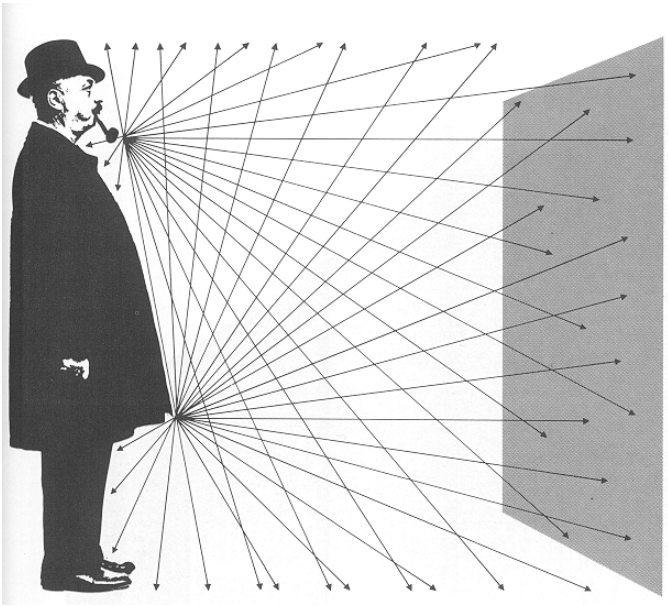
The Light Field

Last lecture: Radiometry and photometry

This lecture: Light field = radiance function on rays

- **Conservation of radiance**
- **Measurement equation**
- **Throughput and counting rays**
- **Irradiance calculations**

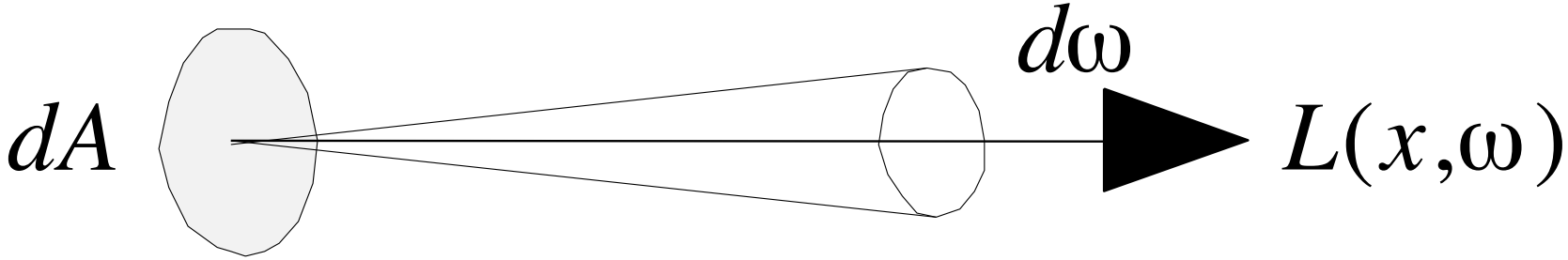


From London and Upton

Subtitle: "Lines and Light"

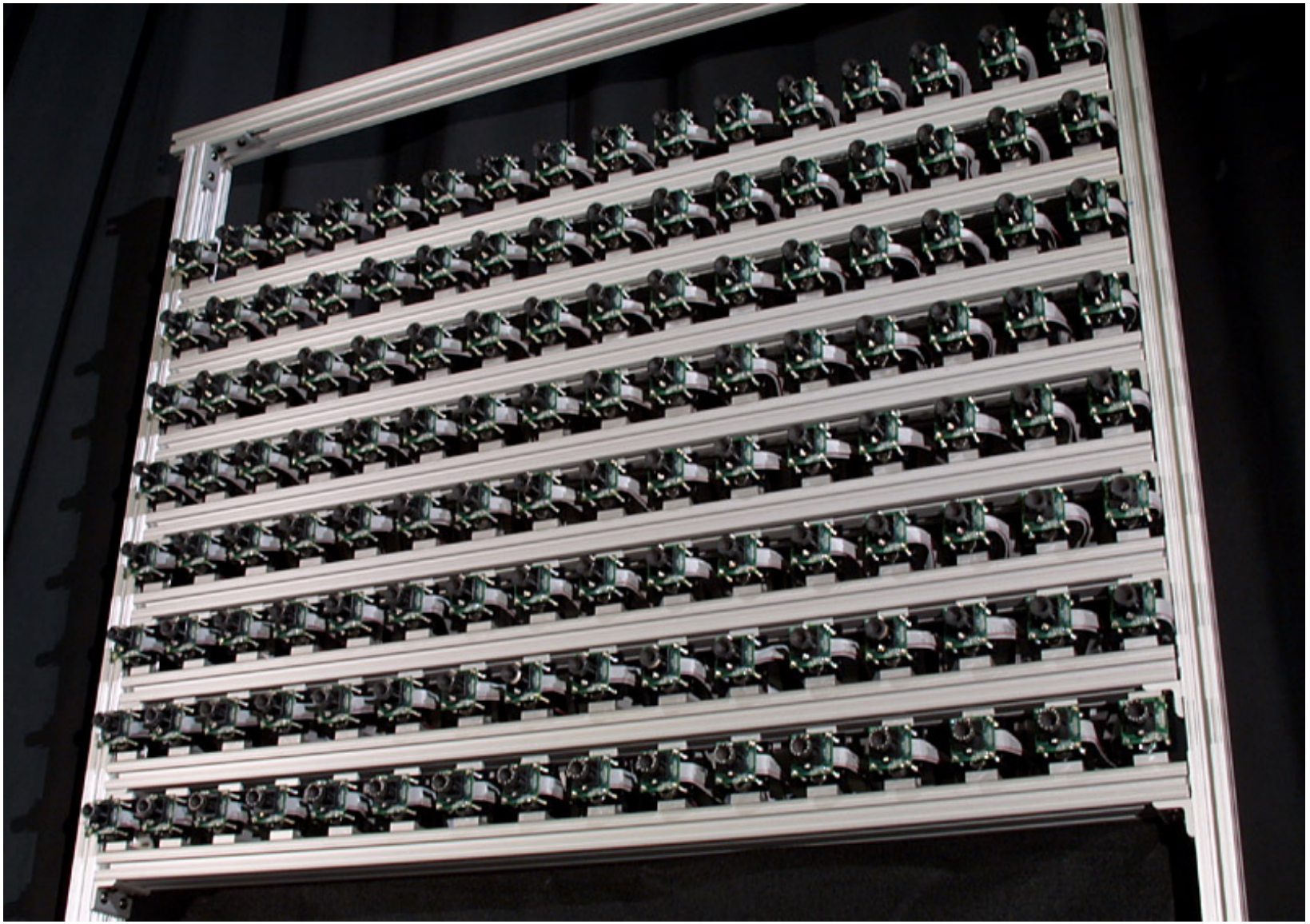
Field Radiance or Light Field

Definition: The field radiance (luminance) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction

