Systematically Accelerating the Fracture Simulation Workflow using Shape Matching Techniques

Yulin Jin

Energy Resources Engineering

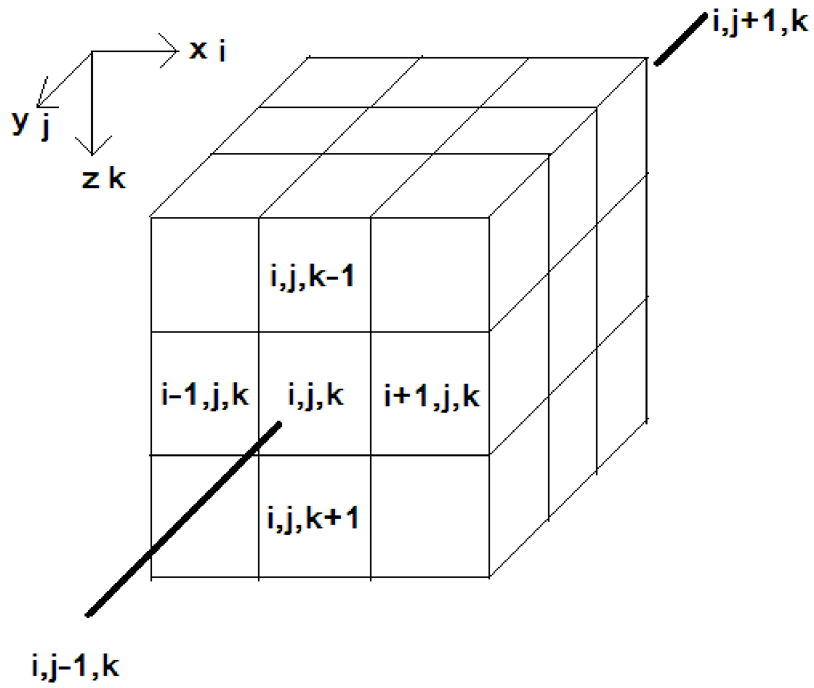
CS 468 Project Presentation

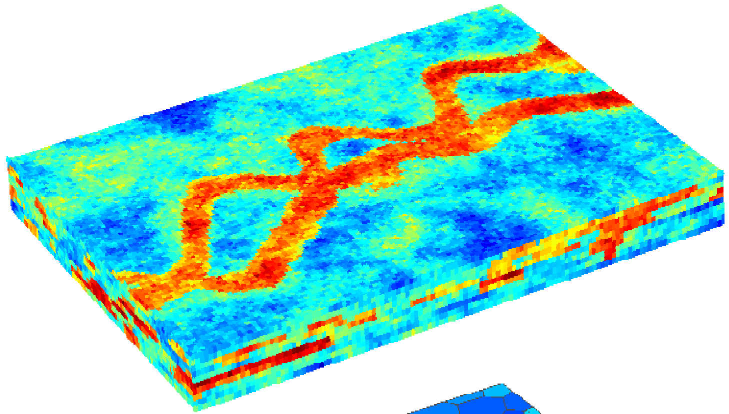
Content

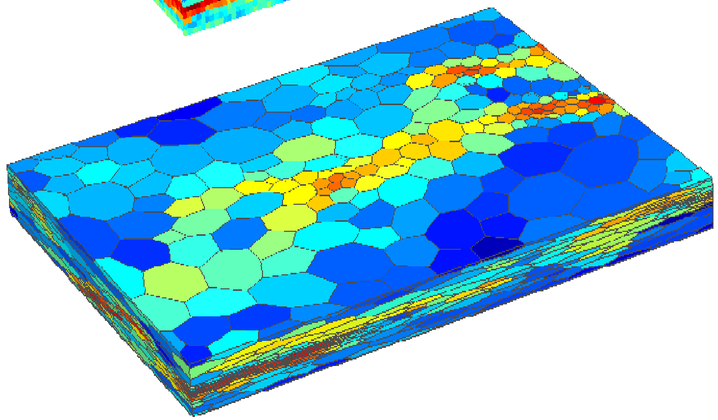
Reservoir Simulation

Finite Volume Methods

- Computational cost is very expensive
- Computational time increases dramatically when the number of cells increases

