Adaptive Dynamics of Articulated Bodies

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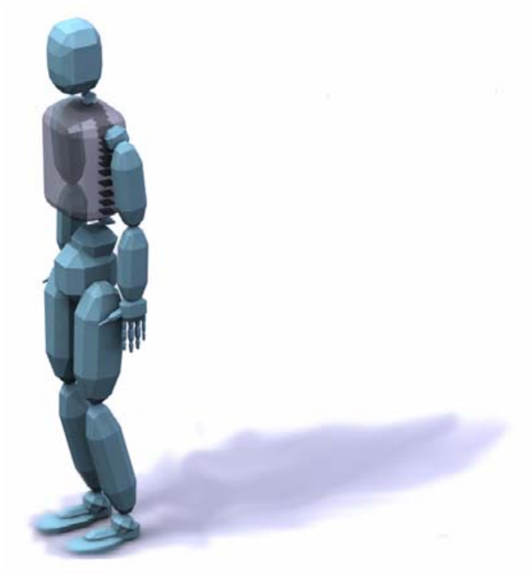
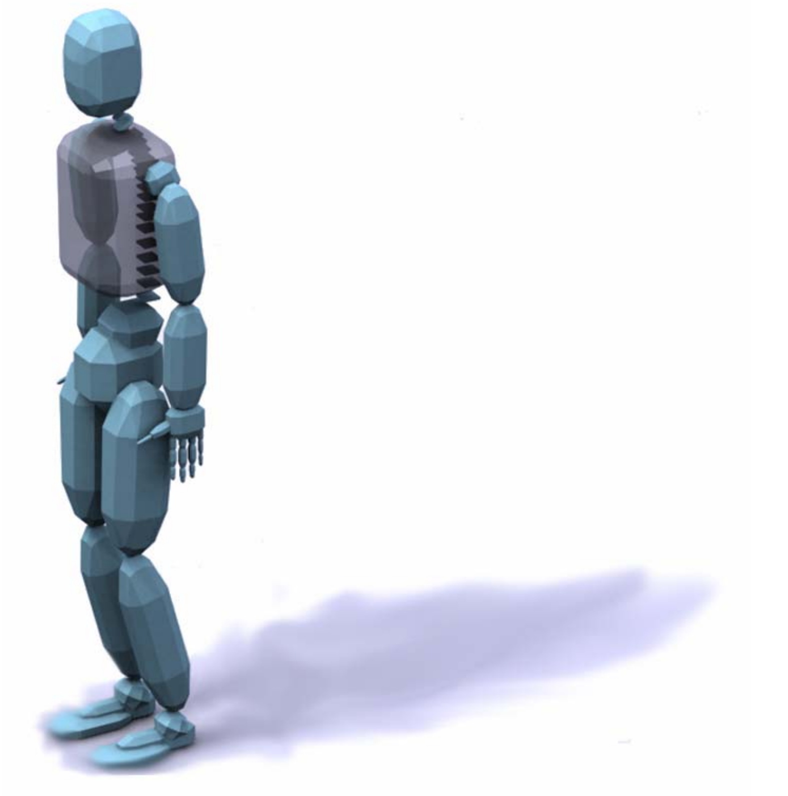


Motivation

- Articulated bodies in Computer Graphics
  - Humans, hair, animals
  - Trees, forests, grass
  - Deformable bodies
  - Molecular graphics
  - …



Motivation

- Forward dynamics
- Optimal solutions are linear
- Optimal forward dynamics methods are **too slow** for numerous or complex articulated bodies

Contributions

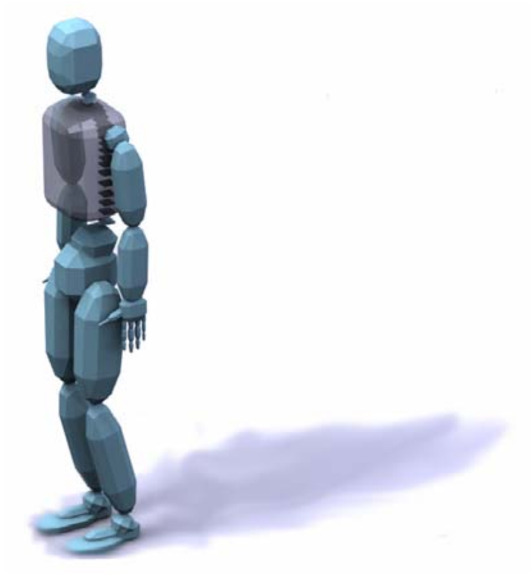
- ~~Forward dynamics~~
- **Adaptive forward dynamics**
  - Specify the number of degrees of freedom
  - Only this number of degrees of freedom is simulated
  - The most relevant degrees of freedom are automatically found

Contributions

- Hybrid bodies
  - Articulated-body representation
  - To reduce the number of degrees of freedom
- Adaptive joint selection
  - Customizable motion metrics
  - To determine the most relevant degrees of freedom
- Adaptive update mechanisms

Outline

- **Related work**
- Hybrid bodies
- Adaptive joint selection
- Adaptive update mechanisms
- Results



Related work

Forward dynamics of articulated bodies

- Optimal algorithms
- Parallel algorithms
- Human motion
- Plant motion
- Hair modeling

- View-dependent dynamics
- Articulated-body motion simplification
  - Faure 1999
  - Redon and Lin 2005: Adaptive quasi-statics

This paper: adaptive simplification using customizable motion error metrics

Outline

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- **Hybrid bodies**
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- Adaptive update mechanisms
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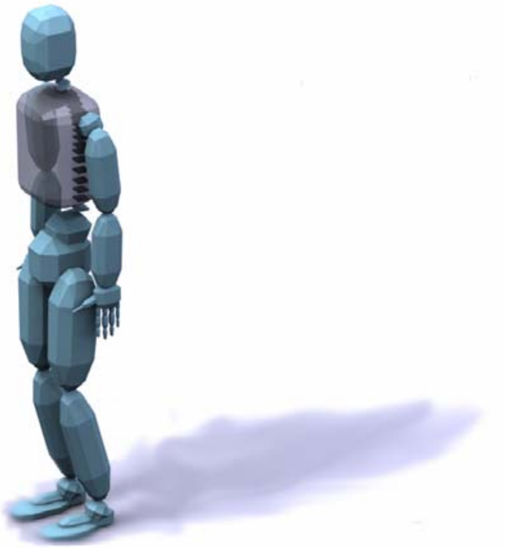
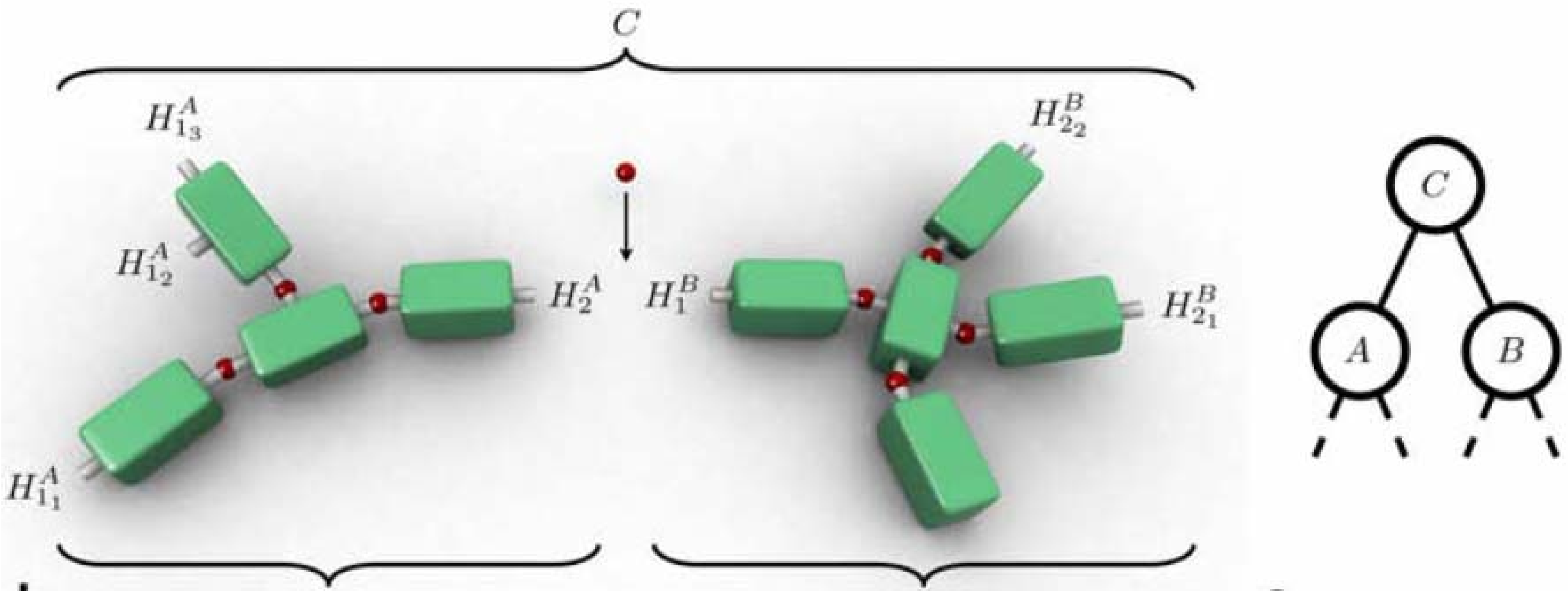


Articulated-body

Definition

- An articulated-body is a rigid-body system with one or more handles;
- A handle is a specified location within an articulated body to which external forces may be applied and which responds with an observable acceleration.

