Moving Least Squares

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presented by Niloy J. Mitra

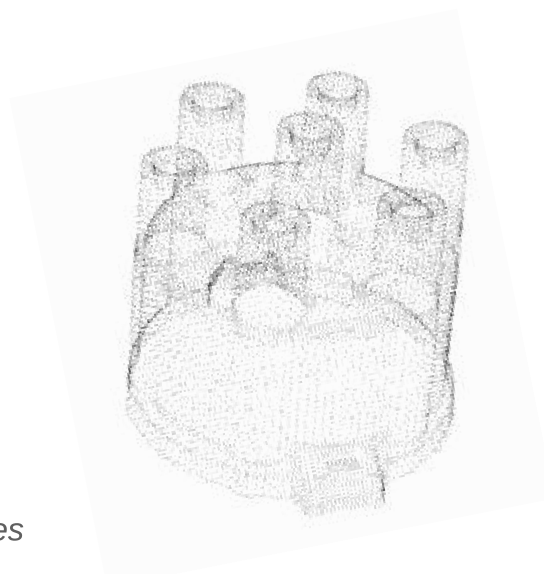
Outline

- The Approximation Power of Moving Least-Squares
  D. Levin
- Mesh-Independent Surface Interpolation
  D. Levin

- Defining point-set surfaces
  N. Amenta and Y. Kil

Problem

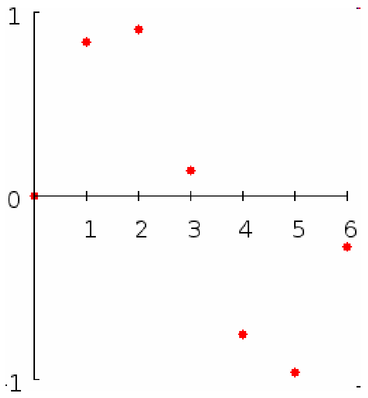
- Collection of point
  - Source of data : laser scanner
- Points are unorganized
- Usually no information about normal
  - But not always the case (next paper)

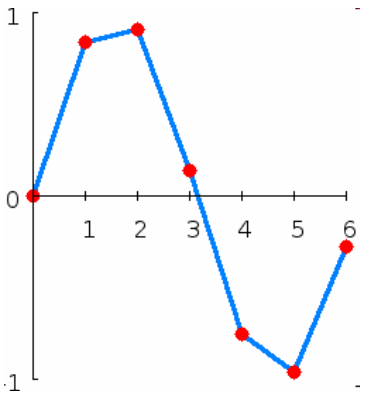


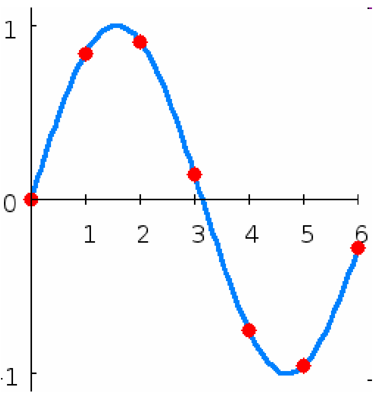
Applications

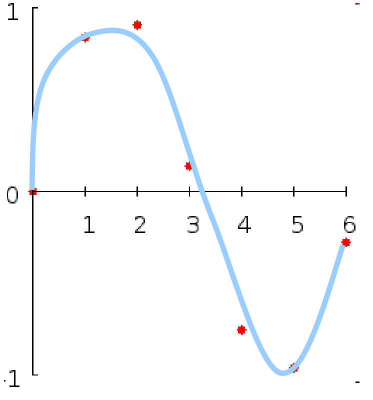
- Implicit surface definition
  - Projection operator
- Noise removal / Thinning
- Upsampling
- Ray tracing