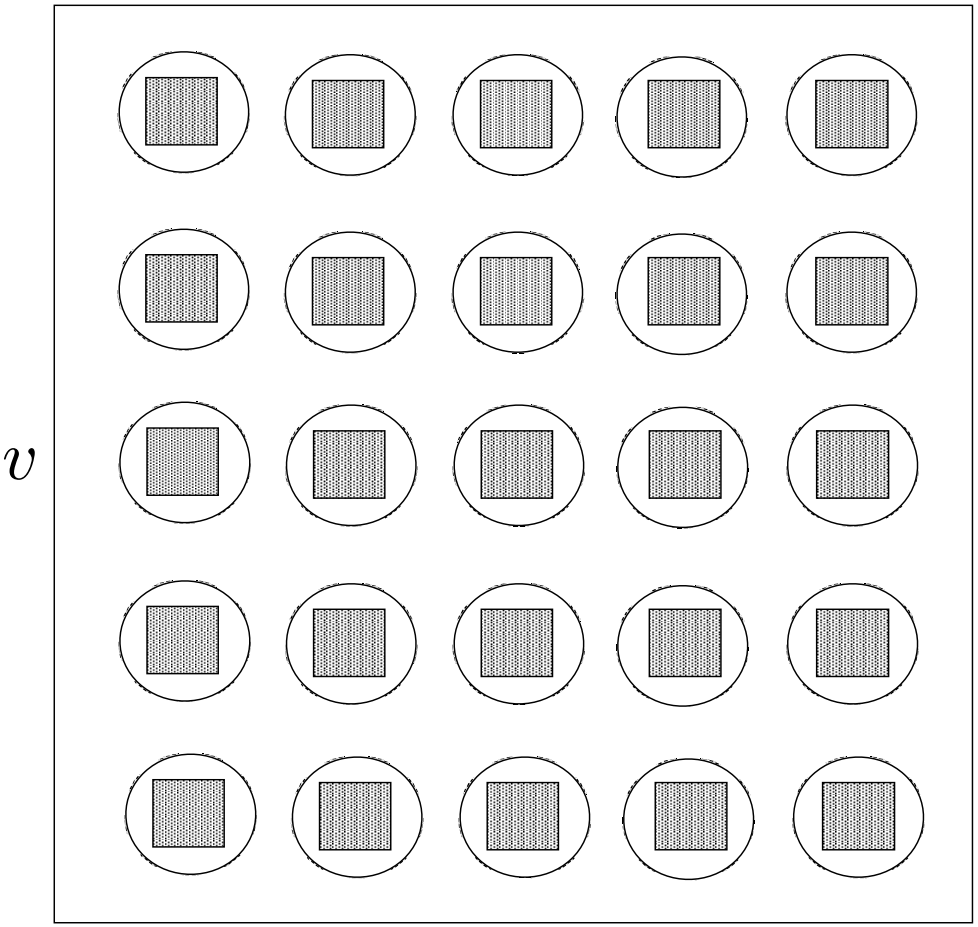


$$L(x,\omega) \equiv \frac{d^2\Phi(x,\omega)}{dx d\omega}$$

Multi-Camera Array



Two-Plane 4D Light Field

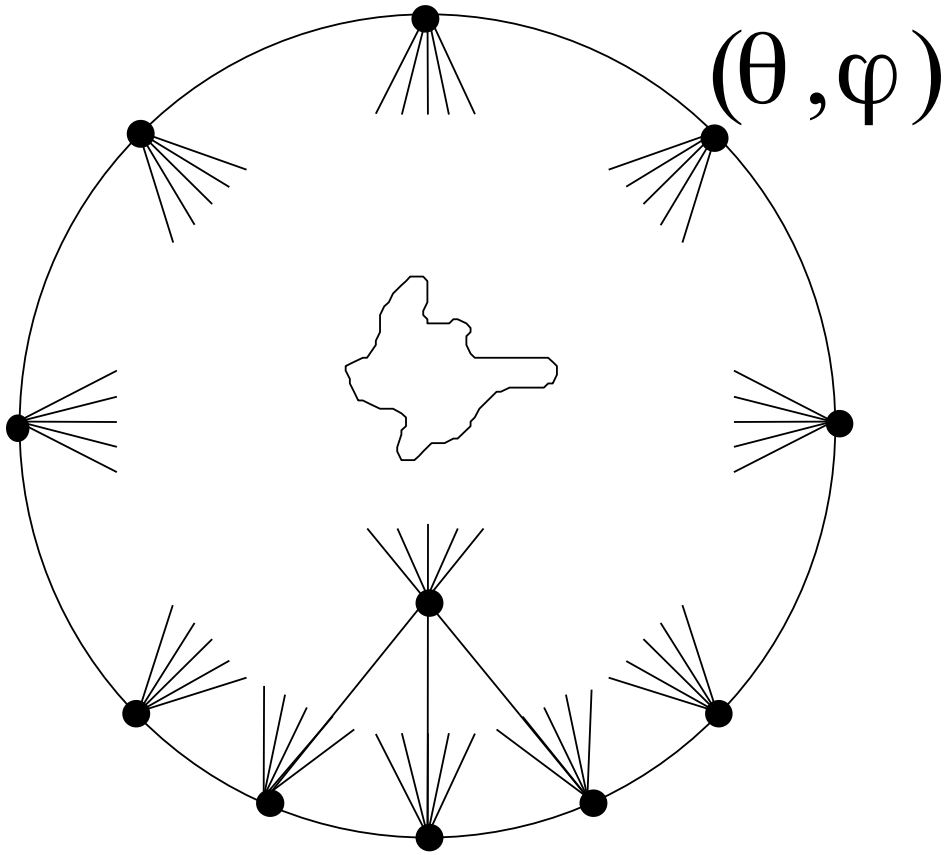


2D Array of Cameras **2D Array of Images**

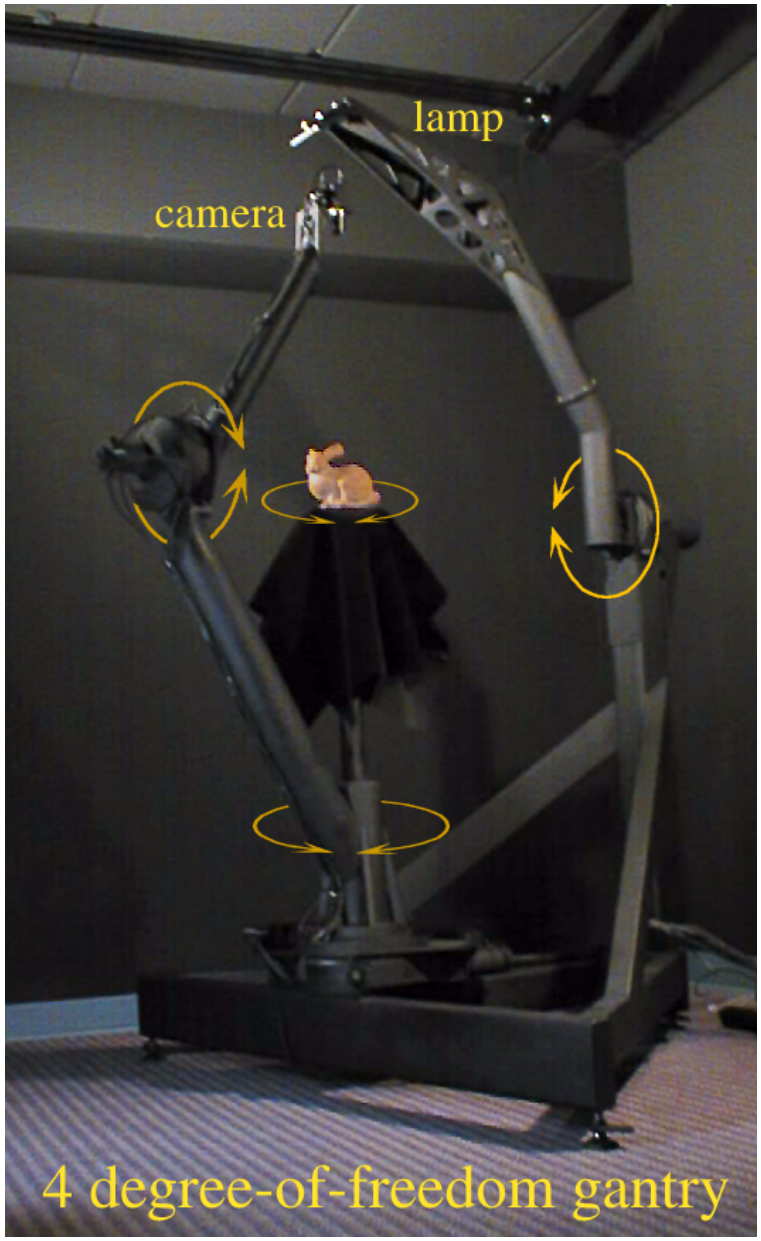
$$L(u,v,s,t)$$

Spherical Gantry ⇒ 4D Light Field

$L(x, y, \theta, \varphi)$



Captures all the light leaving an object - like a hologram

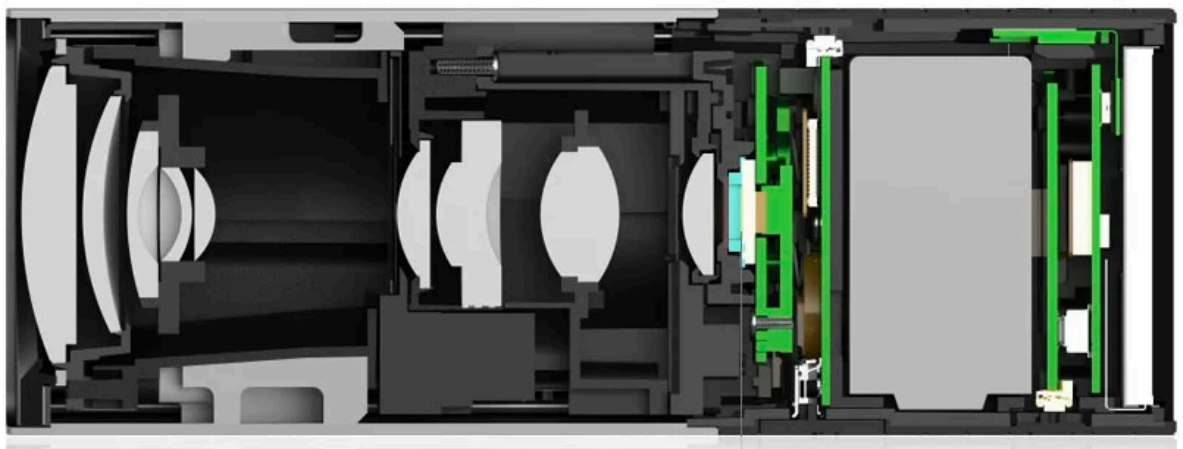


Ren Ng
Light Field Camera

Lens
The Lytro Light Field Camera starts with an 8X optical zoom, f/2 aperture lens. The aperture is constant across the zoom range allowing for unheard of light capture.

Light Field Engine 1.0
The Light Field Engine replaces the supercomputer from the lab and processes the light ray data captured by the sensor.

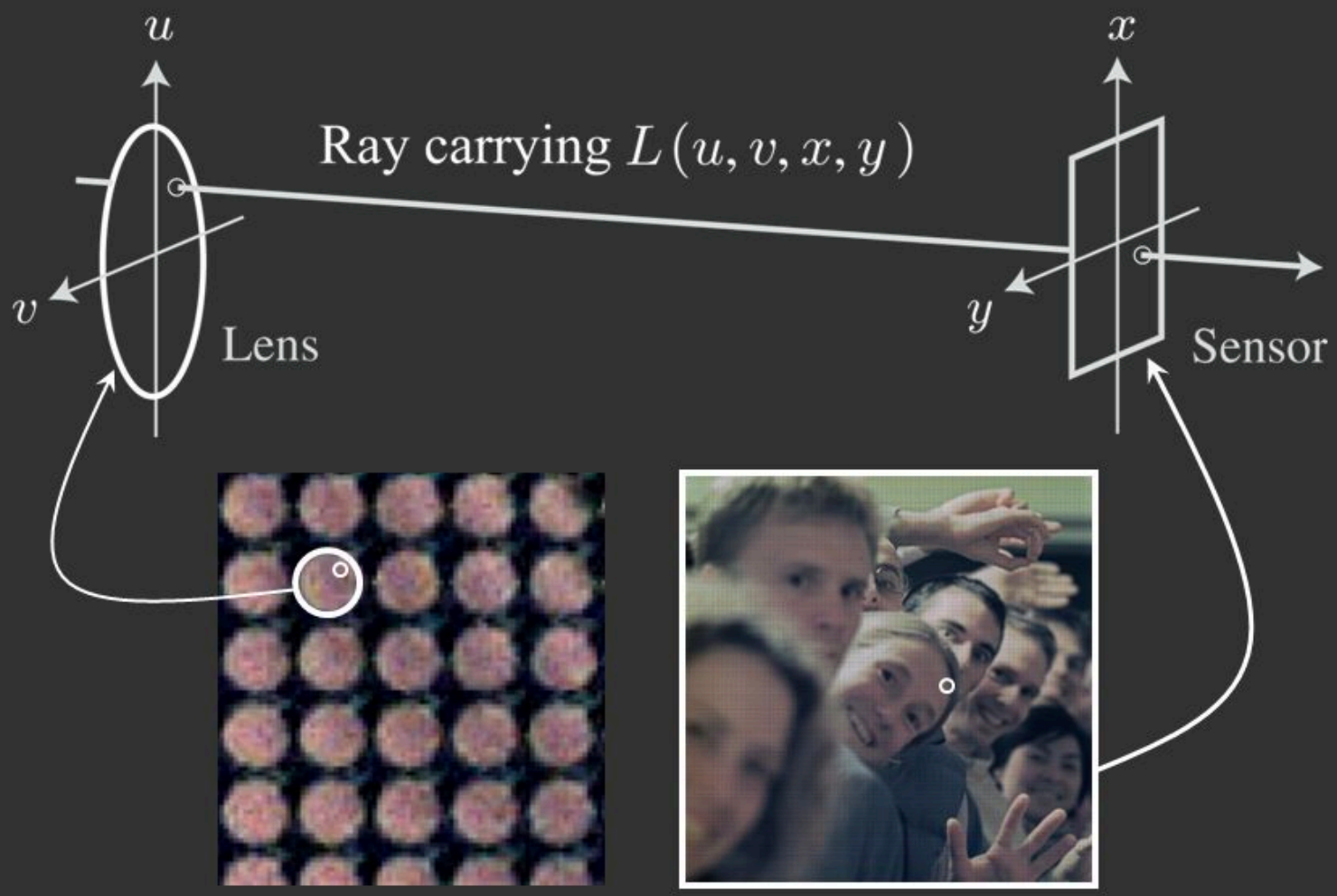
The Light Field Engine travels with every living picture as it is shared, letting you refocus pictures right on the camera, on your desktop and online.

