Defining Point-Set Surfaces

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Point-Set Surfaces

- Surface *S* implied by the point cloud *P*
  - No connectivity
- Surface properties
  - Does *x* belong to *S*
  - Project *x* to *S*

MLS Surface

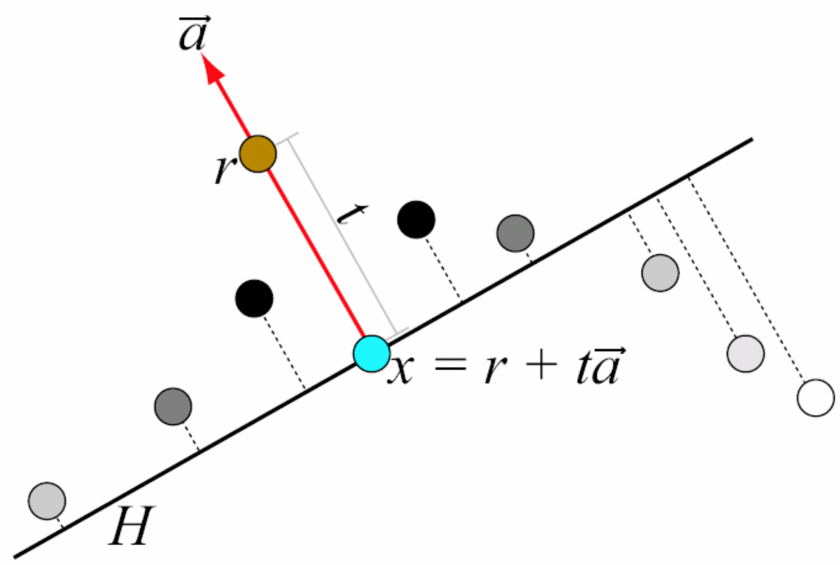
- Given a point cloud P and a point r near P

1. Find (a, t) that minimizes

Weights depend on an unknown plane

$$e_{MLS}(\vec{a},t) = \sum_{p_i \in P} \left(\langle \vec{a}, p_i \rangle - \langle \vec{a}, r + t\vec{a} \rangle\right)^2 \theta(r + t\vec{a}, p_i)$$

$$\theta(x, p_i) = e^{\frac{-d^2(x-p_i)}{h^2}}$$

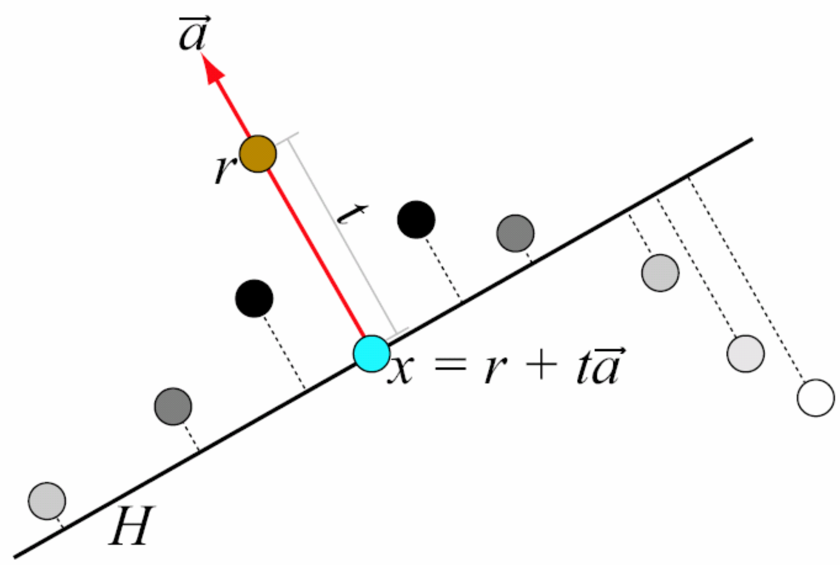


MLS Surface

- Local minima over $\mathbb{S}^2 \times \mathbb{R}$ occur at discrete set of inputs (*a*, *t*)