Interpolation vs Smoothing



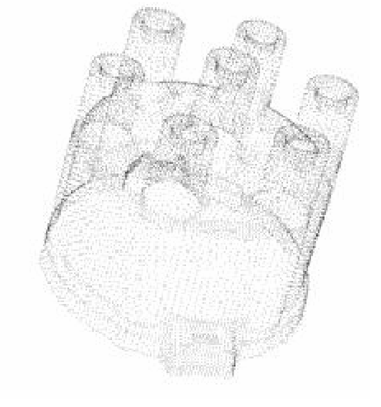




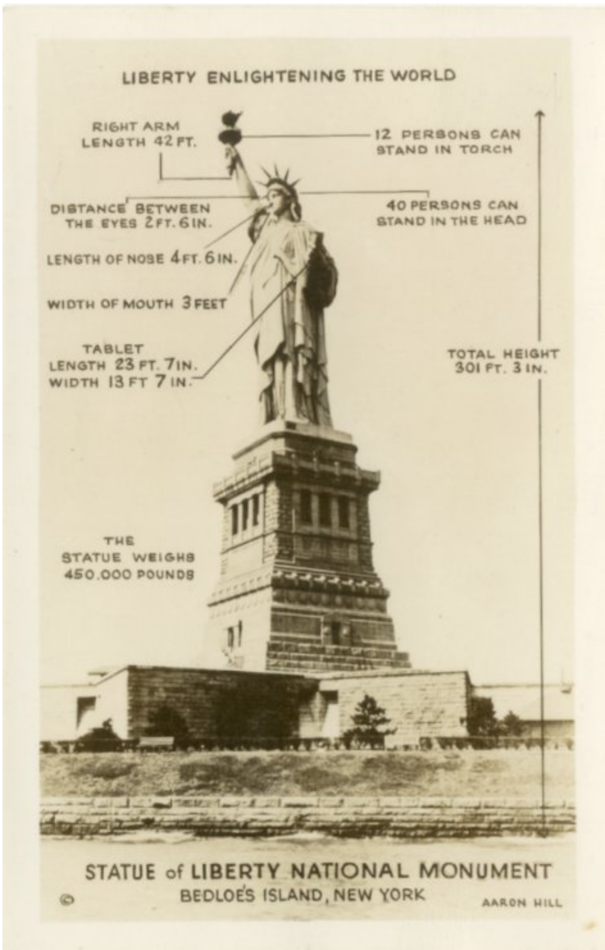


One Approach (Mesh based)

- Smooth interpolation by joining local patches each being an approximation in local reference domain.
- Piecewise polynomial patches.
- In most cases, result depends on the mesh defining the patches.



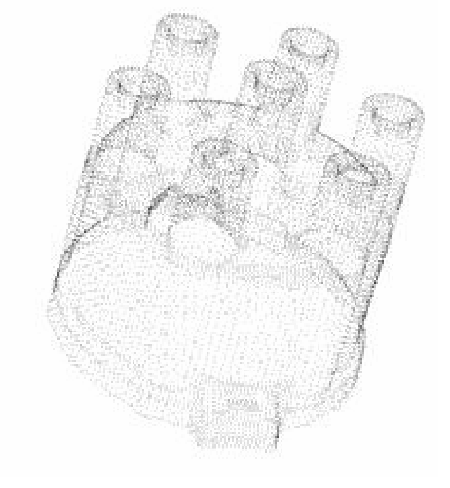
Example



350 pieces/patches

Alternative Approach (Meshless)

- Implicit definition of surface.
- $S = f(\{p_i\})$



Roadmap

Given

$R = \{x_i\}$

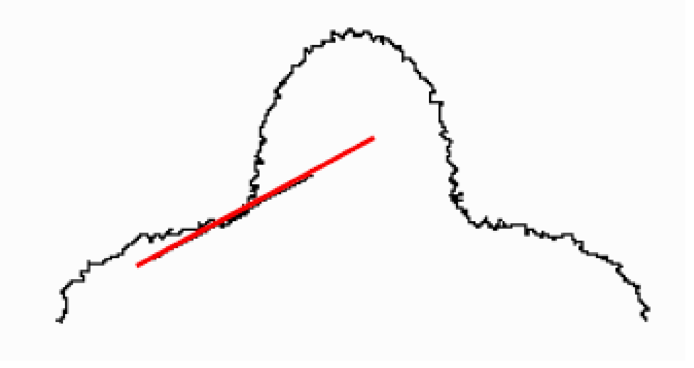
Goal

- Define a projection operator P
  - $x \in \Re^d \quad P : x \to P(x) \in S$
- Unique manifold $S \equiv \{x \mid P(x) = x\}$

MLS Approach

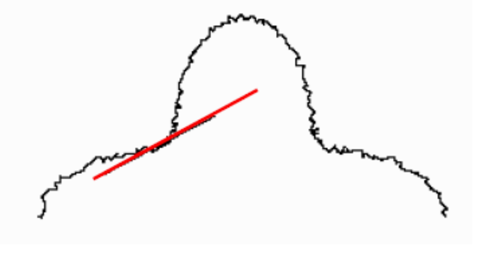
- Step 1
  - Define a local/reference domain (like a tangent plane)