Articulated-bodies
Featherstone's DCA

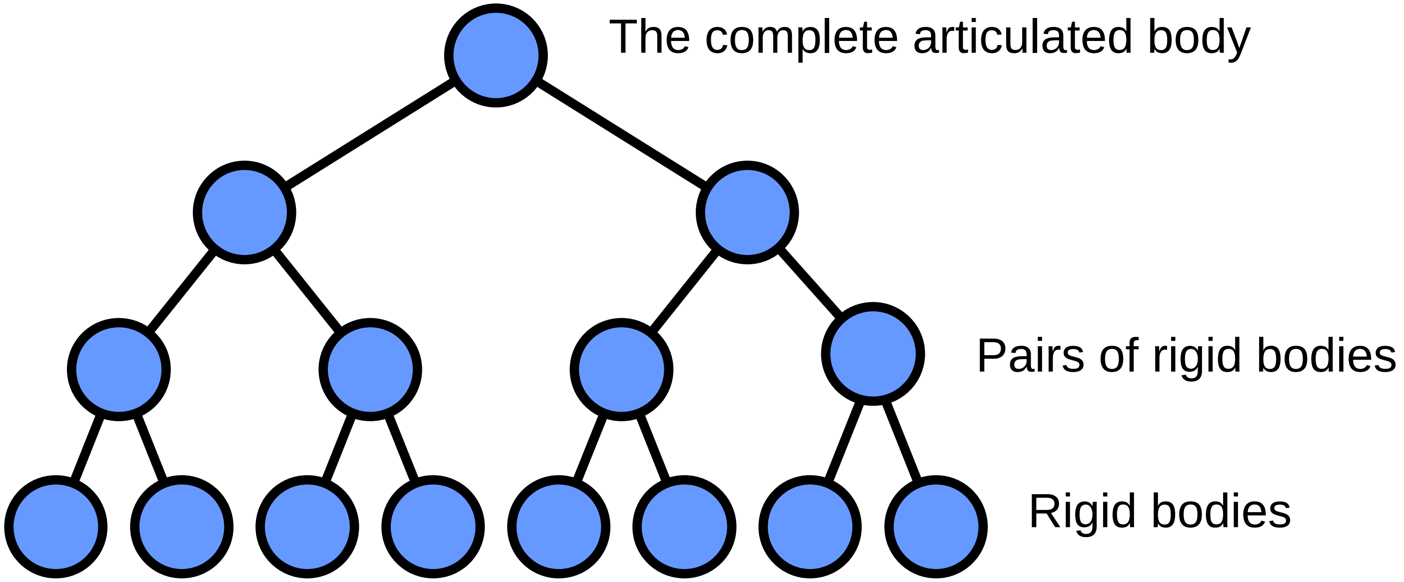


A   B

An articulated body is recursively defined as a pair of articulated bodies connected by a joint

Articulated bodies
Featherstone's DCA

- Recursive definition



The assembly tree of an articulated body

Articulated-body Dynamics

Featherstone's DCA

Articulated-body equations of motion:

$$\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_m \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_{12} & \cdots & \mathbf{\Phi}_{1m} \\ \mathbf{\Phi}_{21} & \mathbf{\Phi}_2 & \cdots & \mathbf{\Phi}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Phi}_{m1} & \mathbf{\Phi}_{m2} & \cdots & \mathbf{\Phi}_m \end{bmatrix} \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_m \end{bmatrix} + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_m \end{bmatrix}$$

Body Accelerations

Inverse inertias and cross-inertias

Applied Forces

Bias accelerations

Articulated-body Dynamics
Featherstone's DCA

Articulated-body equations

the effect of a force $\mathbf{f}_2$ applied to body 2, on the acceleration $\mathbf{a}_1$ of body 1

$$\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_m \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_{12} & \cdots & \mathbf{\Phi}_{1m} \\ \mathbf{\Phi}_{21} & \mathbf{\Phi}_2 & \cdots & \mathbf{\Phi}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Phi}_{m1} & \mathbf{\Phi}_{m2} & \cdots & \mathbf{\Phi}_m \end{bmatrix} \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_m \end{bmatrix} + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_m \end{bmatrix}$$

The bias acceleration $\mathbf{b}_1$ is the acceleration of body 1 when no forces are applied

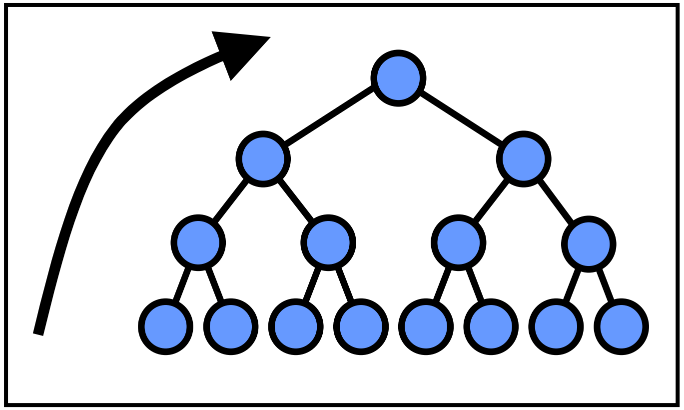
Featherstone's DCA

Two main passes

1. The main pass: Compute the articulated-body coefficients (↑)

Inverse inertias

$$\mathbf{\Phi}_1^C = \mathbf{\Phi}_1^A - \mathbf{\Phi}_{12}^A \mathbf{W} \mathbf{\Phi}_{21}^A$$
$$\mathbf{\Phi}_2^C = \mathbf{\Phi}_2^B - \mathbf{\Phi}_{21}^B \mathbf{W} \mathbf{\Phi}_{12}^B$$
$$\mathbf{\Phi}_{21}^C = \mathbf{\Phi}_{21}^B \mathbf{W} \mathbf{\Phi}_{21}^A = (\mathbf{\Phi}_{12}^C)^T$$



Bias accelerations

$$\mathbf{b}_1^C = \mathbf{b}_1^A - \mathbf{\Phi}_{12}^A \gamma$$
$$\mathbf{b}_2^C = \mathbf{b}_2^B + \mathbf{\Phi}_{21}^B \gamma$$

Leaf-node coefficients

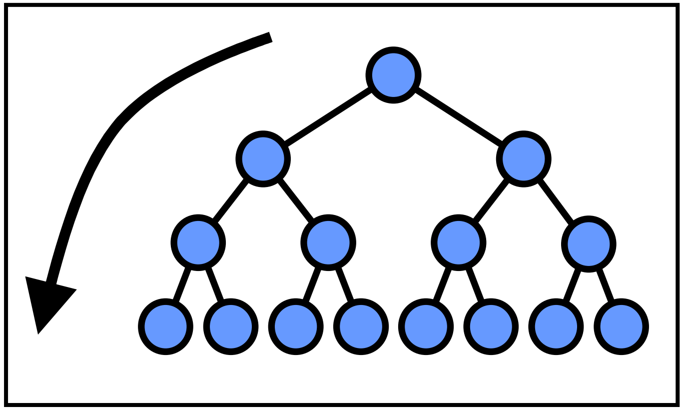
$$\mathbf{\Phi}_i = \mathbf{\Phi}_{ij} = \mathbf{I}^{-1} \qquad \mathbf{b}_i = \mathbf{I}^{-1}(\mathbf{f}_k - \mathbf{v} \times \mathbf{I}\mathbf{v})$$

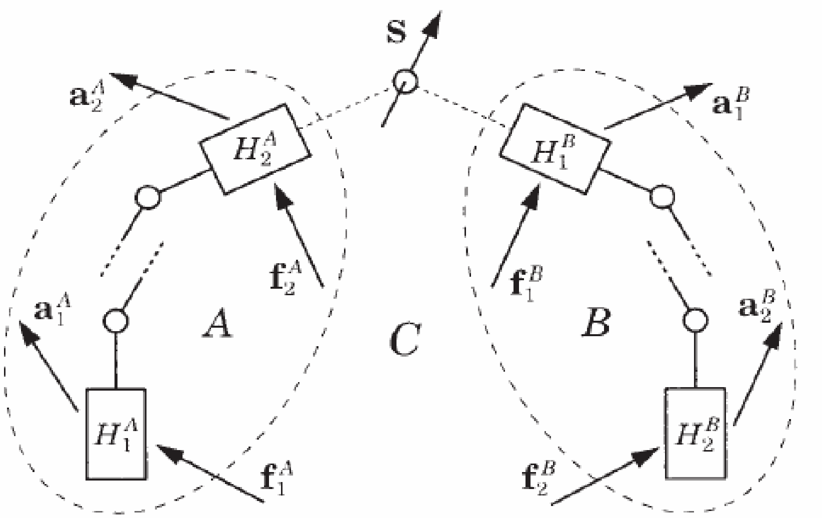
↑ Acceleration-independent external force applied to the rigid body

Featherstone's DCA

Two main passes

2. The back-substitution pass: the kinematic constraint forces are propagated down the tree to compute all the joint accelerations (↓).





Joint acceleration

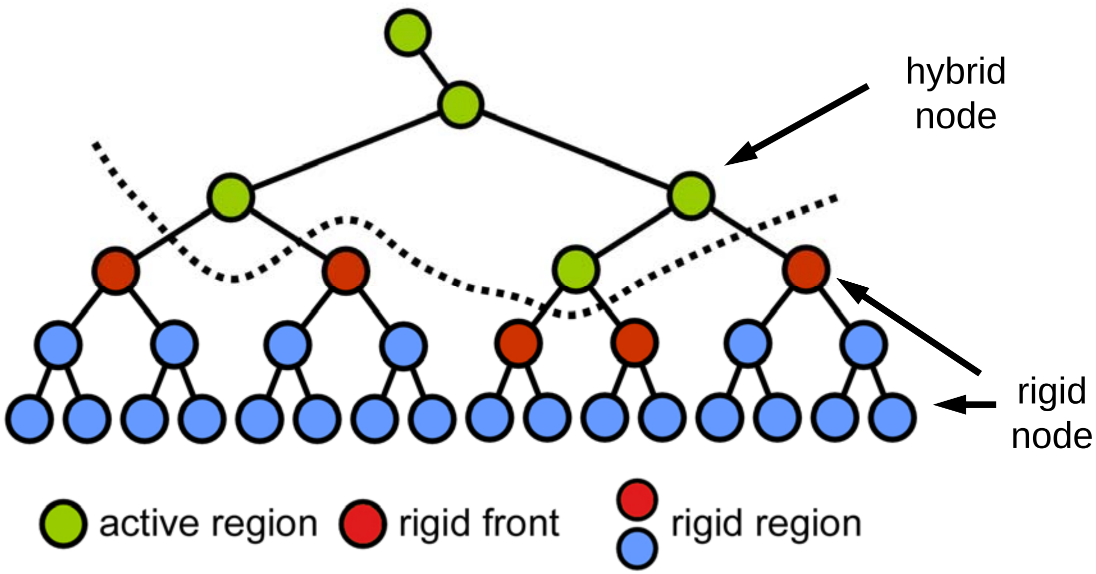
$$\ddot{\mathbf{q}} = (\mathbf{S}^T\mathbf{V}\mathbf{S})^{-1}(\mathbf{Q} - \mathbf{S}^T\mathbf{V}(\mathbf{\Phi}_{21}^A\mathbf{f}_1^A - \mathbf{\Phi}_{12}^B\mathbf{f}_2^B + \boldsymbol{\beta}))$$

Kinematic constraint forces

$$\mathbf{f}_1^B = \mathbf{W}\mathbf{\Phi}_{21}^A\mathbf{f}_1^A - \mathbf{W}\mathbf{\Phi}_{12}^B\mathbf{f}_2^B + \gamma$$
$$\mathbf{f}_2^A = -\mathbf{f}_1^B$$

Hybrid bodies
Definitions

- Active region
- Goal: to simply the dynamics
- Means: select a subset of joints to simulate (the complement set of nodes are *rigidified*)

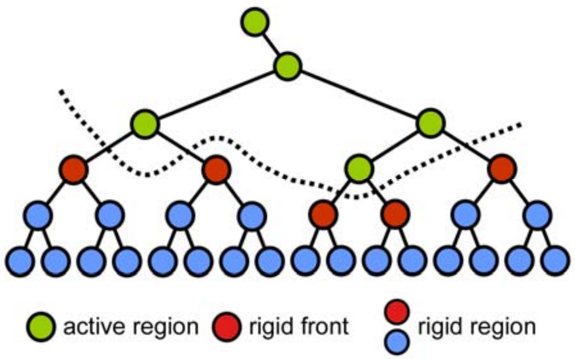


The active region contains the mobile joints

Hybrid body – an articulated body whose set of active joints is a sub-tree of the assembly tree, with an identical root.

