Radiometry and Photometry

Measuring spatial properties of light

- Radiant power
- Radiant intensity
- Irradiance
  - Inverse square law and cosine law
- Radiance
- Radiant exitance (radiosity)

Goal is to perform lighting calculations in the physically correct way
Radiant Energy and Power

Power: Watts (radiometry)
$\Phi$ vs. Lumens (photometry)
- Spectral efficacy
- Energy efficiency

Energy: Joules vs. Talbot
- Exposure
  - Film response
  - Skin - sunburn

$Y = \int V(\lambda) L(\lambda) d\lambda$
Radiant Intensity
Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle emanating from a point source.

\[ I(\omega) \equiv \frac{d\Phi}{d\omega} \]

\[
\begin{bmatrix}
\frac{W}{sr} \\
\frac{lm}{sr} = cd = \text{candela}
\end{bmatrix}
\]
Angles and Solid Angles

**Angle**
\[ \theta = \frac{l}{r} \]

⇒ circle has 2 \( \pi \) radians

**Solid angle**
\[ \Omega = \frac{A}{R^2} \]

⇒ sphere has 4 \( \pi \) steradians
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) = r^2 \sin \theta \, d\theta \, d\phi \]
Differential Solid Angles

\[ dA = (r \, d\theta)(r \sin \theta \, d\phi) \]
\[ = r^2 \sin \theta \, d\theta \, d\phi \]

\[ d\omega = \frac{dA}{r^2} = \sin \theta \, d\theta \, d\phi \]
Differential Solid Angles

\[ d\omega = \sin \theta \, d\theta \, d\phi \]

\[ \Omega = \int d\omega \]

\[ = \int\int_{S^2} \sin \theta \, d\theta \, d\phi \]

\[ = \int\int_{0 \to 1} \int_{0 \to 2\pi} \sin \theta \, d\theta \, d\phi \]

\[ = \int\int_{-1 \to 0} \int_{0 \to 2\pi} d\cos \theta \, d\phi \]

\[ = 4\pi \]
Isotropic Point Source

\[ \Phi = \int_{S^2} I \, d\omega \]

\[ = 4\pi I \]

\[ I = \frac{\Phi}{4\pi} \]
\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s \]
\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s \]

\[ \Phi = \int_0^{2\pi} \int_0^1 I(\omega) d\cos\theta d\varphi \]
\( I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s \)

\[
\Phi = \int_0^{2\pi} \int_0^1 I(\omega) \, d\cos \theta \, d\varphi = 2\pi \int_0^1 \cos^s \theta \, d\cos \theta = \frac{2\pi}{s + 1}
\]
\[ I(\omega) = \cos^s \theta = (\vec{\omega} \cdot \hat{A})^s \]

\[
\Phi = \int_0^{2\pi} \int_0^1 I(\omega) \, d\cos\theta \, d\varphi = 2\pi \int_0^1 \cos^s \theta \, d\cos\theta = \frac{2\pi}{s+1}
\]

\[ I(\omega) = \Phi \frac{s+1}{2\pi} \cos^s \theta \]
Light Source Goniometric Diagrams

<table>
<thead>
<tr>
<th>3</th>
<th>Porcelain-enameded ventilated standard dome with incandescent lamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%↑</td>
<td>83 1/2%↓</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>Concentric ring unit with incandescent silvered-bowl lamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>83%↓</td>
<td>3 1/2%↓</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Pendant diffusing sphere with incandescent lamp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>35 1/2%↑</td>
</tr>
<tr>
<td></td>
<td>45%↓</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>R-40 flood with specular anodized reflector skirt; 45° cutoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%↑</td>
<td>85%↓</td>
</tr>
</tbody>
</table>
Irradiance
Irradiance

Definition: The irradiance (illuminance) is the power per unit area incident on a surface.

\[ E(x) \equiv \frac{d\Phi_i}{dA} \]

\[
\begin{bmatrix}
\frac{W}{m^2} \\
\frac{lm}{m^2} = lux
\end{bmatrix}
\]

Sometimes referred to as the radiant (luminous) incidence.
## Typical Values of Illuminace [lm/m²]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunlight plus skylight</td>
<td>100,000 lux</td>
</tr>
<tr>
<td>Sunlight plus skylight (overcast)</td>
<td>10,000</td>
</tr>
<tr>
<td>Interior near window (daylight)</td>
<td>1,000</td>
</tr>
<tr>
<td>Artificial light (minimum)</td>
<td>100</td>
</tr>
<tr>
<td>Moonlight (full)</td>
<td>0.02</td>
</tr>
<tr>
<td>Starlight</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
Beam Power in Terms of Irradiance

\[ \Phi = EA \]

\[ E = \frac{\Phi}{A} \]
Beam Power Falling on the Surface

\[ \Phi' = E' A' \]

\[ E' = \frac{\Phi'}{A'} \]
Projected Area

\[ A = A' \cos \theta \]
Lambert’s Cosine Law

\[ A = A' \cos \theta \]
\[ \Phi = \Phi' \]
\[ E' = \frac{\Phi'}{A'} = \frac{\Phi}{A} \cos \theta = E \cos \theta \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ d\Phi = I \, d\omega \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ d\omega = \frac{\cos \theta}{r^2} \, dA \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ I \, d\omega = \frac{\Phi}{4\pi} \frac{\cos\theta}{r^2} \, dA \]
Irradiance: Isotropic Point Source

\[ I = \frac{\Phi}{4\pi} \]

\[ I \, d\omega = \frac{\Phi}{4\pi} \frac{\cos\theta}{r^2} \, dA = E \, dA \]

\[ E = \frac{\Phi}{4\pi} \frac{\cos\theta}{r^2} \]
The Invention of Photometry

Bouguer’s classic experiment
■ Compare a light source and a candle
■ Move until they both appear equally bright
■ Intensity is proportional to ratio of distances squared