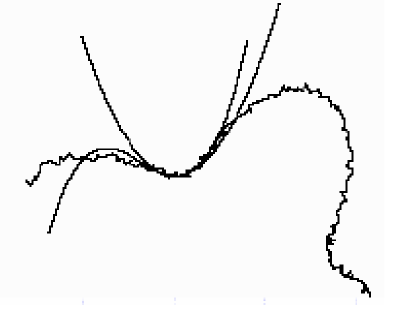
  - Local parameterization



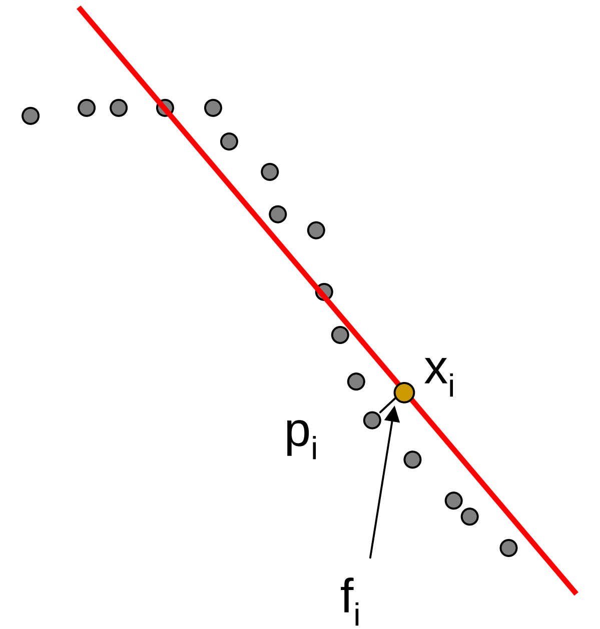
MLS Approach

- Step 1
  - Define a local/reference domain
- Step 2
  - MLS approximation wrt reference domain (polynomial fitting)





Fitting Functions



Given (*functional setting*)

$\{x_i, f_i\}$

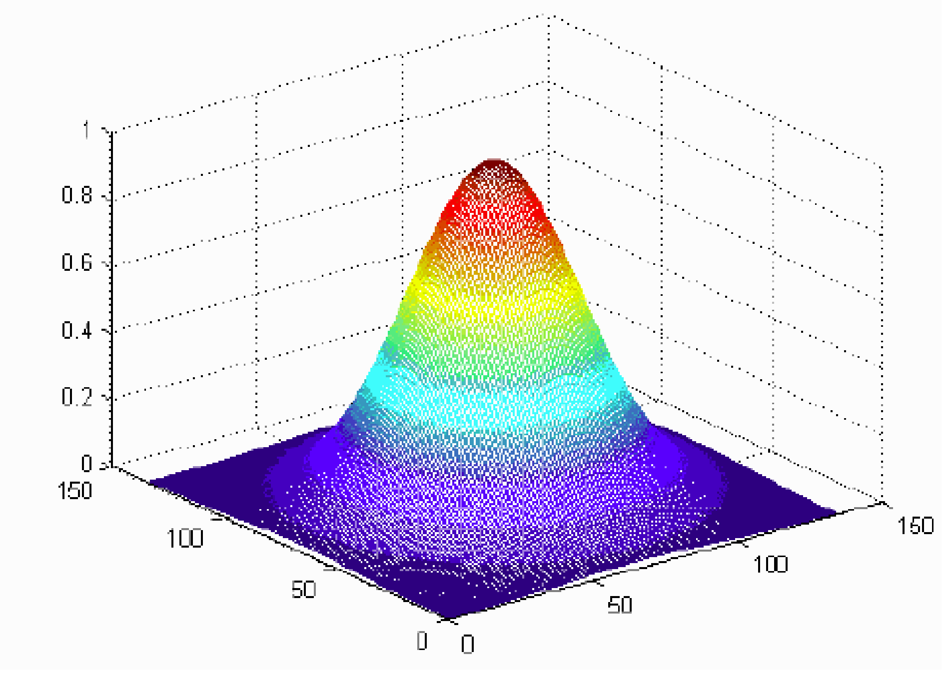
Goal

Find p in Π_m such that $\{x_i, f_i\}$ satisfies

$$\min_{p \in \prod_m^{d-1}} \sum_i \underbrace{(p(x_i) - f_i)^2}_{\text{error}} \underbrace{\theta(\| x_i \|)}_{\text{weight}}$$