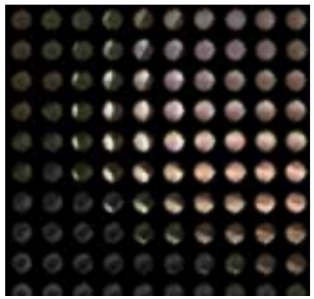


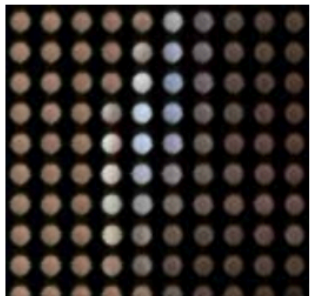
Light Field Sensor
From a roomful of cameras to a micro-lens array specially adhered to a standard sensor, the Lytro's Light Field Sensor captures 11 million light rays.

Lytro Camera

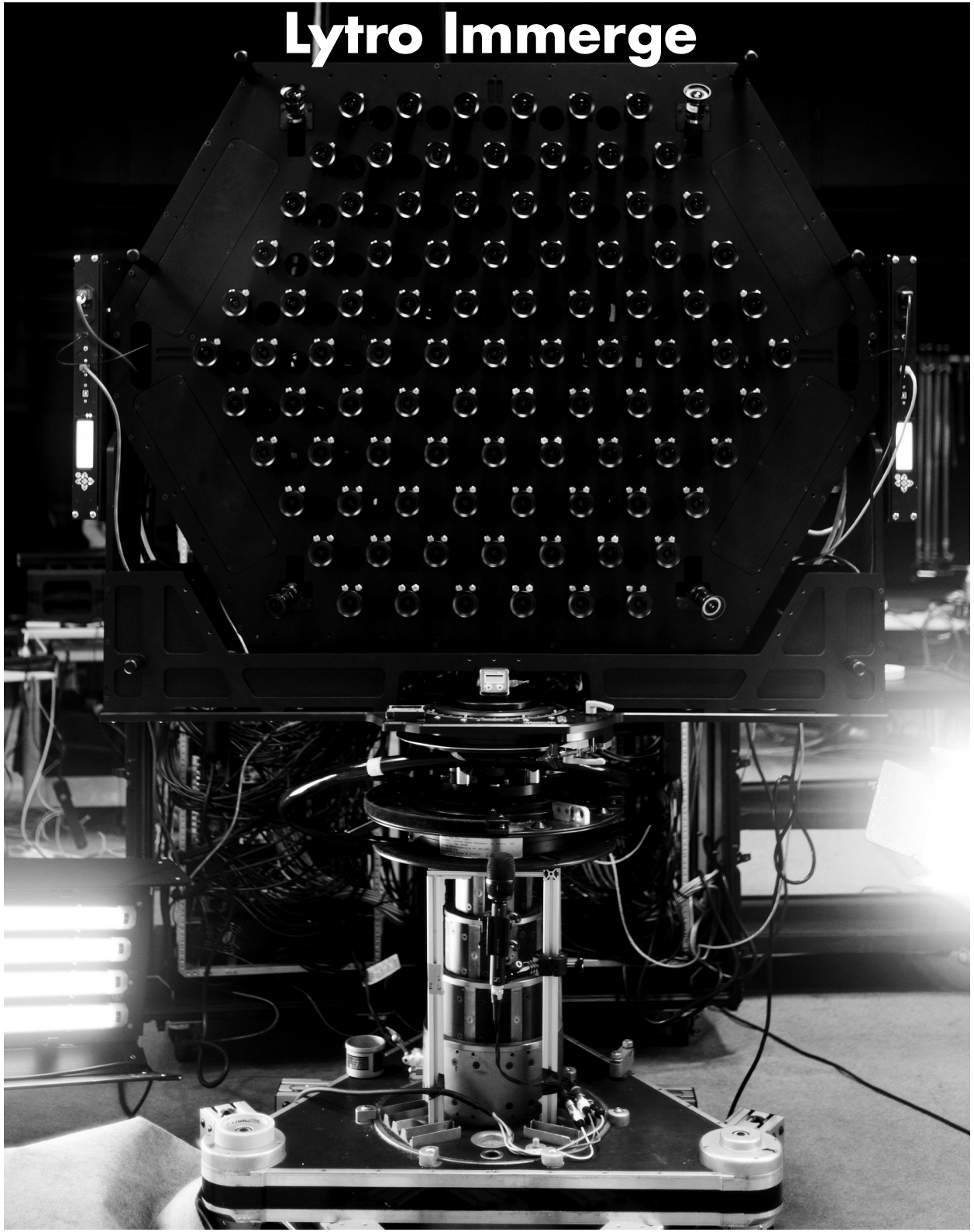
Light Field Inside the Camera Body



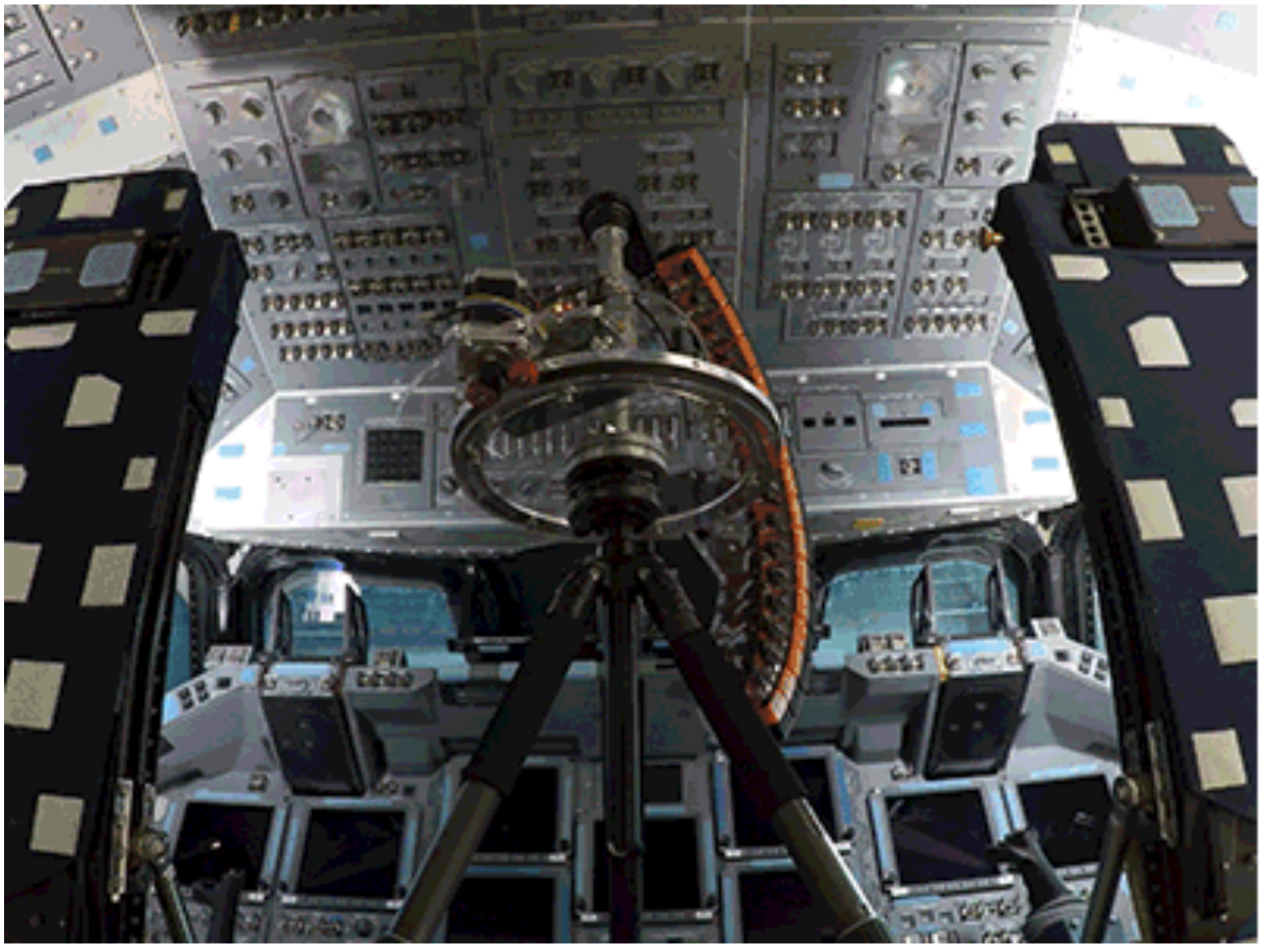


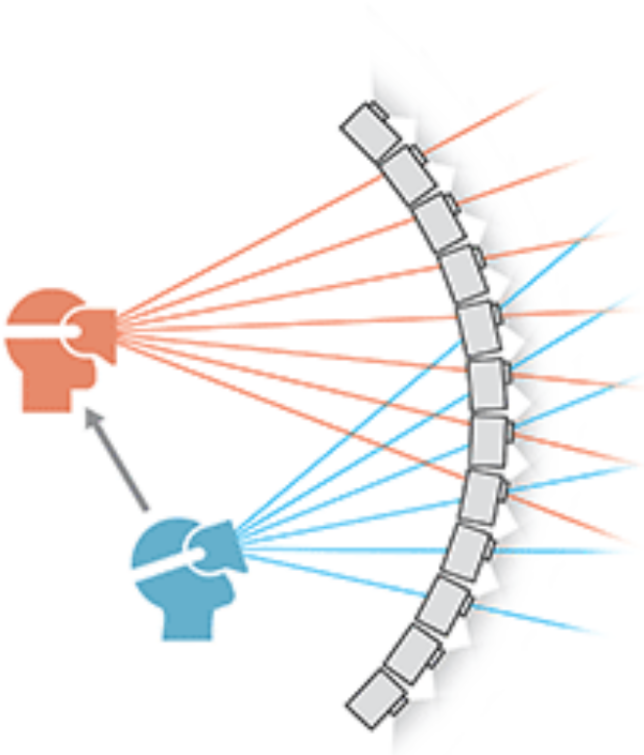


Lytro Immerge



Google VR180





Left: A time lapse video of recording a spherical light field on the flight deck of Space Shuttle Discovery.
Right: Light field rendering allows us to synthesize new views of the scene anywhere within the spherical volume by sampling and interpolating the rays of light recorded by the cameras on the rig.

