Reflection Models III

Previous lectures

■ Ideal materials: mirrors, glass, matte
■ Glossy materials
■ Light transport: surfaces & volumes

Today

■ Subsurface reflection
■ Hair
Subsurface Reflection
Translucent Materials: Jade
Translucent Materials: Marble
Subsurface Scattering

Skin

Given a ray path Li → θi → θr → Lr,s → Lr,v → θt → Lt,v, where

- Li is the incident light
- θi is the angle of incidence
- θr is the angle of reflection
- Lr,s is the light scattered in the skin
- Lr,v is the light scattered in the vascular system
- θt is the angle of transmission
- Lt,v is the light scattered in the tissue

We can analyze the interaction of light with skin layers to understand subsurface scattering effects in the skin.
Volume Scattering Recap

Absorption coefficient $\sigma_a$
- probability of absorption per unit length

Scattering coefficient $\sigma_s$
- probability of scattering per unit length

Albedo $\alpha = \sigma_s / (\sigma_a + \sigma_s)$
- probability of scattering at an interaction

Asymmetry parameter $g \in [-1, 1]$
- distribution of scattered light
Whole Milk

Scattering coefficient (RGB): (2.55, 3.21, 3.77)
Absorption coefficient: (0.0011, 0.0024, 0.014)
One Problem...

Albedo: (.9996, .9993, .9963)

\[ .9996^n \]
Traditional Reflection Models: BRDF

\[
BRDF: \quad f_r(p, \omega_i, \omega_o) = \frac{dL(p, \omega_o)}{dE(p, \omega_i)}
\]
Subsurface Scattering: BSSRDF

BSSRDF: \[ S(p_i, \omega_i, p_o, \omega_o) = \frac{dL(p_o, \omega_o)}{d\Phi_i(p_i, \omega_i)} \]
Evaluating the BSSRDF

\[ L_o(p, \omega) = \int_A \int_{H^2} S(p_i, \omega_i, p_o, \omega_o) L_i(p_i, \omega_i) \cos \theta_i \, d\omega_i \, dp_i \]
Rendering with BSSRDFs

\[
L(p_1 \rightarrow p) = \sum_i \left[ \int_{A^i} T_j(\bar{p}) L_e(p_{j+2} \rightarrow p_{j+1}) \, dp_{j+2} \cdots dp_2 \right]
\]

\[
T_2(\bar{p}) = f_r(p_2 \rightarrow p_1 \rightarrow p) \, G(p_1 \leftrightarrow p_2) \, S(p_2, p_1 - p_2, p_3, p_4 - p_3)
\]
$L_o(p, \omega_o) \approx \frac{S(p_o, \omega_o, p_i, \omega_i)}{p(p_i)p(\omega_i)} \frac{L_i(p_i, \omega_i) \cos \theta_i}{r_{max}}$
Whither S?

Four main ideas

- Isotropization from multiple scattering
- Similarity principles
- The diffusion approximation
- Outgoing radiance @ the boundary: the dipole
$g = 0.9, \ n = 1$
$g = 0.9, \ n = 10$
g = 0.9, n = 100
Forward-Scattering Path
Backward-Scattering Path
Similarity Principles

Approximate anisotropic \((g \neq 0)\) media as isotropic

**Reduced scattering coefficient** \(\sigma'_s = (1 - g)\sigma_s\)

**Reduced extinction coefficient** \(\sigma'_t = \sigma_a + \sigma'_s\)

**Reduced albedo** \(\alpha' = \sigma'_s / (\sigma_a + \sigma'_s)\)
Equation of transfer approximation

Apply similarity principles:

\[ \frac{\partial}{\partial t} L(p + t\omega, \omega) = \sigma_t' L(p, \omega) + L_e(p, \omega) + \frac{\sigma_s'}{4\pi} \int L(p, \omega') \, d\omega' \]

Two-term approximation to the radiance function

\[ L(p, \omega) = \frac{1}{4\pi} \phi(p) + \frac{3}{4\pi} \omega \cdot \vec{E}(p) \]

\[ \phi(p) = \int L(p, \omega) \, d\omega \]

\[ \vec{E}(p) = \int \omega L(p, \omega) \, d\omega \]
Solution: point source at origin

\[ \phi(r) = \frac{1}{4\pi D} \frac{e^{-r\sqrt{\sigma_a/D}}}{r} \]

\[ D = \frac{1}{3\sigma_t} \]
**Boundary condition:**
no scattering into surface

\[ S_d(x_i, \omega_i, x_o, \omega_o) \propto -F(\hat{n} \cdot \nabla(\phi_1(r) - \phi_2(r))) \]

**Negative source**
Final BSSRDF

\[
S(p_o, \omega_o, p_i, \omega_i) \approx (1 - F_r(\cos \theta_o)) S_p(p_o, p_i) S_\omega(\omega_i)
\]

\[
\approx (1 - F_r(\cos \theta_o)) S_r(\|p_o - p_i\|) S_\omega(\omega_i)
\]

Further details (see book):

- Non-Classical diffusion
- Fresnel reflection at the surface
- Beam solution
- Single-scattering
Marble: BDRF versus BSSDRF

BRDF  BSSRDF
Diffuse Milk
Skim Milk
Whole Milk
Dipole
Beam diffusion
Scattering From Hair and Fur
Hair Appearance
Hair Appearance
Direct Lighting Only
Global Illumination
Herbert the Teddy Bear
Diffuse Reflection from a Cylinder

$$\sin \theta_i = \sqrt{1 - (\hat{t} \cdot \omega_i)^2}$$
Specular Reflection from a Cylinder

$R_{\bar{n}}(\omega_i)$
Far Field Model

Near Field Model

[Yan et al. 2017]
Hair Scattering Measurements

Black hair

Blond hair

Synthetic

\[ \theta_r \]

\[ \theta_r \]

\[ \theta_r \]
Hair Fibers

Measurements for cosmetics industry

- [Stamm 1977; Bustard & Smith 1994]

Wei 2006

[Robbins ’94]
Simulate Directly?

$\omega_i$
Fiber Model

Marschner et al. 2003
Fiber + Scales
Angles

Longitudinal

Azimuthal
Azimuthal Scattering

\[ \Phi(p, h) = 2p\gamma_t - 2\gamma_i + p\pi \]
Fresnel and Absorption

\[ A_0 = f \]
Fresnel and Absorption

\[(1 - f) \quad T \quad (1 - f)\]
Fresnel and Absorption

\[ A_p = (1 - f)^2 T^p f^{p-1} \]
Marschner et al.  

Energy Conserving(*)

d’Eon et al. 2011
Varying Roughness

Top: longitudinal, bottom: azimuthal
Importance Sampling: d’Eon et al. 2013
Non-Separable Lobes

From left: MC simulation, new model, previous

d’Eon et al. 2014
Friendly Parameterization

Chiang et al. 2016
Yan et al. 2017
Marschner model

Yan et al. 2016
Multi-Scale Model

Yan et al. 2017