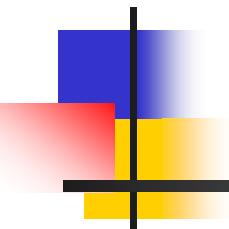
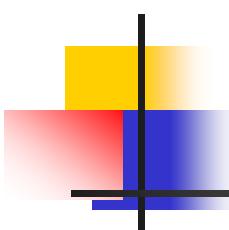


# Sensor Placement for Isotropic Source Localization



Tamir Hegazy and George  
Vachtsevanos

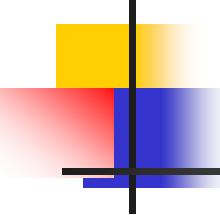
Georgia Institute of Technology



# Motivation

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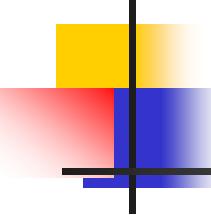
- Source localization
  - Surveillance
  - object tracking
  - fault detection
- Sensor placement for point source localization
- Minimize the localization error



# Goals

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- What is a good placement strategy for sensors?
  - Minimize localization error
- Minimize the number of sensors needed



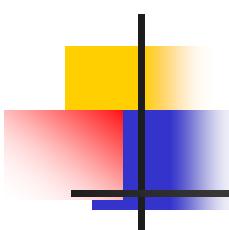
# Assumptions

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- $p_s$  current point source position, unknown
- $k$  intensity of source, known
- $N$  sensors located at known  $p_i$ 's
- Observation model

$$\hat{u}_i = u_i + e_i = k / \| p_i - p_s \|^2 + e_i$$

- $e_i$  is bounded,  $|e_i| \leq \varepsilon$
- Ignore propagation delay



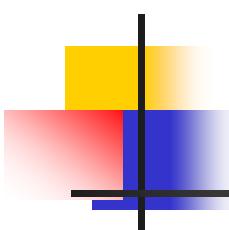
# Goals

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- Minimize N
- Minimize localization error

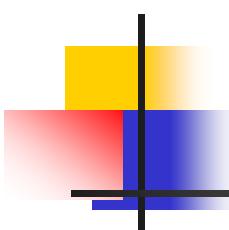
$$\|\partial p_s\| = \|\hat{p}_s - p_s\|$$

- $\min_{p_i} [\|\partial p_s\| / \|p_s\|, N]$



# Simple case (no measurement error)

- $e_i = 0, \forall i \in \{1, \dots, N\}$
- $\|p_i - p_s\|^2 = k/u_i$
- $\|p_i\|^2 + \|p_s\|^2 - 2p_i \cdot p_s = k/u_i$
- $-2(p_i - p_j) \cdot p_s = k \left( \frac{1}{u_i} - \frac{1}{u_j} \right) - (\|p_i\|^2 - \|p_j\|^2)$
- linear equation!!



# Linear Equation

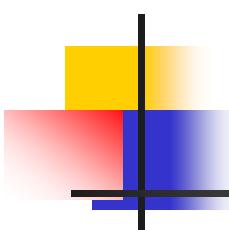
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- Place  $j^{\text{th}}$  sensor at origin,  $p_j = 0$
- Simplified equation,  $-2 p_i \cdot p_s = k \left( \frac{1}{u_i} - \frac{1}{u_j} \right) - \|p_i\|^2$
- Linear equation,  $A p_s = b$
- Need atleast 4 sensors?

# Sensor Placement

- Assume  $|\varepsilon/u_i| \ll 1$
- $A(p_s + \partial p_s) = b + \partial b$
- Estimation error bounded as,

$$\|\partial b\| \leq \varepsilon \left\| \begin{pmatrix} \frac{1}{u_1^2} + \frac{1}{u_0^2} \\ \frac{1}{u_2^2} + \frac{1}{u_0^2} \\ \frac{1}{u_3^2} + \frac{1}{u_0^2} \end{pmatrix} \right\|$$