Definition of a candela
■ Originally a “standard” candle
■ Currently 550 nm laser with 1/683 W/sr
■ 1 of 6 fundamental SI units
Radiance
Definition: The surface radiance (luminance) is the intensity per unit area leaving a surface

\[ \Phi(x, \omega) = \frac{dI(x, \omega)}{dA} \]

\[ = \frac{d^2 \Phi(x, \omega)}{d\omega \, dA} \]

\[ \left[ \frac{W}{sr \, m^2} \right] \left[ \frac{cd}{m^2} = \frac{lm}{sr \, m^2} = \text{nit} \right] \]
## Typical Values of Luminance [cd/m²]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Surface of the sun</strong></td>
<td>2,000,000,000 nit</td>
</tr>
<tr>
<td><strong>Sunlight clouds</strong></td>
<td>30,000</td>
</tr>
<tr>
<td><strong>Clear sky</strong></td>
<td>3,000</td>
</tr>
<tr>
<td><strong>Overcast sky</strong></td>
<td>300</td>
</tr>
<tr>
<td><strong>Moon</strong></td>
<td>0.03</td>
</tr>
</tbody>
</table>
Directional Power Leaving a Surface

\[ d^2 \Phi_o (x, \omega) = L_o (x, \omega) \cos \theta \, dA \, d\omega \]

Same \( dA \) for all directions
Radiant Exitance
(Radiosity)
Radiant Exitance

Definition: The radiant (luminous) exitance is the energy per unit area leaving a surface.

\[ M(x) \equiv \frac{d\Phi_o}{dA} \]

\[
\begin{bmatrix}
\frac{W}{m^2} \\
\frac{lm}{m^2} = \text{lux}
\end{bmatrix}
\]

In computer graphics, this quantity is usually referred to as the radiosity \((B)\)
Area Light Source

\[ d^2 \Phi_0 (x, \omega) = L_0 (x, \omega) \cos \theta dAd\omega \]
Area Light Source

\[ dM(x, \omega) = \frac{d^2 \Phi_o(x, \omega)}{dA} = L_o(x, \omega) \cos \theta \, d\omega \]
Area Light Source

\[ M = \int_{H^2} dM(x, \omega) = \int_{H^2} L_o(x, \omega) \cos \theta \, d\omega \]
Uniform Diffuse Emitter

\[ M = \int_{H^2} L_o \cos \theta \, d\omega \]

\[ = L_o \int_{H^2} \cos \theta \, d\omega \]

Uniform means \( L_o \) is not a function of direction

\[ L_o(x, \omega) = L_o \]
Projected Solid Angle

\[ \Omega = \int_{\Omega} \cos \theta \, d\omega \]
Projected Solid Angle

\[ \tilde{\Omega} \equiv \int_{\Omega} \cos \theta \, d\omega \]

\[ \tilde{\Omega} = \int_{H^2} \cos \theta \, d\omega = \pi \]
Uniform Diffuse Emitter

\[ M = \int \limits_{H^2} L_o \cos \theta \, d\omega \]

\[ = L_o \int \limits_{H^2} \cos \theta \, d\omega \]

\[ = \pi L_o \]

\[ L_o = \frac{M}{\pi} \]
Radiometry and Photometry

Summary
## Radiometric and Photometric Terms

<table>
<thead>
<tr>
<th>Physics</th>
<th>Radiometry</th>
<th>Photometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>Radiant Energy</td>
<td>Luminous Energy</td>
</tr>
<tr>
<td>Flux (Power)</td>
<td>Radiant Power</td>
<td>Luminous Power</td>
</tr>
<tr>
<td>Flux Density</td>
<td>Irradiance</td>
<td>Illuminance</td>
</tr>
<tr>
<td></td>
<td>Radiosity</td>
<td>Luminosity</td>
</tr>
<tr>
<td>Angular Flux Density</td>
<td>Radiance</td>
<td>Luminance</td>
</tr>
<tr>
<td>Intensity</td>
<td>Radiant Intensity</td>
<td>Luminous Intensity</td>
</tr>
</tbody>
</table>
## Photometric Units

<table>
<thead>
<tr>
<th>Photometry</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MKS</td>
</tr>
<tr>
<td></td>
<td>CGS</td>
</tr>
<tr>
<td></td>
<td>British</td>
</tr>
<tr>
<td>Luminous Energy</td>
<td>Talbot</td>
</tr>
<tr>
<td>Luminous Power</td>
<td>Lumen</td>
</tr>
<tr>
<td>Illuminance</td>
<td>Lux</td>
</tr>
<tr>
<td>Luminosity</td>
<td>Phot</td>
</tr>
<tr>
<td>Luminance</td>
<td>Stilb</td>
</tr>
<tr>
<td>Luminance</td>
<td>Lambert</td>
</tr>
<tr>
<td>Luminous Intensity</td>
<td>Candela (Candle, Candlepower, Carcel, Hefner)</td>
</tr>
</tbody>
</table>

“Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?”, James Kajiya