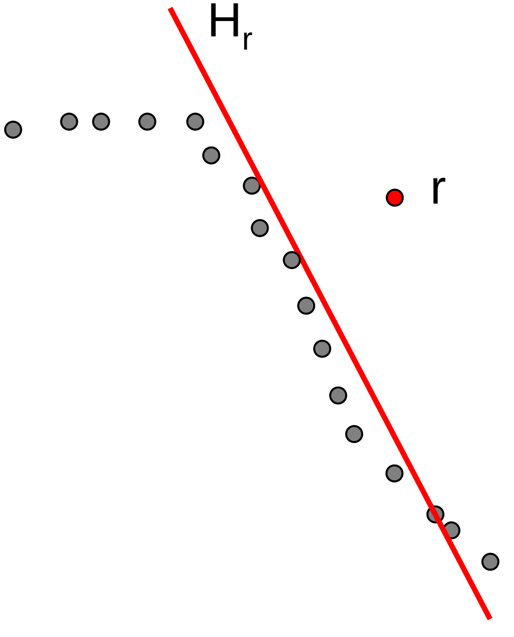
θ : The Weight Function

- Non-negative decaying function
- Typical example
  - Gaussian kernel $\theta(d) = \exp(-d^2/h^2)$



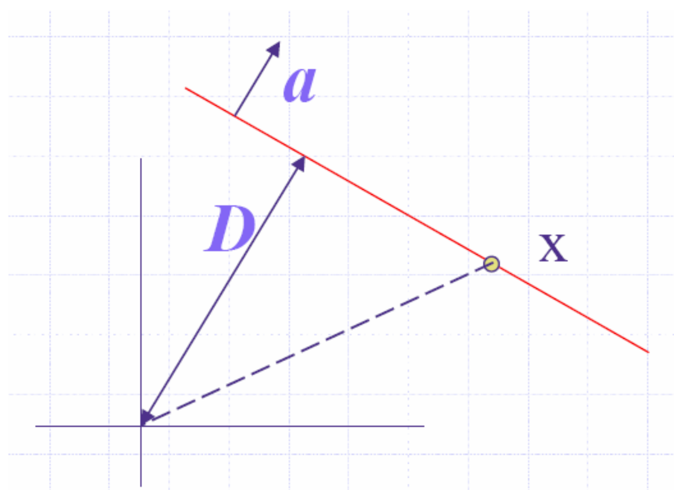
Basic MLS

- For a given point r near R, define a local approximating hyper-planer H_r



Equation of a line

$$H_{a,D} = \{x \mid \langle a, x \rangle - D = 0, x \in \Re^d\}, a \in \Re^d, \| a \| = 1$$



Basic MLS

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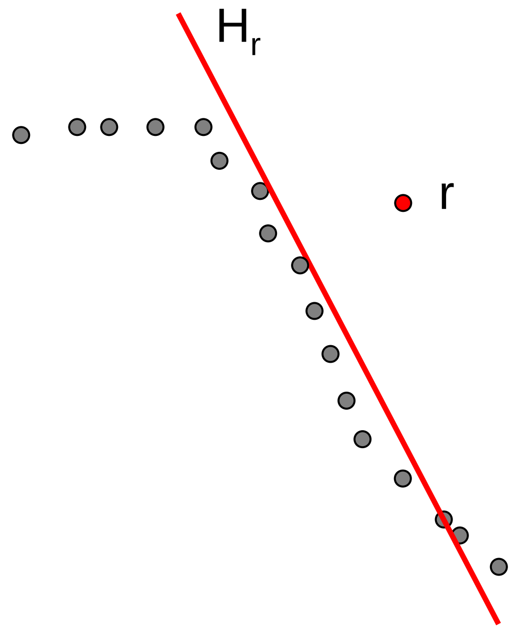


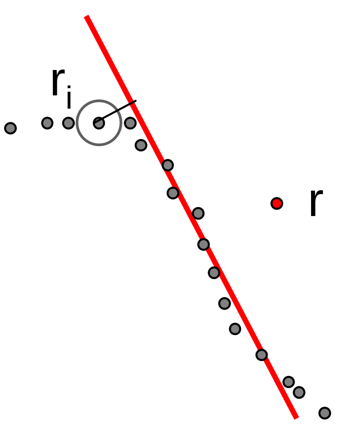
Basic MLS

- For a given point r near R, define H_r

Non-linear optimization

$$\min_{a,D} \sum_i (\langle a, r_i \rangle - D)^2 \theta(\| r_i - r \|)$$

- In case of multiple local minima, the plane closest to r is chosen.

