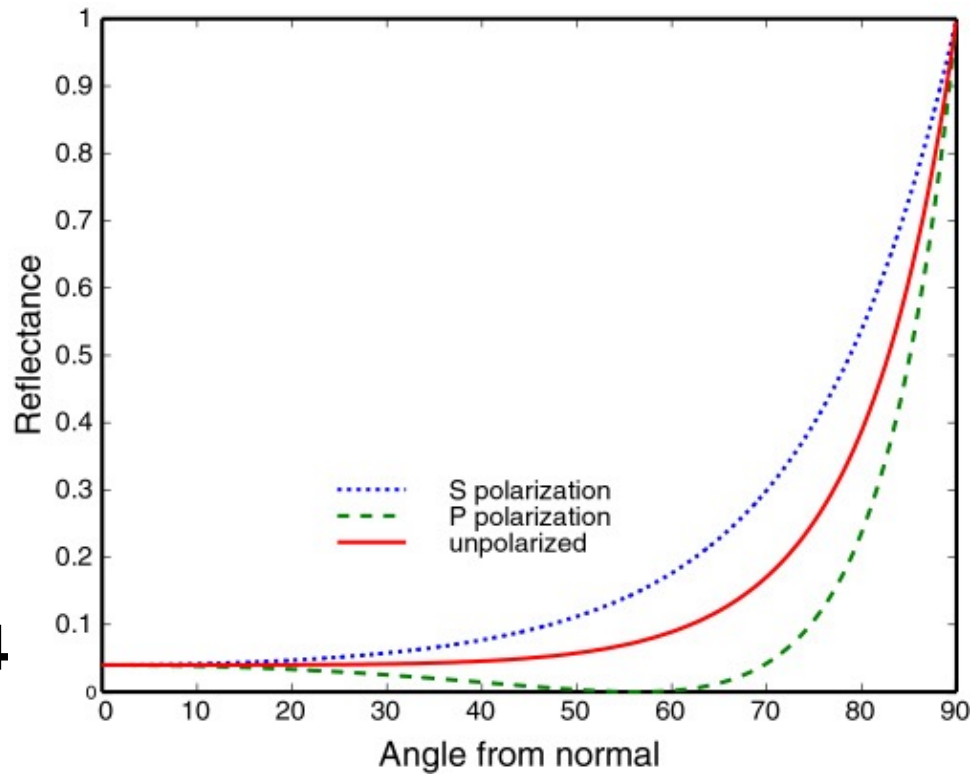
From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Reflection is greater at glancing angles

Fresnel Reflectance

Dielectric (Glass n=1.5)

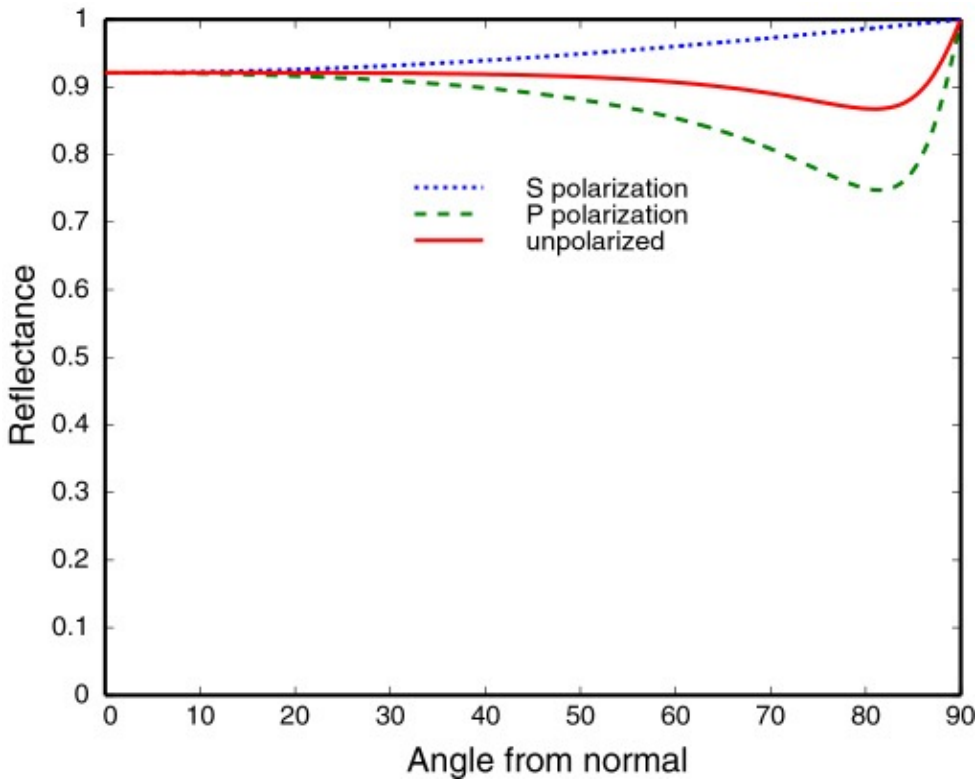
F(0)=0.04



Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

Fresnel Reflectance

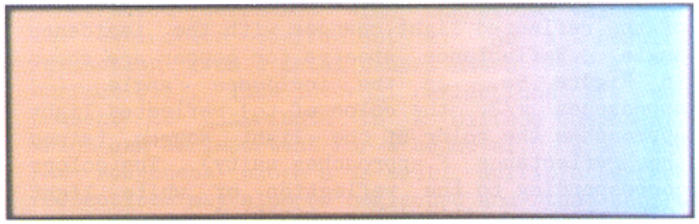
Metal (Aluminum)



