

Ray Tracing

- Viewpoint, Image plane, etc.
 - viewpoint, pixels define rays
- Forward vs. Backward tracing
 - the path is the same either way
- Ray Casting?
- Recursive Ray Tracing

Recursive ray tracing

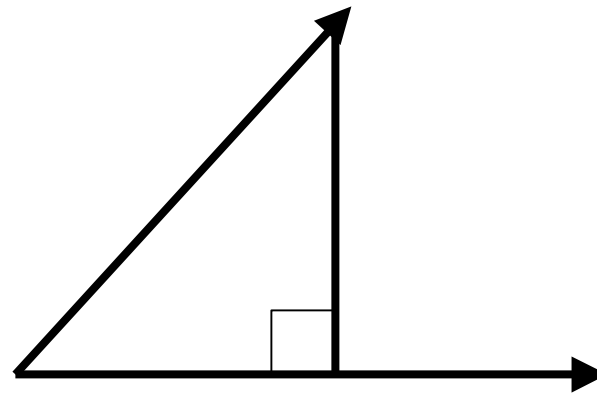
- Intersection testing
- Ray propagation
 - how far?
- Shading
 - texture coordinates

Intersection Testing

- Spheres
- Ray-Sphere interactions
 - Miss, Graze, Enter/Exit
- Implicit/Algebraic calculation
- Vector/Geometric calculation

Vector Operations

- Dot product
 - scalar product of vectors
 - $\mathbf{A} \cdot \mathbf{B} = A_x * B_x + A_y * B_y + A_z * B_z$
 - $|\mathbf{A}| |\mathbf{B}| \cos \theta$
 - Geometric meaning
 - $<0?$, $>0?$, $=0?$



Vector operations

- Magnitude

- $|\mathbf{A}| = \sqrt{A_x * A_x + A_y * A_y + A_z * A_z}$

- Normalization

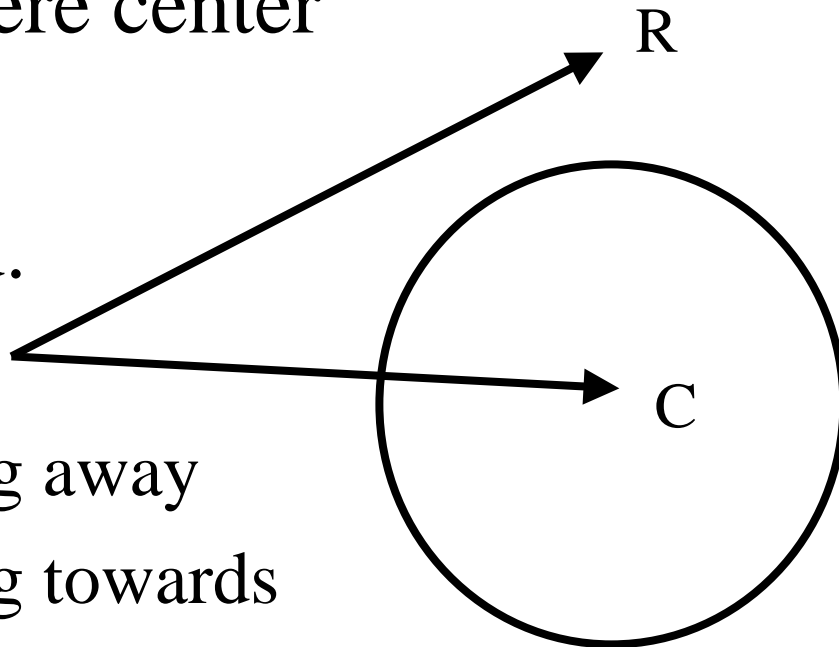
- $\text{Normal}(\mathbf{A}) = \mathbf{A} / |\mathbf{A}|$

Intersecting a Sphere

- Is the ray headed toward the sphere?
- Where is the ray closest to the center?
- Find the intersections