Hybrid bodies
Hybrid-body coefficients

- Hybrid nodes use the articulated-body equations
- A rigidified node behaves like a rigid body



$$\mathbf{\Phi}_1^C = \mathbf{\Phi}_1^A - \mathbf{\Phi}_{12}^A \mathbf{W} \mathbf{\Phi}_{21}^A$$
$$\mathbf{\Phi}_2^C = \mathbf{\Phi}_2^B - \mathbf{\Phi}_{21}^B \mathbf{W} \mathbf{\Phi}_{12}^B$$
$$\mathbf{\Phi}_{21}^C = \mathbf{\Phi}_{21}^B \mathbf{W} \mathbf{\Phi}_{21}^A = (\mathbf{\Phi}_{12}^C)^T$$

$$\mathbf{b}_1^C = \mathbf{b}_1^A - \mathbf{\Phi}_{12}^A \gamma$$
$$\mathbf{b}_2^C = \mathbf{b}_2^B + \mathbf{\Phi}_{21}^B \gamma$$

Articulated-body coefficients

Rigidify joint

$$\mathbf{\Phi}^C = \mathbf{\Phi}_i^C = \mathbf{\Phi}_{ij}^C = \mathbf{\Phi}^B (\mathbf{\Phi}^A + \mathbf{\Phi}^B)^{-1} \mathbf{\Phi}^A$$
$$\mathbf{b}^C = \mathbf{b}_i^C = \mathbf{b}^A - \mathbf{\Phi}^A (\mathbf{\Phi}^A + \mathbf{\Phi}^B)^{-1} (\mathbf{b}^A - \mathbf{b}^B)$$

Rigidified-body coefficients

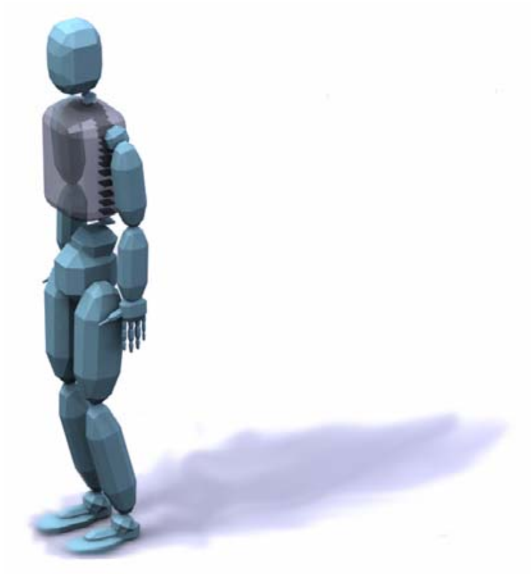
Hybrid bodies

Hybrid-body simulation

- Same steps as articulated-body simulation
- Computations restricted to a sub-tree in the back-substitution pass (and consequently in updates of the velocities and positions)

Outline

- Related work
- Hybrid bodies
- **Adaptive joint selection**
- Adaptive update mechanisms
- Results



Adaptive joint selection

To *predict* which joints should be activated so as *to best approximate the motion* of the articulated body, *without* computing the accelerations of all the joints in the articulated-body.

Adaptive joint selection
Motion metrics

- Acceleration metric

$$\mathscr{A}(C) = \sum_{i \in C} \ddot{\mathbf{q}}_i^T \mathbf{A}_i \ddot{\mathbf{q}}_i$$

- Velocity metric

$$\mathscr{V}(C) = \sum_{i \in C} \dot{\mathbf{q}}_i^T \mathbf{V}_i \dot{\mathbf{q}}_i$$

Symmetric, PSD

usually identity matrix

Adaptive joint selection

Motion metrics

- **Theorem**

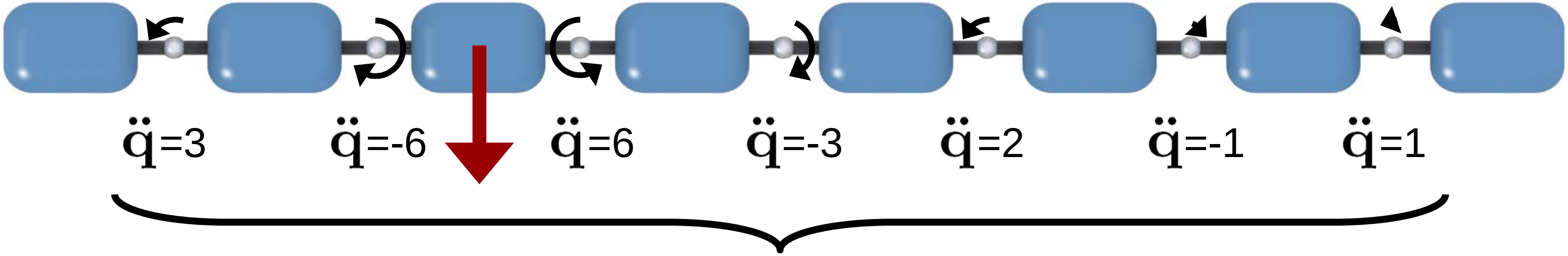
  The acceleration metric value of an articulated body can be computed **before** computing its joint accelerations

$$\mathscr{A}(C) = \begin{bmatrix} \mathbf{f}_1^A \\ \mathbf{f}_2^B \end{bmatrix}^T \begin{bmatrix} \mathbf{\Psi}_1^C & \mathbf{\Psi}_{12}^C \\ \mathbf{\Psi}_{21}^C & \mathbf{\Psi}_2^C \end{bmatrix} \begin{bmatrix} \mathbf{f}_1^A \\ \mathbf{f}_2^B \end{bmatrix} + \begin{bmatrix} \mathbf{f}_1^A \\ \mathbf{f}_2^B \end{bmatrix}^T \begin{bmatrix} \mathbf{p}_1^C \\ \mathbf{p}_2^C \end{bmatrix} + \eta^C$$

Computed in a bottom-up fashion just like the computation of articulated-body coefficients

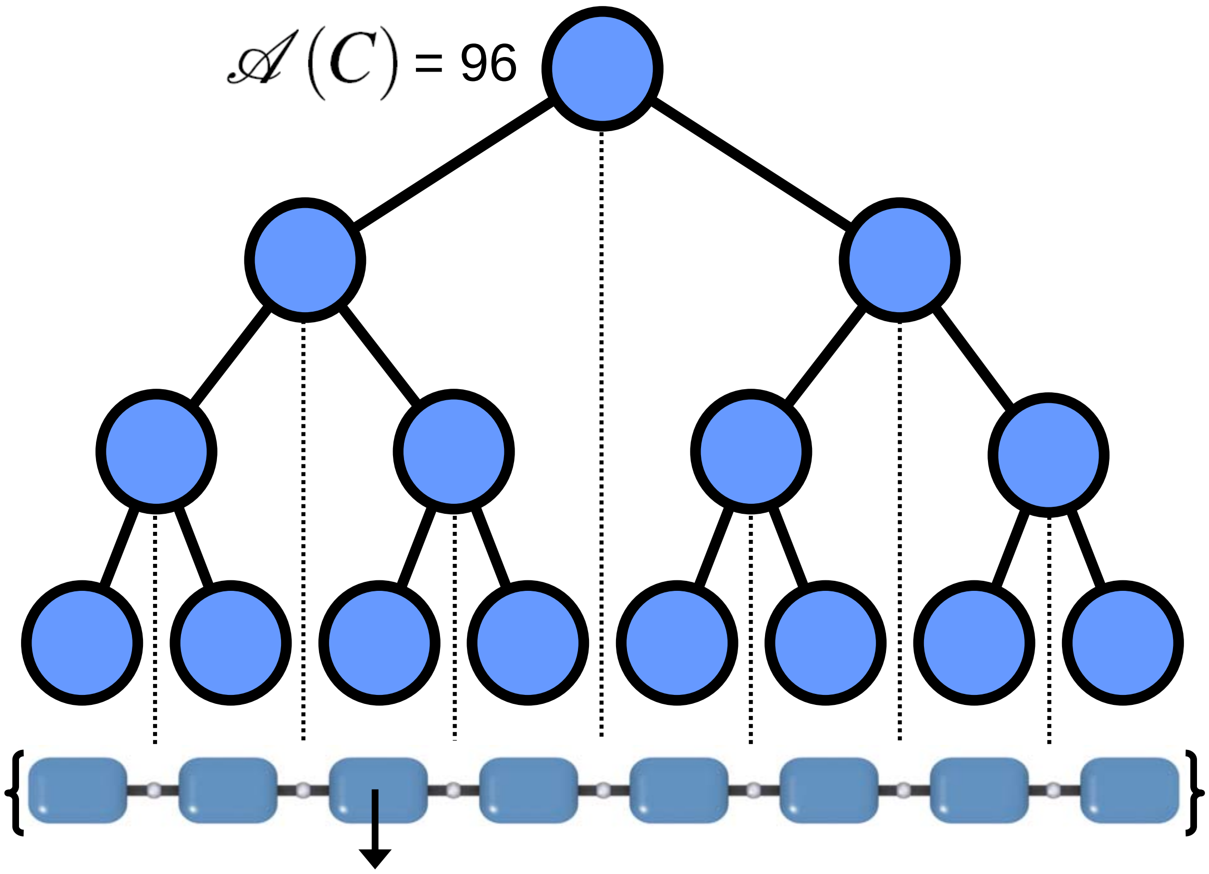
Adaptive joint selection
Acceleration update

