CS 348C: COMPUTER GRAPHICS: ANIMATION AND SIMULATION, PROF. JAMES, TUES OCT 4, 2016

Particle systems with holonomic constraints

Due: *Thurs Oct 13, 2016 (at start of class)*

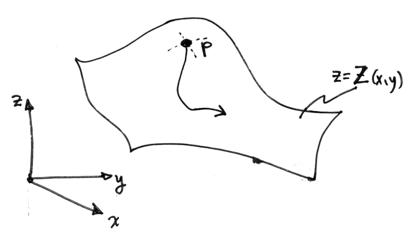
The following problems consider a 3D particle system (of n particles of mass m) subject to gravitational acceleration $\boldsymbol{g}$, and specified holonomic constraints of the form $\mathbf{C}(\mathbf{p}) = \mathbf{0}$. In each case, derive specific mathematical expressions for the following:

1. the constraint(s), $\mathbf{C}$, and their total number, N_c;
2. the constraint Jacobian, $\mathbf{J}$, its derivative $\dot{\mathbf{J}}$, and the matrix dimensions; and
3. an analytical expression for the constraint force $\mathbf{f}_c$ expressed as the product of three quantities:

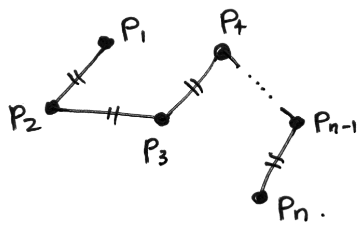
$$\mathbf{f}_c = \mathbf{J}^\top \underbrace{\left(\mathbf{J}\mathbf{W}\mathbf{J}^\top\right)^{-1} \left(-\dot{\mathbf{J}}\mathbf{v} - \mathbf{J}\mathbf{W}\mathbf{f}\right)}_{\lambda}$$

   where $\mathbf{W} = \mathbf{M}^{-1}$, and $\mathbf{f}$ is the external force due to gravity. Be as explicit as possible when deriving the two quantities in () brackets, but do not multiply the full expression, or calculate the matrix inverse.

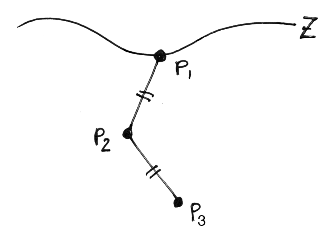
Note: there are different values depending on how you formulate your constraints, but the same physical $\mathbf{f}_c$.



Problem 1



Problem 2



Problem 3

PROBLEM 1. Particle on a height field: Consider a particle with position $\mathbf{p} = (x, y, z)^\top$, and a surface specified by a heightfield function, $z = Z(x, y)$. The particle is constrained to be attached to the heightfield surface at all times.

PROBLEM 2. Inextensible n-particle chain: Consider n particles $\mathbf{p}_1$, $\mathbf{p}_2$, ..., $\mathbf{p}_n$ with massless rods of length ℓ attached between each consecutive particle, i.e., the $n - 1$ line segments $\overline{\mathbf{p}_1\mathbf{p}_2}$, ..., $\overline{\mathbf{p}_{n-1}\mathbf{p}_n}$. (Assume that $n \geq 2$.)

PROBLEM 3. Double pendulum on a height field: In this problem, you will combine the constraints from the previous two problems for the case of $n = 3$ particles with positions $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ and the following constraints:

- particle $\mathbf{p}_1$ is constrained to the height field from question 1, and
- there are rigid link constraints (of length ℓ) on the line segments $\overline{\mathbf{p}_1\mathbf{p}_2}$ and $\overline{\mathbf{p}_2\mathbf{p}_3}$.