Robust Distributed Network Localization with Noisy Range Measurements

David Moore, John Leonard, Daniela Rus, Seth Teller
(MIT Computer Science and Artificial Intelligence Laboratory)

Presented by Emilio Antúnez

Motivation

- Many algorithms require sensor locations.
- Absolute positions may be unknown.
- We may only have distances between nodes.
- We want to recover *relative* 2D positions.

Problems / Pitfalls

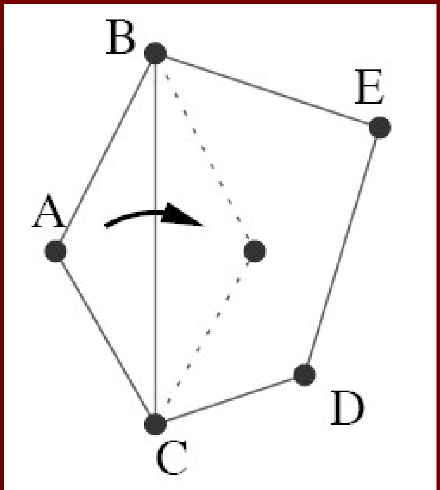
- Insufficient graph connectivity
- Noisy distance measurements
- Discontinuous errors
  - Flip ambiguities
  - Flex ambiguities

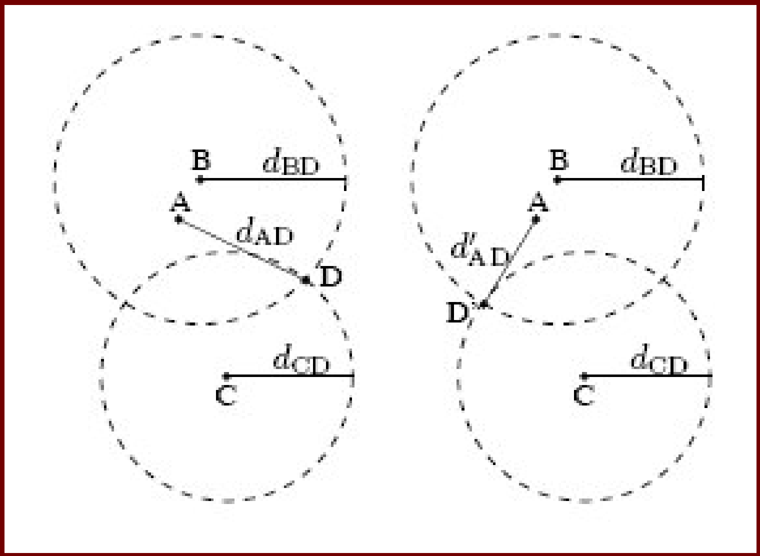
Goal

- Recover the relative 2D Euclidean position of a node.
- Resist noise in distance measurements.
- Assign a node location only if we’re confident in its accuracy.

Flip Ambiguities

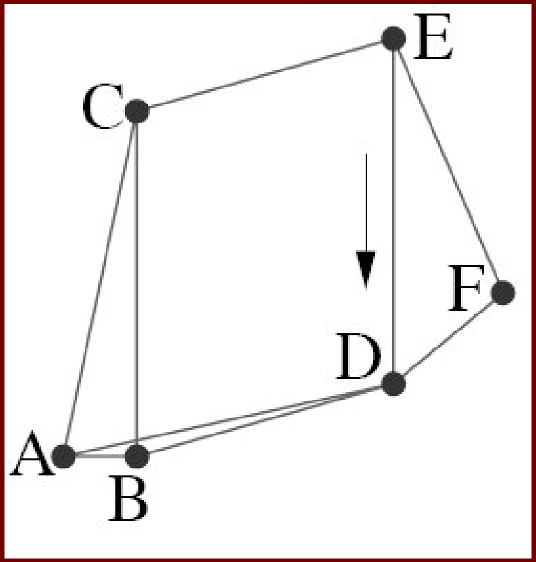
- Node with only two distance constraints may appear on either side of an edge.
- A third (noisy) measurement may not resolve this ambiguity correctly.