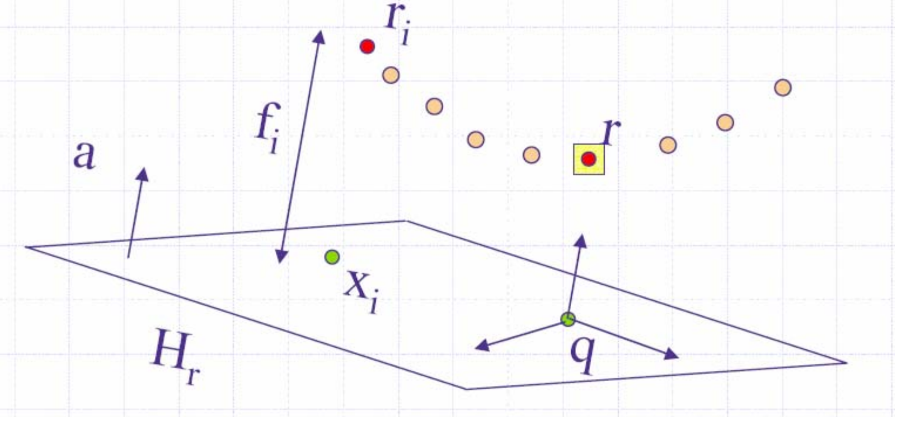


Basic MLS

- For a given point r near R, define H_r

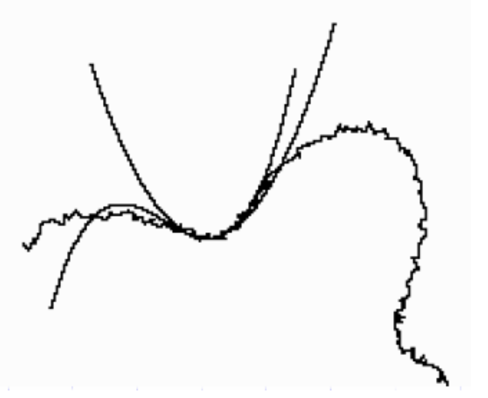
  Find a polynomial approx. of degree m

$$\min_{p \in \prod_m^{d-1}} \sum_i (p(x_i) - f_i)^2 \theta(\| r_i - r \|)$$



MLS

Step 1



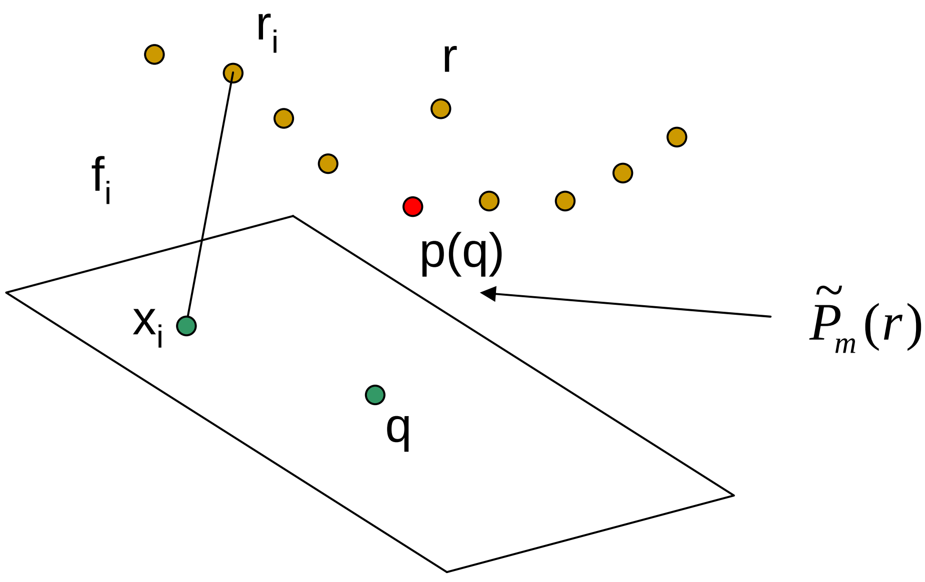
Step 2

Projection?

