Camera Simulation

Effect | Cause
--- | ---
Field of view | Film size, focal length
Depth of field | Aperture, focal length
Exposure | Film speed, aperture, shutter
Motion blur | Shutter

References

*Photography*, B. London and J. Upton
*Optics in Photography*, R. Kingslake
*The Camera, The Negative, The Print*, A. Adams
Cameras (5D integral)

\[ R = \int_{T} \int_{\Omega} \int_{A} L(x, \omega, t) \ P(x) \ S(t) \ \cos \theta \ \, dA \ d\omega \ dt \]

- **Motion Blur**
- **Depth of Field**

Cook, Porter, Carpenter, 1984

Mitchell, 1991

Cook, Porter, Carpenter, 1984

Mitchell, 1991
Topics

Lenses

Depth of field

Exposure

Light field cameras
Lenses
Pinhole Camera

Figure 1.1. A Pinhole Camera.

From Pharr et al. Physically Based Rendering (3rd Edition)
Thin Lens Demonstration

http://graphics.stanford.edu/courses/cs178-10/applets/gaussian.html
Lenses Focus!

\[ \frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \]

Rays from a point in object space intersect at a point in image space.

These are conjugate points.

Points at infinity converge at the focal point or focus.

Points on different object planes focus on different image planes.

The position of the point of focus can be changed by moving the lens.
Gauss’ Ray Tracing Construction

- Parallel Ray
- Focal Ray
- Chief Ray
- Focal Point
- Object
- Image
Snell’s Law

\[ n_i \sin \theta_i = n_t \sin \theta_t \]

\[ n_i \hat{N} \times \hat{I} = n_t \hat{N} \times \hat{T} \]

\[ \varphi_t = \varphi_i \pm \pi \]
Law of Refraction

\[ \hat{T} = \mu \hat{I} + \gamma \hat{N} \]

\[ \hat{T}^2 = 1 = \mu^2 + \gamma^2 + 2 \mu \gamma \hat{I} \cdot \hat{N} \]

\[ \gamma = -\mu \hat{I} \cdot \hat{N} \pm \sqrt{1 - \mu^2 \left(1 - (\hat{I} \cdot \hat{N})^2\right)} \]

\[ = \mu \cos \theta_i \pm \sqrt{1 - \mu^2 \sin^2 \theta_i} \]

\[ = \mu \cos \theta_i \pm \cos \theta_t \]

\[ = \mu \cos \theta_i - \cos \theta_t \]

Total internal reflection:

\[ 1 - \mu^2 (1 - (\hat{I} \cdot \hat{N})^2) < 0 \]
Total internal reflection

\[ n_w = \frac{4}{3} \]

From Livingston and Lynch
Refraction at a Spherical Lens

$n$ : index of refraction
Snell’s Law

\[ n' \sin I' = n \sin I \]
The sum of the interior angles is equal to the exterior angle.

\[ I = U + \phi \]
Angle of Refraction

\[ I' = U' + \phi \]

Opposite angles are equals
Snell's Law

\[ n' \sin(U' + \phi) = n \sin(U + \phi) \]

\[ I = U + \phi \]

\[ I' = U' + \phi \]
Paraxial Approximation: Small Angles

\[ n' \sin(U' + \phi) = n \sin(U + \phi) \]

\[ \sin A = a = \tan A \]

Rays deviate only slightly from the axis
Paraxial Approximation: Small Angles

\[ n'(u' + \phi) = n(u + \phi) \]

\[ \sin A = a = \tan A \]
Angles to Slopes

\[ u = \frac{h}{z} \quad u' = \frac{h}{z'} \quad \phi = -\frac{h}{R} \]

\[ a = \tan A \]
Gauss’ Formula

Paraxial approximation to Snell’s Law

\[ n'(u' + \phi) = n(u + \phi) \]

Ray coordinates

\[ u = \frac{h}{z} \quad u' = \frac{h}{z'} \quad \phi = -\frac{h}{R} \]

\[ n'(\frac{h}{z'} - \frac{h}{R}) = n(\frac{h}{z} - \frac{h}{R}) \]

\[ \frac{n'}{z'} = \frac{n}{z} + \frac{(n' - n)}{R} \quad \text{Holds for any height, any ray!} \]
Perspective Transformation

How does $z$ transform?