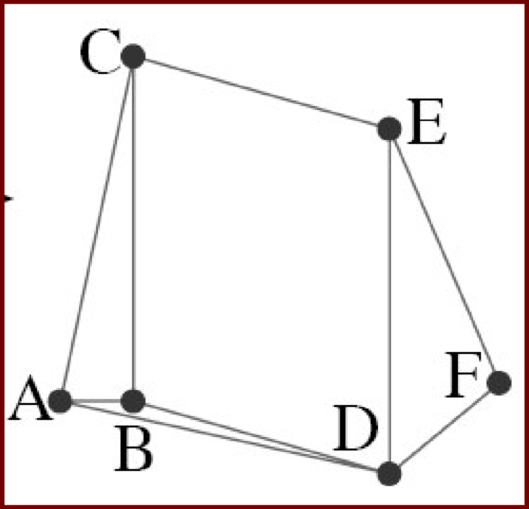




Discontinuous Flex Ambiguities

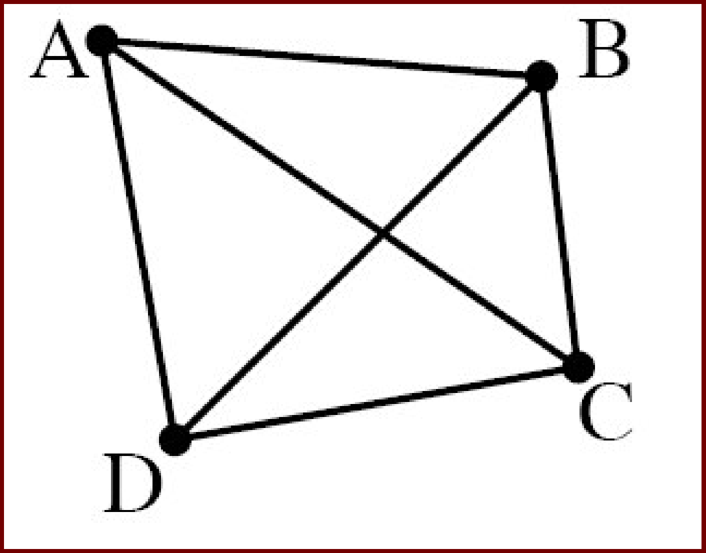
- Removal of a single edge makes graph nonrigid
- Graph can be flexed to a different configuration, and edge reinserted at same length





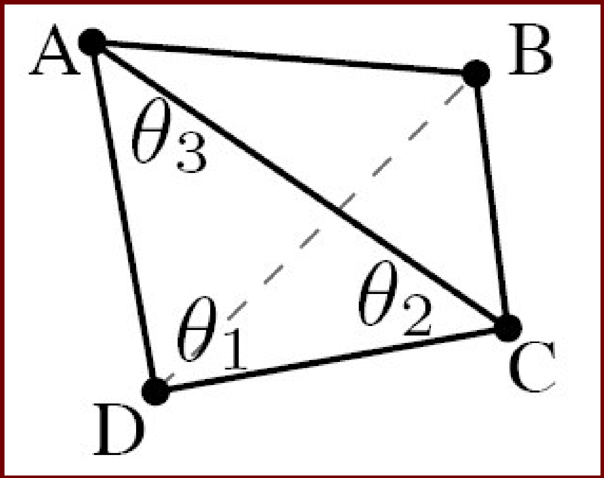
Quadrilaterals

- Estimate positions using quads instead of triangulation
- Fully-connected quad stays rigid even if one edge is removed
- No flex ambiguities
- Still susceptible to flip ambiguities



