Participating Media & Vol. Scattering

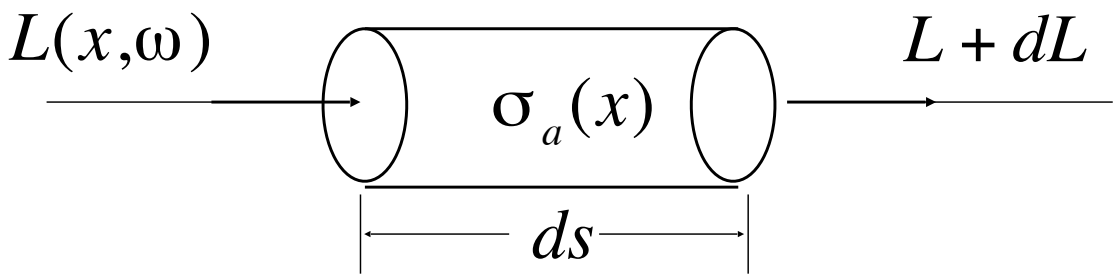
Applications

- Clouds, smoke, water, ...
- Subsurface scattering: paint, skin, ...
- Scientific/medical visualization: CT, MRI, ...

Topics

- Absorption and emission
- Scattering and phase functions
- Volume rendering equation
- Homogeneous media
- Ray tracing volumes