Example:



$$\mathscr{A}(C) = 96$$

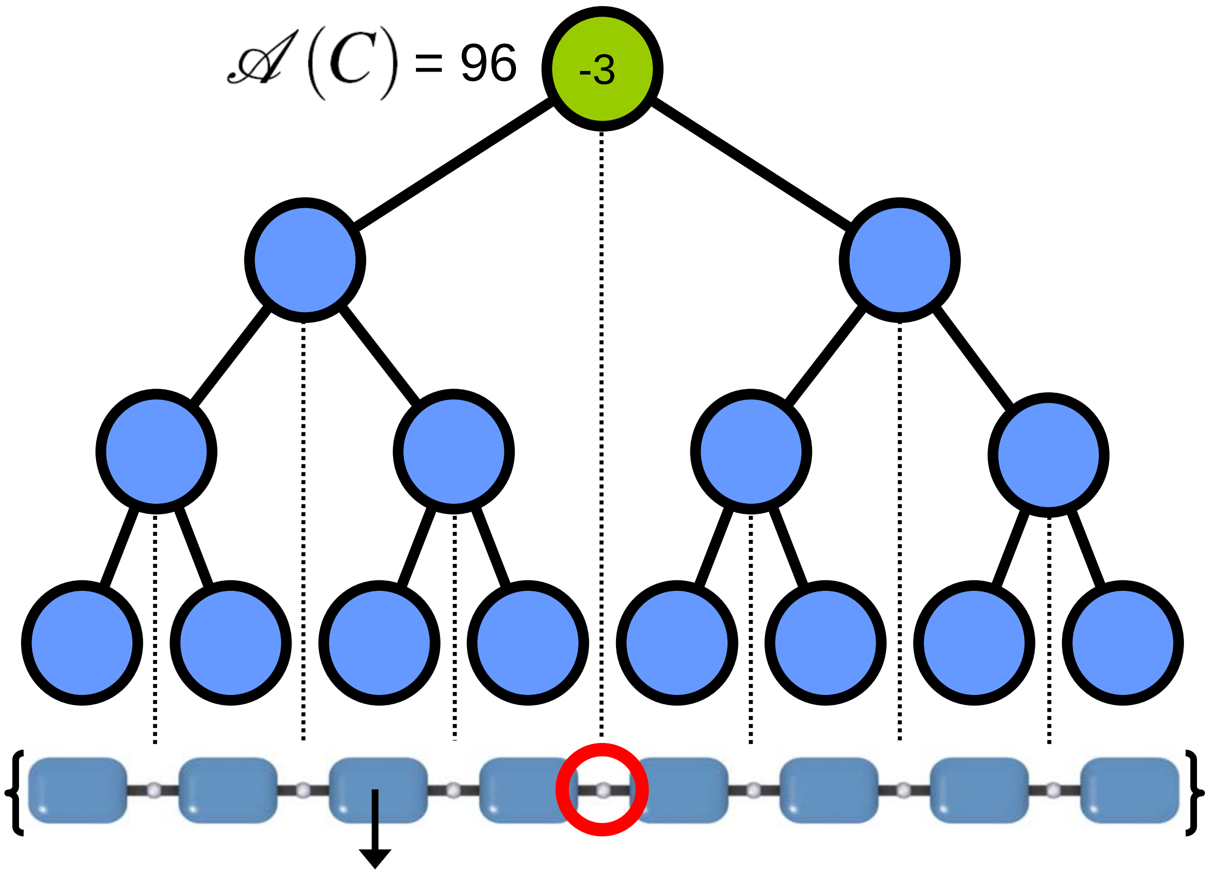
Adaptive joint selection
Acceleration simplification



Compute the acceleration metric value of the root

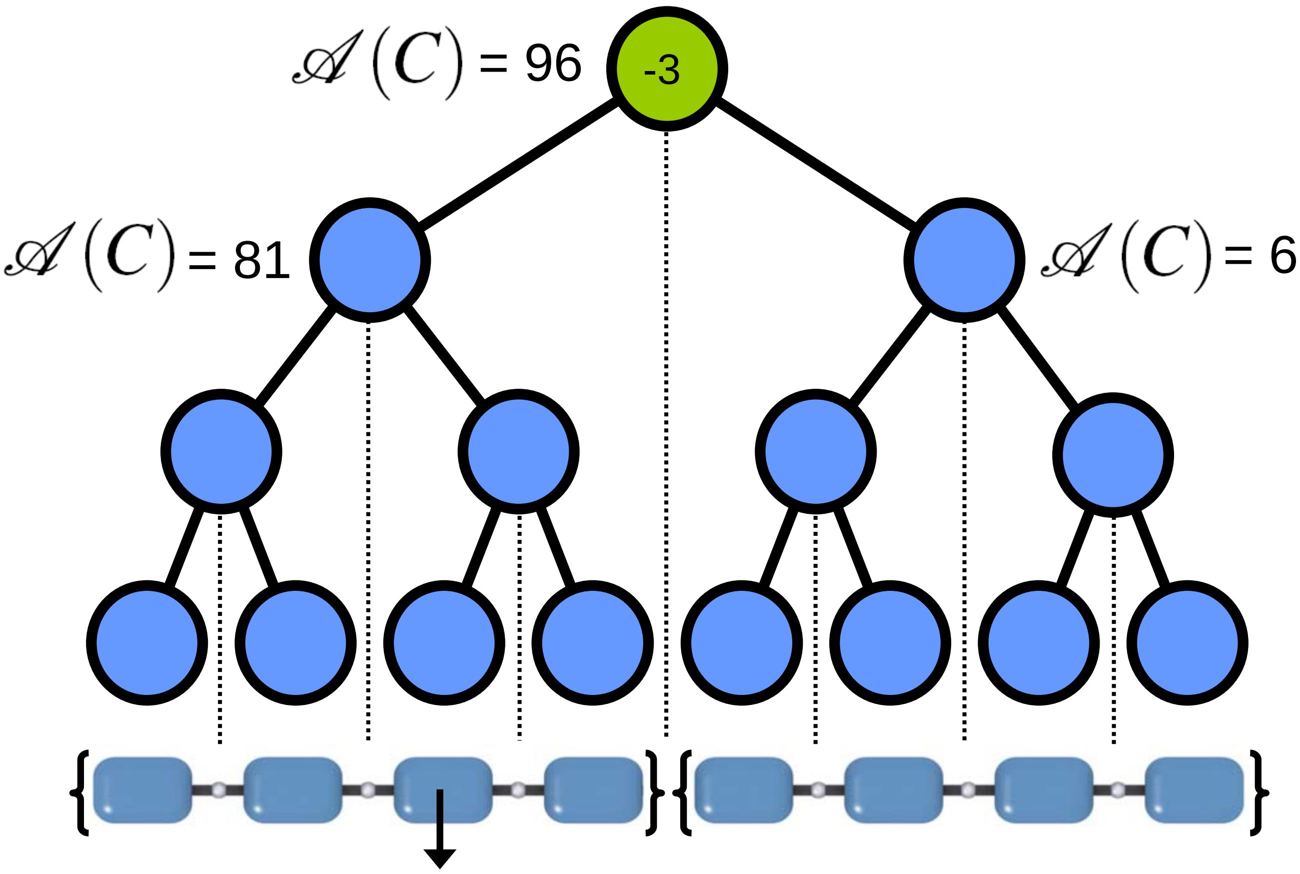
Adaptive joint selection

Acceleration simplification



Compute the joint acceleration of the root

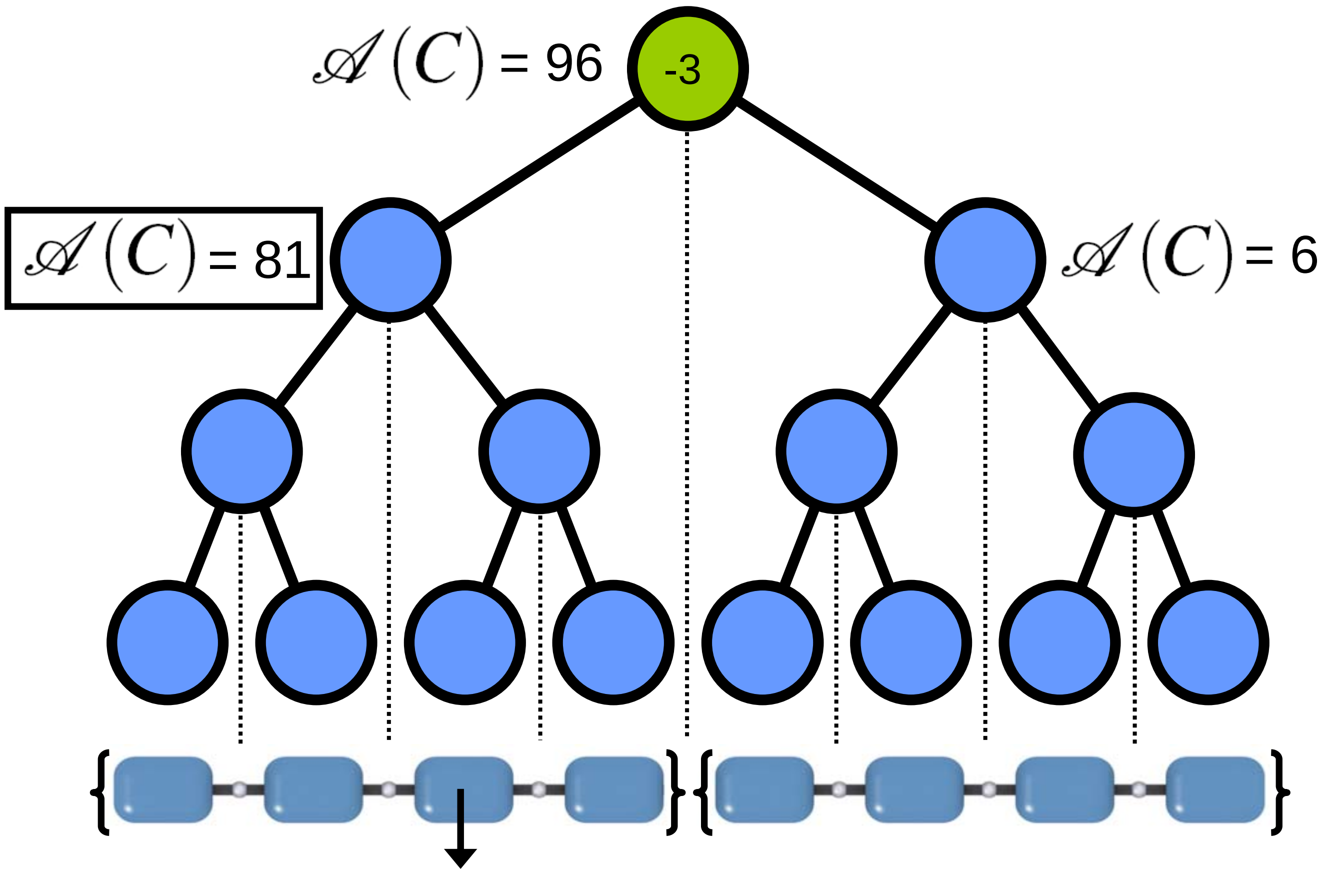
Adaptive joint selection
Acceleration simplification