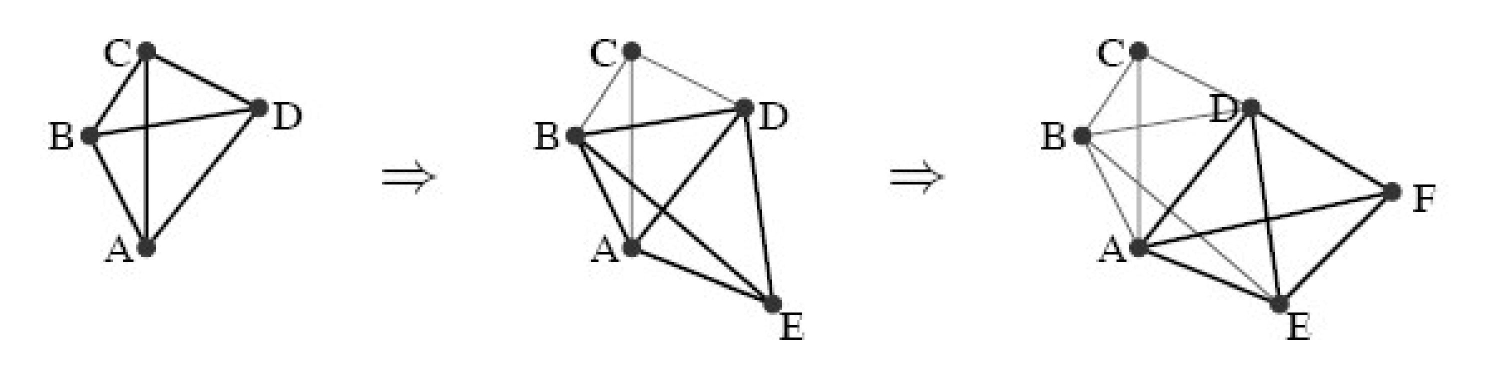
Robust Quadrilaterals

- d_{min} = largest manageable error
- b = triangle's shortest side length
- θ = smallest angle in a triangle
- Triangle is "robust" if $b \cdot \sin^2\theta > d_{min}$
- Quad is robust if all four triangles defined by its edges are robust.



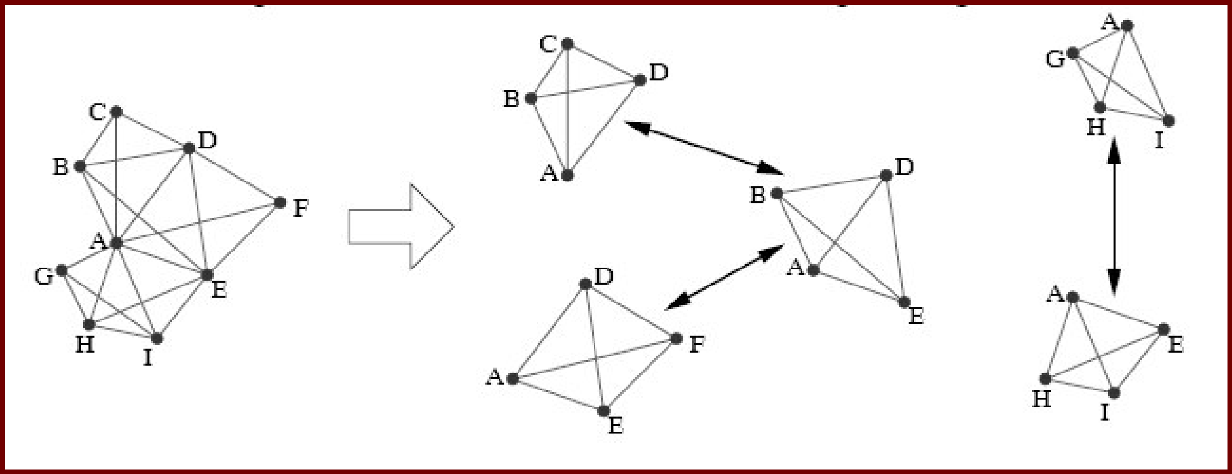
Clusters

- Quads that share three nodes can be merged.
- Many quads can be merged into a “cluster”.



Phase I. Cluster Localization

- Each node:
  - Finds all robust quadrilaterals which include it
  - Finds the largest disconnected subgraph
  - Computes distances for all nodes in that subgraph



Phase II. Cluster Optimization (Optional)

- Locations found in phase I are based on trilateration (three edges).
- Positions can be refined by optimizing the fit over all distance constraints.

Phase III. Cluster Transformation

- Coordinate systems of adjacent clusters may disagree by rotation / translation / reflection.
- Adjacent clusters merged using quaternion-based global registration scheme.
- Registration is possible if adjacent clusters have at least three nodes in common.
- If three nodes form a robust triangle, same resistance to ambiguities is guaranteed.