Absorption



$$dL(x,\omega) = -\sigma_a(x)L(x,\omega)\,ds$$

Absorption cross-section: $\sigma_a(x)$

Probability of being absorbed per unit length

Transmittance

$$dL(x,\omega) = -\sigma_a(x)L(x,\omega)\,ds$$

$$\frac{dL(x,\omega)}{L(x,\omega)} = -\sigma_a(x)\,ds$$

$$\ln L(x+s\omega,\omega) = -\int_0^s \sigma_a(x+s'\omega)ds' = -\tau(s)$$

Optical distance or depth

$$\tau(s) = \int_0^s \sigma_a(x+s'\omega)\,ds'$$

Homogenous media: constant σ_a

$$\sigma_a \rightarrow \tau(s) = \sigma_a s$$

Transmittance and Opacity

$$dL(x,\omega) = -\sigma_a(x)L(x,\omega)\,ds$$

$$\frac{dL(x,\omega)}{L(x,\omega)} = -\sigma_a(x)\,ds$$

$$\ln L(x+s\omega,\omega) = -\int_0^s \sigma_a(x+s'\omega)ds' = -\tau(s)$$

$$L(x+s\omega,\omega) = e^{-\tau(s)}L(x,\omega) = T(s)L(x,\omega)$$

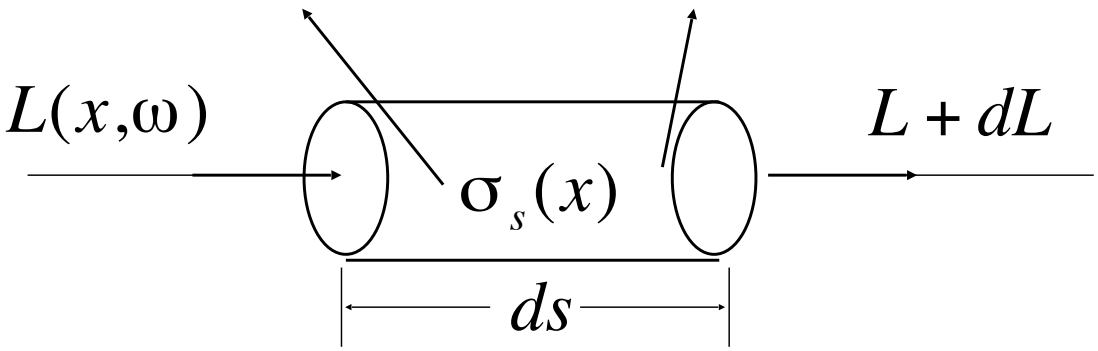
Transmittance

$$T(s) = e^{-\tau(s)}$$

Opacity

$$\alpha(s) = 1 - T(s)$$

Out-Scatter

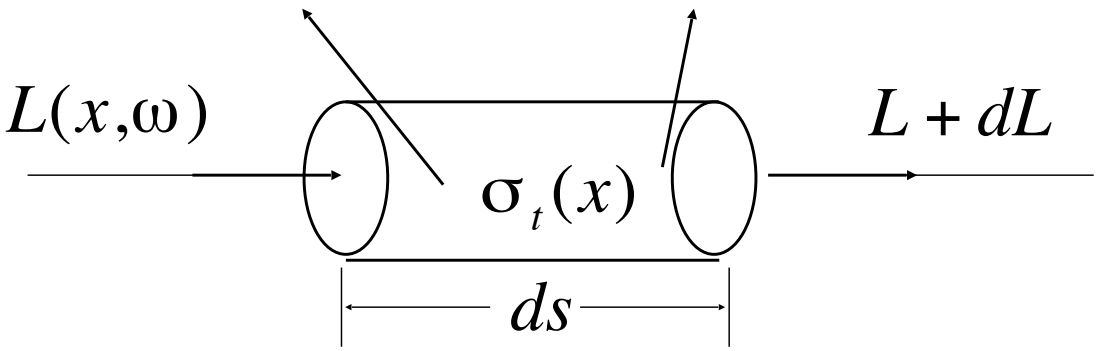


$$dL(x,\omega) = -\sigma_s(x)L(x,\omega)\,ds$$

Scattering cross-section: $\sigma_s(x)$

Probability of being scattered per unit length

Extinction



$$dL(x,\omega) = -\sigma_t(x) L(x,\omega)\, ds$$

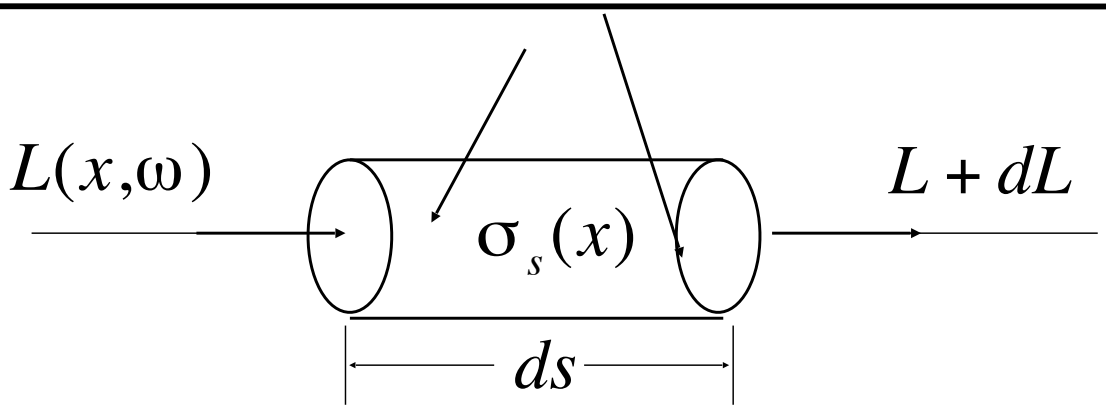
Total cross-section $\sigma_t = \sigma_a + \sigma_s$

Albedo $W = \dfrac{\sigma_s}{\sigma_t} = \dfrac{\sigma_s}{\sigma_a + \sigma_s}$

White and Black Clouds

From Greenler, Rainbows, halos and glories

In-Scatter



$$S(x,\omega) = \sigma_s(x) \int_{S^2} p(\omega' \rightarrow \omega) L(x,\omega')\, d\omega'$$

Phase function $p(\omega' \rightarrow \omega)$

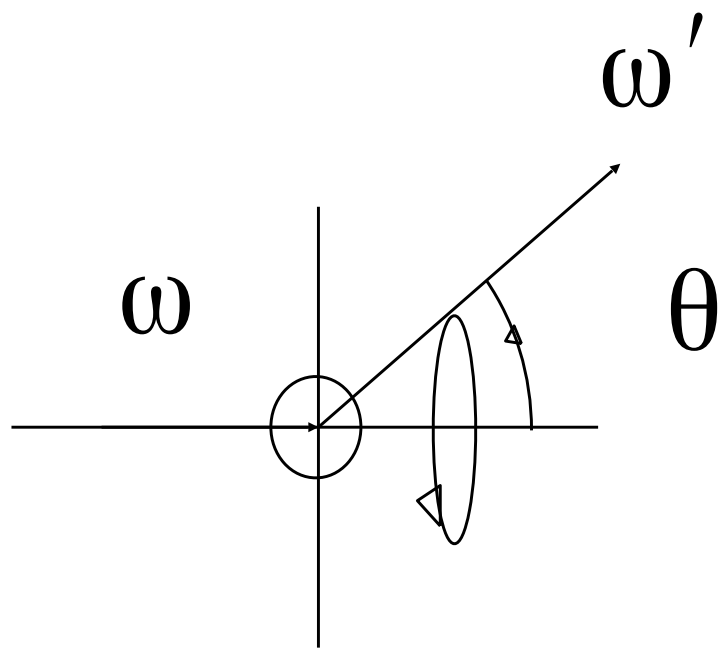
Phase Functions

Phase angle $\cos\theta = \omega \bullet \omega'$

Phase functions (from the phase of the moon)

1. **Isotropic** $p(\cos\theta) = \dfrac{1}{4\pi}$

2. **Rayleigh** $p(\cos\theta) = \dfrac{3}{4}\dfrac{1+\cos^2\theta}{\lambda^4}$
   - **Molecules**