2. Define *f(r)* to be *x* nearest to *r*

3. Stationary points of *f* form the MLS surface

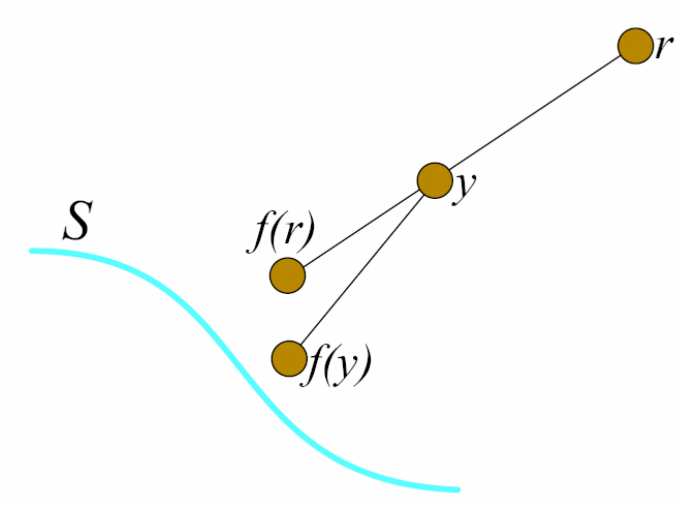


MLS Surface

- Need to solve optimization problem to find *f(r)*
  - Expensive
  - Original paper proposed that one iteration is enough

MLS Projection

- Levin's method, minimizes $e_{MLS}(\vec{a},t)$

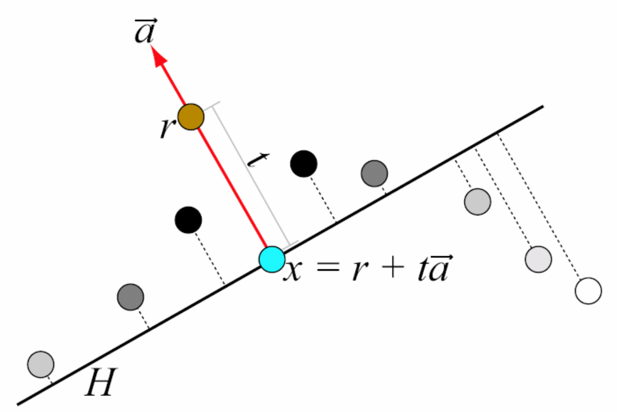


- Not a projection

MLS Energy

- Rewrite in terms of a point and a direction

$$e_{MLS}(x,a) = \sum_{p_i \in P} \left(\langle a, p_i \rangle - \langle a, x \rangle \right)^2 \theta(x, p_i)$$

