

In[2]:=  $\mathbf{p}_0 := \{x_0, y_0\}$

In[3]:=  $\mathbf{p}_1 := \{x_1, y_1\}$

In[4]:=  $\mathbf{p}_2 := \{x_2, y_2\}$

In[11]:=  $\mathbf{a} := \mathbf{p}_1 - \mathbf{p}_0$

In[6]:=  $\mathbf{b} := \mathbf{p}_2 - \mathbf{p}_1$

Out[18]=  $-\frac{\text{Normalize}[a] \cdot \text{Normalize}[b]}{2}$

In[30]:=  $\mathbf{aHat} = \mathbf{a} / \sqrt{\mathbf{a} \cdot \mathbf{a}}$

Out[30]=  $\left\{ \frac{-x_0 + x_1}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2}}, \frac{-y_0 + y_1}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2}} \right\}$

In[31]:=  $\mathbf{bHat} = \mathbf{b} / \sqrt{\mathbf{b} \cdot \mathbf{b}}$

Out[31]=  $\left\{ \frac{-x_1 + x_2}{\sqrt{(-x_1 + x_2)^2 + (-y_1 + y_2)^2}}, \frac{-y_1 + y_2}{\sqrt{(-x_1 + x_2)^2 + (-y_1 + y_2)^2}} \right\}$

In[33]:=  $\text{Energy} = -\frac{k}{2} (\mathbf{aHat} \cdot \mathbf{bHat})$

Out[33]=  $-\frac{1}{2} k \left( \frac{(-x_0 + x_1)(-x_1 + x_2)}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2} \sqrt{(-x_1 + x_2)^2 + (-y_1 + y_2)^2}} + \frac{(-y_0 + y_1)(-y_1 + y_2)}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2} \sqrt{(-x_1 + x_2)^2 + (-y_1 + y_2)^2}} \right)$

In[48]:=  $\mathbf{f}_0 = \text{FullSimplify}[D[\text{Energy}, \{\mathbf{p}_0\}]]$

Out[48]=  $\left\{ \frac{k(y_0 - y_1)(x_2(y_0 - y_1) + x_0(y_1 - y_2) + x_1(-y_0 + y_2))}{2((x_0 - x_1)^2 + (y_0 - y_1)^2)^{3/2} \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}, \right.$   
 $\left. - \frac{k(x_0 - x_1)(x_2(y_0 - y_1) + x_0(y_1 - y_2) + x_1(-y_0 + y_2))}{2((x_0 - x_1)^2 + (y_0 - y_1)^2)^{3/2} \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} \right\}$

In[50]:=  $\mathbf{f}_1 = \text{FullSimplify}[D[\text{Energy}, \{\mathbf{p}_1\}]]$

$$\text{Out[50]} = \left\{ \left( k (x_2 (-y_0 + y_1) + x_1 (y_0 - y_2) + x_0 (-y_1 + y_2)) (x_2^2 (y_0 - y_1) + x_1^2 (y_0 - y_2) + (x_0^2 + (y_0 - y_1)(y_0 - y_2)) (y_1 - y_2) + 2 x_1 (x_2 (-y_0 + y_1) + x_0 (-y_1 + y_2))) \right) / \right. \\ \left. \left( 2 \left( (x_0 - x_1)^2 + (y_0 - y_1)^2 \right)^{3/2} \left( (x_1 - x_2)^2 + (y_1 - y_2)^2 \right)^{3/2} \right), \right. \\ \left. \left( k (x_2 (y_0 - y_1) + x_0 (y_1 - y_2) + x_1 (-y_0 + y_2)) (x_0^2 (x_1 - x_2) + x_1^2 x_2 - x_2 (y_0 - y_1)^2 + x_0 (-x_1^2 + x_2^2 + (y_1 - y_2)^2) - x_1 (x_2^2 - (y_0 - y_2)(y_0 - 2 y_1 + y_2))) \right) / \right. \\ \left. \left( 2 \left( (x_0 - x_1)^2 + (y_0 - y_1)^2 \right)^{3/2} \left( (x_1 - x_2)^2 + (y_1 - y_2)^2 \right)^{3/2} \right) \right\}$$

In[49]:=  $\mathbf{f}_2 = \text{FullSimplify}[D[\text{Energy}, \{\mathbf{p}_2\}]]$

$$\text{Out[49]} = \left\{ \frac{k (y_1 - y_2) (x_2 (y_0 - y_1) + x_0 (y_1 - y_2) + x_1 (-y_0 + y_2))}{2 \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2} \left( (x_1 - x_2)^2 + (y_1 - y_2)^2 \right)^{3/2}}, \right. \\ \left. \frac{k (x_1 - x_2) (x_2 (-y_0 + y_1) + x_1 (y_0 - y_2) + x_0 (-y_1 + y_2))}{2 \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2} \left( (x_1 - x_2)^2 + (y_1 - y_2)^2 \right)^{3/2}} \right\}$$

In[51]:=  $\mathbf{f1Sum} = -\mathbf{f}_0 - \mathbf{f}_2$

$$\text{Out[51]} = \left\{ -\frac{k (y_0 - y_1) (x_2 (y_0 - y_1) + x_0 (y_1 - y_2) + x_1 (-y_0 + y_2))}{2 \left( (x_0 - x_1)^2 + (y_0 - y_1)^2 \right)^{3/2} \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} - \right. \\ \frac{k (y_1 - y_2) (x_2 (y_0 - y_1) + x_0 (y_1 - y_2) + x_1 (-y_0 + y_2))}{2 \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2} \left( (x_1 - x_2)^2 + (y_1 - y_2)^2 \right)^{3/2}}, \\ \left. \frac{k (x_0 - x_1) (x_2 (y_0 - y_1) + x_0 (y_1 - y_2) + x_1 (-y_0 + y_2))}{2 \left( (x_0 - x_1)^2 + (y_0 - y_1)^2 \right)^{3/2} \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} - \right. \\ \left. \frac{k (x_1 - x_2) (x_2 (-y_0 + y_1) + x_1 (y_0 - y_2) + x_0 (-y_1 + y_2))}{2 \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2} \left( (x_1 - x_2)^2 + (y_1 - y_2)^2 \right)^{3/2}} \right\}$$

In[63]:=  $\mathbf{D}[\mathbf{aHat}, \mathbf{x}_0]$

$$\text{Out[63]} = \left\{ \frac{(-x_0 + x_1)^2}{\left( (-x_0 + x_1)^2 + (-y_0 + y_1)^2 \right)^{3/2}} - \frac{1}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2}}, \frac{(-x_0 + x_1) (-y_0 + y_1)}{\left( (-x_0 + x_1)^2 + (-y_0 + y_1)^2 \right)^{3/2}} \right\}$$