$$\tilde{P}_m(\tilde{P}_m(r)) \neq \tilde{P}_m(r)$$

$$\theta(\| r_i - r \|)$$

$$\min_{a,D} \sum_i (< a, r_i > -D)^2 \theta(\| r_i - r \|)$$



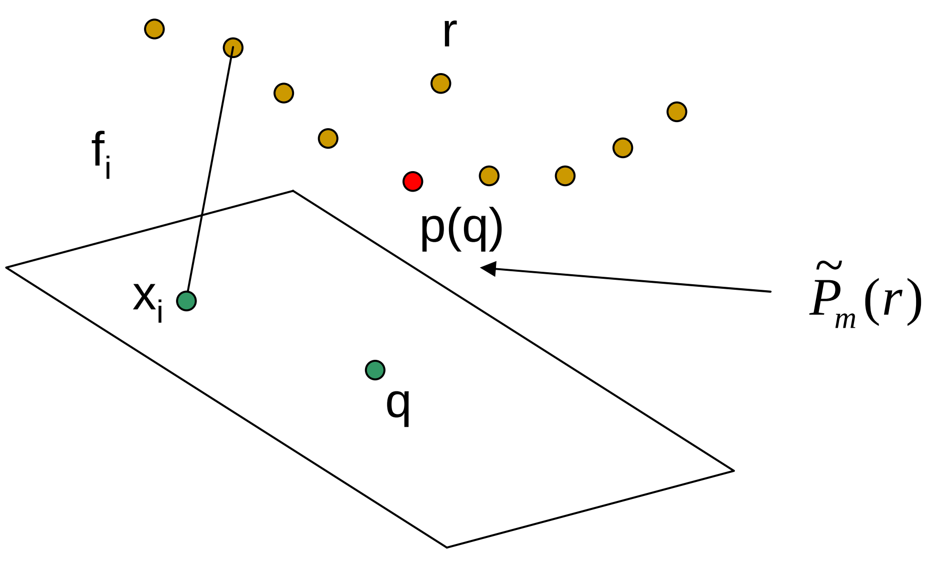
Basic MLS

$$\tilde{P}_m(\tilde{P}_m(r)) \neq \tilde{P}_m(r)$$

- Doesn’t project points to a (d-1)-dim manifold.
- Doesn’t define a surface.

Simple fix

$$\theta(\| r_i - r \|) \qquad \theta(\| r_i - q \|)$$



Non-linear Optimization

$$I(q,a) = \min_{a,D} \sum_i (< a, r_i > - D)^2 \theta(\| r_i - q \|)$$

constraints

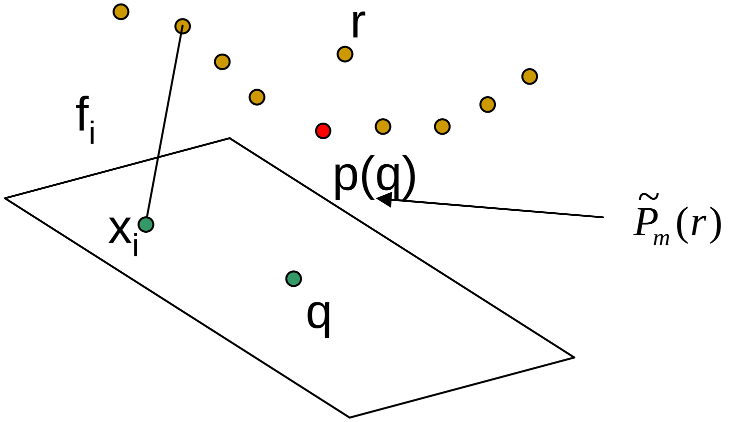
$$\begin{cases} (r-q) \parallel a(q) \\ J(q) = I(q, a(q)) \quad \partial_{a(q)} J(q) = 0 \end{cases}$$

Basic MLS

- For a given point r near R, define H_r

$$\min_{a,D} \sum_i (< a, r_i > -D)^2 \theta(\| r_i - q \|)$$

- In case of multiple local minima, the plane closest to r is chosen.