Compute the acceleration metric values of the two children

Adaptive joint selection

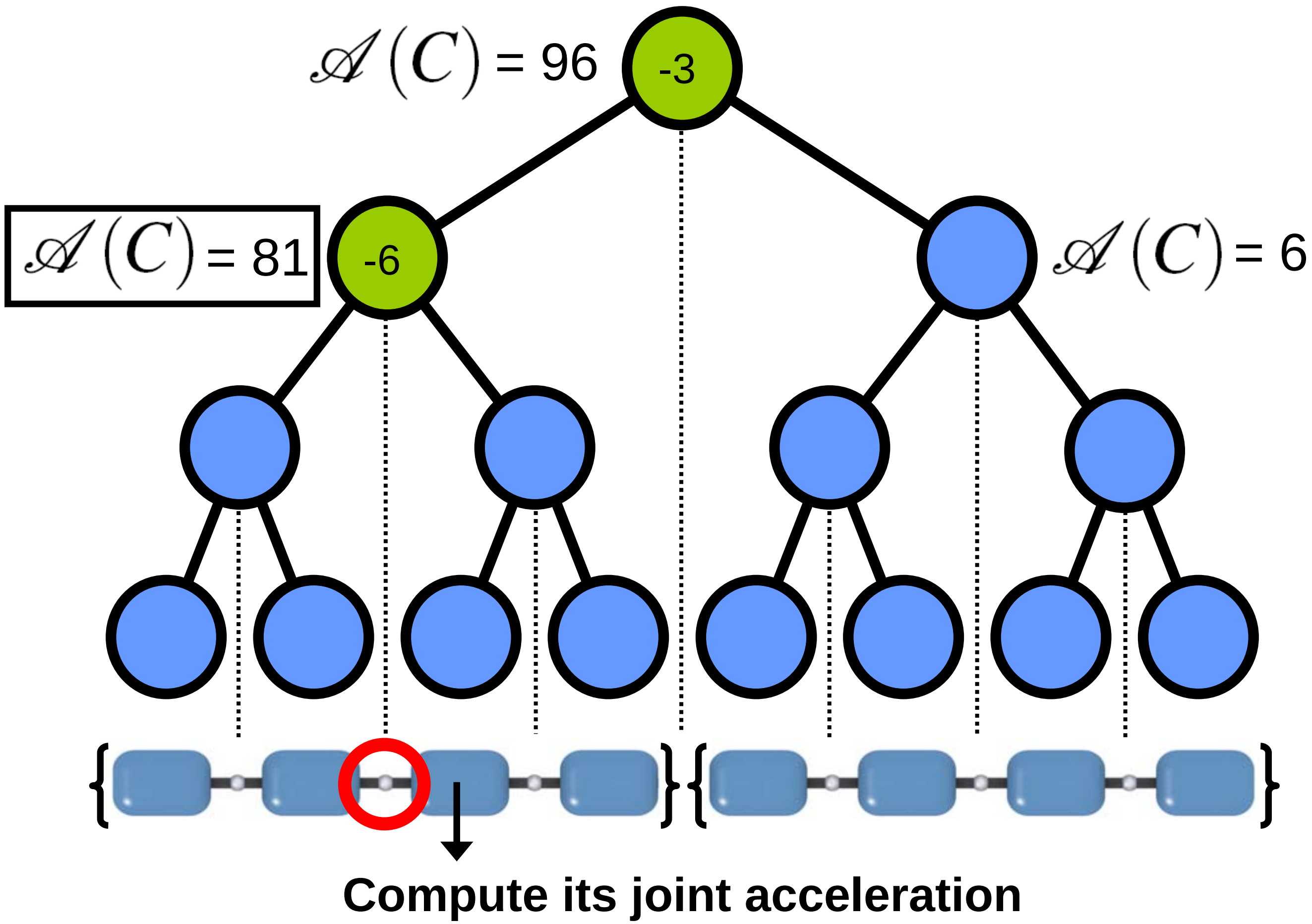
Acceleration simplification



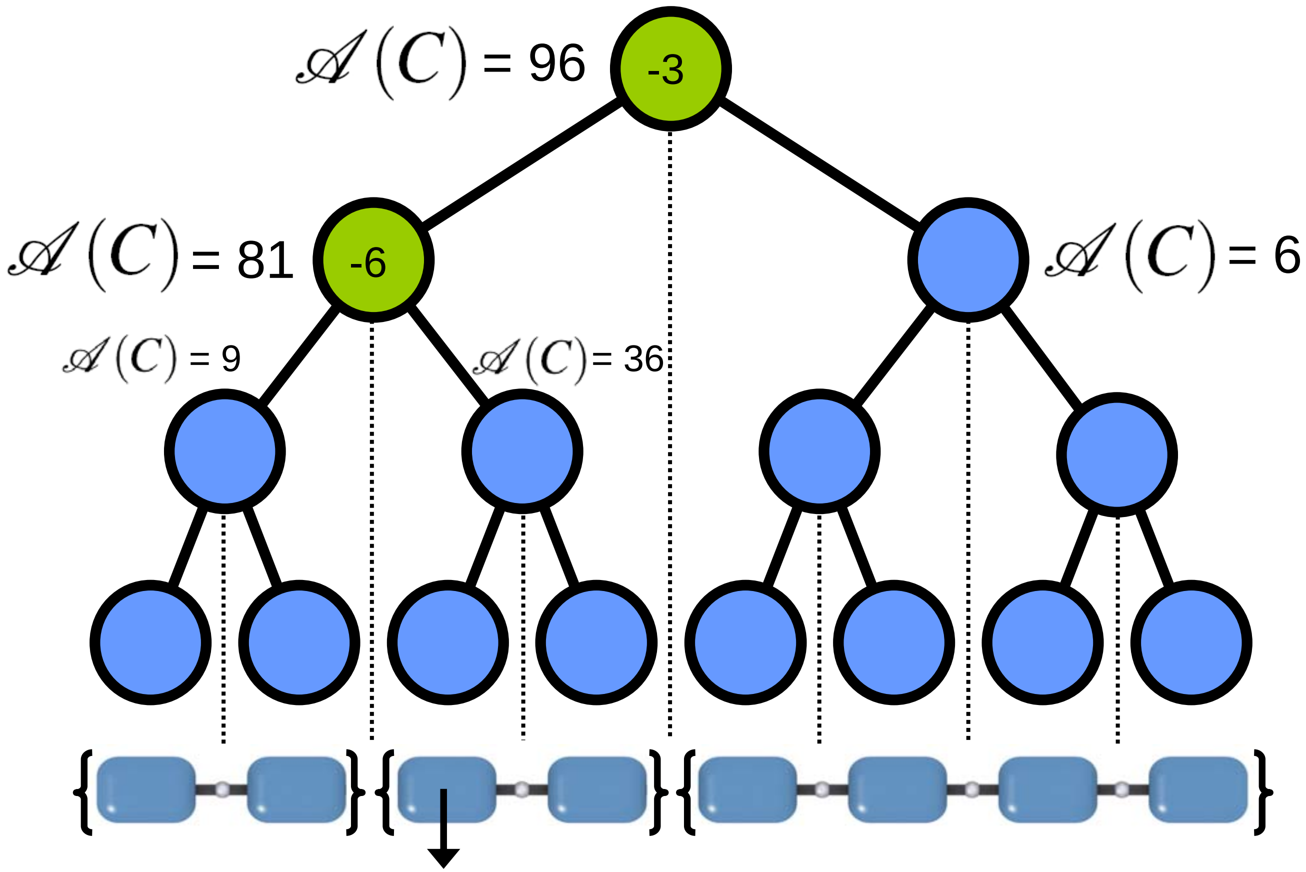
Select the node with the highest acceleration metric value

Adaptive joint selection

Acceleration simplification



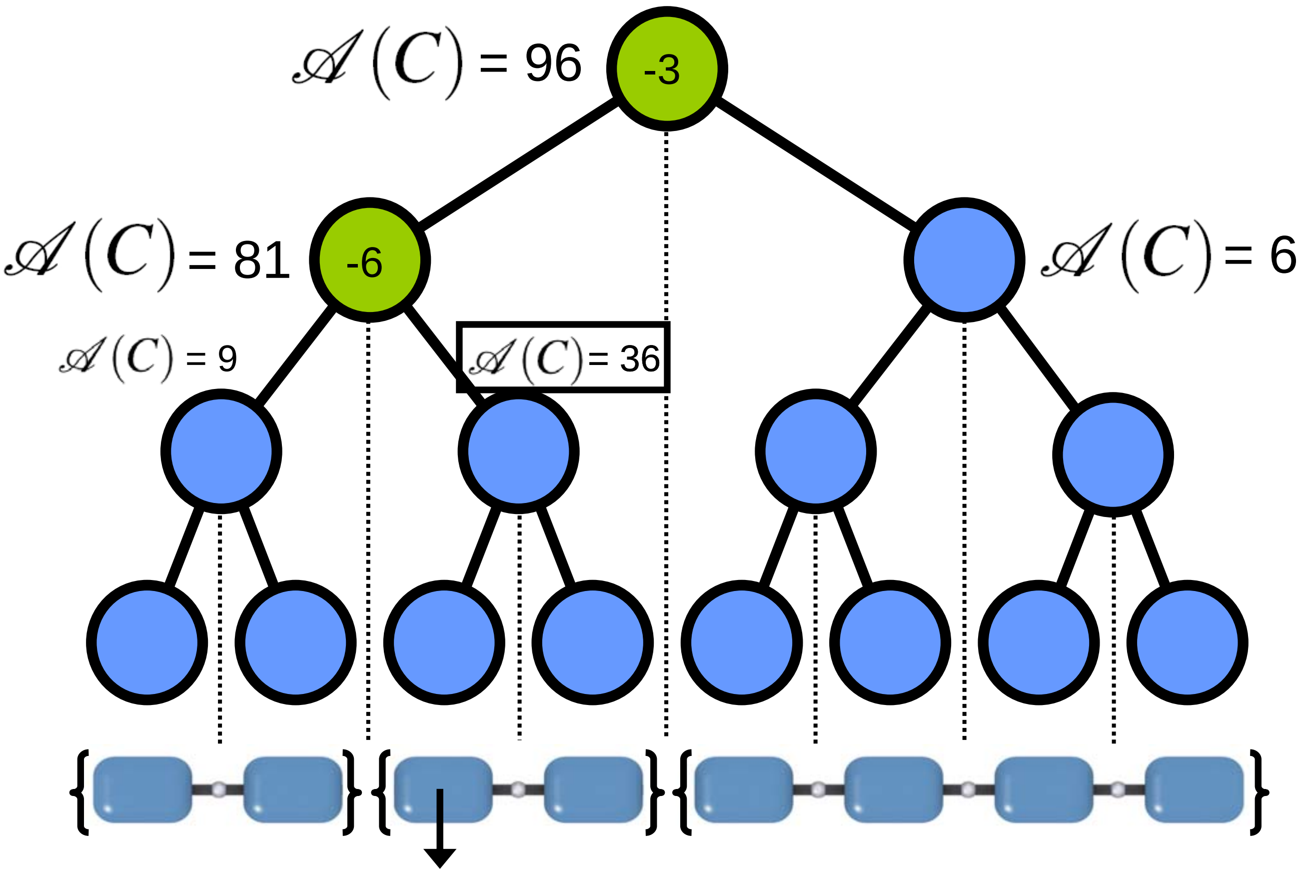
Adaptive joint selection
Acceleration simplification



