Photographic optics

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Outline

- pinhole cameras
- thin lenses
  - graphical constructions, algebraic formulae
- lenses in cameras
  - focal length, sensor size
- thick lenses
  - stops, pupils, perspective transformations
- exposure
  - aperture, shutter speed (ISO comes later)
- depth of field
- aberrations...
Cutaway view of a real lens

Vivitar Series 1 90mm f/2.5
Cover photo, Kingslake, *Optics in Photography*
Lens quality varies

- Why is this toy so expensive?
  - EF 70-200mm f/2.8L IS USM
  - $1700

- Why is it better than this toy?
  - EF 70-300mm f/4-5.6 IS USM
  - $550

- Why is it so complicated?
Stanford Big Dish
Panasonic GF1

Panasonic 45-200/4-5.6 zoom, at 200mm f/4.6
$300

Leica 90mm/2.8 Elmarit-M prime, at f/4
$2000
Zoom lens versus prime lens

Canon 100-400mm/4.5-5.6 zoom, at 300mm and f/5.6
$1600

Canon 300mm/2.8 prime, at f/5.6
$4300
Why not use sensors without optics?

- each point on sensor would record the integral of light arriving from every point on subject
- all sensor points would record similar colors
Pinhole camera
(a.k.a. camera obscura)
Pinhole photography

- no distortion
  - straight lines remain straight
- infinite depth of field
  - everything is in focus
Large pinhole causes geometric blur

Photograph made with small pinhole

Photograph made with larger pinhole
Small pinhole causes diffraction blur

- smaller aperture means more diffraction
- due to wave nature of light

(Hecht)
Examples

- large pinhole → geometric blur
- small pinhole → diffraction blur
- optimal pinhole → very little light

(Hecht)
Replacing the pinhole with a lens

Photograph made with small pinhole

Photograph made with lens

(London)
Physical versus geometrical optics

- light can be modeled as traveling waves
- the perpendiculatrs to these waves can be drawn as rays
- diffraction causes these rays to bend, e.g. at a slit
- *geometrical optics* assumes
  - $\lambda \rightarrow 0$
  - no diffraction
  - in free space, rays are straight (a.k.a. rectilinear propagation)
Snell’s law of refraction

- as waves change speed at an interface, they also change direction

- index of refraction $n$ is defined as the ratio between the speed of light in a vacuum / speed in some medium

$$\frac{x_i}{x_t} = \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}$$

$n \sin i = n' \sin i'$
Typical refractive indices \((n)\)

- **air** = 1.0
- **water** = 1.33
- **glass** = 1.5 - 1.8

- when transiting from air to glass, light bends towards the normal
- when transiting from glass to air, light bends away from the normal
- light striking a surface perpendicularly does not bend

(Hecht)
Q. What shape should an interface be to make parallel rays converge to a point?

A. a hyperbola

✦ so lenses should be hyperbolic!
Spherical lenses

- two roughly fitting curved surfaces ground together will eventually become spherical
- spheres don’t bring parallel rays to a point
  - this is called spherical aberration
  - nearly axial rays (paraxial rays) behave best
Paraxial approximation

object $P$

e
'image $P'$

✧ assume $e \approx 0$
Paraxial approximation

- assume $e \approx 0$
- assume $\sin u = h/l \approx u$ (for $u$ in radians)
- assume $\cos u \approx z/l \approx 1$
- assume $\tan u \approx \sin u \approx u$

If we express $u$ in radians, we can say these expressions are approximately equal ($\approx$), rather than merely proportional ($\propto$) as I said in class. For example, $\sin(10^\circ) = 0.1736$ and $10^\circ$ in radians is $0.1745$. See how close these values are? In keeping with this cleaner explanation, I've changed all "$=\" and "\propto\" to "$\approx\" in this sequence of slides.
The paraxial approximation is a.k.a. first-order optics

✦ assume first term of $\sin \phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots$
  • i.e. $\sin \phi \approx \phi$

✦ assume first term of $\cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \ldots$
  • i.e. $\cos \phi \approx 1$
  • so $\tan \phi \approx \sin \phi \approx \phi$
Paraxial focusing

Snell’s law:

\[ n \sin i = n' \sin i' \]

Paraxial approximation:

\[ n i \approx n' i' \]
Given object distance $z$, what is image distance $z'$?