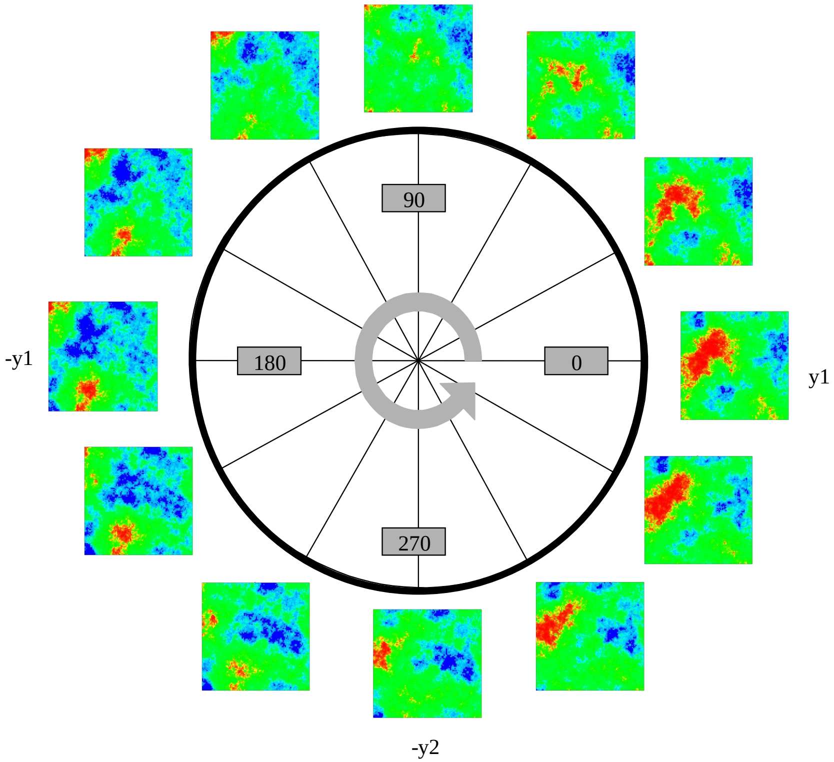


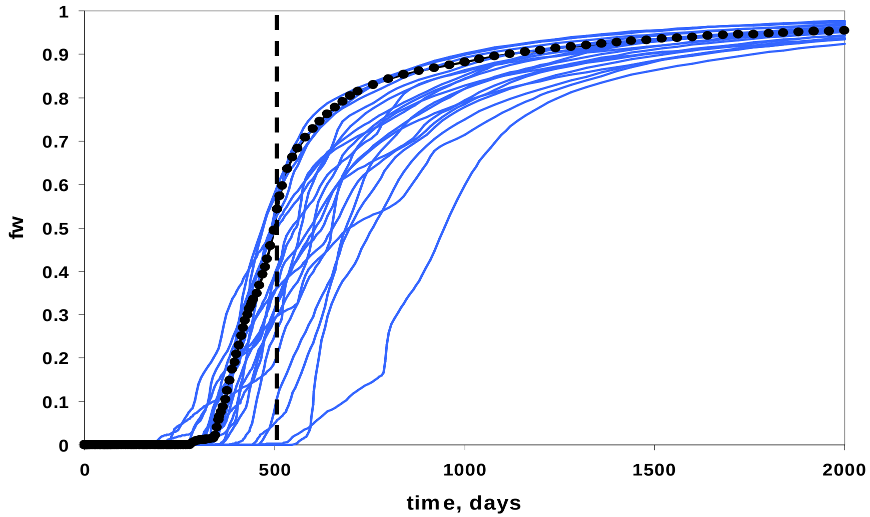




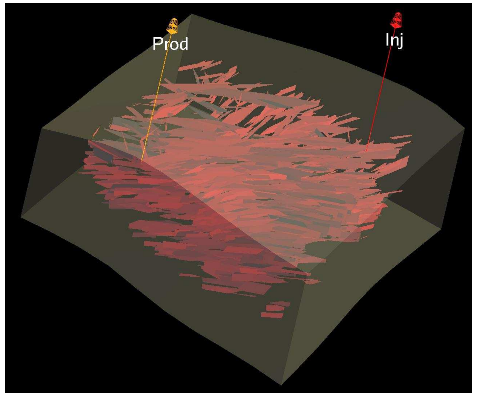
Goal of Simulation: P10-P50-P90 Analysis

- Geostatistics
- Run multiple simulations to consider the uncertainty

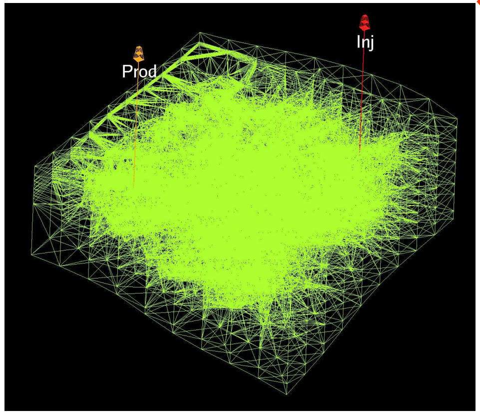




Fracture Modeling Workflow