Computational Complexity

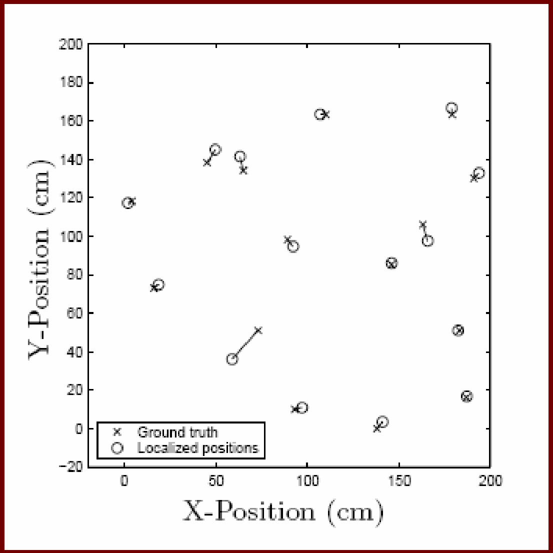
- If maximum node degree, m, varies:
  - Algorithm runs in worst-case $O(m^3)$ time.
  - Work per node is $O(m^3)$.
  - Communication cost across network is $O(m^2)$.
- Performance is generally much better than worst-case, since network is not fully connected.
- For constant number of neighbors, running time is linear, and work per node is constant.

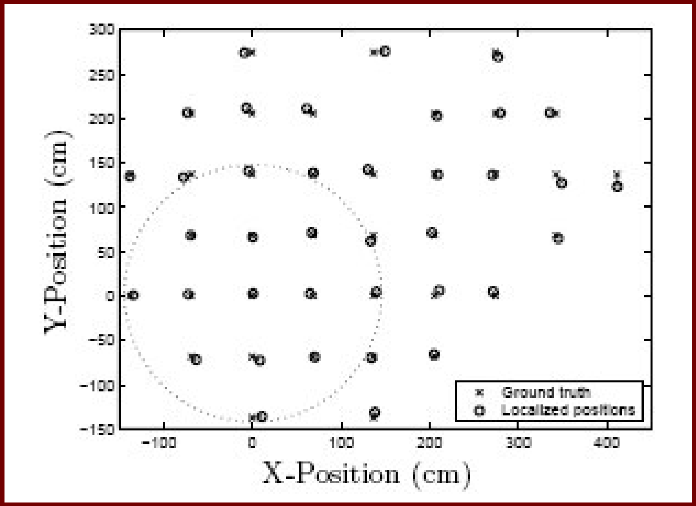
Experimental Results

- Algorithm tested on small hardware network
- Larger network simulated in software
- Evaluated based on:
  - Measurement error (MSE), σ_d
  - Location error (MSE), σ_p
  - Average localization rate per cluster, R_bar
  - Fraction of nodes localized, R_tilde

Hardware Tests

- Conducted using MIT's "Crickets", which have ultrasonic ranging devices (error ~5cm)