Compute the acceleration metric values of its two children

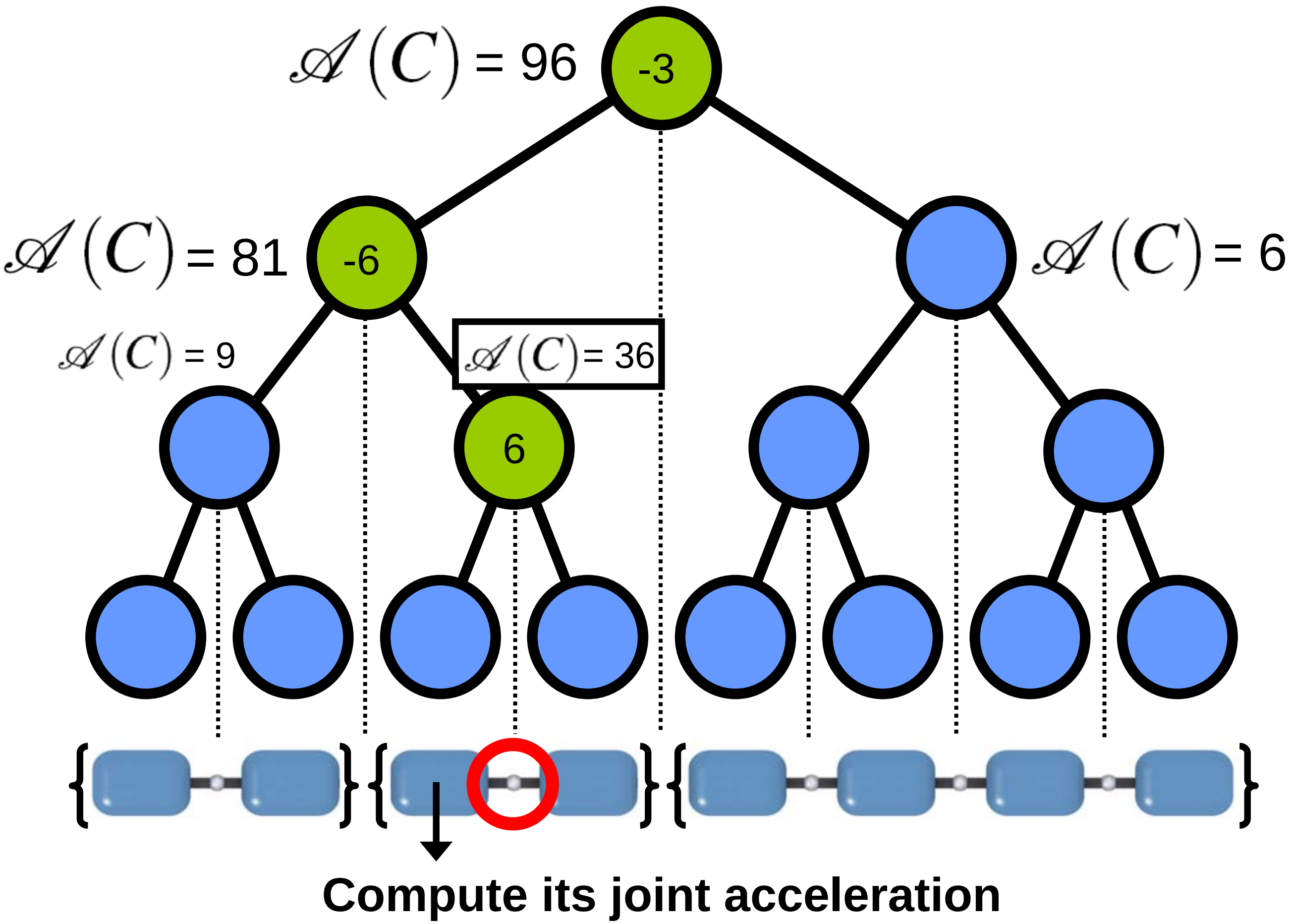
Adaptive joint selection

Acceleration simplification

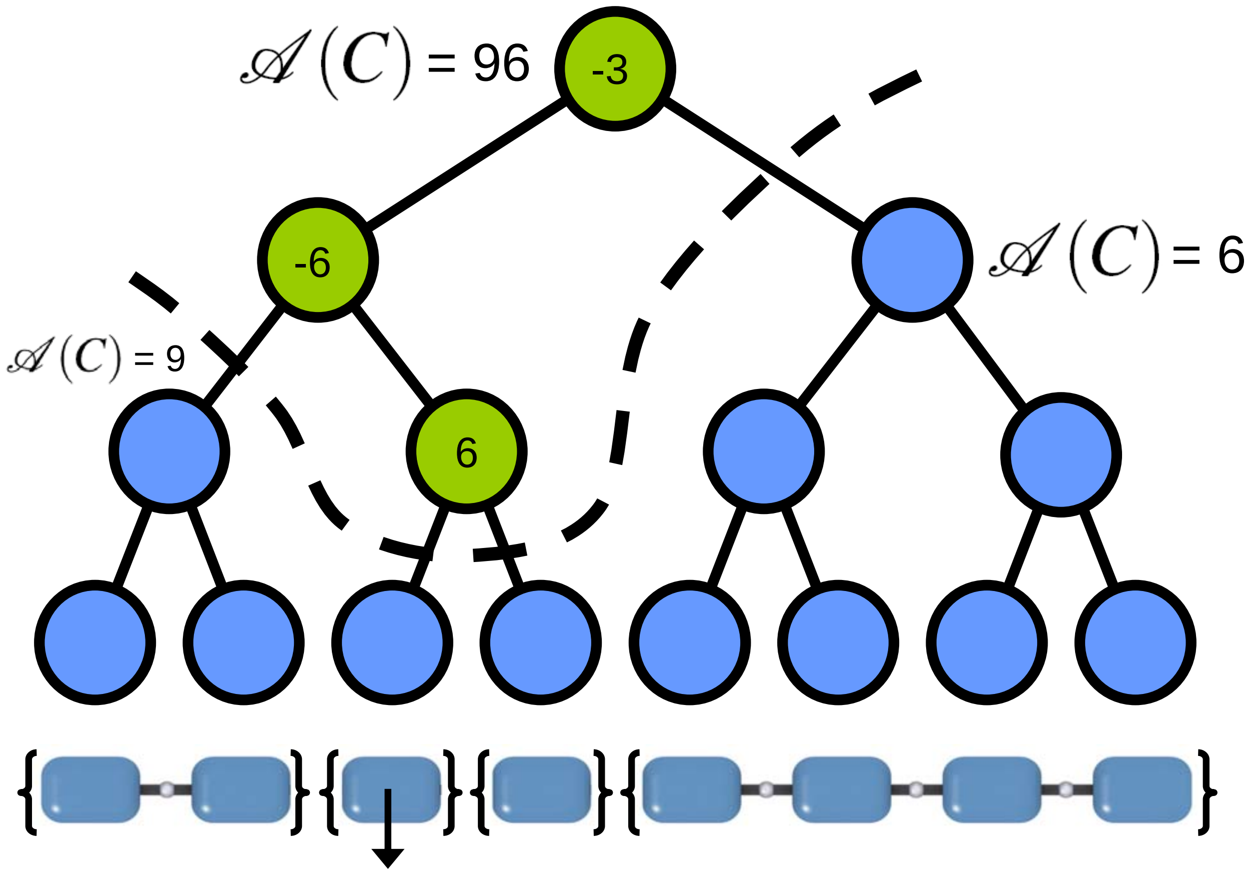


Select the node with the highest acceleration metric value

Adaptive joint selection
Acceleration simplification

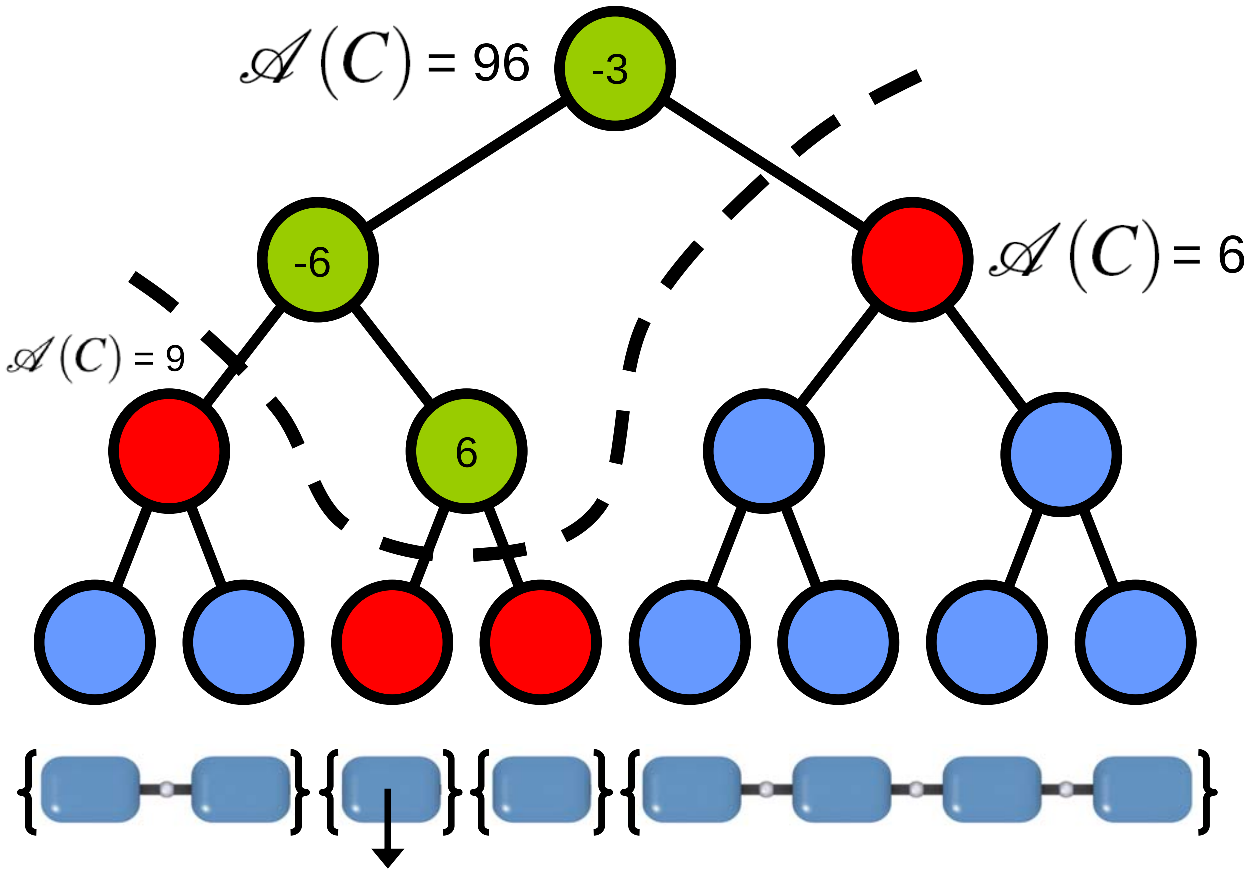


Adaptive joint selection
Acceleration simplification



Stop because a user-defined sufficient precision has been reached

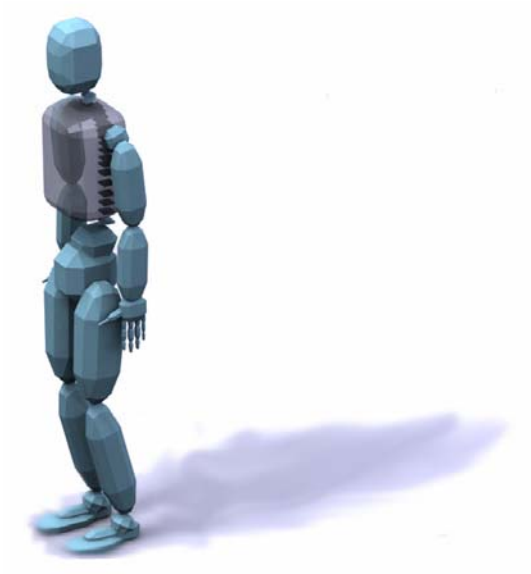
Adaptive joint selection
Acceleration simplification



Four subassemblies with joint accelerations implicitly set to zero

Outline

- Related work
- Hybrid bodies
- Adaptive joint selection
- **Adaptive update mechanisms**
- Results

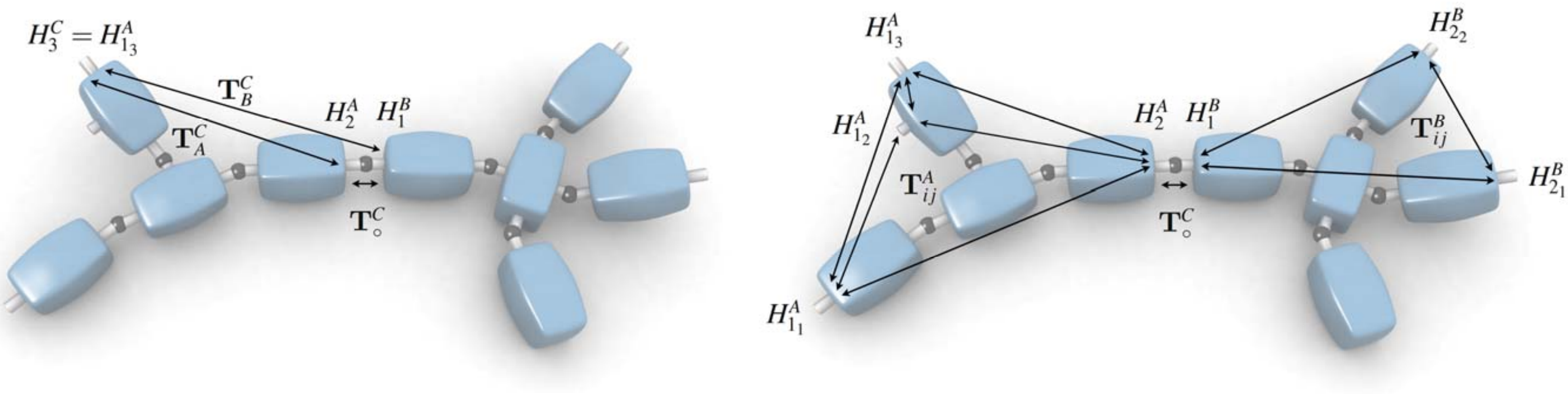


Adaptive update mechanisms
Handling two types of coefficients