https://www.blog.google/products/google-vr/experimenting-light-fields/

Properties of Radiance

Properties of Radiance

Fundamental field quantity that characterizes the distribution of light in an environment

- Radiance is the quantity associated with a ray
- Ray tracers compute the radiance

Radiance is invariant along a ray

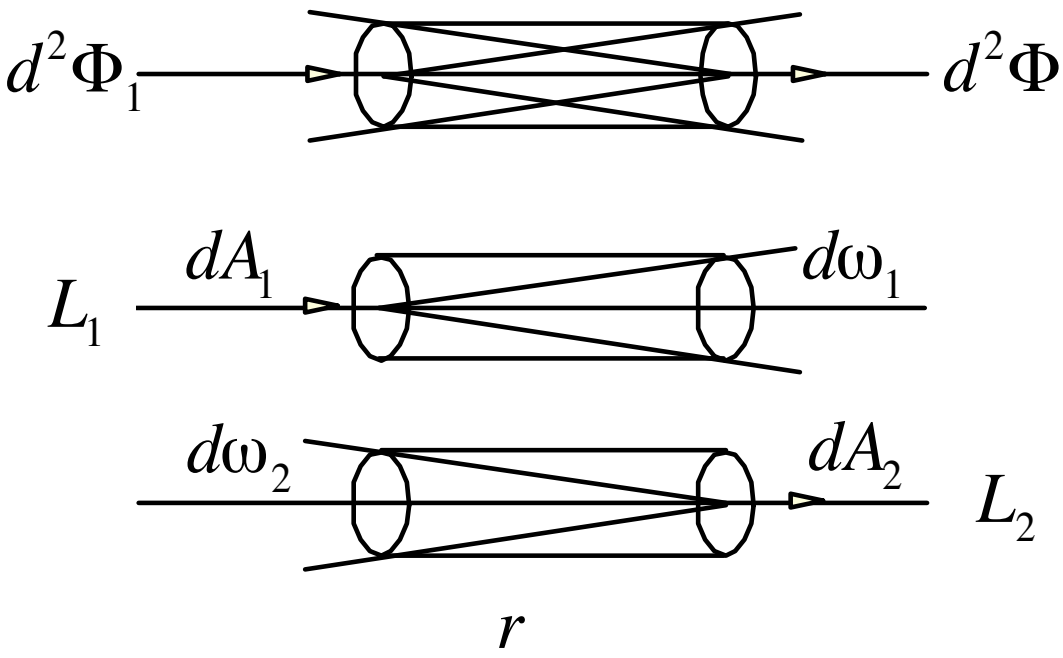
- Reduces parameters from 6D to 4D

Response of a sensor proportional to the radiance

- Cameras measure radiance

1st Law: Conservation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates



$$d^2\Phi_1 = d^2\Phi_2$$

$$d^2\Phi_1 = L_1 d\omega_1 dA_1$$

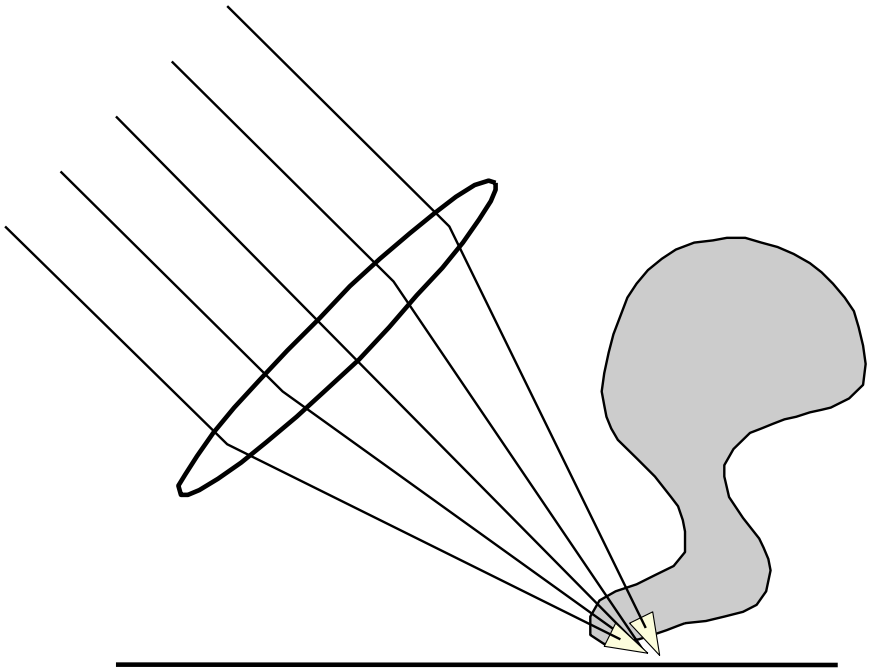
$$d^2\Phi_2 = L_2 d\omega_2 dA_2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

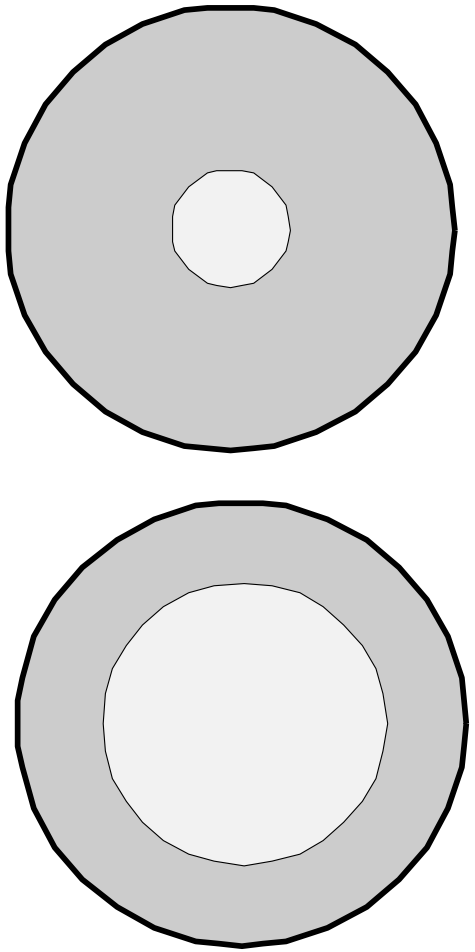
$$\therefore L_1 = L_2$$

Quiz

Does radiance increase under a magnifying glass?



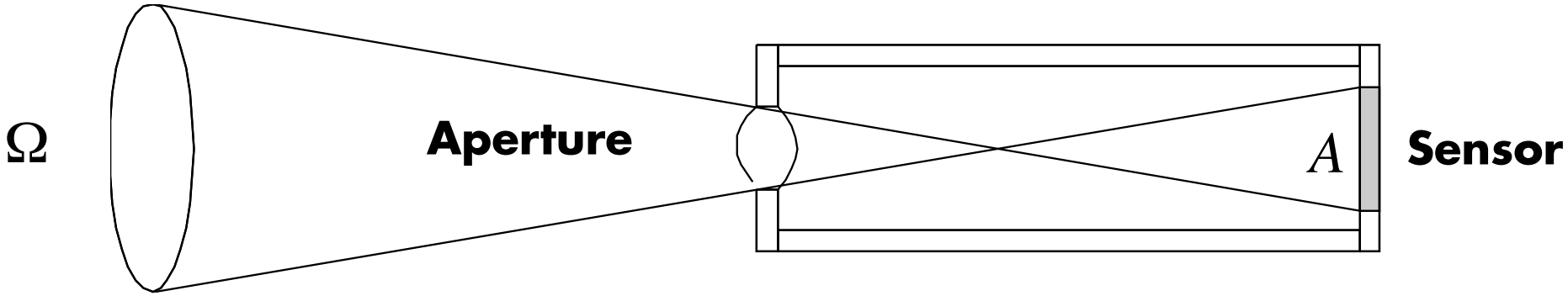
No!!



The Measurement Equation

Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.



$$R = \int_A \int_\Omega L \, d\omega \, dA = \bar{L} T \qquad T = \int_A \int_\Omega d\omega \, dA$$