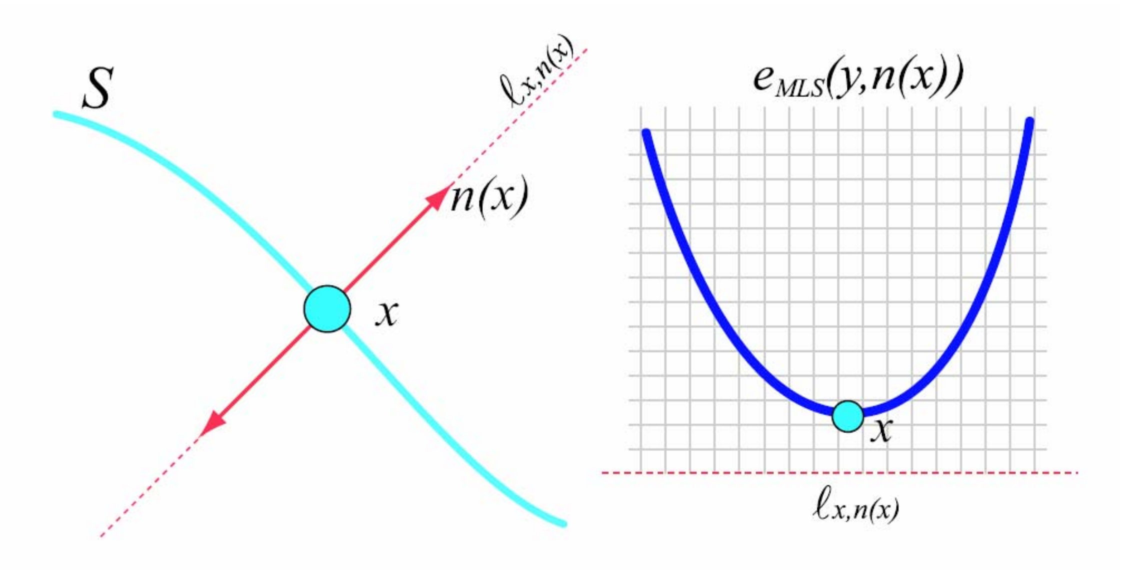


  - Domain is $\mathbb{R}^3 \times \mathbb{P}^2$

Explicit MLS Definition

- Define $n(x) = \mathrm{argmin}_a \, e_{MLS}(x, a)$
- MLS surface consists of points *x* such that

$$x \in arglocalmin_{y \in \ell_{x,n(x)}} e_{MLS}(y, n(x))$$

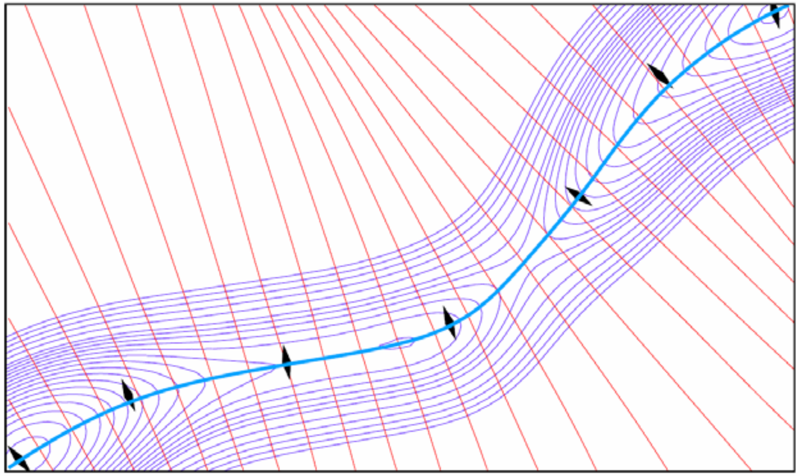


Extremal Surfaces

Definition 1 *For any functions* $n : \mathbb{R}^3 \to \mathbb{P}^2$ *and* $e : \mathbb{R}^3 \times \mathbb{P}^2 \to \mathbb{R}$, *let*

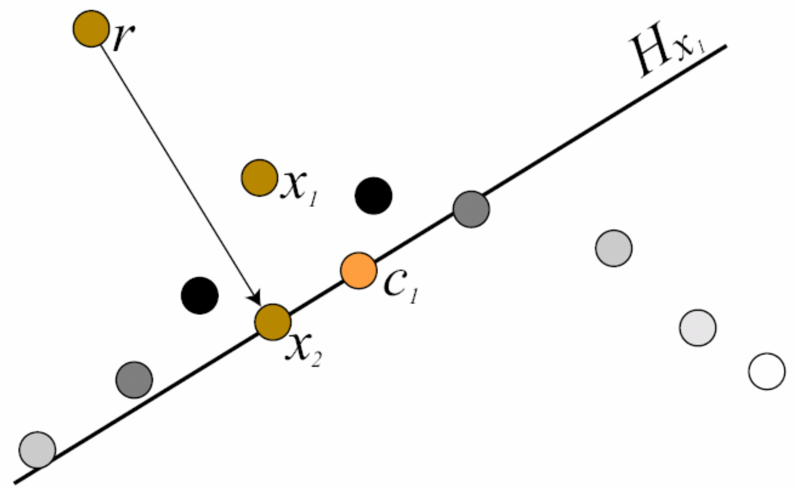
$$S = \{x \mid x \in arglocalmin_{y \in \ell_{x,n(x)}} e(y, n(x))\}$$

be the extremal surface of n and e.