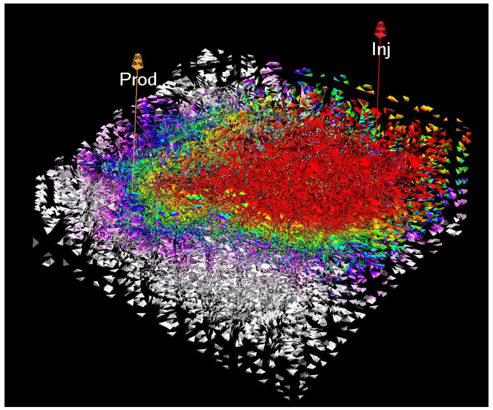
Fracture Characterization



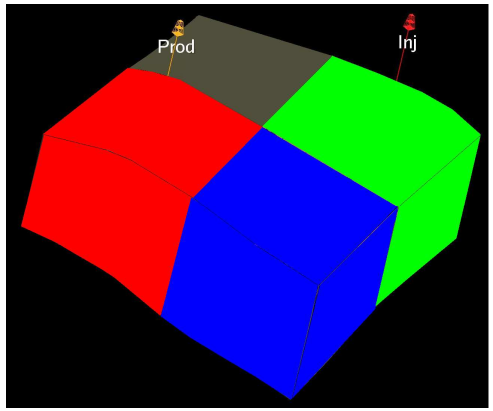
Gridding of Unstructured DFM

3M cells

Save 72h/2h = 36 times!



Simulation on DFM



MSR & DPDP

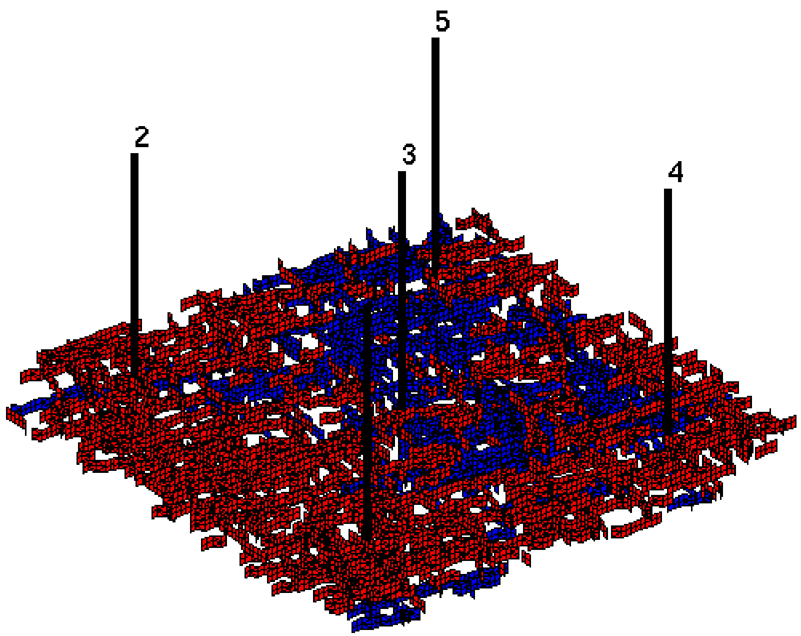
Systematically Accelerating the Fracture Simulation Workflow using Shape Matching Techniques