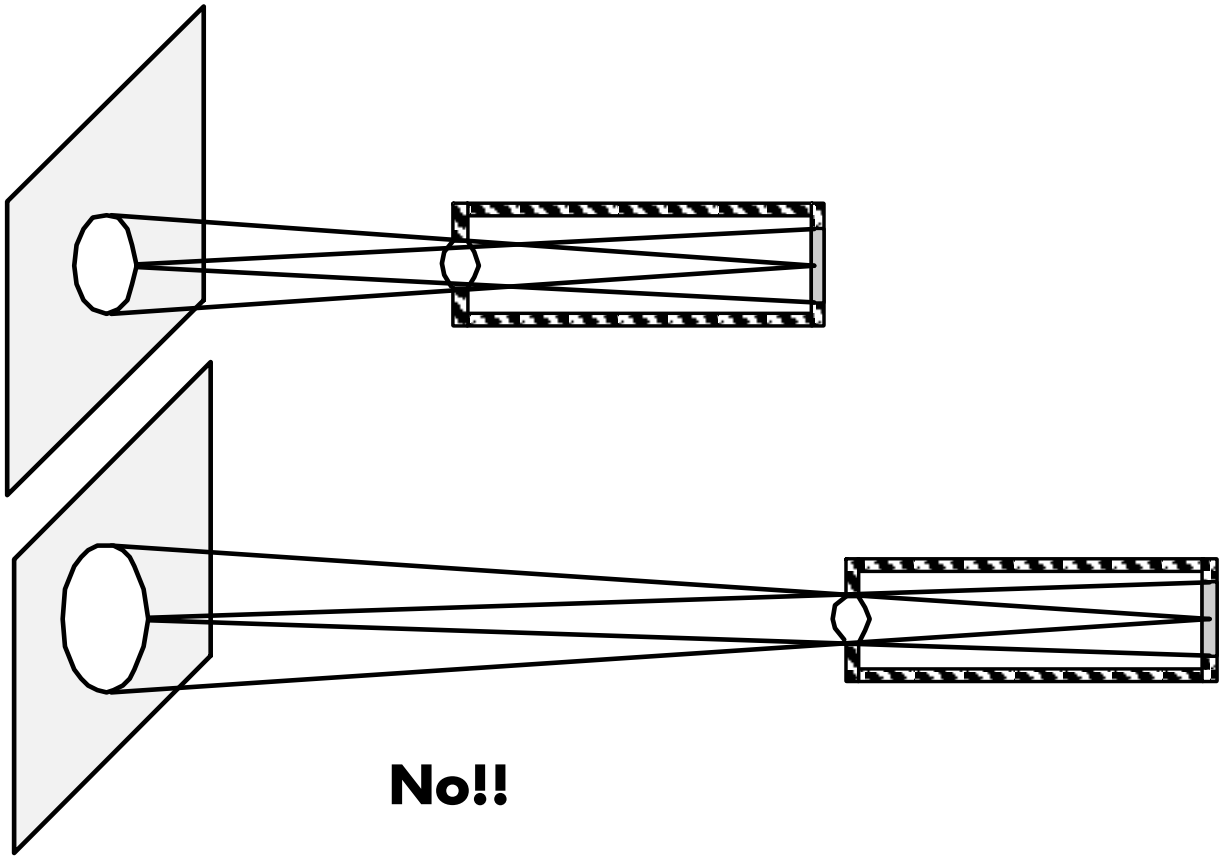
***T* (throughput) quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered**

T* is constant, sensor response *R* proportional to average radiance *L

Quiz

Assume a wall has constant radiance

Does the response of a sensor when pointed a wall depend on the distance to the wall?



No!!

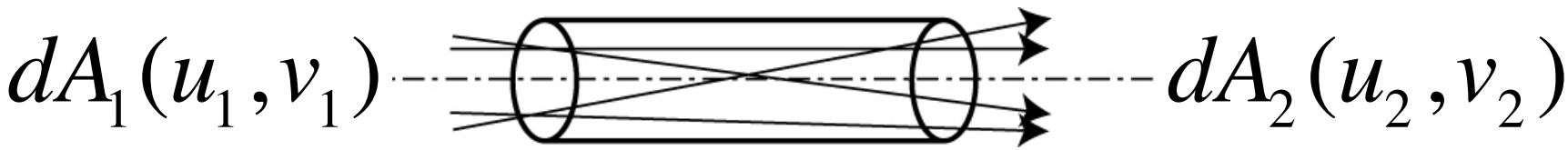
Throughput

Measuring the Number of Rays

Beam of Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

$$r(u_1, v_1, u_2, v_2)$$



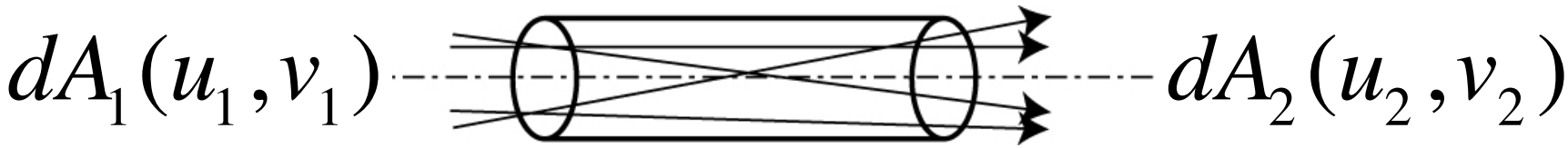
How many rays are in this beam?

Defn: The throughput is the number of rays in a beam

Number of Rays in a Beam

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

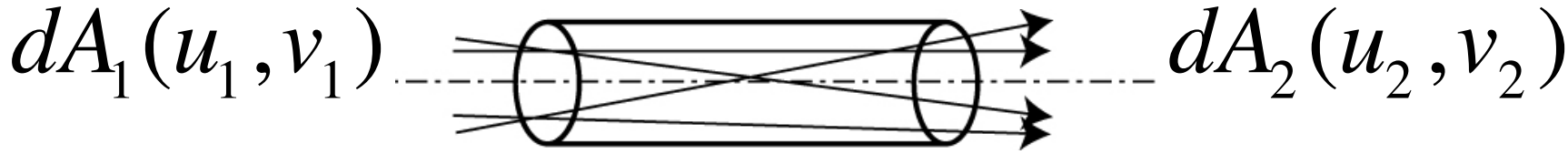
$$r(u_1, v_1, u_2, v_2)$$



This many:

$$d^2T = \frac{dA_1 dA_2}{\left|x_1 - x_2\right|^2}$$

Why is this the Right Measure?



$$d^2T = \frac{dA_1 dA_2}{\left|x_1 - x_2\right|^2}$$

Suppose we double the area dA_1 (or dA_2), then the number of rays in the beam should double

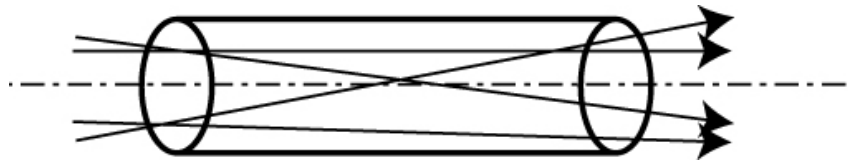
Suppose we half the distance $|x_1\text{-}x_2|^2$, then the number of rays also doubles

Alternative Definition of Radiance

Two physical laws

- **Energy is conserved** $\Delta\Phi$
- **The size of a beam** ΔT **does not change as rays propagate**

Beam

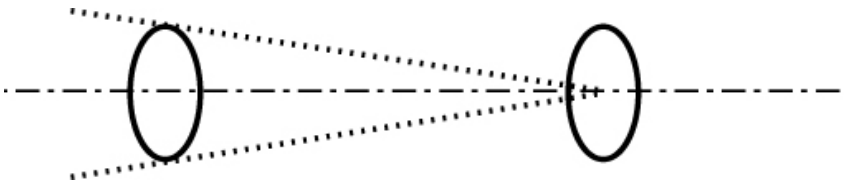


$$L(r) = \lim_{\Delta T \to r} \frac{\Delta\Phi}{\Delta T}$$

Radiance is the ratio of 2 conserved quantities, therefore, the radiance is also conserved

Changing Ray Coordinates

Parameterize rays wrt to receiver $r(u_2, v_2, \theta_2, \phi_2)$

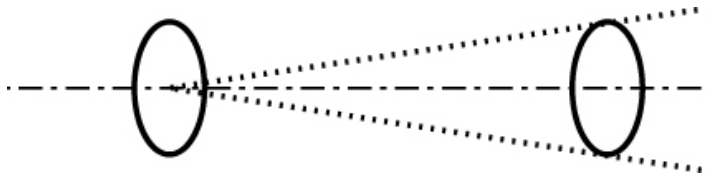
$d\omega_2(\theta_2, \phi_2)$  $dA_2(u_2, v_2)$

$$d^2T = \frac{dA_1}{\left|x_1 - x_2\right|^2} dA_2 = d\omega_2 dA_2$$

Same value for the throughput, different formula

Changing Ray Coordinates

Parameterize rays wrt to source $r(u_1, v_1, \theta_1, \phi_1)$

$dA_1(u_1, v_1)$  $d\omega_1(\theta_1, \phi_1)$

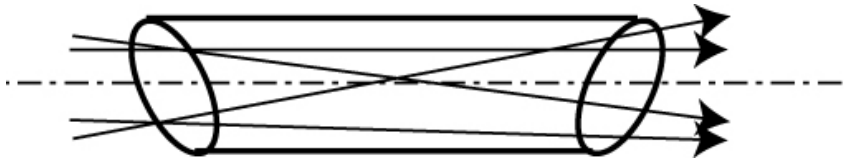
$$d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1$$

Changing Ray Coordinates Again

Tilting the surfaces re-parameterizes the rays!

$$r(u_1, v_1, u_2, v_2)$$

$$dA_1(u_1, v_1)$$



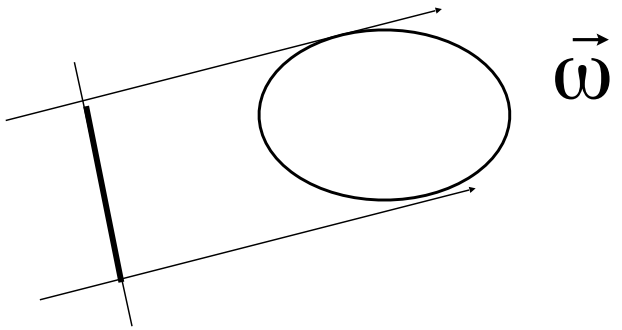
$$dA_2(u_2, v_2)$$

$$d^2T = \frac{\cos\theta_1 \cos\theta_2}{\left|x_1 - x_2\right|^2} dA_1 dA_2$$

Number of Rays Hitting a Shape

Parameterize rays by $r(x, y, \theta, \phi)$

Projected area $\tilde{A}(\vec{\omega})$



Measuring the number or rays that hit a shape

$$T = \int_{S^2} d\omega(\theta, \varphi) dA(x, y) = \int_{S^2} d\omega(\theta, \varphi) \int_{R^2} dA(x, y)$$

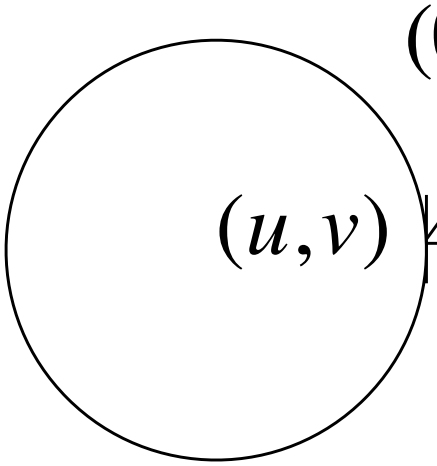
$$= \int_{S^2} \tilde{A}(\theta, \varphi)\, d\omega(\theta, \varphi)$$

$$= 4\pi \bar{\tilde{A}}$$

Sphere: $T = 4\pi \bar{\tilde{A}} = 4\pi^2 R^2$

Calculated Another Way

Parameterize rays by $r(u,v,\theta,\phi)$



$$T = \underbrace{\left[\int_{H^2(\vec{\mathbf{N}})} \cos\theta \, d\omega(\theta,\varphi)\right]}_{\pi} \underbrace{\left[\int_{M^2} dA(u,v)\right]}_{S}$$