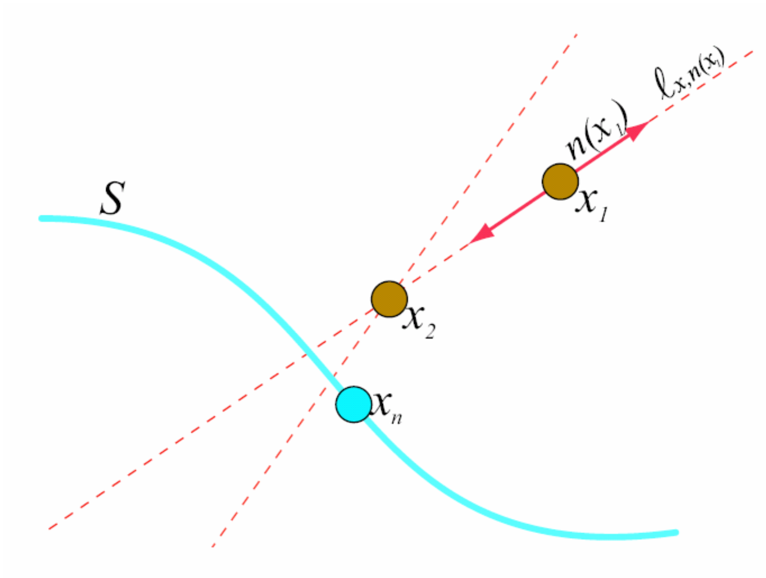
MLS Projection

- PointShop method (linear approximation)
  - Alternate searching for *x* and best-fit plane