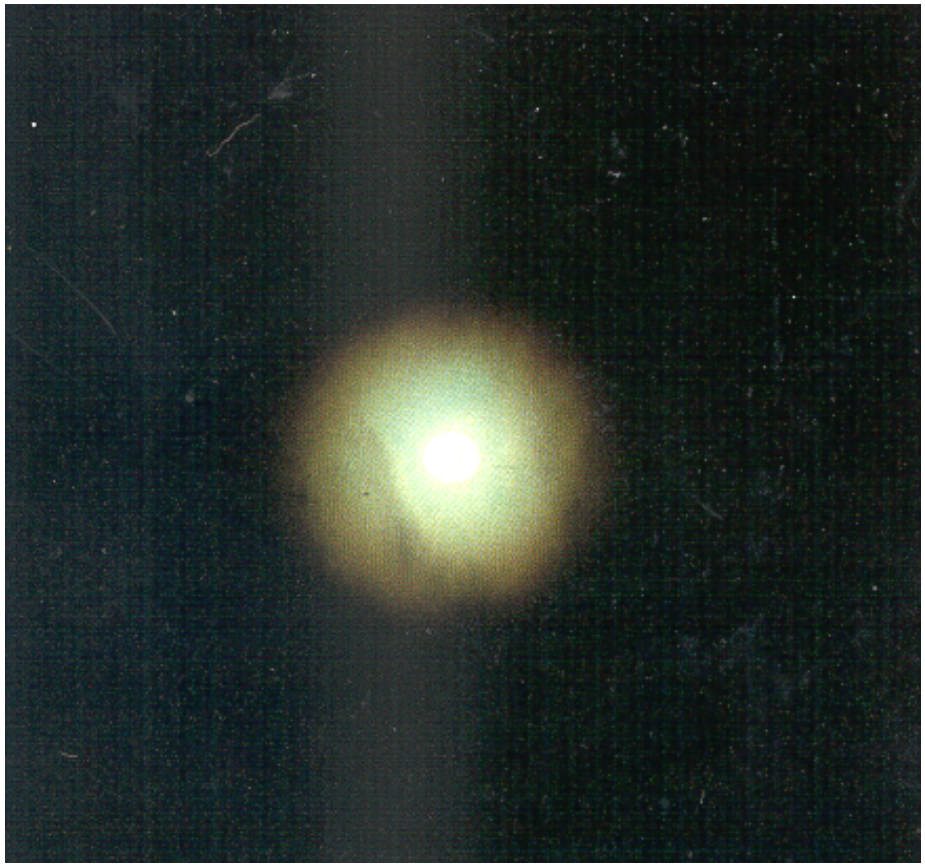


...

Blue Sky = Red Sunset

From Greenler, Rainbows, halos and glories

Coronas and Halos



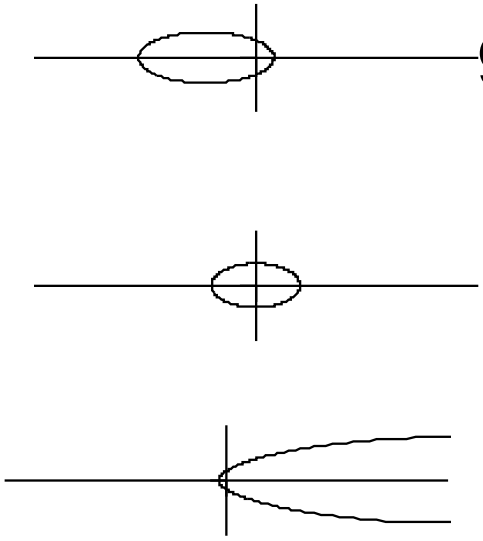
Moon Corona

Sun Halos

From Greenler, "Rainbows, halos and glories"

Henyey-Greenstein Phase Function

Empirical phase function



$$p(\cos\theta) = \frac{1}{4\pi}\frac{1-g^2}{\left(1+g^2-2g\cos\theta\right)^{3/2}}$$

$$2\pi\int_0^\infty p(\cos\theta)\cos\theta\, d(\cos\theta) = g$$

***g*: average phase angle**

Properties of Phase Functions

Phase Function $p(\omega' \rightarrow \omega)$

Properties:

1. Reciprocity $p(\omega \rightarrow \omega') = p(\omega' \rightarrow \omega)$

2. Energy conserving $\int_{S^2} p(\omega' \rightarrow \omega)\, d\omega' = 1$

The Volume Rendering Equation

Integro-differential equation:

$$\frac{\partial L(\mathrm{p}, \omega)}{\partial s} = -\sigma_t L(\mathrm{p}, \omega) + S(\mathrm{p}, \omega)$$

Integro-integral equation:

$$L(\mathrm{p}, \omega) = \int_0^\infty T(\mathrm{p} + s'\omega) S(\mathrm{p} + s'\omega, \omega)\, \mathrm{d}s'$$

Transmission: attenuation due to absorption and out-scattering

$$\mathrm{e}^{-\int_0^{s'} \sigma_t(\mathrm{p} + s''\omega)\, \mathrm{d}s''}$$

Source: emission and in-scattering

$$\sigma_s(\mathrm{p} + s'\omega) \int_{S^2} p(\omega' - \to \omega)\, L(\mathrm{p} + s'\omega, \omega')\, \mathrm{d}\omega'$$

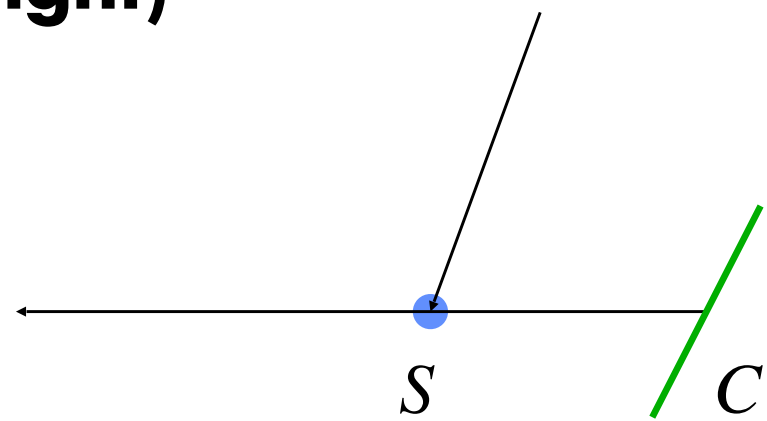
Simple Atmosphere Model

Assumptions

- Homogenous media
- Constant source term (airlight)

$$\frac{\partial L(s)}{\partial s} = -\sigma_t L(s) + S$$

$$L(s) = \left(1 - e^{-\sigma_t s}\right) S \quad + e^{-\sigma_t s} C$$



Applications: Fog and haze

The Sky

Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)