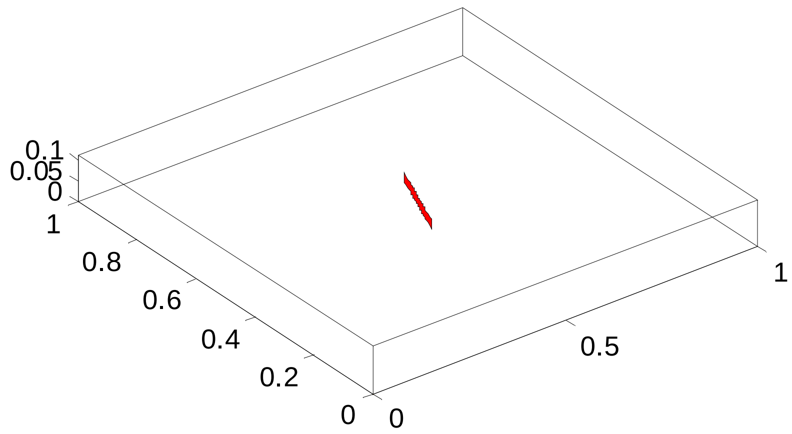
Content

- Motivation and Background Information
- Connectivity Analysis and Sampling
- Heterogeneous Case
- Homogeneous Case
- Instinct Methods
- Performance

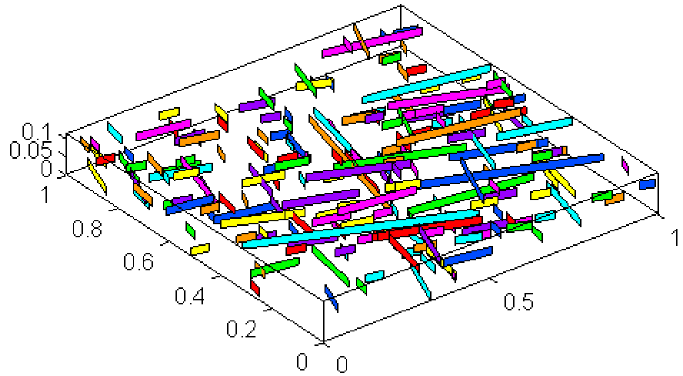
Assumptions

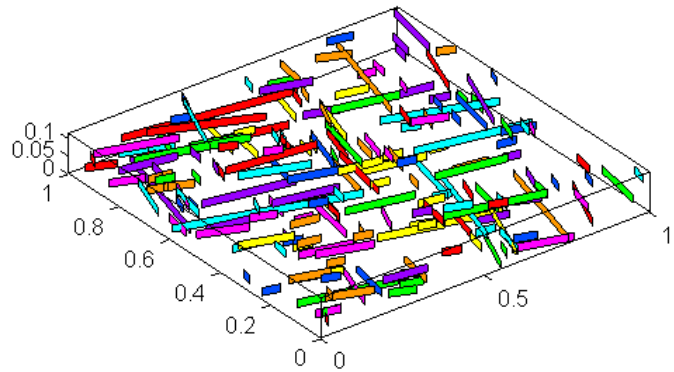
- 1 Injector in the center, 4 producers on the corners
- 200 fractures
- Injector connects to 1 fracture