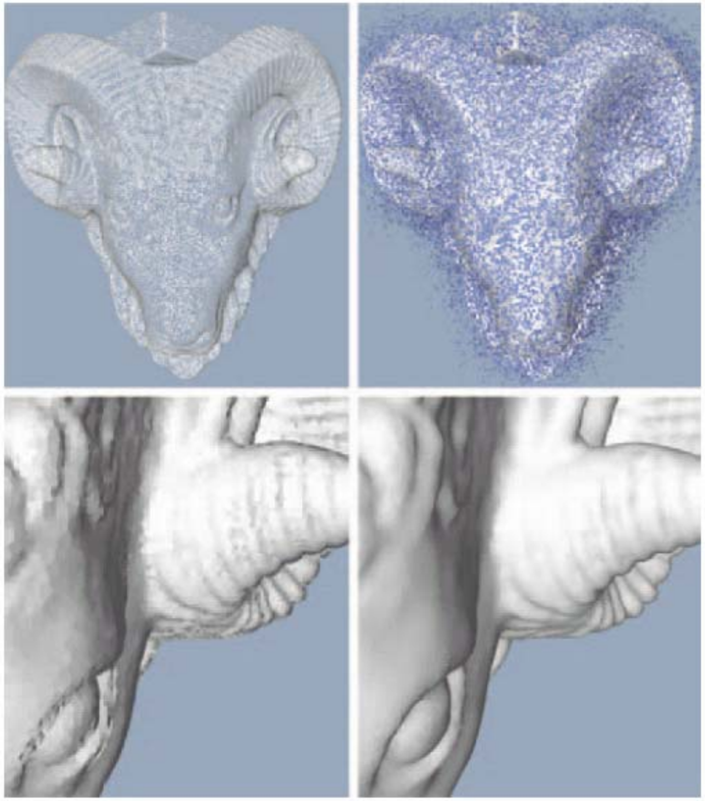
New MLS Projection

- Search for local minima along $l_{x,\, n(x)}$

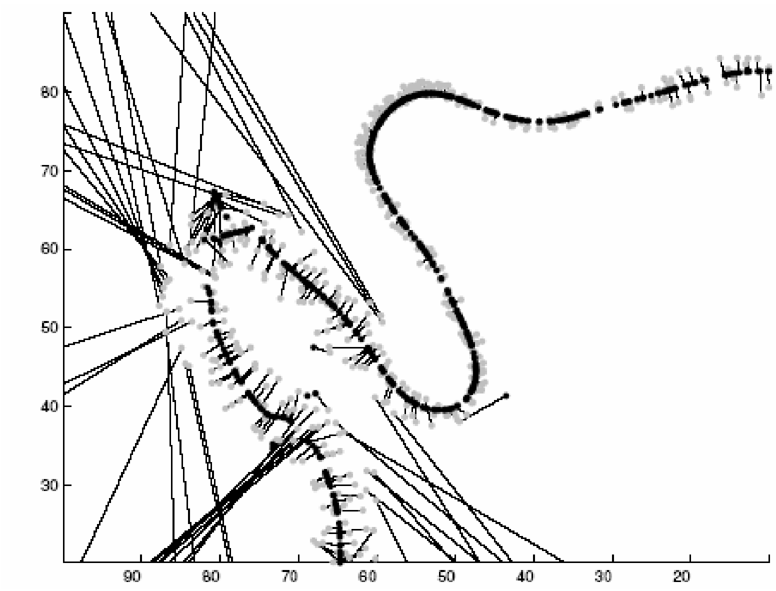


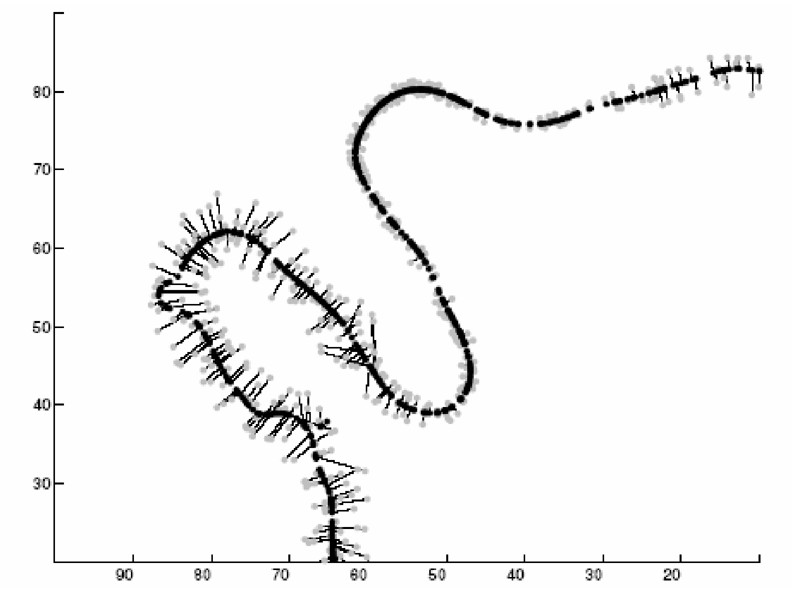
  - If this process converges, it produces a point on S