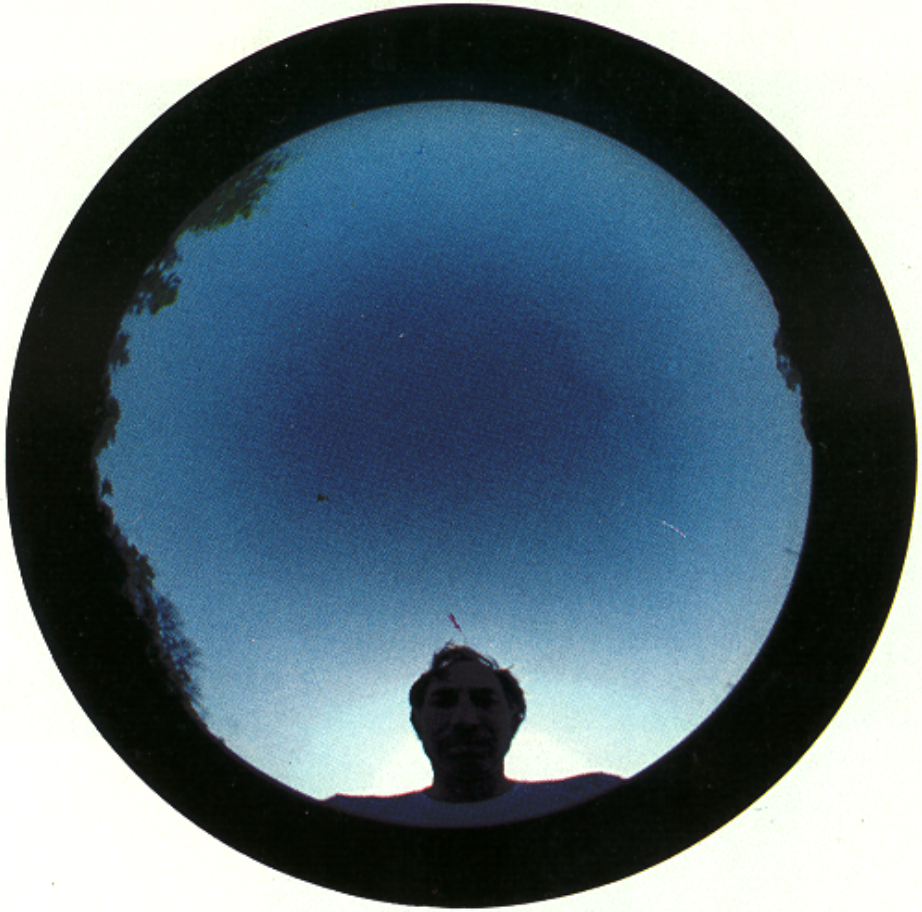


Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

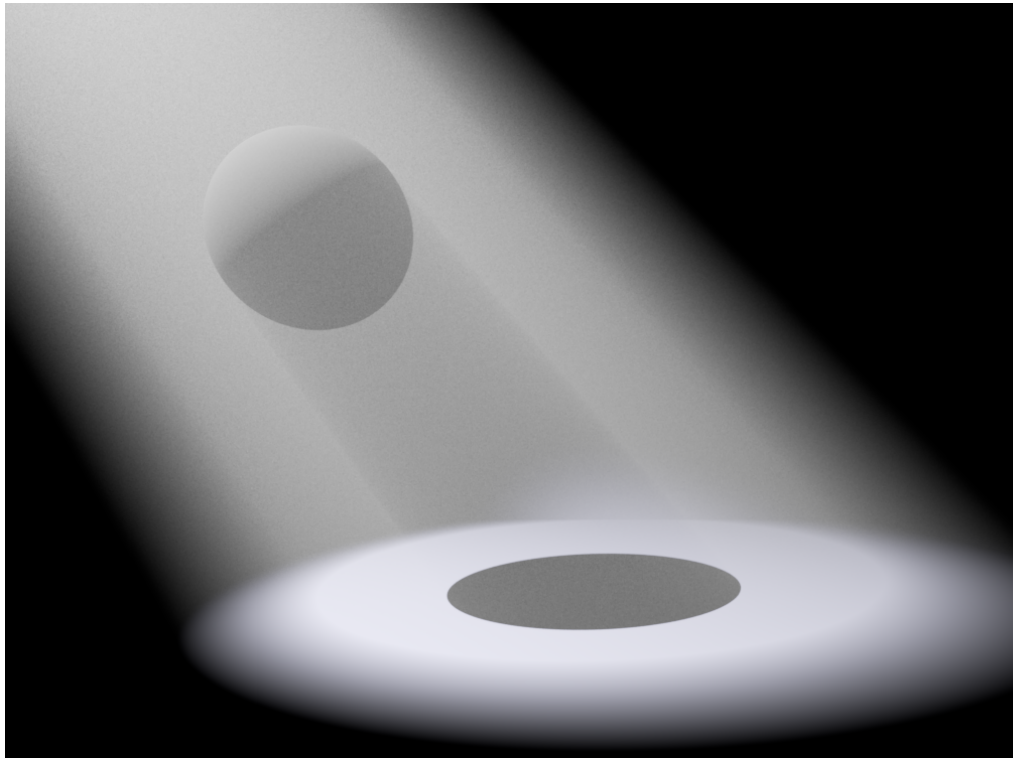
From Greenler, Rainbows, halos and glories