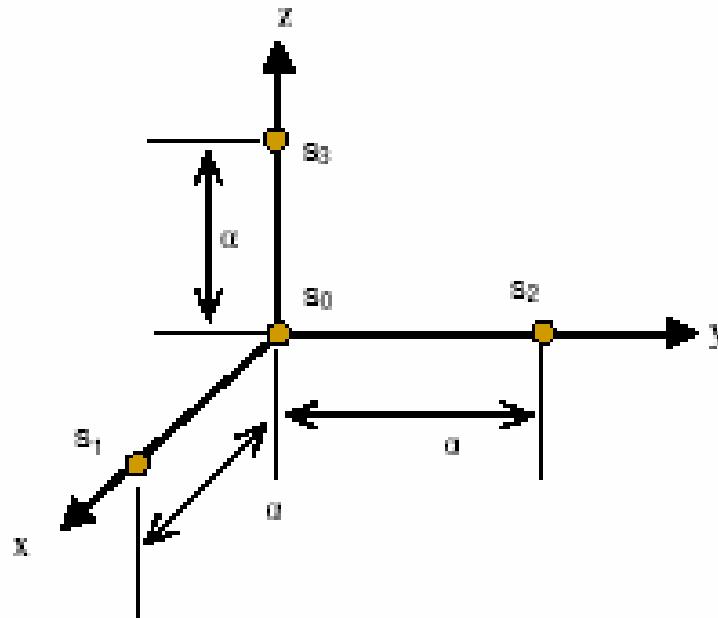


# Sensor Placement(cont.)

- $\frac{\|\partial p_s\|}{\|p_s\|} \leq \kappa(A) \frac{\|\partial b\|}{\|b\|}$
- $\kappa(A)$  ratio of largest to smallest sing. Values
- To minimize  $\kappa(A)$ ,
  - choose  $A = U\Sigma V^T = I(\alpha I)I^T = \alpha I$

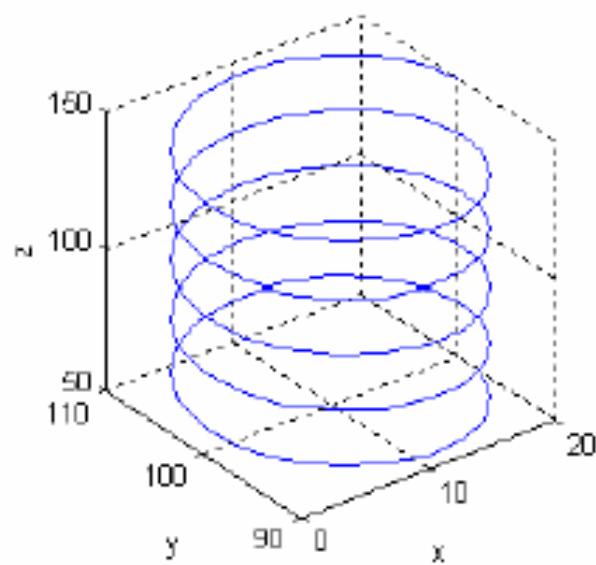
# BCAP

- $A = 2\alpha I$
- Best Conditioned Aligned Pyramid (BCAP)