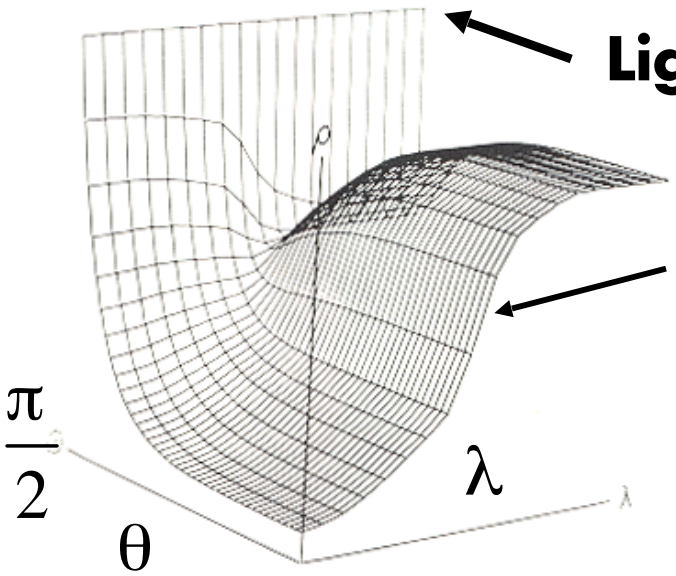
Gold F(0)=0.82
Silver F(0)=0.95

Reflection from Metals

Reflectance of Copper as a function of wavelength and angle of incidence



Measured Reflectance



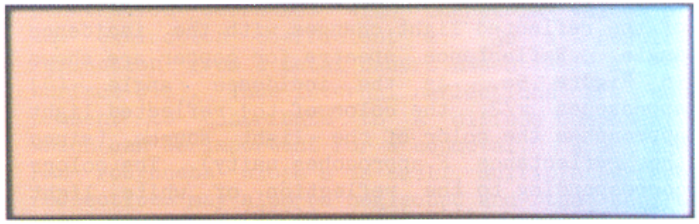
Light spectra

Copper spectra

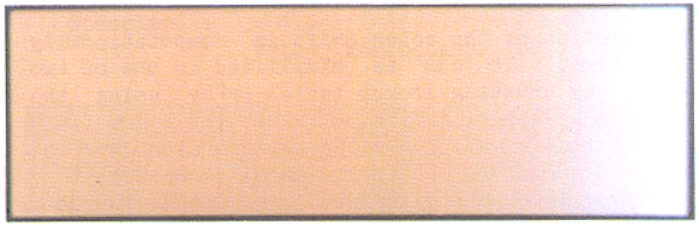
Cook-Torrance Reflection Model

Interpolate between color measured at normal incidence and the light color

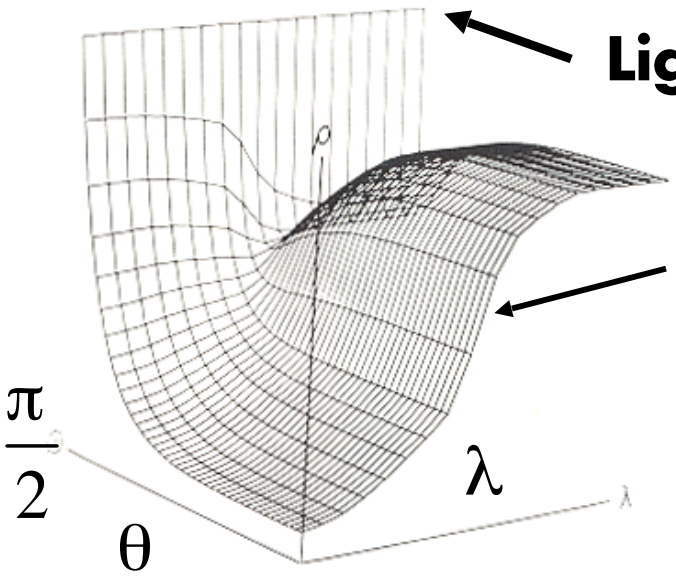
Use Fresnel formula to interpolate



Measured Reflectance



Approximated Reflectance

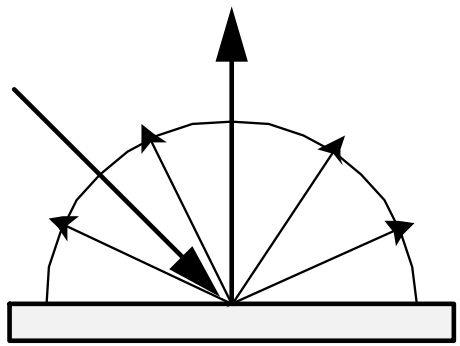


Cook-Torrance approximation

$$R(\theta) = R(0) + (R(\pi/2) - R(0)) \max(0, \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)})$$

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction



$$f_{r,d} = c$$

$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos\theta_i \, d\omega_i$$

$$= f_{r,d} \int L_i(\omega_i) \cos\theta_i \, d\omega_i$$

$$= f_{r,d} E$$

$$M = \int L_r(\omega_r) \cos\theta_r \, d\omega_r = L_r \int \cos\theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

"Diffuse" Reflection

Theoretical

- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

Experimental

- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

Paint manufactures attempt to create ideal diffuse