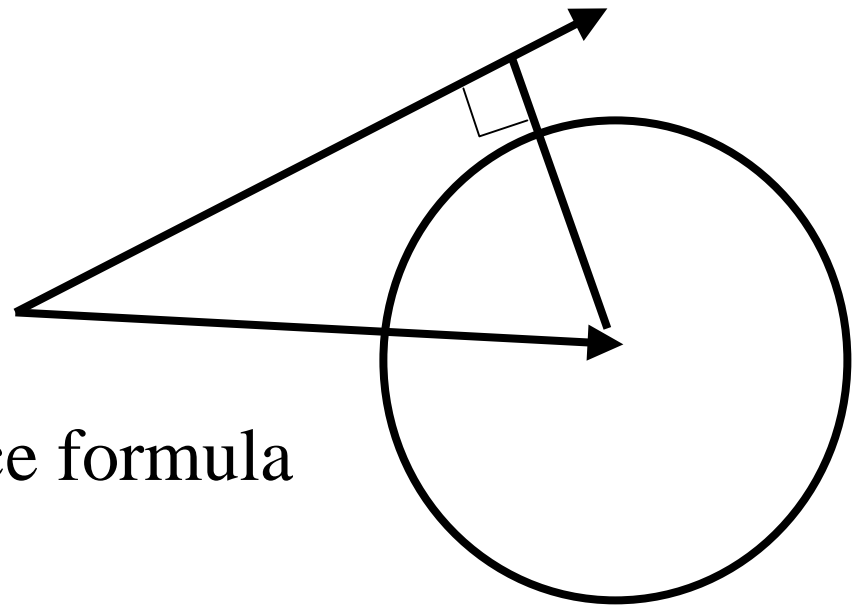
Headed toward the Sphere?

- Vector to sphere center
- Ray
- Take dot prod.
 - $\langle \cdot, \cdot \rangle = 0$?
 - If < 0 looking away
 - If > 0 looking towards
 - If $= 0$ looking at right angles



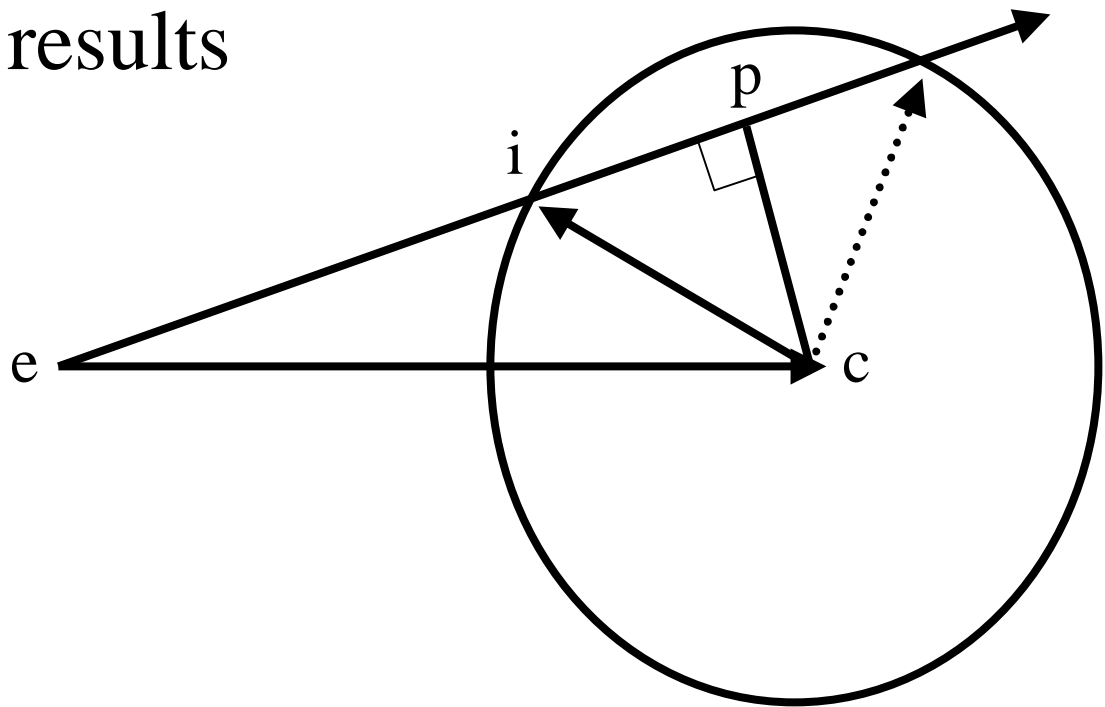
Closest point?