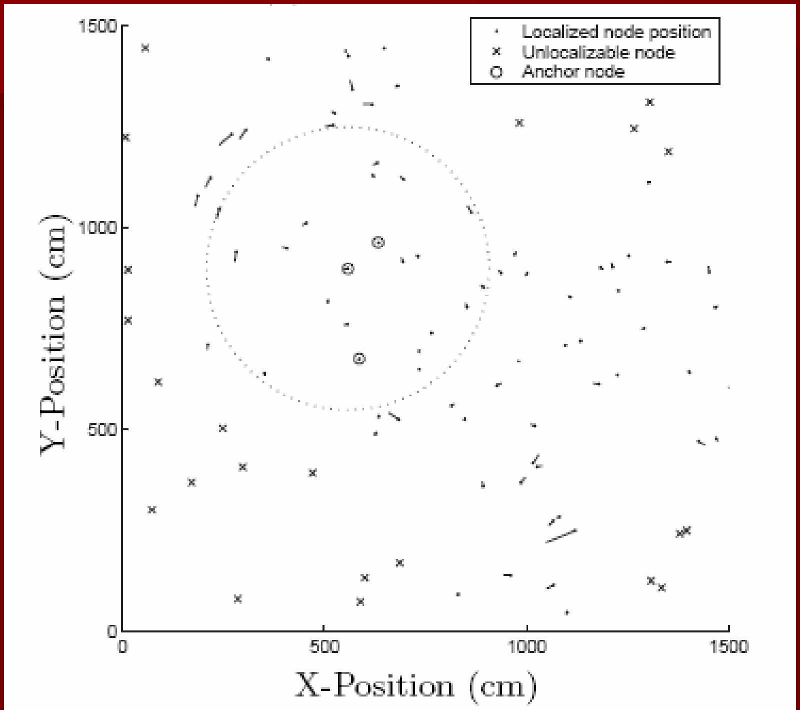


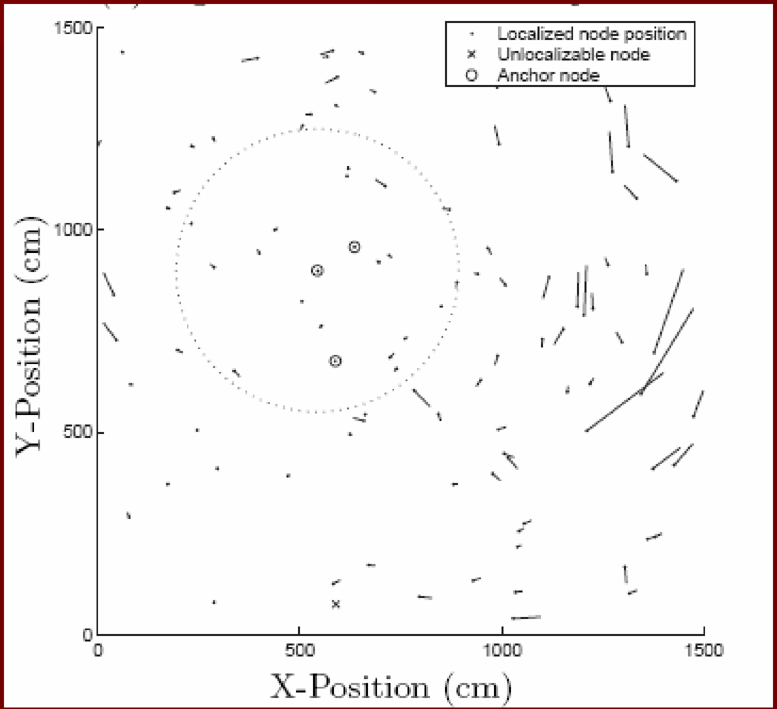


metric	value
σ_d	5.18 cm
σ_p	7.02 cm
$\bar{R}$	$15/16 = 0.94$
$\tilde{R}$	$15/16 = 0.94$