Sphere: $T = \pi S = 4\pi^2 R^2$

Crofton's Theorem: $4\pi \bar{\tilde{A}} = \pi S \Rightarrow \bar{\tilde{A}} = \frac{S}{4}$

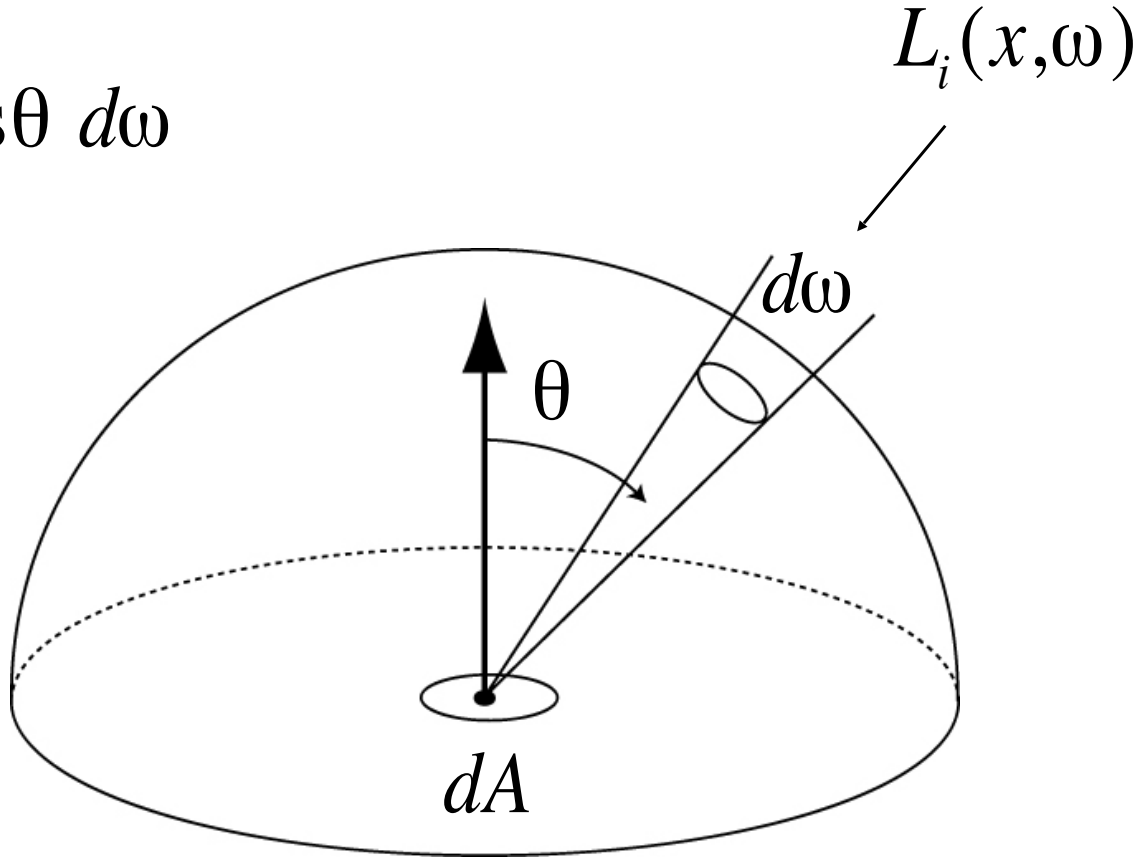
Irradiance
from a
Uniform Area Light Source

Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta\, dA\, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta\; d\omega$$

Light meter



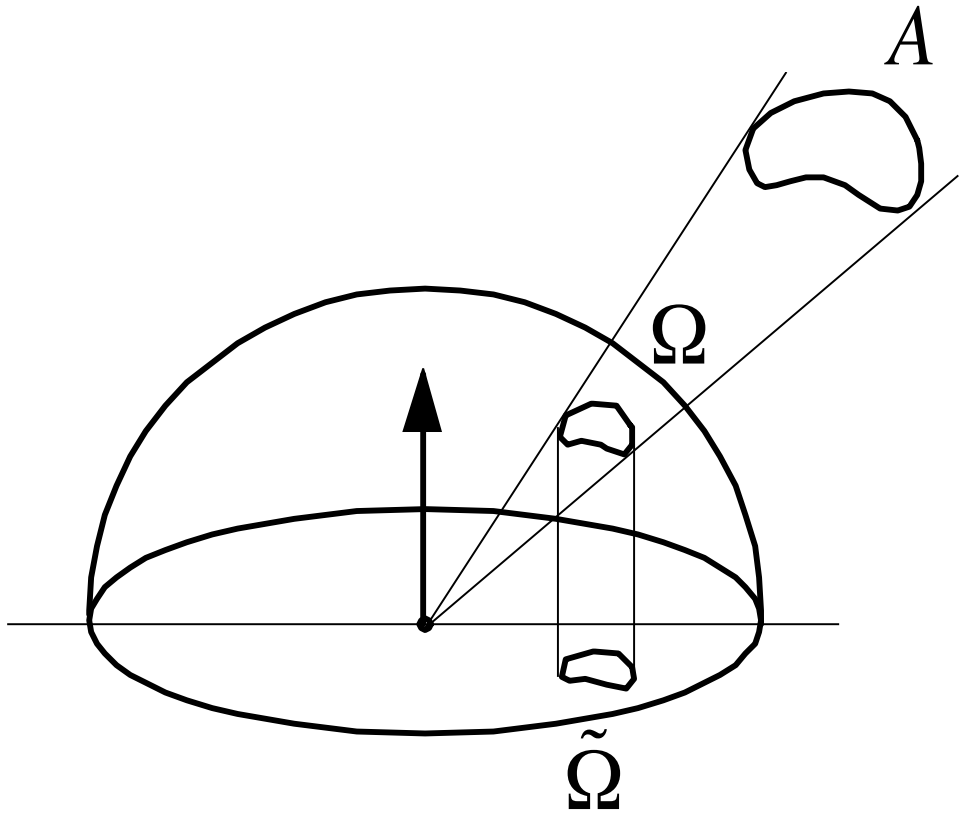
$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta\; d\omega$$

Irradiance from a Uniform Area Source

$$E(x) = \int_{H^2} L \cos\theta \, d\omega$$

$$= L \int_{\Omega} \cos\theta \, d\omega$$

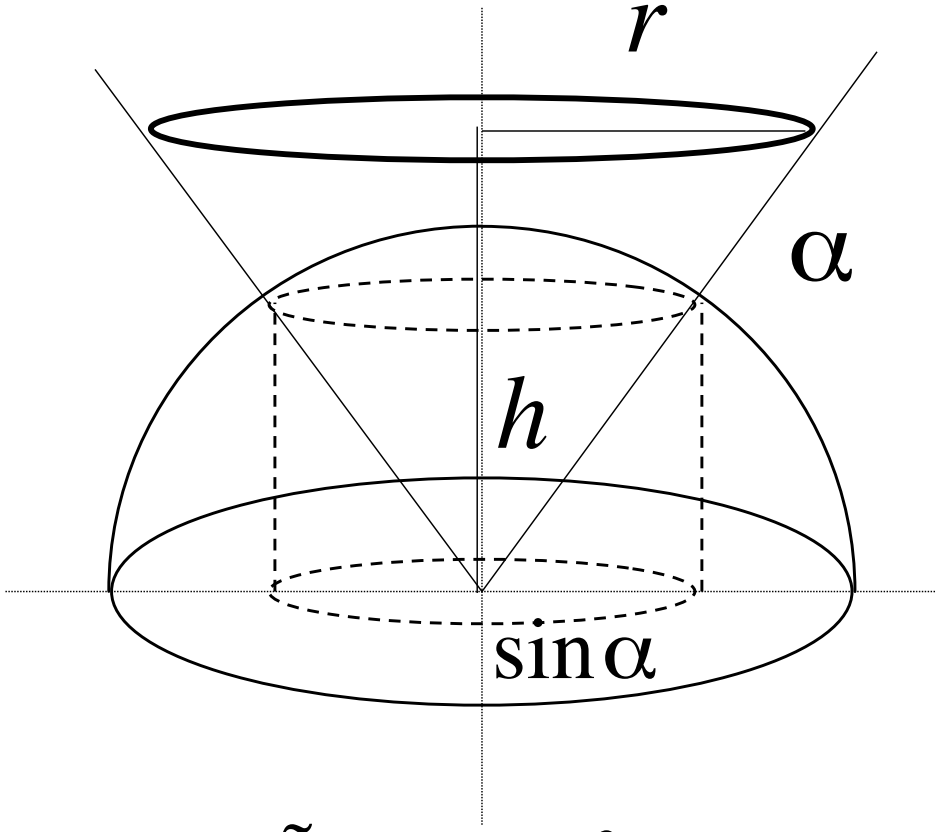
$$= L\tilde{\Omega}$$



Direct Illumination

Uniform Disk Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int_0^{2\pi} \int_1^{\cos\alpha} \cos\theta \, d\phi \, d\cos\theta$$

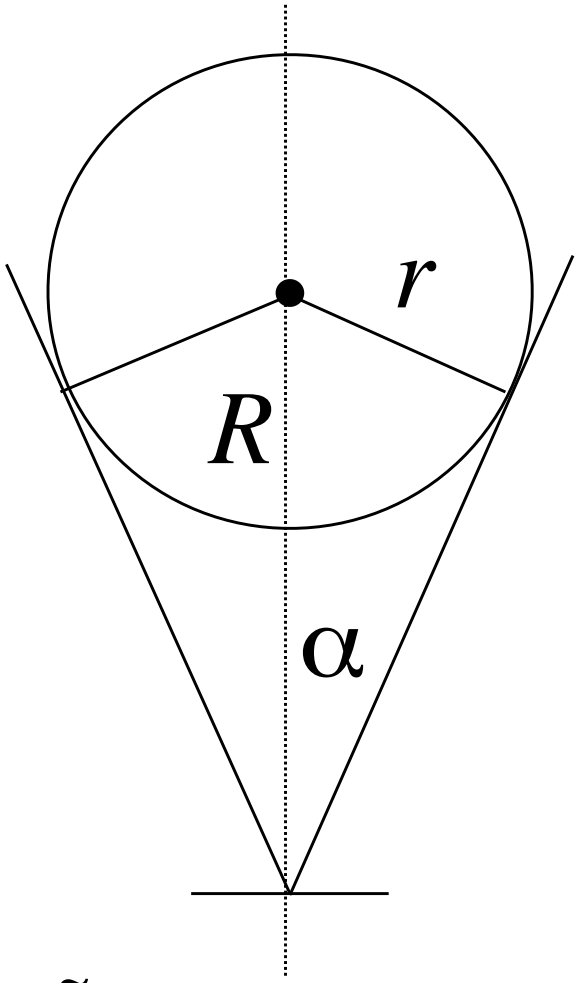
$$= 2\pi \left. \frac{\cos^2 \theta}{2} \right|_1^{\cos\alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$