$n i \approx n' i'$
Paraxial focusing

\[ i = u + a \]
\[ a = u' + i' \]
\[ u \approx h / z \]
\[ a \approx h / r \]
\[ u' \approx h / z' \]

\[ n (u + a) \approx n' (a - u') \]
\[ n (h / z + h / r) \approx n' (h / r - h / z') \]
\[ n i \approx n' i' \]
\[ n / z + n / r \approx n' / r - n' / z' \]

\[ h \] has canceled out, so any ray from \( P \) will focus to \( P' \)
Focal length

What happens if $z$ is $\infty$?

- $n / z + n / r \approx n' / r - n' / z'$
- $n / r \approx n' / r - n' / z'$
- $z' \approx (r \, n') / (n' - n)$

- $f \triangleq \text{focal length} = z'$
Focusing of rays versus waves

rays from infinity ≡ plane waves

rays converging to a focus ≡ spherical waves
Lensmaker’s formula

- using similar derivations, one can extend these results to two spherical interfaces forming a lens in air

- as \( d \to 0 \) (thin lens approximation), we obtain the lensmaker’s formula

\[
\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]
Gaussian lens formula

- Starting from the lensmaker’s formula
  \[
  \frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right),
  \]  
  (Hecht, eqn 5.15)

- and recalling that as object distance \( s_o \) is moved to infinity, image distance \( s_i \) becomes focal length \( f_i \), we get
  \[
  \frac{1}{f_i} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right).
  \]  
  (Hecht, eqn 5.16)

- Equating these two, we get the Gaussian lens formula
  \[
  \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_i}.
  \]  
  (Hecht, eqn 5.17)
Gauss’ ray tracing construction

- assume that parallel rays converge to a point located at focal length $f$ from lens
  
  ![Gauss' ray tracing diagram]

- and rays going through center of lens are not deviated
  - hence same perspective as pinhole

![Gauss' ray tracing diagram]
Gauss’ ray tracing construction

- rays coming from points on a plane parallel to the lens are focused on another plane parallel to the lens
From Gauss’s ray construction to the Gaussian lens formula

- Positive $s_i$ is rightward, positive $s_o$ is leftward
- Positive $y$ is upward
From Gauss’s ray construction to the Gaussian lens formula

\[ \frac{|y_i|}{y_o} = \frac{s_i}{s_o} \]
From Gauss’s ray construction to the Gaussian lens formula

\[ \frac{|y_i|}{y_o} = \frac{s_i}{s_o} \quad \text{and} \quad \frac{|y_i|}{y_o} = \frac{s_i - f}{f} \]

\[ \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \]
Changing the focus distance

✧ to focus on objects at different distances, move sensor relative to lens

\[
\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}
\]
Changing the focus distance

- to focus on objects at different distances, move sensor relative to lens

- at $s_o = s_i = 2f$
  we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

In 1:1 imaging, if the sensor is 36mm wide, an object 36mm wide will fill the frame.
Changing the focus distance

- to focus on objects at different distances, move sensor relative to lens

- at \( s_o = s_i = 2f \) we have 1:1 imaging, because

\[
\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}
\]

- can’t focus on objects closer to lens than its focal length \( f \)

\[
\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}
\]
Changing the focal length

- weaker lenses have longer focal lengths

- to stay in focus, move the sensor further back

(Kingslake)
Changing the focal length

- weaker lenses have longer focal lengths
- to stay in focus, move the sensor further back
- if the sensor size is constant, the field of view becomes smaller

\[ FOV = 2 \arctan \left( \frac{h}{2f} \right) \]
Focal length and field of view

*FOV* measured diagonally on a 35mm full-frame camera (24 × 36mm)
Focal length and field of view

FOV measured diagonally on a 35mm full-frame camera (24 × 36mm)
Changing the sensor size

- if the sensor size is smaller, the field of view is smaller too

- smaller sensors either have fewer pixels, or noiser pixels

(Kingslake)
Full-frame 35mm versus APS-C

- Full-frame sensor is $24 \times 36\text{mm}$ (same as 35mm film)
- APS-C sensor is $14.8 \times 22.2\text{mm}$ (Canon DSLRs)
- Conversion factor is $1.6 \times$

- Switching camera bodies
  - Object occupies the same number of pixels, but takes up more of frame

- Switching lenses
  - Objects occupies fewer pixels, but composition stays the same
Sensor sizes

38x24mm (35mm format)

23.7x19.1mm (EOS 1D) = 1.20x magnification factor

APS-C sized sensors (EOS 1D, Nikon D100, Pentax *ist D, etc) = 1.5x - 1.6x

16x13.5mm (4/3" system - Olympus E-1)

8.8x6.6mm (2/3"