Atmospheric Perspective

From Greenler, Rainbows, halos and glories

Direct Lighting with Shadows

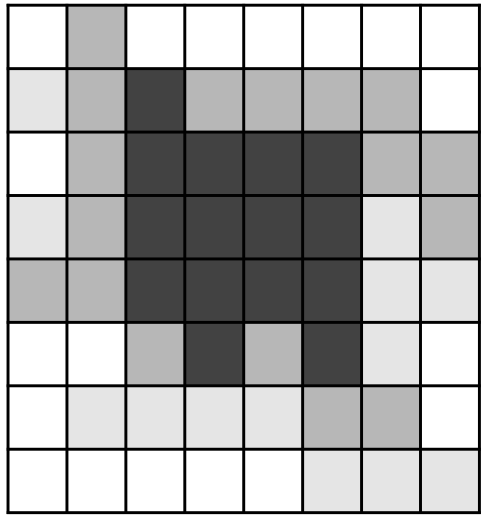
From Minneart, "Light in the Open Air"



pbrt: Spot-Lit ball in fog

Volume Representations

Scalar volume grids



+ flexible – useful for rendering results from physical simulation / measured data

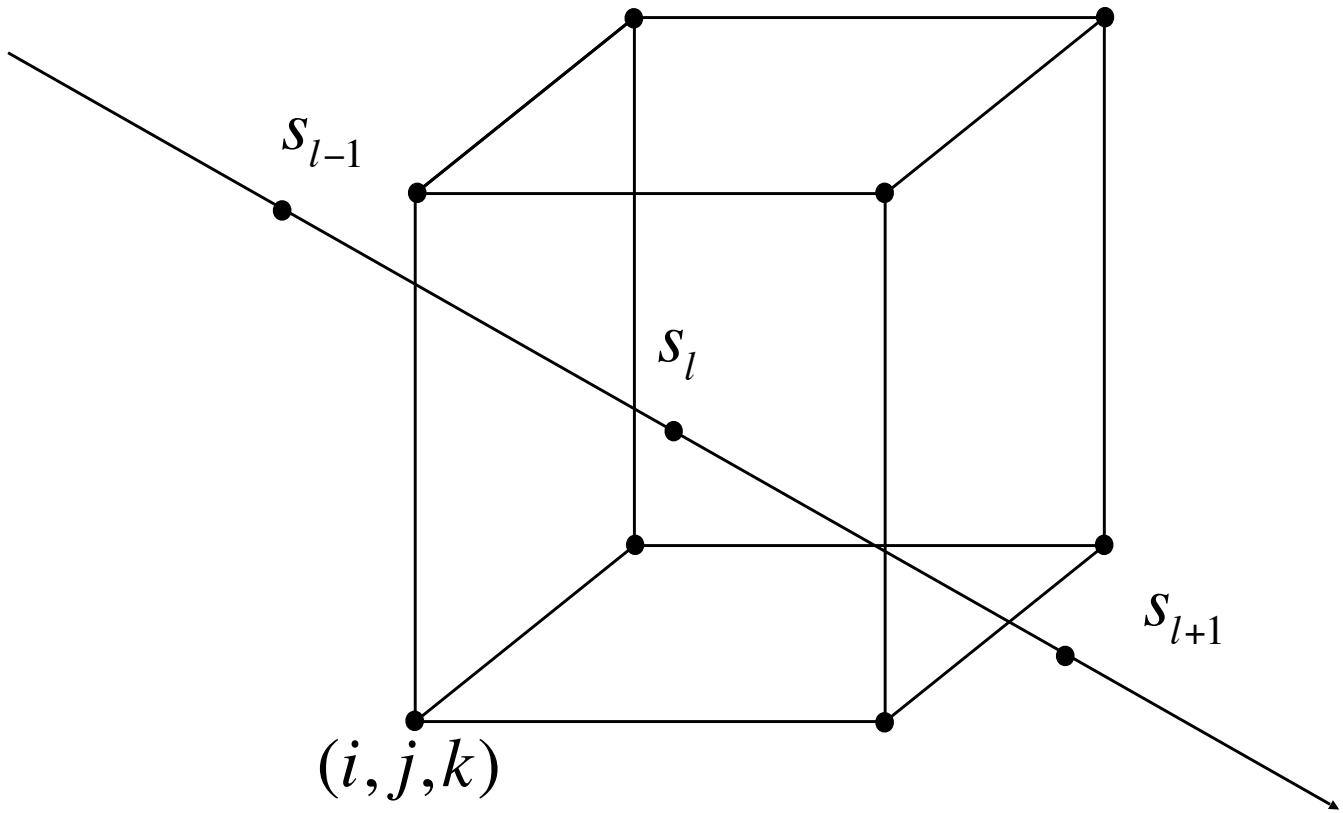
- fixed resolution, may have high memory requirements

Procedural models

+ minimal storage, infinite detail

- can be hard to control

Interpolate Scalar Volume Grids



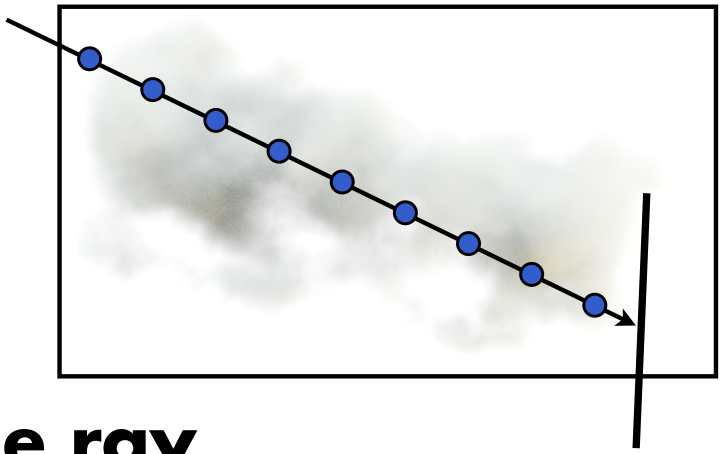
Interpolation $v(s_l) = \text{trilinear}(v, i, j, k, x(s_l))$