Spherical Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int \cos\theta \, d\omega$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{R^2}$$

The Sun

Solar constant (normal incidence at zenith)

Irradiance **1353 W/m²**

Illuminance **127,500 lm/m² = 127.5 kilolux**

Solar angle

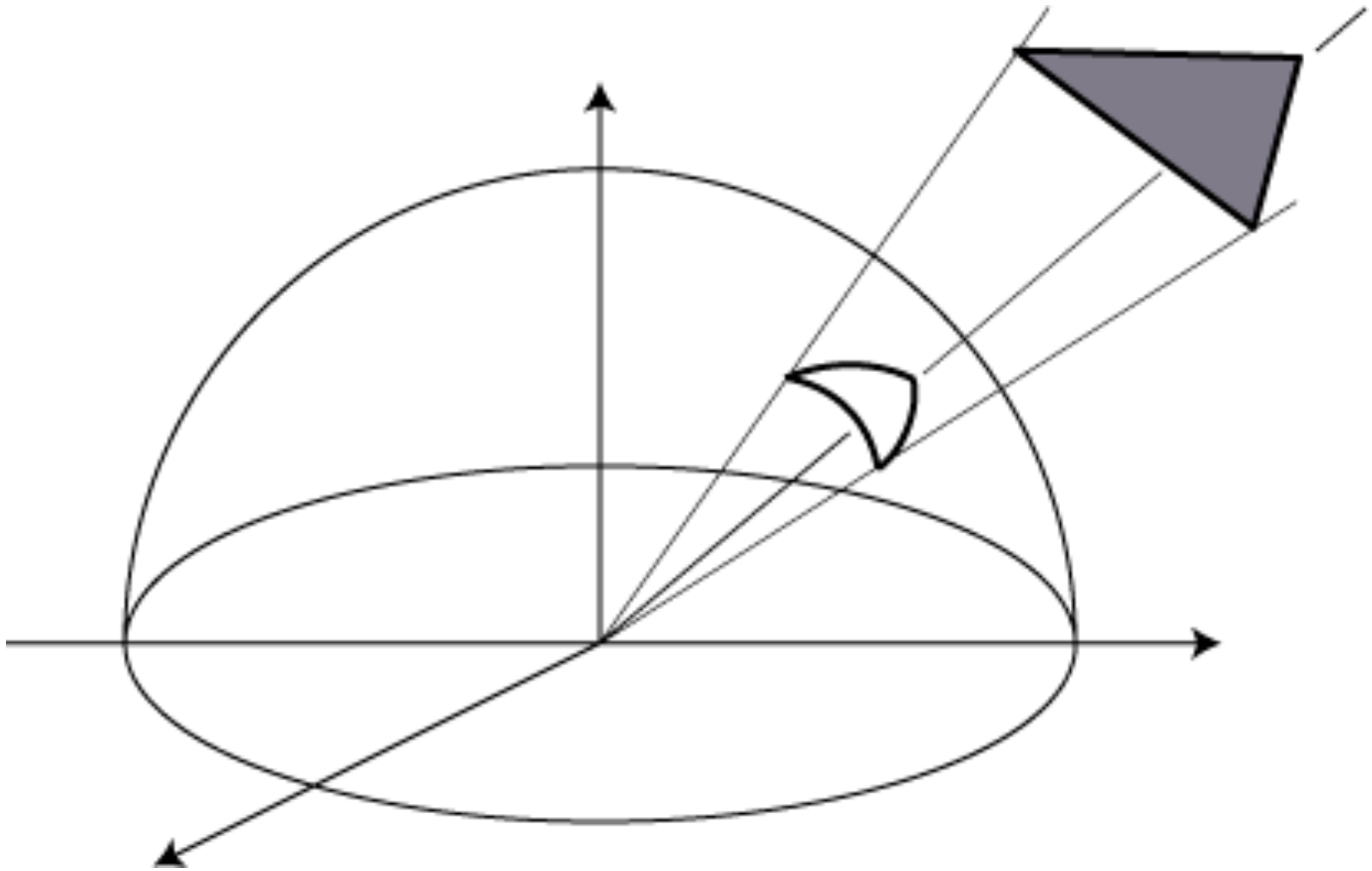
α **= .25 degrees = .004 radians (half angle)**

$\tilde{\Omega} = \pi \sin^2 \alpha \approx \pi\alpha^2$ **= 6 x 10-5 steradians**

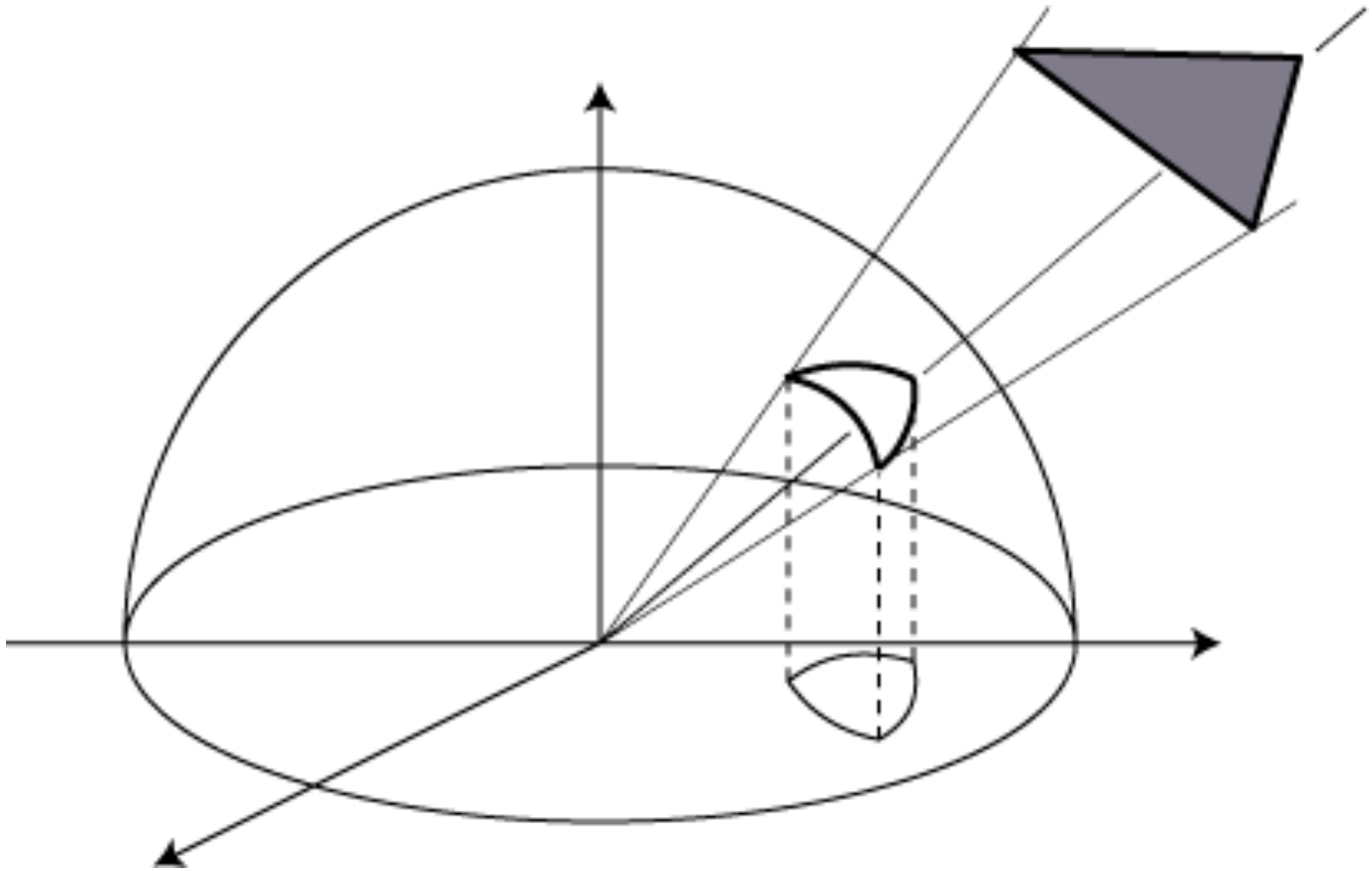
Solar radiance

$$L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \, W/m^2}{6 \times 10^{-5} \, sr} = 2.25 \times 10^7 \, \frac{\mathrm{W}}{m^2 \cdot sr}$$