Projection Results



Comparison with Levin's Method

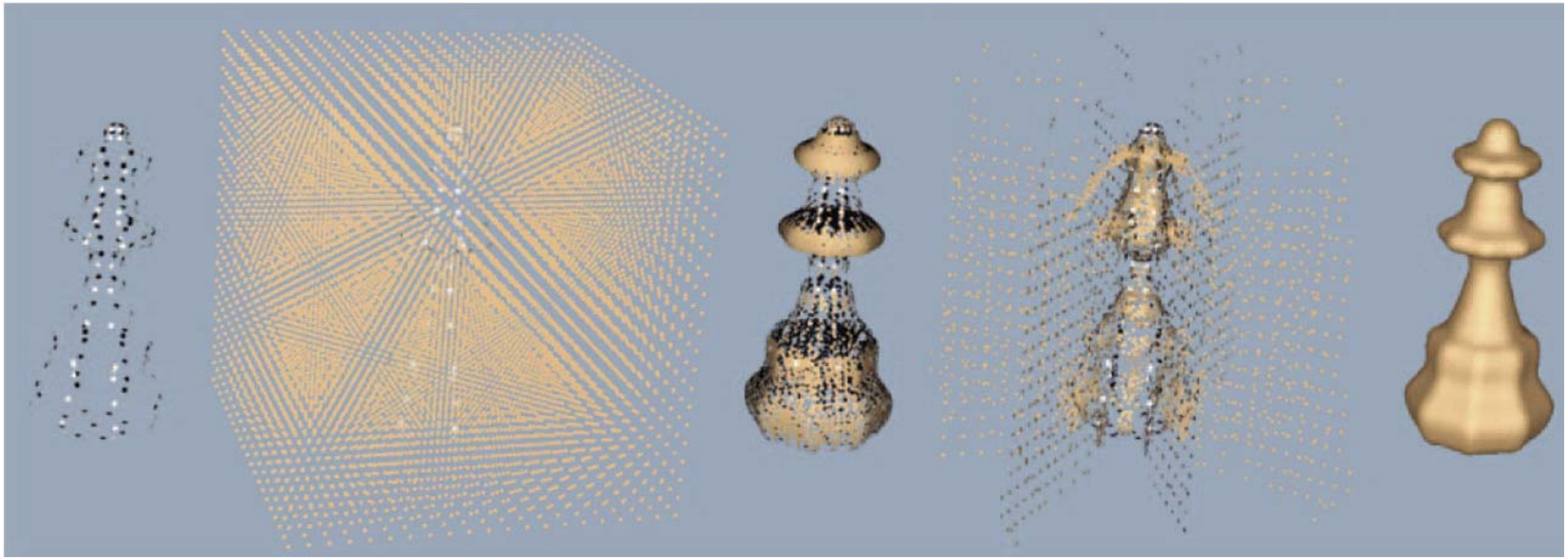




MLS for Surfels

- Input (p_i, a_i)

$$n(x) = \sum_i a_i\, \theta_N(x, p_i) \qquad \theta_N(x, p_i) = \frac{e^{-d^2(x,p_i)/h^2}}{\sum_j e^{-d^2(x,p_j)/h^2}}$$

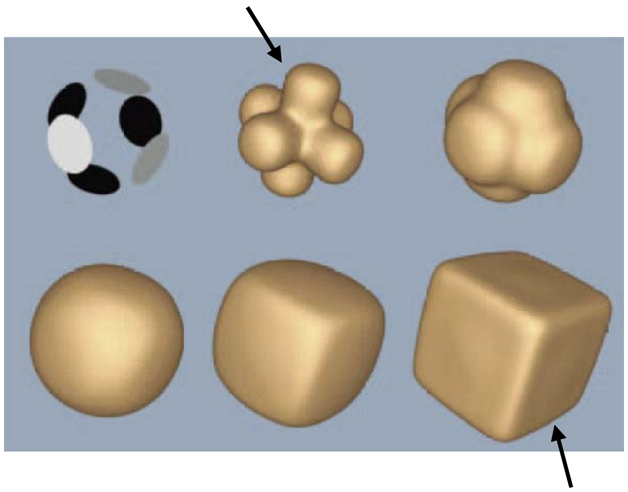


Varying Normal Weight

- Distance function favors the normal component

$$d_M(p_i, a_i, x) = \langle (x - p_i), a_i \rangle^2 + c \, || \, (x - p_i) \; - \; \langle (x - p_i), a_i \rangle \, a_i \, ||^2$$

$$e(x, a) = e(x) = \sum_i d_M(p_i, a_i, x) \theta_N(x, p_i)$$



MLS for Weighted Point Clouds

- Separate weight for each point

$$\theta_N(x, p_i) = \frac{e^{-d^2(x,p_i)/h_i^2}}{\sum_j e^{-d^2(x,p_j)/h_j^2}}$$