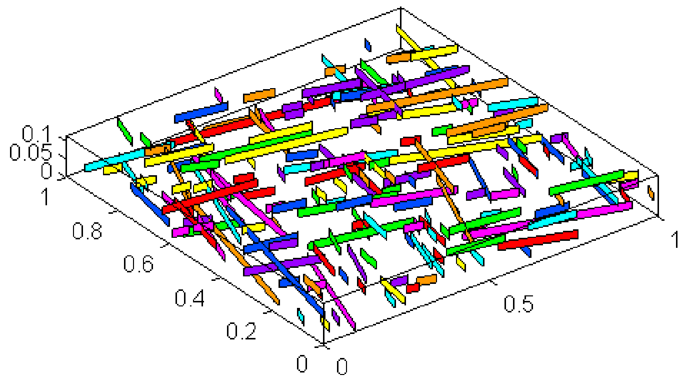


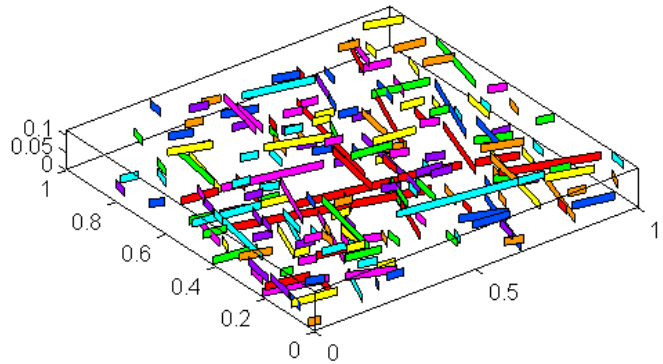


Different Fracture realizations with the Same Distribution