MLS

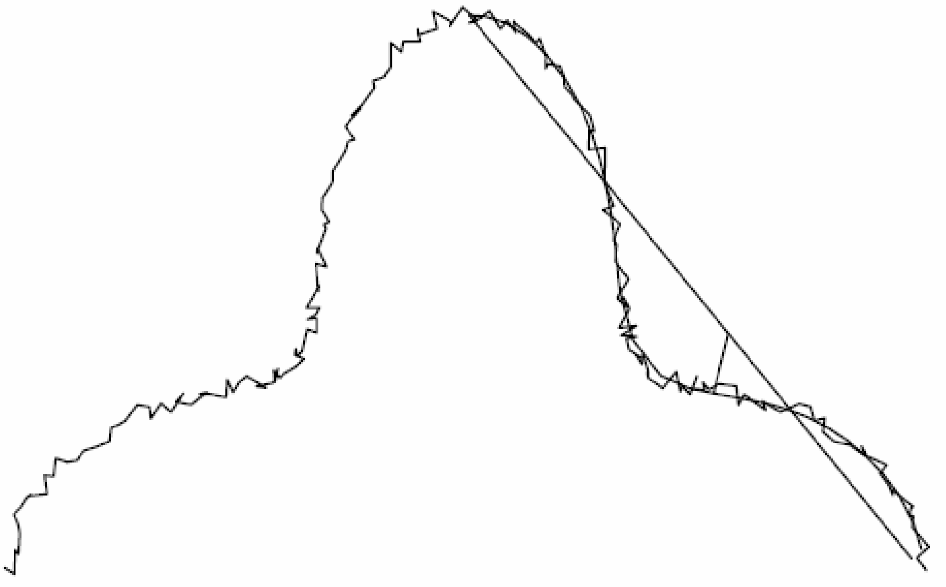
Given

$R = \{x_i\}$

MLS

- Define a projection operator $P(P(x))=P(x)$
- Unique manifold $S \equiv \{x|P(x)=x\}$
- Conjecture S is C^∞

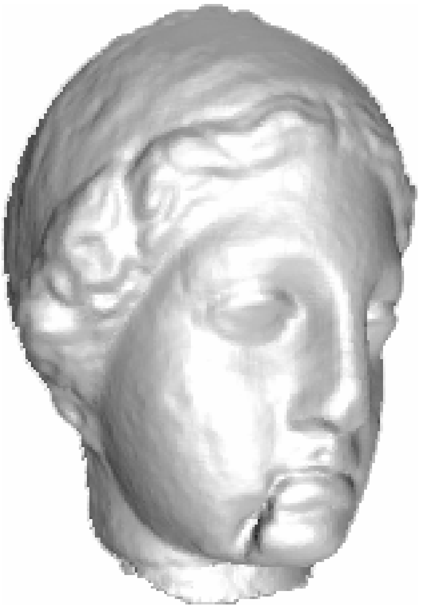
MLS surface



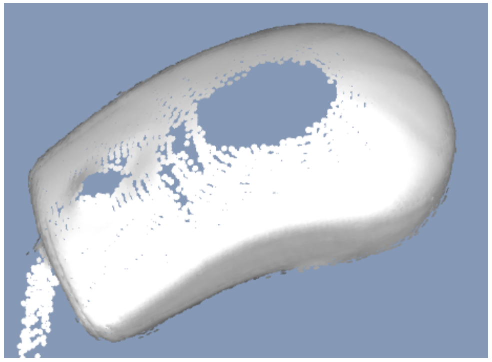
Computing H_r and p

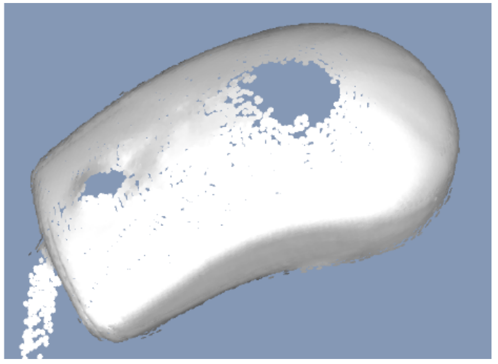
- Computing hyper-plane H_r
- Non-linear optimization problem
- Computed iteratively
- Computing $\theta()$: time consuming step
  - O(N) for each iteration step
  - Approximate by doing a hierarchical clustering
- Fitting a polynomial p(.), given Hr
  - Solve a linear system
    - Size depends on the order of approximation (m)