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \Rightarrow z' = \frac{fz}{z + f}$$

$$\Rightarrow x' = \frac{fx}{z + f}$$

$$\Rightarrow y' = \frac{fy}{z + f}$$

Represent lens transformation as a 4x4 matrix
Ray Tracing: Finite Aperture

1. Pick a point on image plane $x'$
2. Pick a point on the lens $u$
3. Map $x'$ to $x$ on the focal plane; form ray $(u, x-u)$
Real Lens

Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, Optics in Photography
## Double Gauss

Data from W. Smith, Modern Lens Design, p 312

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Ray Tracing Through Lenses

200 mm telephoto

35 mm wide-angle

50 mm double-gauss

16 mm fisheye

From Kolb, Mitchell and Hanrahan (1995)
Ray Tracing Through Lenses

\[ R = \text{Ray(point on film plane, point on rear-most element)} \]
For each lens element \( E_i \), from rear to front,
\[ p = \text{intersection of } R \text{ and } E_i \]
If \( p \) is outside clear aperture of \( E_i \)
ray is blocked
Else if medium on far side of \( E_i \) \( \neq \) medium on near side
compute new direction for \( R \) using Snell’s law

\[
E(x') = \frac{1}{Z^2} \int_{x'' \in D} L(x'', x') \cos^4 \theta' \, dA''
\]

From Kolb, Mitchell and Hanrahan (1995)
Depth of Field
Depth of Field

less depth of field

more depth of field

wider aperture

smaller aperture

From London and Upton
Circle of confusion
proportional to the size of the aperture

\[
\frac{c}{a} = \frac{d'}{z'} = \frac{s' - z'}{z'}
\]
Resolving Power

Diffraction limit

\[ c = 1.22 \frac{f}{a} \lambda \quad [= 1.22 \times 64 \times .500\mu m=0.040\ mm] \]

35mm film (Leica standard)

\[ c = 0.025\ mm \]

CCD/CMOS pixel aperture

\[ c = 0.0116\ mm\ (Nikon\ D1) \]
Bokeh

Photo credit: Wikimedia's The Photographer
Image Irradiance

\[ E = \int_{\Omega} L \cos \theta \, d\omega = L \pi \sin^2 \theta = L \frac{\pi}{4} \left( \frac{a}{f} \right)^2 \]
Uniform Disk Source

Geometric Derivation

Algebraic Derivation

\[ \Omega = \pi \sin^2 \alpha \]

\[ \Omega = \int \int \cos \theta \, d\phi \, d\cos \theta \]

\[ = 2\pi \cos^2 \theta \left|^{\cos \alpha} \right. \]

\[ = \pi \sin^2 \alpha \]

\[ = \pi \frac{r^2}{r^2 + h^2} \]
Relative Aperture or F-Stop

F-Number and exposure: \[ E = L \frac{\pi}{4} \frac{1}{N^2} \]

F-stops: 1.4 2 2.8 4.0 5.6 8 11 16 22 32 45 64

1 stop doubles exposure
Camera Exposure

Exposure \[ H = E \times T \]

Exposure overdetermined

Aperture: f-stop - 1 stop doubles \( H \)
  Decreases depth of field

Shutter: Doubling the open time doubles \( H \)
  Increases motion blur
Aperture vs Shutter

Constant Exposure

f/16 1/8s  f/4 1/125s  f/2 1/500s

From London and Upton

CS348b Lecture 7

Pat Hanrahan / Matt Pharr, Spring 2018
Light Field Cameras
**Lens**
The Lytro Light Field Camera starts with an 8X optical zoom, f/2 aperture lens. The aperture is constant across the zoom range allowing for unheard of light capture.

**Light Field Engine 1.0**
The Light Field Engine replaces the supercomputer from the lab and processes the light ray data captured by the sensor.

The Light Field Engine travels with every living picture as it is shared, letting you refocus pictures right on the camera, on your desktop and online.

**Light Field Sensor**
From a roomful of cameras to a micro-lens array specially adhered to a standard sensor, the Lytro's Light Field Sensor captures 11 million light rays.