- Dot product finds closest pt.
 - Distance along ray
- Distance to Ctr.
 - $<$ radius? - hit
 - $>$ radius? - miss
 - Use Euclidean distance formula

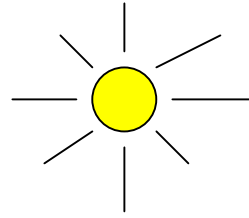


The Intersections

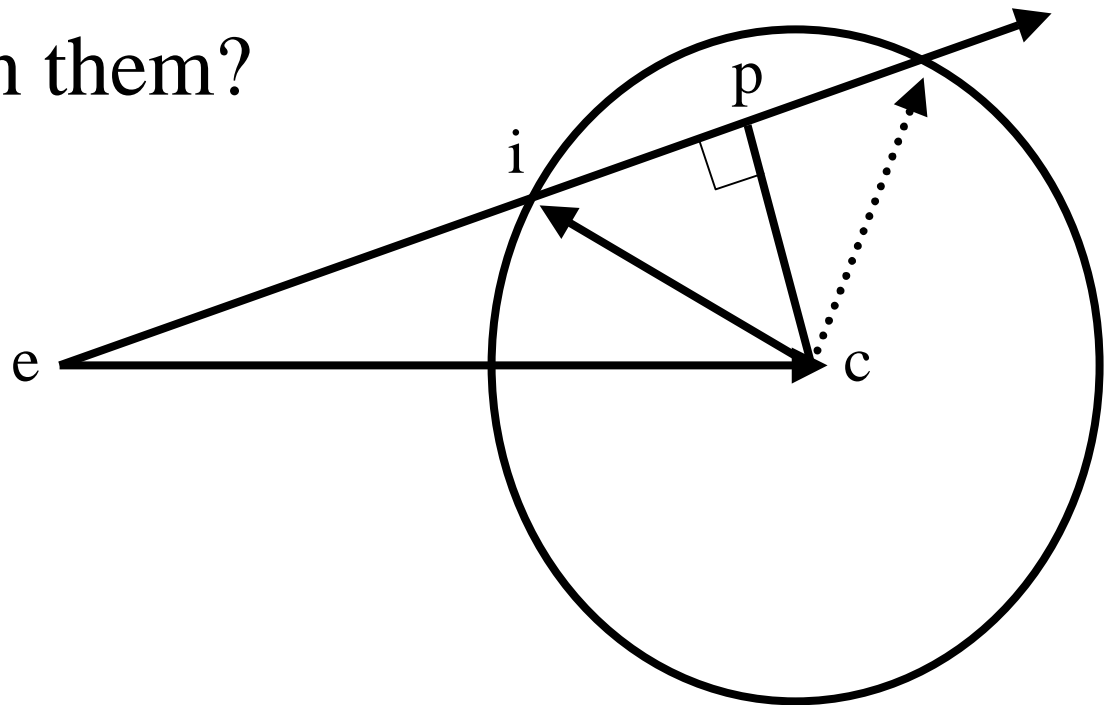
- Right triangles use Pythagorus
- Re-use partial results
 - for efficiency



Shading



- Normal vector?
- Light direction?
- Angle between them?
- Highlights?
- Reflections?



Intersecting a Polygon

- Finding the plane
- Ray intersects plane
- Point of intersection inside polygon?