Limit the update of the coefficients to a subtree

1. Position-dependent coefficients

   Hierarchical state representation [Redon and Lin 2005]



Adaptive update mechanisms

2. Velocity-dependent coefficients

Linear coefficients tensors:

$$(\mathbf{b}_1^C)_a = (\mathbf{B}_1^C)_{abc}(\mathbf{v}_1^C)_b(\mathbf{v}_1^C)_c \qquad (\mathbf{p}_1^C)_a = (\mathbf{P}_1^C)_{abc}(\mathbf{v}_1^C)_b(\mathbf{v}_1^C)_c$$
$$(\mathbf{b}_2^C)_a = (\mathbf{B}_2^C)_{abc}(\mathbf{v}_2^C)_b(\mathbf{v}_2^C)_c \qquad (\mathbf{p}_2^C)_a = (\mathbf{P}_2^C)_{abc}(\mathbf{v}_2^C)_b(\mathbf{v}_2^C)_c$$

$$\eta^C = (\mathbf{E}^C)_{abcd}(\mathbf{v}^C)_a(\mathbf{v}^C)_b(\mathbf{v}^C)_c(\mathbf{v}^C)_d$$

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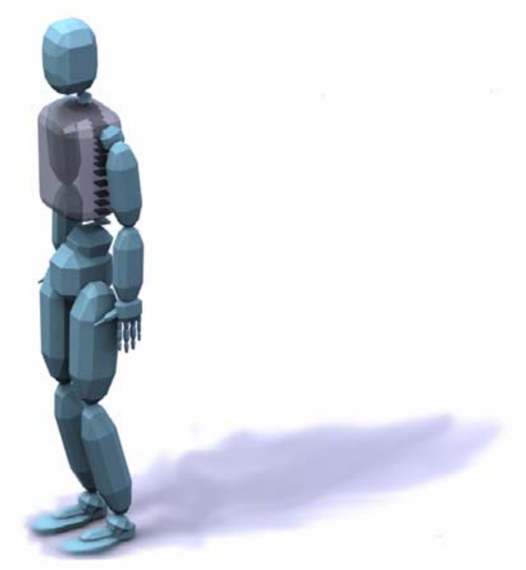
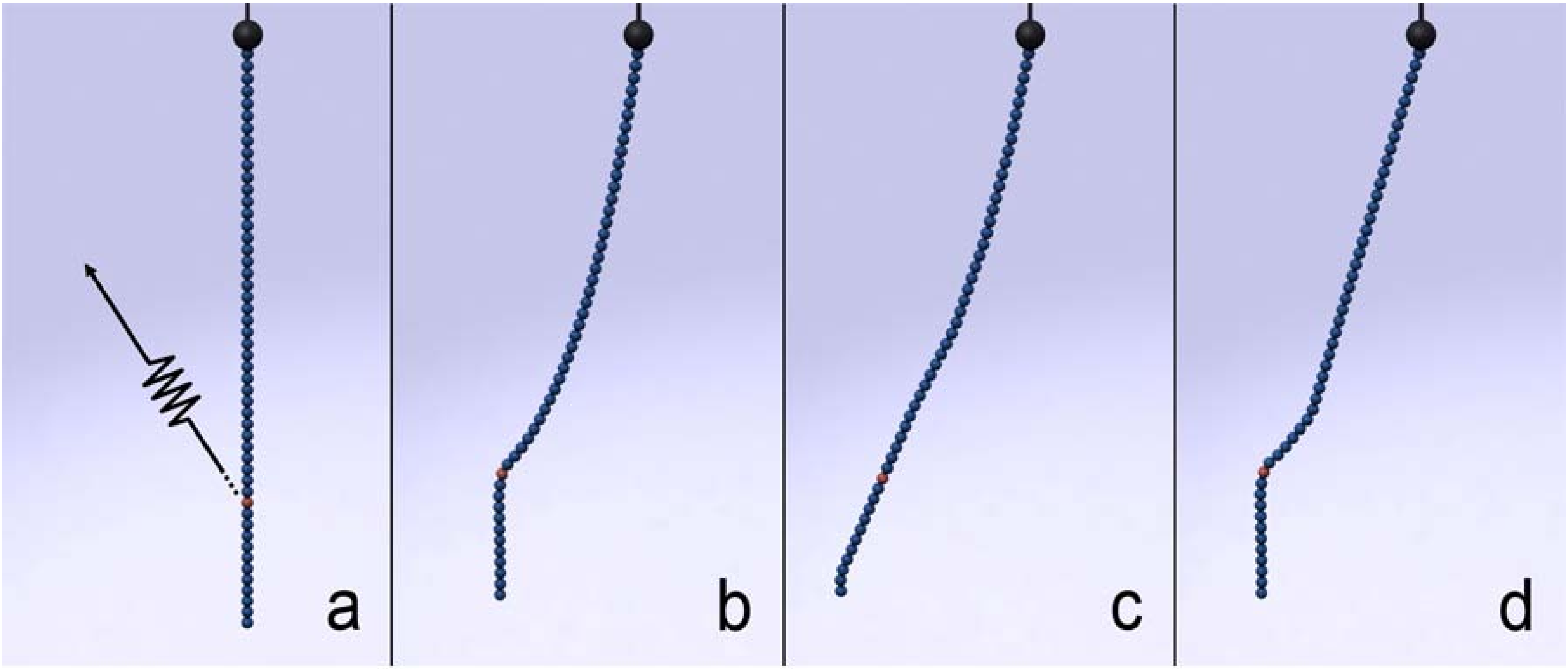
- **Results**