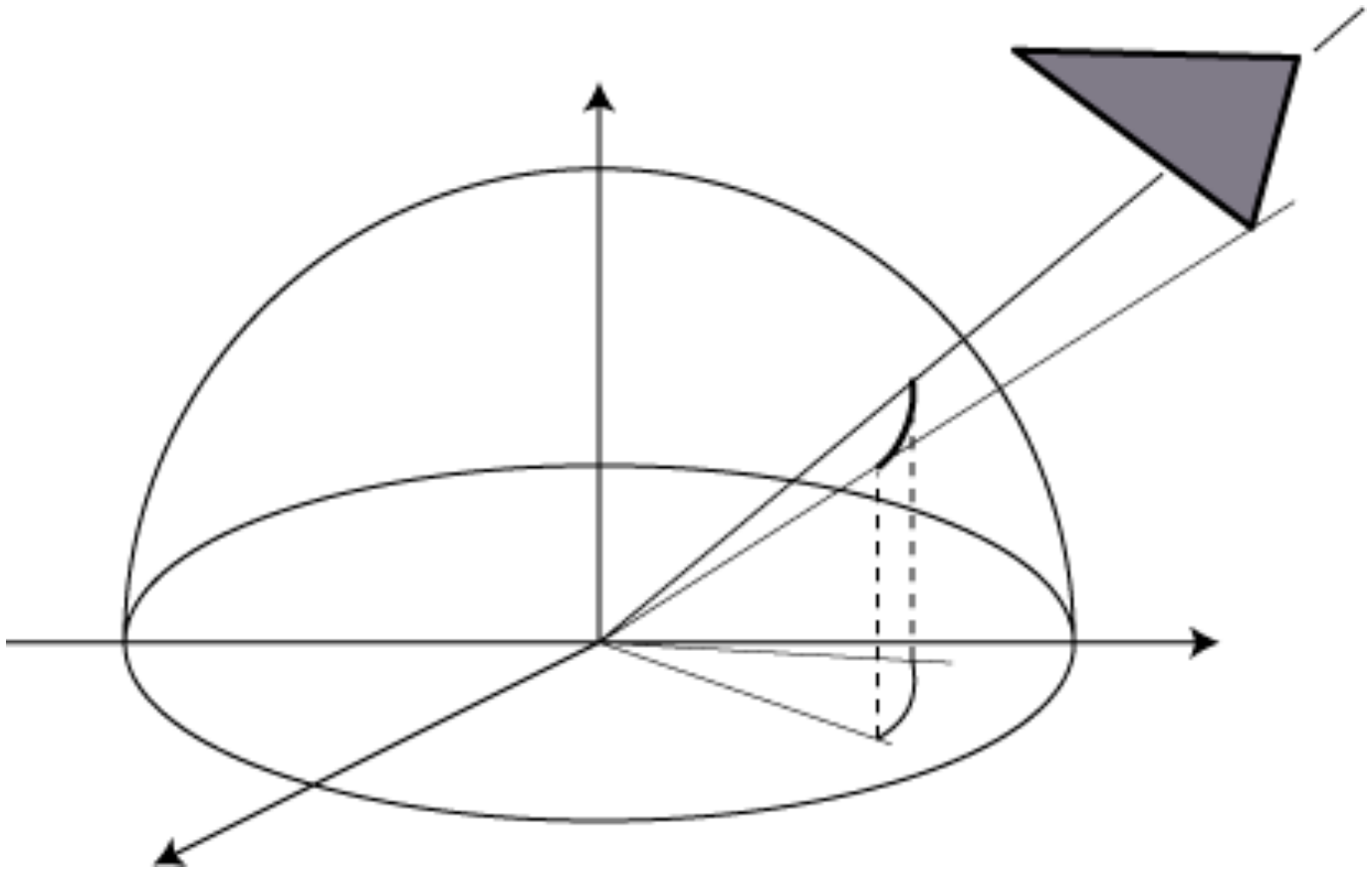
Polygonal Source



Polygonal Source

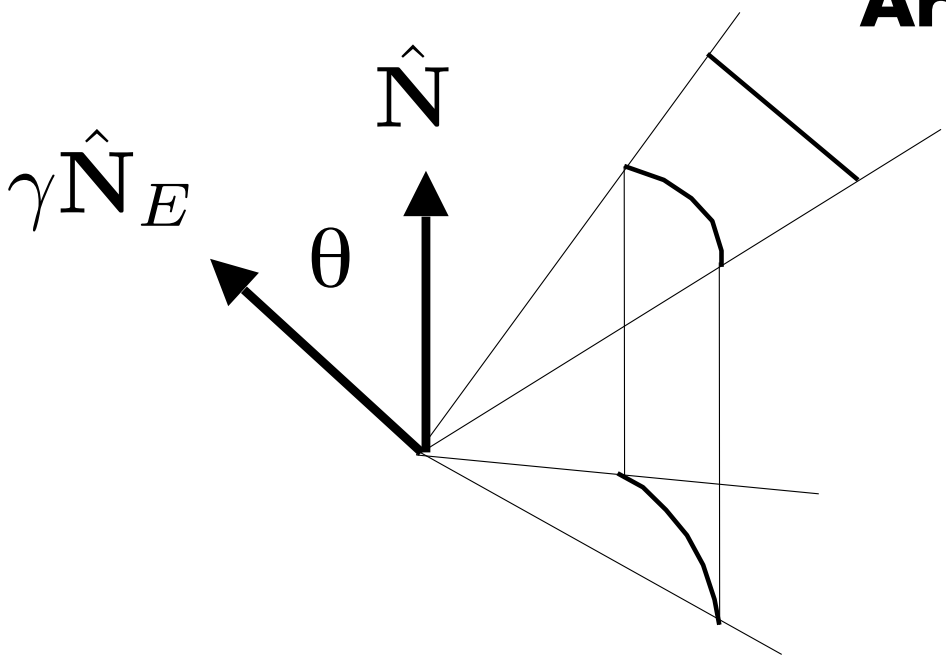


Polygonal Source



Consider 1 Edge

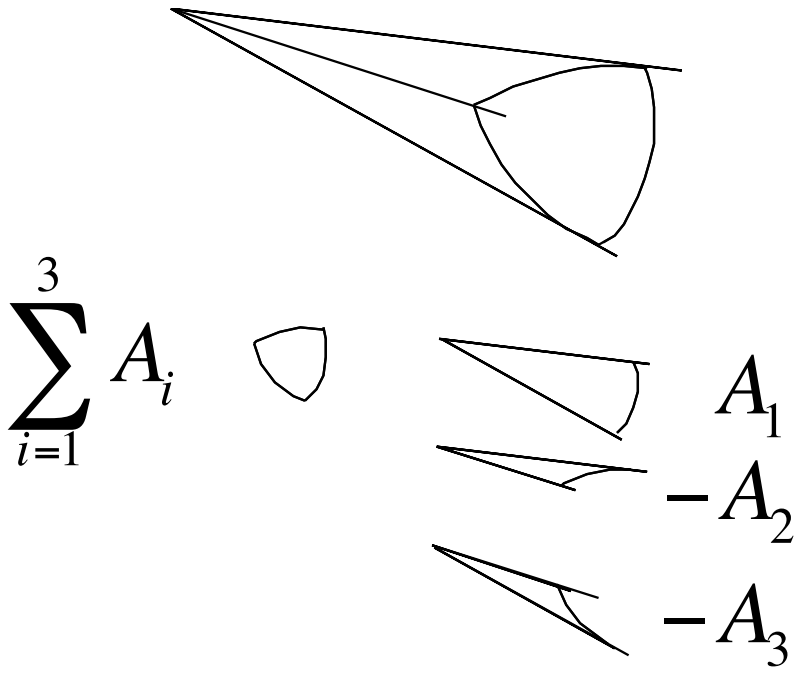
Area of sector γ



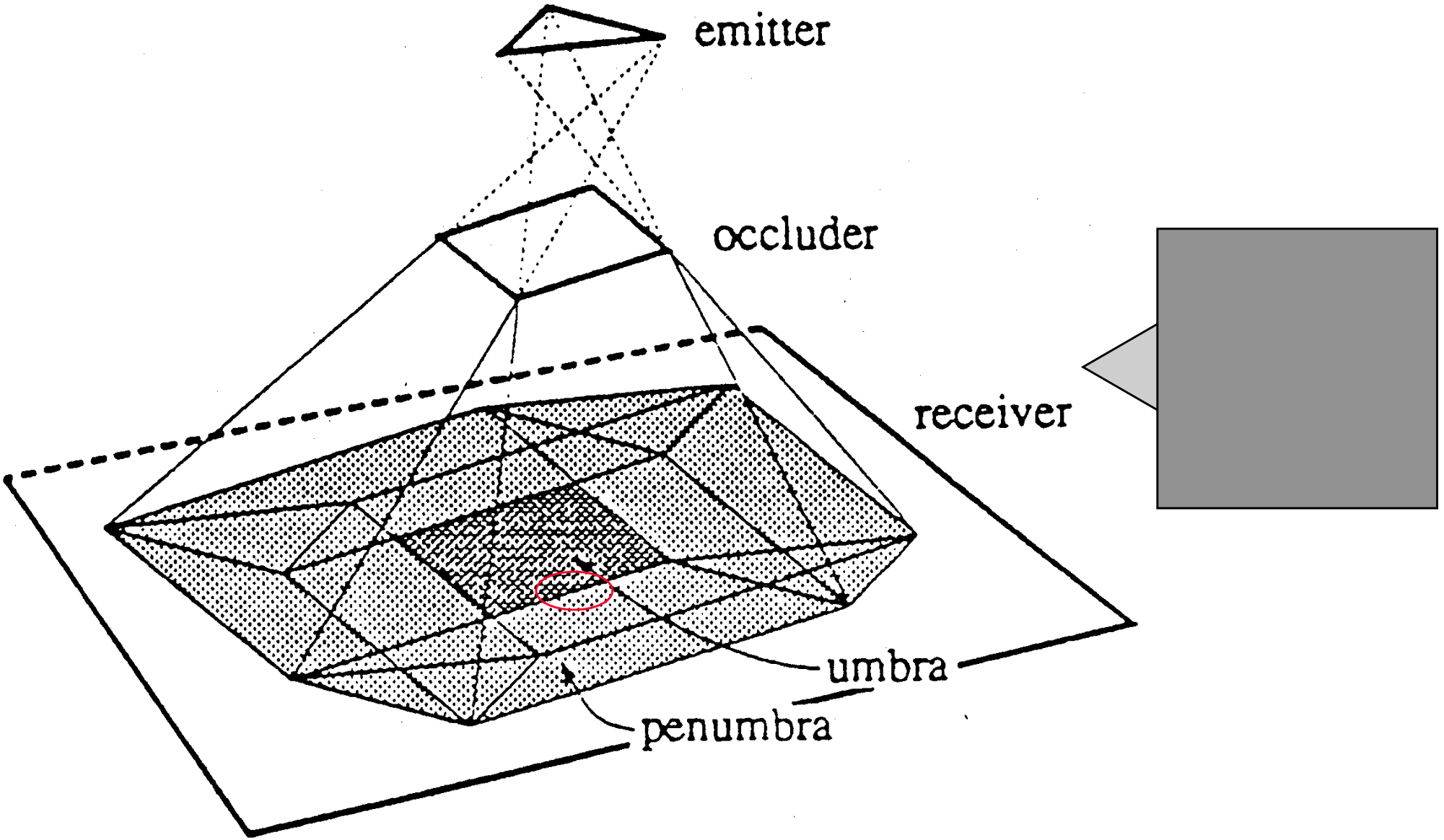
Projected area of sector

$$A = \gamma \cos\theta = \gamma \vec{\mathbf{N}}_E \bullet \vec{\mathbf{N}}$$

Lambert's Formula



$$\sum_{i=1}^{3} A_i \qquad A_1 \qquad -A_2 \qquad -A_3$$

$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$

Penumbras and Umbras