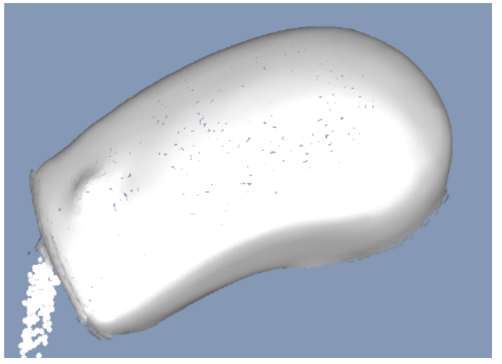
Applications : Denoising



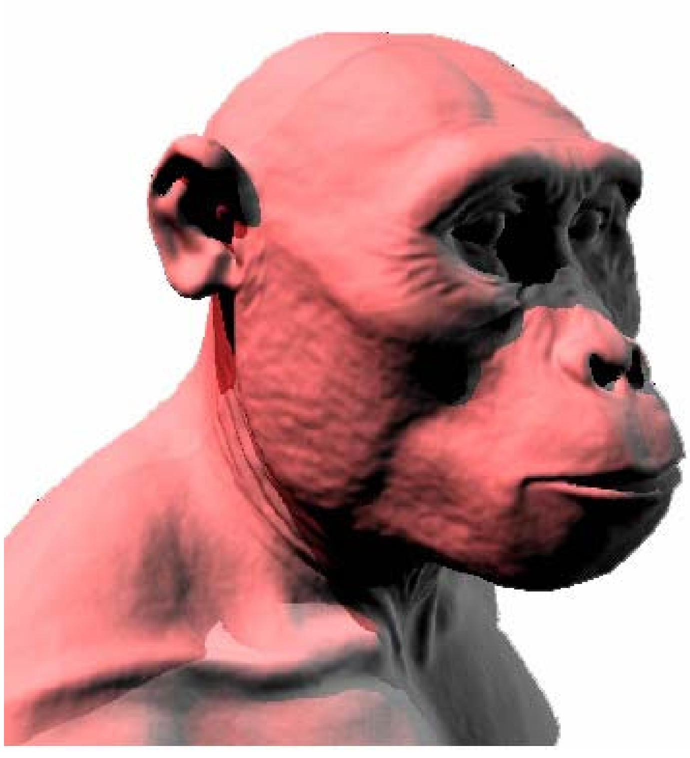
Applications : Upsampling / Hole Filing



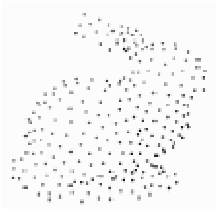


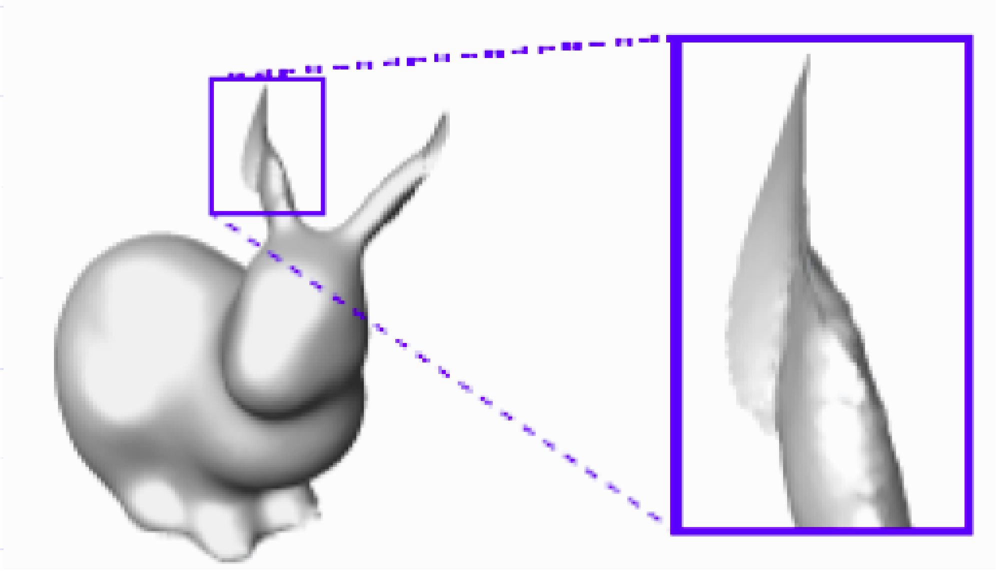


Applications : Ray Tracing



Sampling Condition?

Conclusions

- Surface is smooth and a manifold
- Adjustable feature size h allows to smooth out noise

- The surface changes with addition of points.