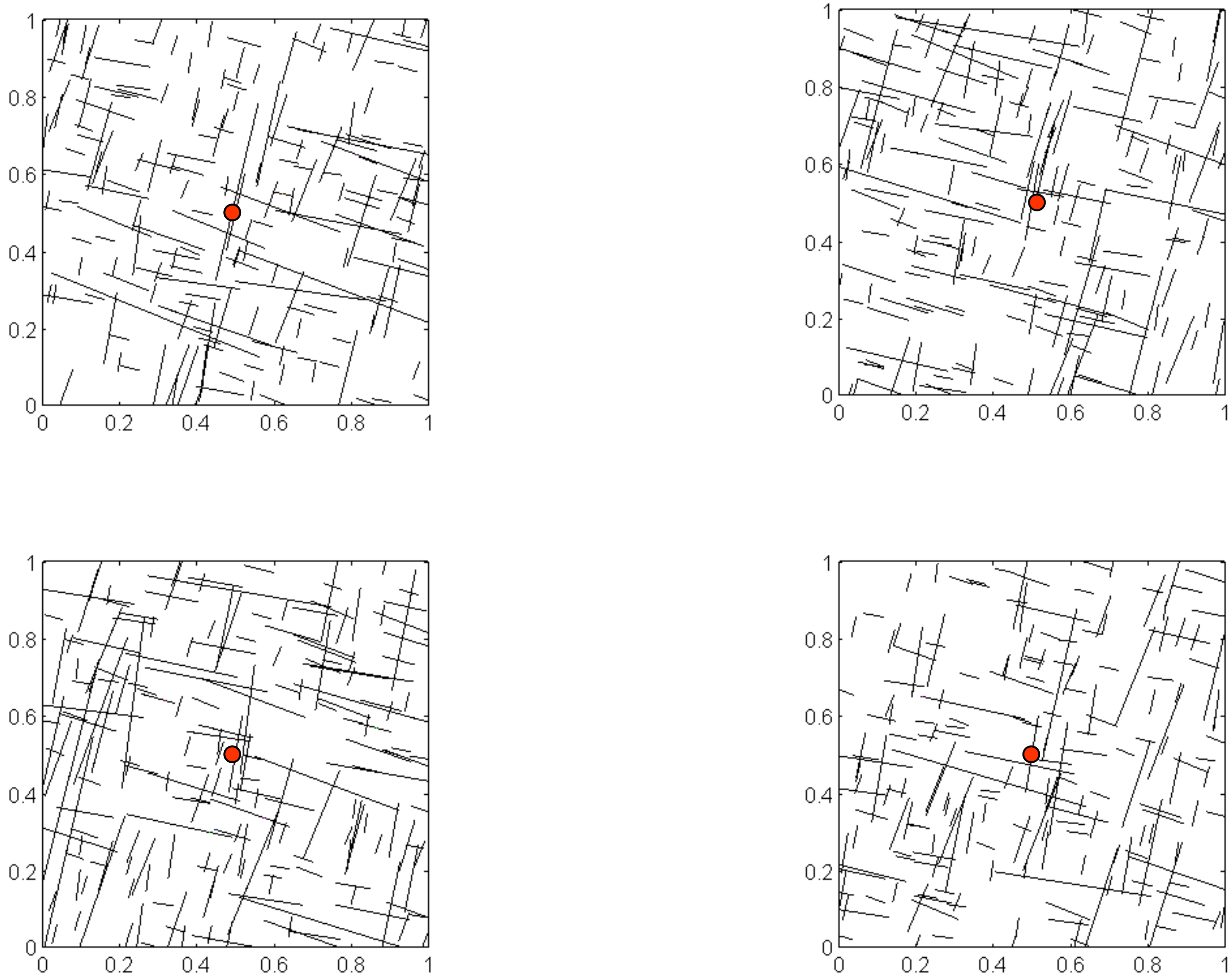




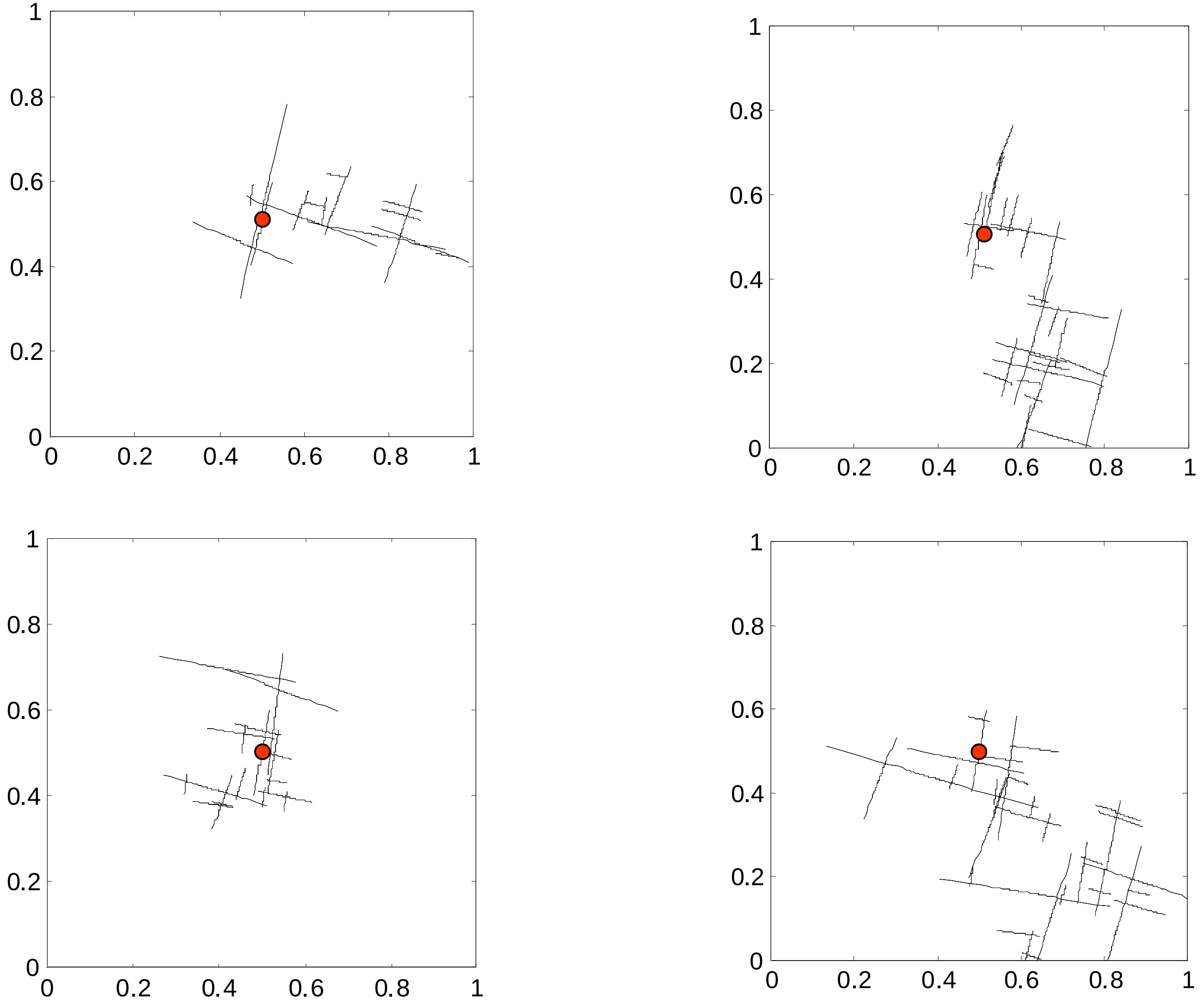




Different Fracture realizations with the Same Distribution

