Reflection Models I: Ideal Materials

Today

- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Next lecture (Thursday)

- Glossy and specular reflection models
- Rough surfaces and microfacet distributions
Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

■ Spectra and Color
■ Polarization
■ Directional distribution
Gonio-reflectometer

4 degree-of-freedom gantry
http://gl.ict.usc.edu/lightstages/
The Reflection Equation

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]
The BRDF

Bidirectional Reflectance-Distribution Function

\[ dL_r(x, \omega_r) \]

\[ \hat{N} \]

\[ \theta_r \]

\[ \theta_i \]

\[ d\omega_i \]

\[ \phi_r \]

\[ \phi_i \]

\[ L_i(x, \omega_i) \]

\[ f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \begin{bmatrix} 1 \\ sr \end{bmatrix} \]
Properties of BRDF’s

1. Linearity

From Sillion, Arvo, Westin, Greenberg
Properties of BRDF’s

2. Reciprocity principle

\[ f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r) \]
3. Isotropic vs. anisotropic

\[ f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) \]

Reciprocity and isotropy

\[ f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|) \]
4. Energy Conservation

Reflectance

\[ \rho = \frac{\Delta \Phi_r}{\Delta \Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r \, d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i \, d\omega_i} \]

\[ 0 \leq \rho \leq 1 \]
Types of Reflection Functions

Ideal Specular
- Reflection Law
- Mirror

Ideal Diffuse
- Lambert’s Law
- Matte

Specular
- Glossy
- Directional diffuse
Law of Reflection

\[ \theta_r = \theta_i \]

\[ \varphi_r = (\varphi_i + \pi ) \text{mod } 2\pi \]

\[ \hat{\mathbf{R}} + (-\hat{\mathbf{I}}) = 2 \cos \theta \hat{\mathbf{N}} = -2(\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})\hat{\mathbf{N}} \]

\[ \hat{\mathbf{R}} = \hat{\mathbf{I}} - 2(\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})\hat{\mathbf{N}} \]
Ideal Reflection (Mirror)

\[ L_i(\theta_i, \varphi_i) \quad \quad \quad L_r(\theta_r, \varphi_r) \]

\[ L_{r,m}(\theta_r, \varphi_r) = L_i(\theta_r, \varphi_r \pm \pi) \]

\[ f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) = \frac{\delta (\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta (\varphi_i - \varphi_r \pm \pi) \]

\[
L_{r,m}(\theta_r, \varphi_r) = \int f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) L_i(\theta_i, \varphi_i) \cos \theta_i \, d \cos \theta_i \, d \varphi_i \\
= \int \frac{\delta (\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta (\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i) \cos \theta_i \, d \cos \theta_i \, d \varphi_i \\
= L_i(\theta_r, \varphi_r \pm \pi) \]
Recall: Snell’s Law

\[ n_i \sin \theta_i = n_t \sin \theta_t \]

\[ n_i \hat{\mathbf{N}} \times \hat{\mathbf{I}} = n_t \hat{\mathbf{N}} \times \hat{\mathbf{T}} \]

\[ \varphi_t = \varphi_i \pm \pi \]
Recall: Law of Refraction

\[
\hat{T} = \mu \hat{I} + \gamma \hat{N}
\]

\[
\gamma = \mu \cos \theta_i - (1 - \mu^2 \sin^2 \theta_i)^{\frac{1}{2}}
\]

Total internal reflection:

\[
1 - \mu^2 (1 - (\hat{I} \cdot \hat{N})^2) < 0
\]
Optical Manhole

Total internal reflection

\[ n_w = \frac{4}{3} \]

From Livingston and Lynch
Experiment

Reflections from a shiny floor

Reflection is greater at glancing angles

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97
Fresnel Reflectance

Dielectric (Glass n=1.5)

\[ F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5 \]

Schlick Approximation

\[ F(0) = 0.04 \]
Fresnel Reflectance

Metal (Aluminum)

Gold \( F(0)=0.82 \)
Silver \( F(0)=0.95 \)
Reflection from Metals

Reflectance of Copper as a function of wavelength and angle of incidence

Measured Reflectance

$\theta$
Cook-Torrance Reflection Model

Reflectance of Copper as a function of wavelength and angle of incidence

Measured Reflectance

θ

Light spectra

Copper spectra
Cook-Torrance Reflection Model

Interpolate between color measured at normal incidence and the light color

Use Fresnel formula to interpolate

Measured Reflectance

Approximated Reflectance

Light spectra

Copper spectra

Cook-Torrance approximation

\[ R(\theta) = R(0) + (R(\pi/2) - R(0)) \max(0, \frac{F(\theta) - F(0)}{F(\pi/2) - F(0)}) \]
Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction

$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos \theta_i \, d\omega_i$$

$$= f_{r,d} \int L_i(\omega_i) \cos \theta_i \, d\omega_i$$

$$= f_{r,d} E$$

$$f_{r,d} = c$$

$$M = \int L_r(\omega_r) \cos \theta_r \, d\omega_r = L_r \int \cos \theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}$$

**Lambert’s Cosine Law**

$$M = \rho_d E = \rho_d E_s \cos \theta_s$$
“Diffuse” Reflection

Theoretical

- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

Experimental

- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

Paint manufactures attempt to create ideal diffuse