Map scalars to optical properties $\sigma_s(v), \sigma_a(v)$

Ray Marching

$$T(s) = \mathrm{e}^{-\tau(s)}$$

$$\tau(s) = \int_0^s \sigma_t(x + s'\omega)\,\mathrm{d}s'$$



Evaluate by stepping along the ray

Monte Carlo not necessary in 1D

$$\tau(s) \approx \frac{s}{N}\sum_i^N \sigma_t(x_i) \qquad x_i = x + \frac{i + 0.5}{N}\omega$$

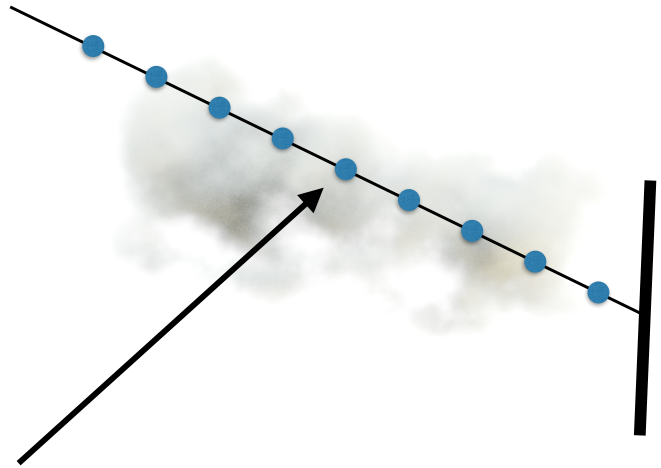
Ray Marching

$$L(x,\omega) = \int_0^s T(x+s',\omega)\, S(x+s'\omega,\omega)\, \mathrm{d}s'$$

Conceptually straightforward:

March along the ray ...

$$L(x,\omega) \approx \frac{s}{N} \sum_i^N T(x_i) S(x_i,\omega)$$



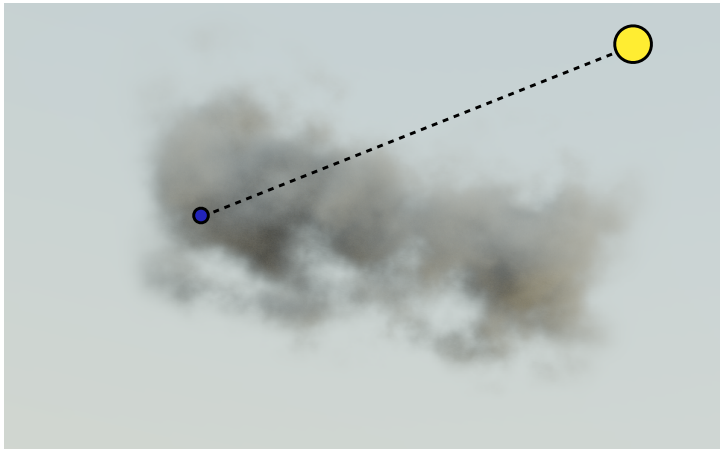
$T(x_i)$ **can be computed incrementally from the product of** $T(x_{i-1})$ **and the additional attenuation** x_i **between** x_{i-1} **and**

Direct Illumination

$$\sigma_s(\mathrm{p}') \int_{S^2} p(\omega' \to \omega)\, L_d(\mathrm{p}', \omega')\, \mathrm{d}\omega'$$

Similar to direct illumination on a surface

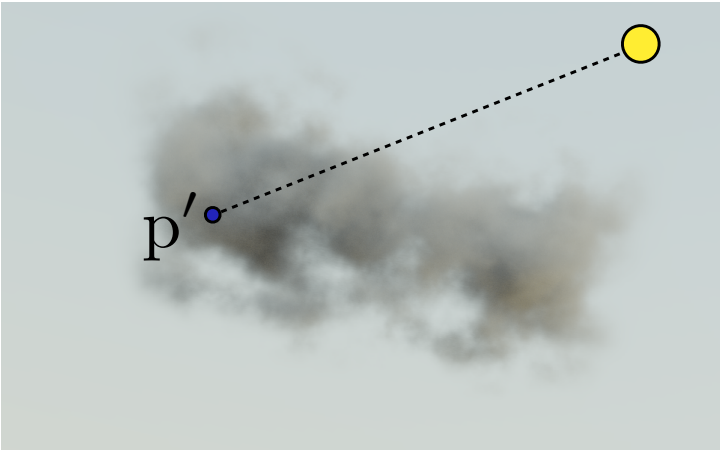
- **Sample directions using phase distribution**
- **Sample light sources**



Can use multiple importance sampling to combine both methods

Direct Illumination

$$\sigma_s(\mathrm{p}')\frac{1}{N}\sum_i^N \frac{p(\omega_i \to \omega)\, L_d(\mathrm{p}', \omega_i)}{p(\omega_i)}$$