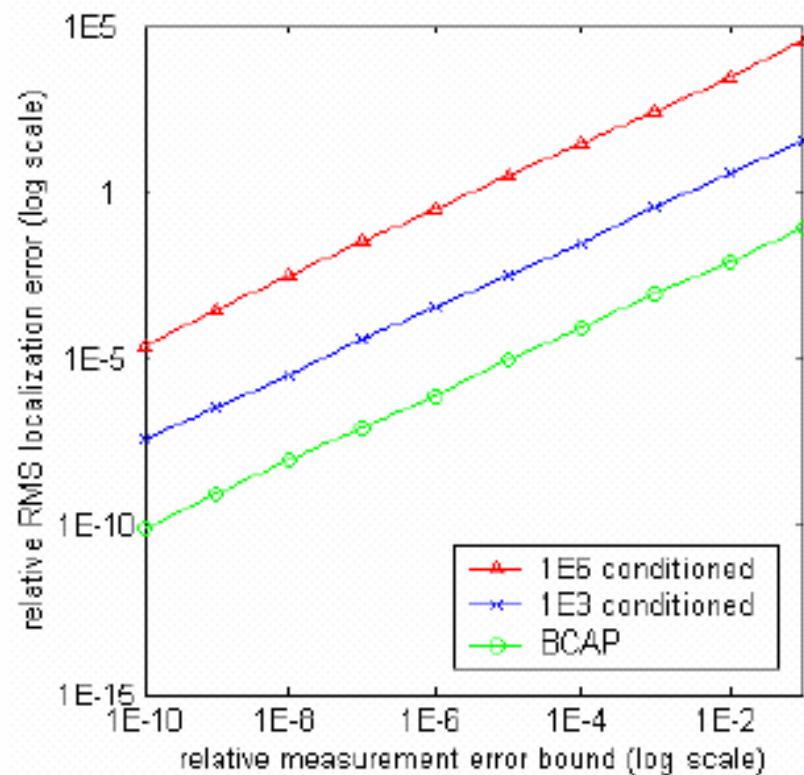
# Experiments

- Setup: slow moving source

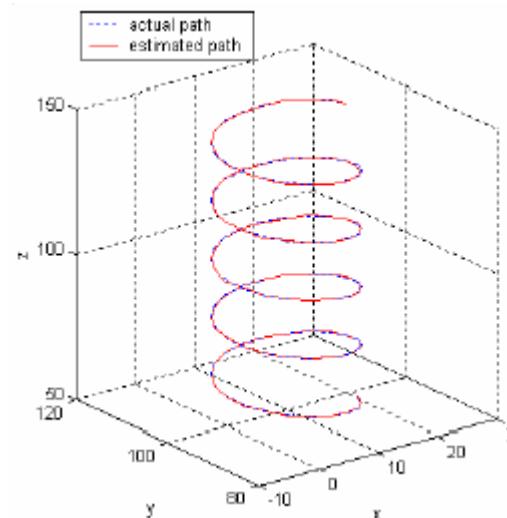
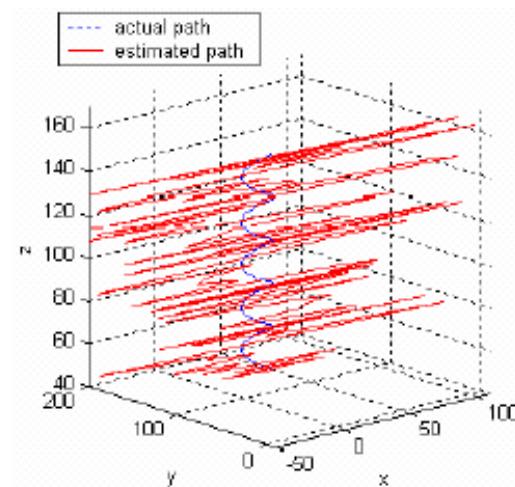
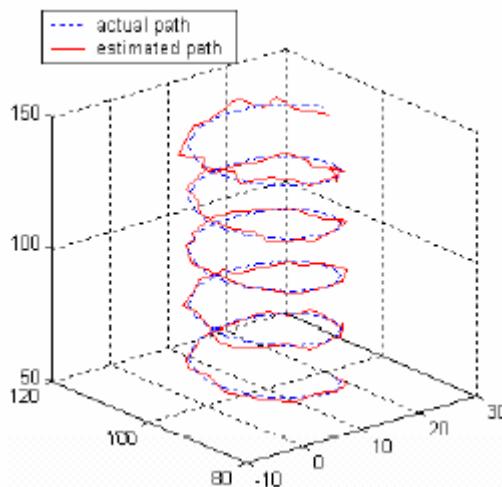


# Experiment 1: vary A

- $A=100I$
- $\kappa(A)=1,000$
- $\kappa(A)=1,000,000$



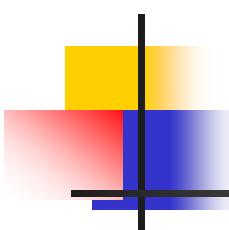
# Experiment 2: vary $\varepsilon$



BACP, 1%

$\kappa(A)=10^3$ , 0.1%

BACP, 0.1%



# Conclusions

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- Solutions in this paper
  - Source localization in linear framework
  - BCAP
- Problems remaining
  - Multiple sources
  - Unknown source signals
  - Larger error
  - Time delay