metric	value
σ_d	4.38 cm
σ_p	6.82 cm
$\bar{R}$	0.97
$\tilde{R}$	$38/40 = 0.95$

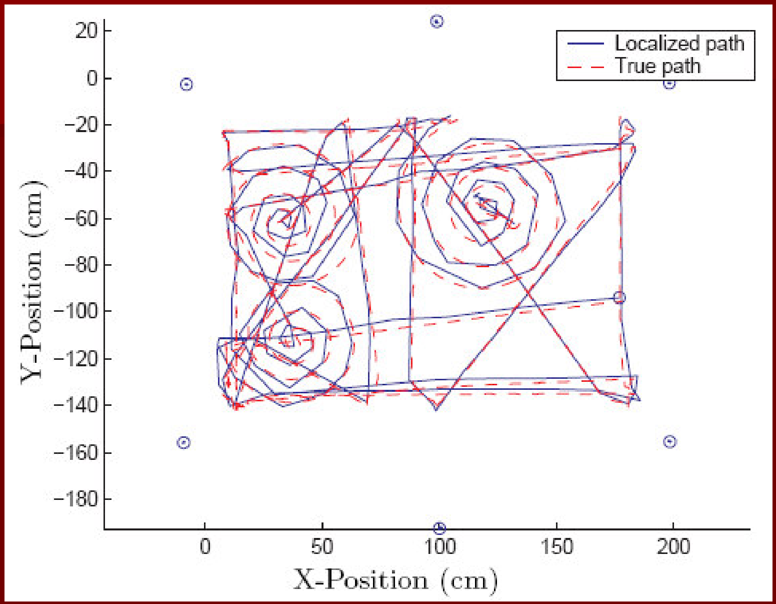
Simulated Tests

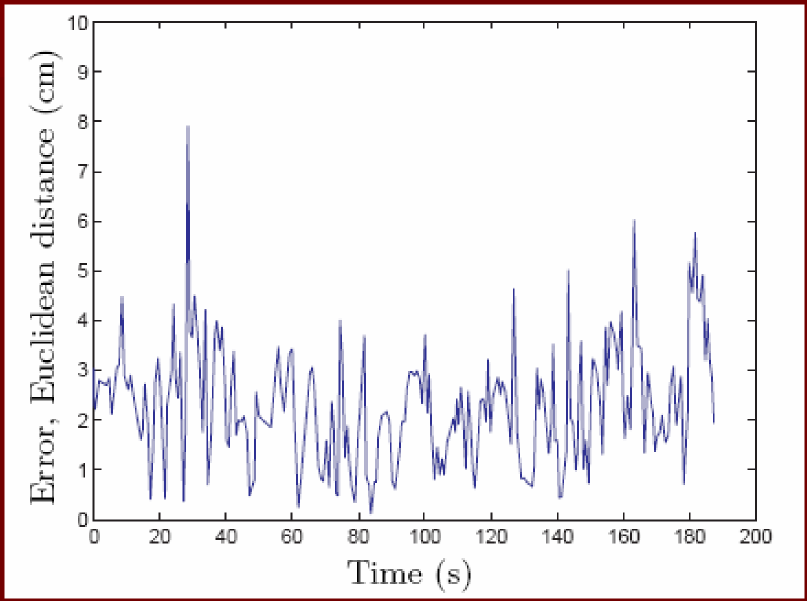


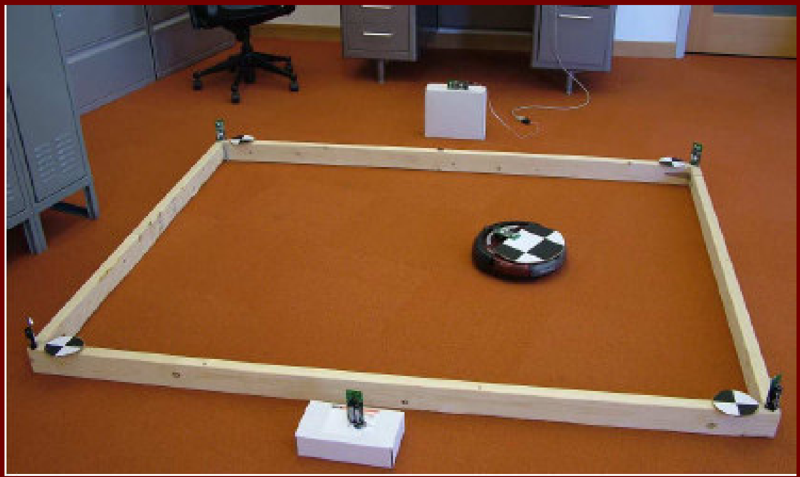


metric	Our algorithm			w/o robust quads
σ_d	**1.0 cm**	**3.0 cm**	**5.0 cm**	**5.0 cm**
σ_p	4.43 cm	14.39 cm	16.22 cm	54.87 cm
$\bar{R}$	0.91	0.85	0.79	0.95
$\tilde{R}$	0.93	0.87	0.75	0.99
Shown in:			Figure 12a	Figure 12b

Mobile Node Test







Conclusion

- Using robust quadrilaterals, can localize most nodes in a well-connected graph.
- Localization error is roughly equal to the inter-node distance measurement error.
- Tunable parameter can trade off accuracy for connectivity.
- Reasonable (linear) time required for constant number of neighbors.