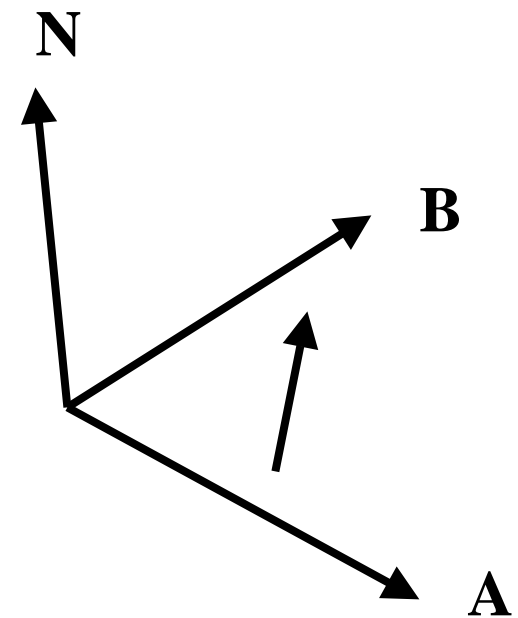
Finding the plane

- Use cross product
 - 2 edges of polygon define plane
 - assuming not collinear

Cross (Vector) Product

- $\mathbf{A} \times \mathbf{B} = [A_y * B_z - A_z * B_y, A_z * B_x - A_x * B_z, A_x * B_y - A_y * B_x]$

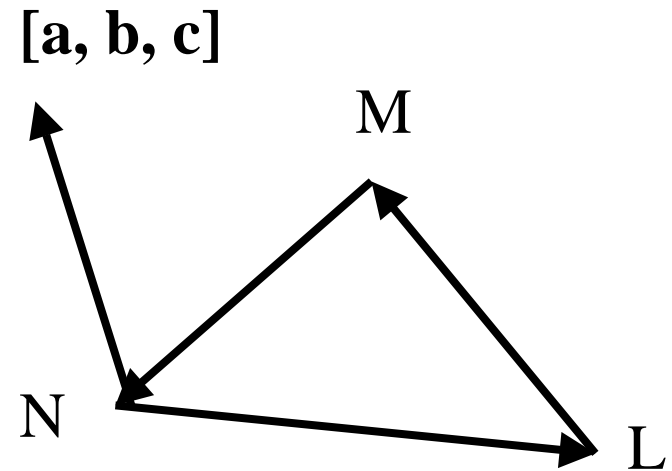
$$\begin{bmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$



Magnitude is $|\mathbf{A}||\mathbf{B}| \sin \theta$

2 Edges define a plane

- Cross product from 2 edges
 - gives normal vector $[a, b, c]$
- Normalize (for comfort)
- Apply known point
 - L, M, or N
 - Plug in $ax + by + cz + d = 0$
 - i.e. $a*L_x + b*L_y + c*L_z = -d$

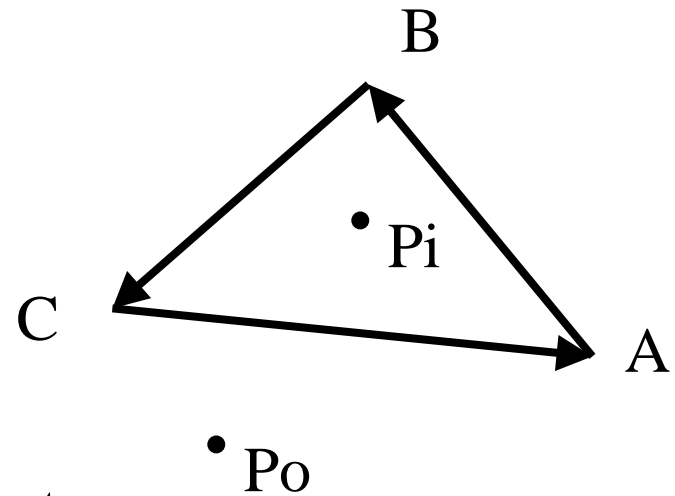


Ray intersects plane

- $ax + by + cz + d = 0$
- $r = [p_x + d_x * t, p_y + d_y * t, p_z + d_z * t]$
- Substitute and solve for t
 - $ar_x + br_y + cr_z + d = 0$
- Evaluate ray at t

Inside Polygon?

- Cross-product test
 - vector to leading vertex on edge
 - crossed with edge vector
 - same result always?
- Looking out from P
 - Do edges all run to left?
 - Outside if edges inconsistent



Reduce to 2 dimensions

- use major direction
- find largest component of plane normal
- collapse onto coordinate plane most orthogonal to normal