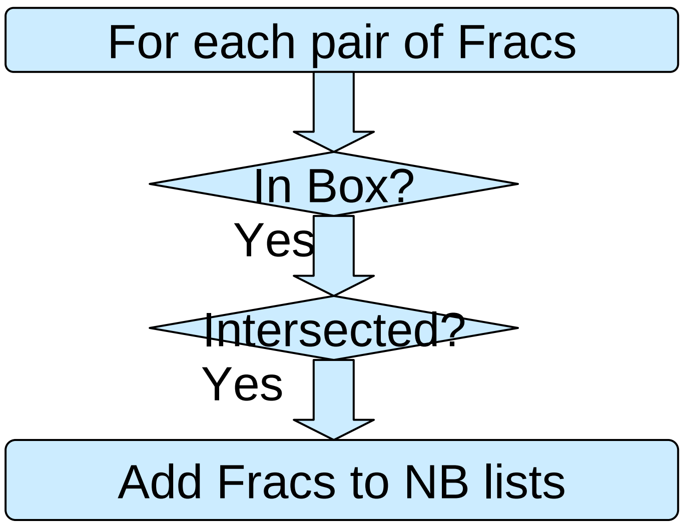


Results of Connectivity Analysis

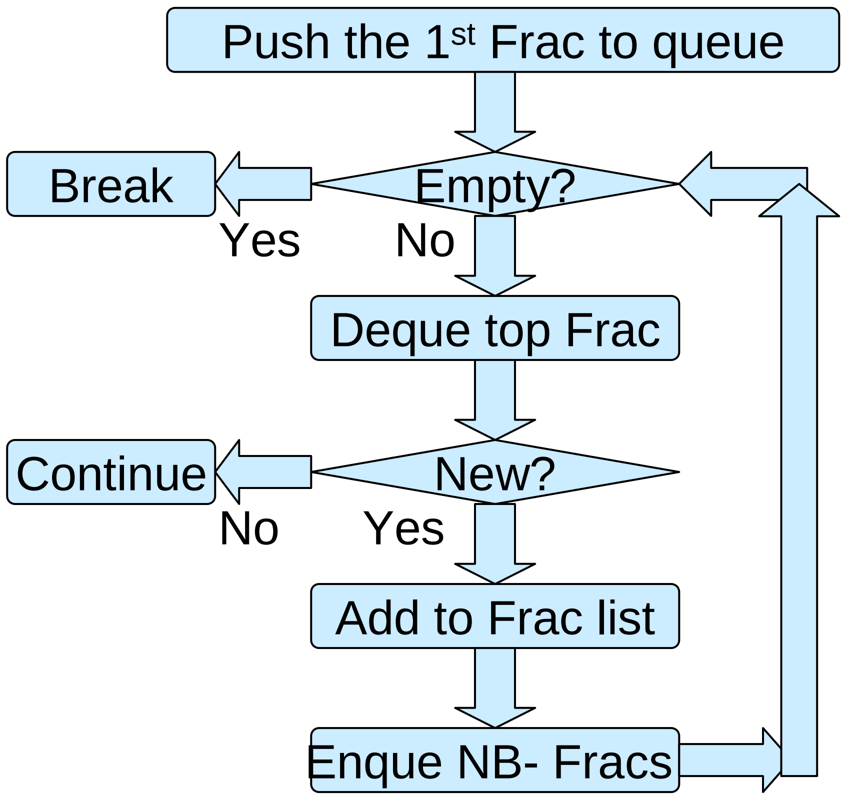


Algorithm of well connectivity analysis