~Canon A590

~Nikon D40

~Panasonic GF1
Changing the focal length versus changing the viewpoint

- changing the focal length lets us move back from a subject, while maintaining its size on the image
- but moving back changes perspective relationships
Convex versus concave lenses

- **positive focal length** $f$ means parallel rays from the left converge to a point on the right
- **negative focal length** $f$ means parallel rays from the left converge to a point on the left (dashed lines above)
Convex versus concave lenses

rays from a convex lens converge

rays from a concave lens diverge

...producing a real image

...producing a virtual image

(Hecht)
Convex versus concave lenses

...producing a real image

...producing a virtual image

(Hecht)
The power of a lens

\[ P = \frac{1}{f} \]

- units are meters\(^{-1}\)
- a.k.a. diopters

- my eyeglasses have the prescription
  - right eye: -0.75 diopters
  - left eye: -1.00 diopters

Q. What’s wrong with me?
A. Myopia
Thick lenses

- an optical system may contain many lenses, but can be characterized by a few numbers

(Smith)
Stops

- in photographic lenses, the *aperture stop* (A.S.) is typically in the middle of the lens system.
- in a digital camera, the *field stop* (F.S.) is the edge of the sensor; no physical stop is needed.
Pupils

- the entrance pupil is the image of the aperture stop as seen from an axial point on the object
- the exit pupil is the image of the aperture stop as seen from an axial point on the image plane
- the center of the entrance pupil is the center of perspective
- you can find this point by following two lines of sight
Lenses perform a 3D perspective transform

- Lenses transform a 3D object to a 3D image; the sensor extracts a 2D slice from that image.
- As an object moves linearly (in Z), its image moves non-proportionately (in Z).
- As you move a lens linearly, the in-focus object plane moves non-proportionately.
- As you refocus a camera, the image changes size!

(Hecht) http://graphics.stanford.edu/courses/cs178/applets/thinlens.html
Exposure

- $H = E \times T$
- exposure = irradiance $\times$ time

- irradiance ($E$)
  - controlled by aperture

- exposure time ($T$)
  - controlled by shutter
Shutters

- quiet
- slow (max 1/500s)
- need one per lens

- loud
- fast (max 1/4000)
- distorts motion
Jacques-Henri Lartigue, Grand Prix (1912)
Shutter speed

- controls how long the sensor is exposed to light
- linear effect on exposure until sensor saturates

- denoted in fractions of a second:
  - 1/2000, 1/1000,...,1/250, 1/125, 1/60,...,15, 30, B(ulb)

- normal humans can hand-hold down to 1/60 second
  - *rule of thumb*: shortest exposure = 1 / *f*
  - e.g. 1/500 second for a 500mm lens
Main side-effect of shutter speed

- motion blur
- halving shutter speed doubles motion blur

(London)
Aperture

- irradiance on sensor is proportional to
  - square of aperture diameter $A$
  - inverse square of distance to sensor ($\sim$ focal length $f$)
- so that aperture values give irradiance regardless of lens, aperture number $N$ is defined relative to focal length

$$N = \frac{f}{A}$$

- $f/2.0$ on a 50mm lens means the aperture is 25mm
- $f/2.0$ on a 100mm lens means the aperture is 50mm
  - low F-number (N) on long zooms require fat lenses
- doubling $N$ reduces $A$ by $2\times$, hence light by $4\times$
  - going from $f/2.0$ to $f/4.0$ cuts light by $4\times$
  - to cut light by $2\times$, increase $N$ by $\sqrt{2}$
How low can N be?

- principal planes are the paraxial approximation of a spherical “equivalent refracting surface”

\[ N = \frac{1}{2 \sin \theta'} \]

- lowest possible N in air is f/0.5
- lowest N in SLR lenses is f/1.0

Canon EOS 50mm f/1.0 (discontinued)
Cinematography by candlelight

Stanley Kubrick,
Barry Lyndon,
1975

- Zeiss 50mm f/0.7 Planar lens
  - originally developed for NASA's Apollo missions
  - very shallow depth of field in closeups (small object distance)
Cinematography by candlelight

Zeiss 50mm f/0.7 Planar lens
- originally developed for NASA’s Apollo missions
- very shallow depth of field in closeups (small object distance)

Stanley Kubrick, Barry Lyndon, 1975
Microscope objectives

- numerical aperture $NA = n \sin \theta$
- for dry objectives, $N \approx \frac{1}{2} NA$
- so $40 \times / 0.95NA$ objective $= f/0.51$ (on object side)!
- $\theta = 71.8^\circ$!
Main side-effect of aperture

- depth of field
- doubling N (two f/stops) doubles depth of field
Trading off motion blur and depth of field