Computing L_d is expensive

- **Not just a shadow ray**
- **Need to compute transmittance between p' and the light source**

Direct Illumination / Single Scattering

Volumetric Path Tracing

Include indirect illumination in source term:

$$S(x,\omega) = \sigma_s(x) \int_{S^2} p(\omega' \to \omega)\, L(x,\omega')\, \mathrm{d}\omega'$$

$$L(x,\omega') = L_d(x,\omega') + L_i(x,\omega')$$

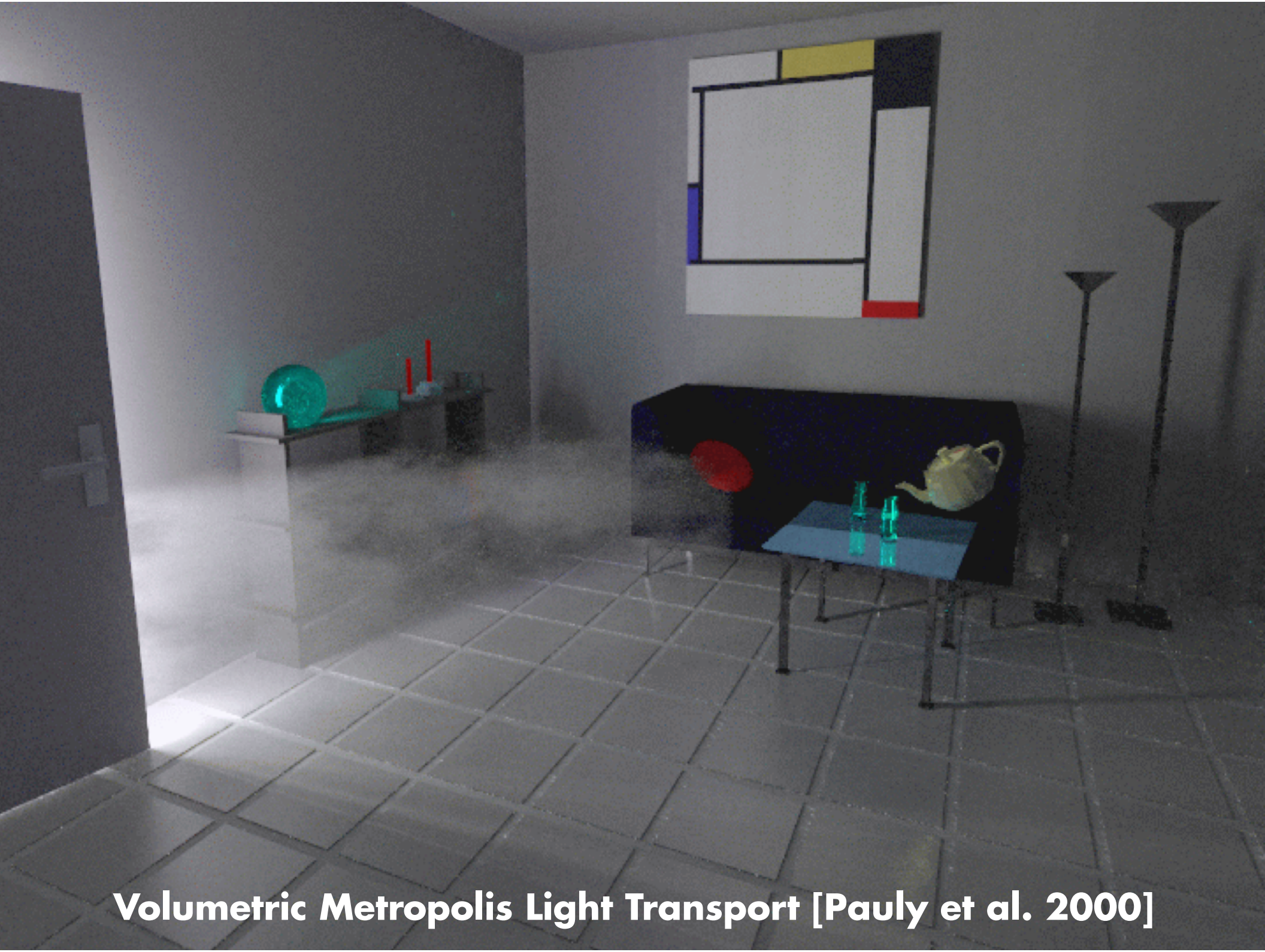
Compute direct lighting as before

- **Sample incident direction using phase distribution**
- **Trace the ray recursively**

$$L_i(x,\omega') \approx \frac{p(\omega'' \to \omega')\, L(x,\omega'')}{p(\omega'')}$$