Results

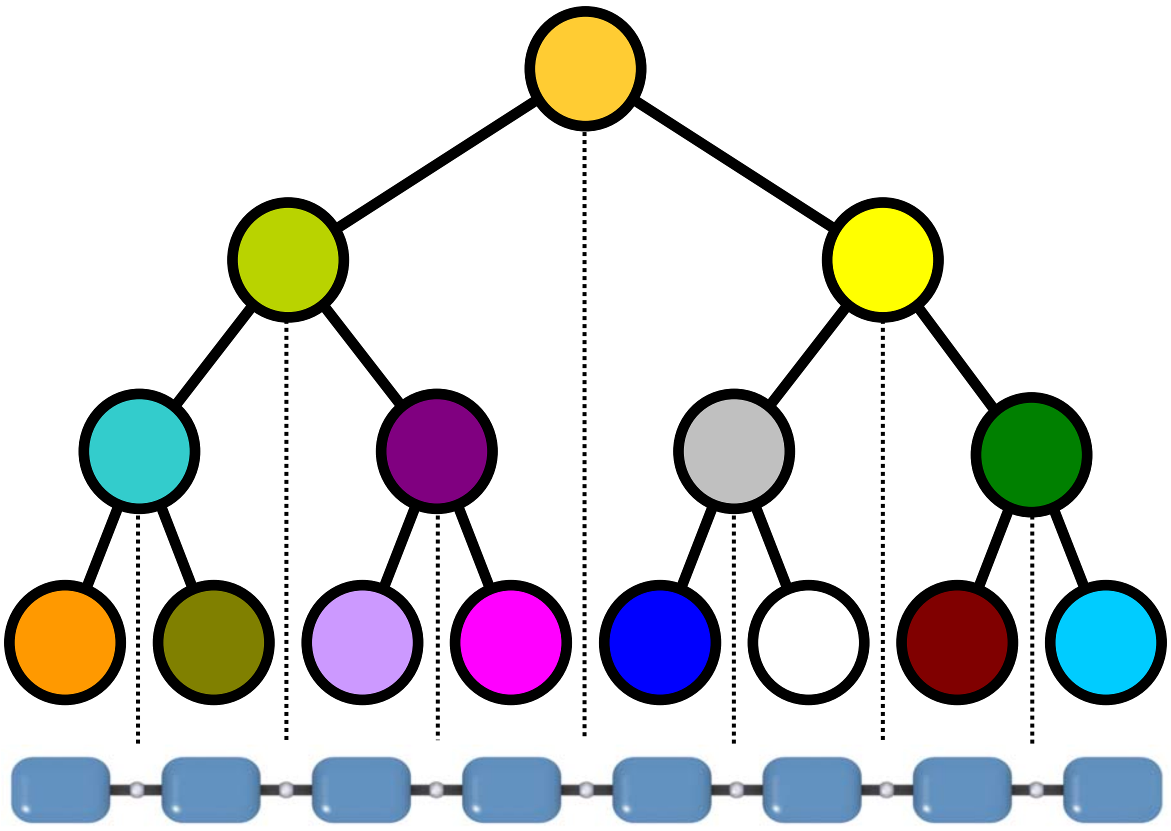
Adaptive joint selection



Adaptive joint selection example (10x speed-up)

Results

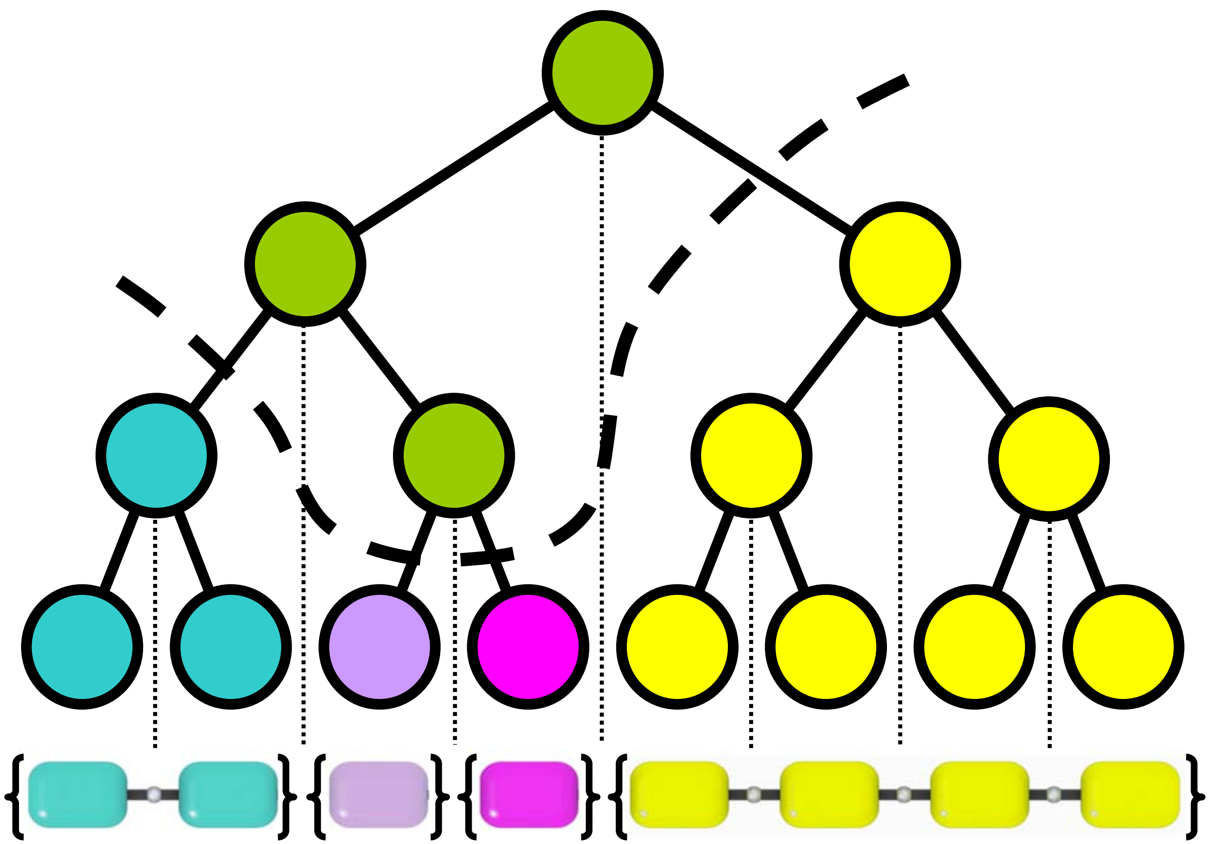
Time-dependent simplification



One color per sub-assembly

Results

Time-dependent simplification



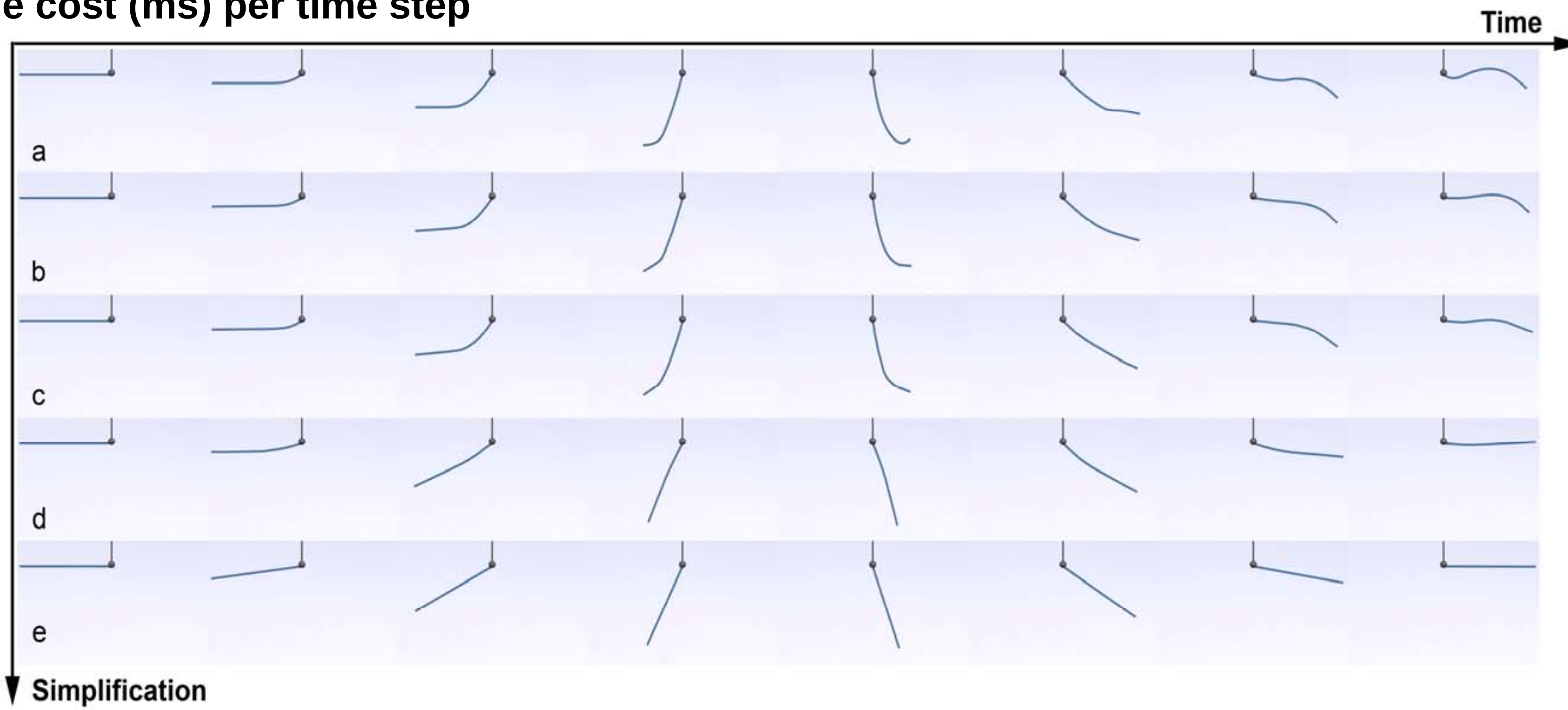
One color per sub-assembly

Results

Progressive simplification of motion

Average cost (ms) per time step

N	Average cost
N=300	5ms
N=100	1.7ms
N=50	0.7ms
N=20	0.25ms
N=1	0.02ms



a 300-link pendulum, N- number of active joints

Results

Precision / Performance trade-off

# of active joints

# of external forces

	N_A						
	Adaptive dynamics						DCA
N_F	1	50	100	150	200	250	300
1	0.02	0.51	1.71	2.97	4.04	5.40	5.89
5	0.07	0.49	1.54	3.21	4.34	5.19	5.89
10	0.16	0.73	1.47	3.03	4.22	5.16	5.88
20	0.40	0.90	1.60	3.00	4.72	5.56	5.90
50	0.98	1.40	2.16	3.58	4.99	5.59	5.92
250	1.83	2.30	2.97	4.28	5.43	6.31	6.04

ms per interation

Results

Test application

MOVIE