Reflection Models I: Ideal Materials

Today

- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Next lecture (Thursday)

- Glossy and specular reflection models
- Rough surfaces and microfacets
Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

■ Spectra and Color
■ Polarization
■ Directional distribution
Gonio-reflectometer

4 degree-of-freedom gantry
The Reflection Equation

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]
The BRDF

Bidirectional Reflectance-Distribution Function

\[ f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \begin{bmatrix} 1 \\ sr \end{bmatrix} \]
Properties of BRDF's

1. Linearity

From Sillion, Arvo, Westin, Greenberg
Properties of BRDF’s

2. Reciprocity principle

\[ f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r) \]
Properties of BRDF’s

3. Isotropic vs. anisotropic

\[ f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) \]

Reciprocity and isotropy

\[ f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|) \]
Properties of BRDF’s

4. Energy Conservation

\[
\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r \, d\omega_r}{\int_{\Omega_i} L_r(\omega_o) \cos \theta_i \, d\omega_i} \leq 1
\]
Types of Reflection Functions

Ideal Specular
- Reflection Law
- Mirror

Ideal Diffuse
- Lambert’s Law
- Matte

Specular
- Glossy
- Directional diffuse
Law of Reflection

\[ \hat{R} + (-\hat{I}) = 2 \cos \theta \, \hat{N} = -2(\hat{I} \cdot \hat{N})\hat{N} \]

\[ \hat{R} = \hat{I} - 2(\hat{I} \cdot \hat{N})\hat{N} \]
Ideal Reflection (Mirror)

\[ L_i(\theta_i, \varphi_i) \quad L_r(\theta_r, \varphi_r) \]

\[ L_{r,m}(\theta_r, \varphi_r) = L_i(\theta_r, \varphi_r \pm \pi) \]

\[ f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) = \frac{\delta (\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta (\varphi_i - \varphi_r \pm \pi) \]

\[
L_{r,m}(\theta_r, \varphi_r) = \int f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) L_i(\theta_i, \varphi_i) \cos \theta_i \, d\cos \theta_i \, d\varphi_i
\]

\[
= \int \frac{\delta (\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta (\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i) \cos \theta_i \, d\cos \theta_i \, d\varphi_i
\]

\[ = L_i(\theta_r, \varphi_r \pm \pi) \]
Recall: Snell’s Law

\[ n_i \sin \theta_i = n_t \sin \theta_t \]

\[ n_i \mathbf{N} \times \mathbf{I} = n_t \mathbf{N} \times \mathbf{T} \]

\[ \varphi_t = \varphi_i \pm \pi \]
Recall: Law of Refraction

\[ \mu = \frac{n_i}{n_t} \]

\[ \hat{T} = \mu \hat{I} + \gamma \hat{N} \]

\[ \hat{T}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \hat{I} \cdot \hat{N} \]

\[ \gamma = -\mu \hat{I} \cdot \hat{N} \pm \{ -\mu^2 (1 - (\hat{I} \cdot \hat{N})^2) \}^{1/2} \]

\[ = \mu \cos \theta_i \pm (1 - \mu^2 \sin^2 \theta_i)^{1/2} \]

\[ = \mu \cos \theta_i \pm \cos \theta_t \]

Total internal reflection:

\[ 1 - \mu^2 (1 - (\hat{I} \cdot \hat{N})^2) < 0 \]
Optical Manhole

Total internal reflection

\[ n_w = \frac{4}{3} \]

From Livingston and Lynch
Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Reflection is greater at glancing angles
Fresnel Reflectance

Dielectric (Glass $n=1.5$)

$F(\theta) = 0.04$

Schlick Approximation  \[ F(\theta) = F(0) + (1 - F(0))(1 - \cos \theta)^5 \]
Fresnel Reflectance

Metal (Aluminum)

Gold \( F(0) = 0.82 \)
Silver \( F(0) = 0.95 \)
Reflection from Metals

Reflectance of Copper as a function of wavelength and angle of incidence

Measured Reflectance

Light spectra

Copper spectra

\[ \frac{\pi}{2} \leq \theta \leq \lambda \]
Cook-Torrance Reflection Model

Interpolate between color measured at normal incidence and the light color

Use Fresnel formula to interpolate

Measured Reflectance

Approximated Reflectance

Cook-Torrance approximation

\[ R(\theta) = R(0) + (R(\pi/2) - R(0)) \max(0, \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)}) \]
**Ideal Diffuse Reflection**

Assume light is equally likely to be reflected in any output direction.

\[
L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos \theta_i \, d\omega_i
\]

\[
= f_{r,d} \int L_i(\omega_i) \cos \theta_i \, d\omega_i
\]

\[
= f_{r,d} E
\]

\[
f_{r,d} = c
\]

\[
M = \int L_r(\omega_r) \cos \theta_r \, d\omega_r = L_r \int \cos \theta_r \, d\omega_r = \pi L_r
\]

\[
\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}
\]

**Lambert’s Cosine Law**

\[
M = \rho_d E = \rho_d E_s \cos \theta_s
\]
“Diffuse” Reflection

Theoretical

■ Bouguer - Special micro-facet distribution
■ Seeliger - Subsurface reflection
■ Multiple surface or subsurface reflections

Experimental

■ Pressed magnesium oxide powder
■ Almost never valid at high angles of incidence

Paint manufactures attempt to create ideal diffuse