Uniform spherical directions: $p(\omega'') = \dfrac{1}{4\pi}$

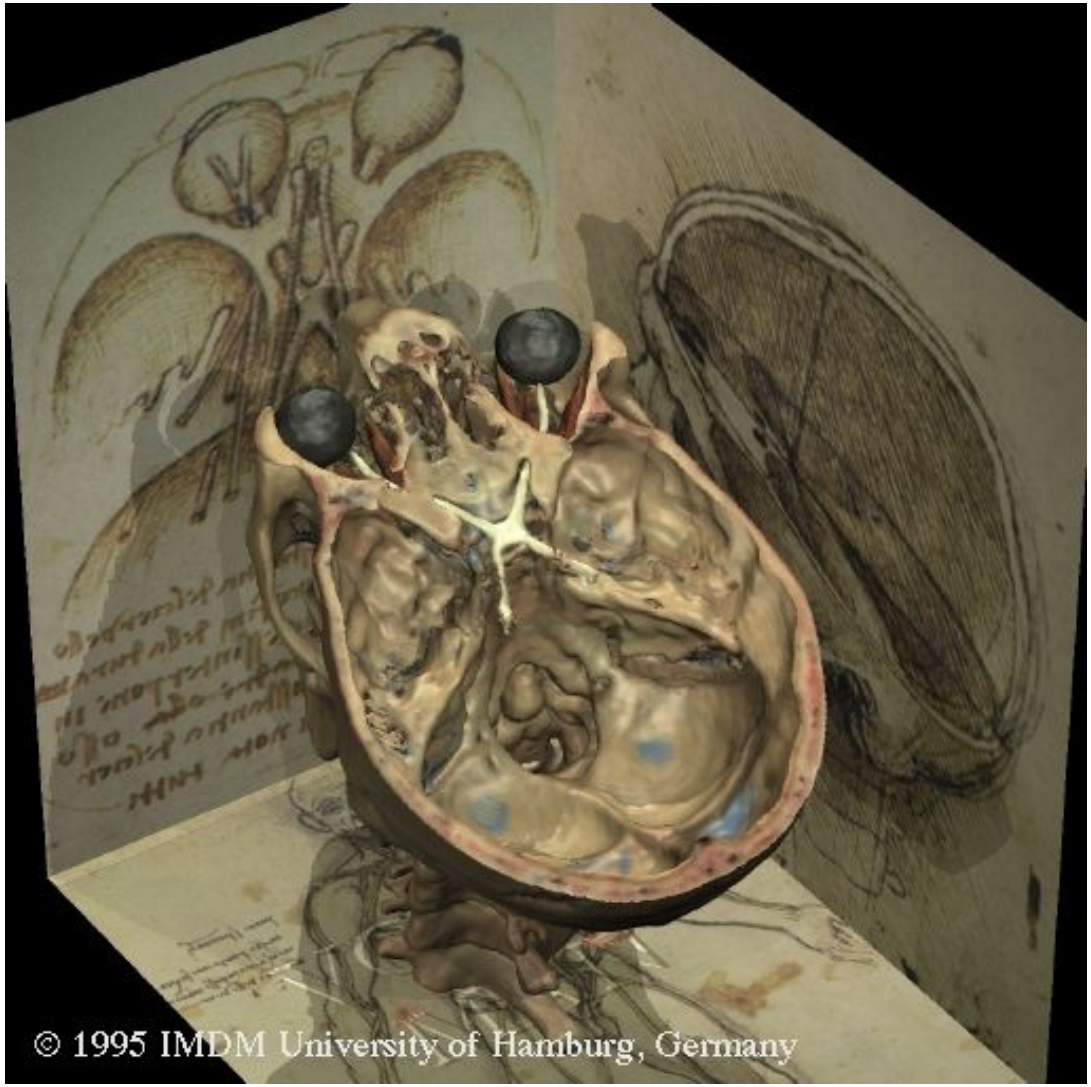
Path Tracing / Multiple Scattering



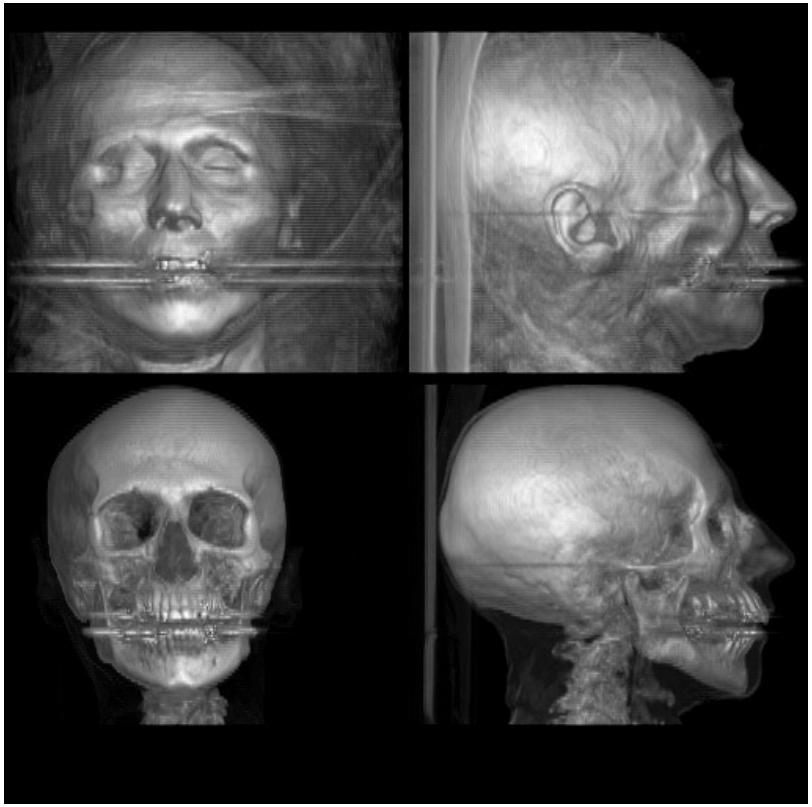
Volumetric Metropolis Light Transport [Pauly et al. 2000]

Volumetric Metropolis Light Transport [Pauly et al. 2000]

Volume Rendering Examples



From Karl Heinz Hoehne



From Marc Levoy