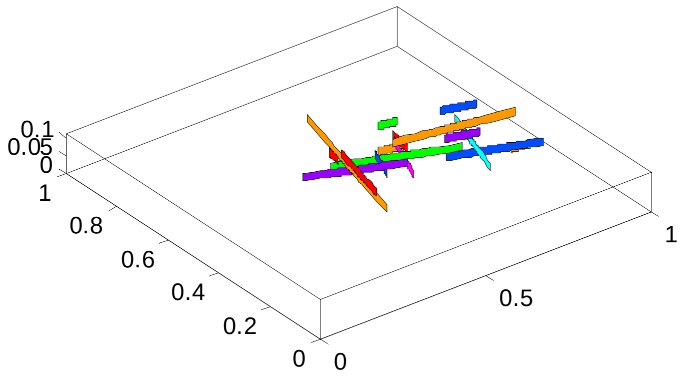
1.Intersection Detection

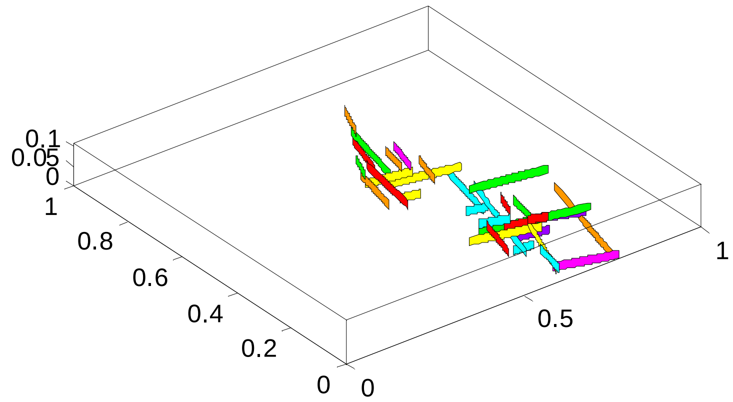


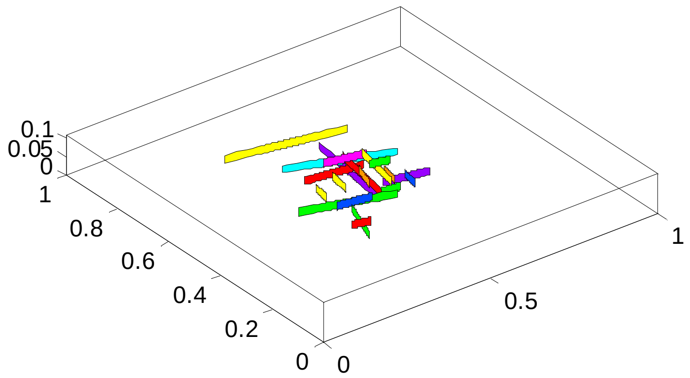
2.Breadth Frist Search

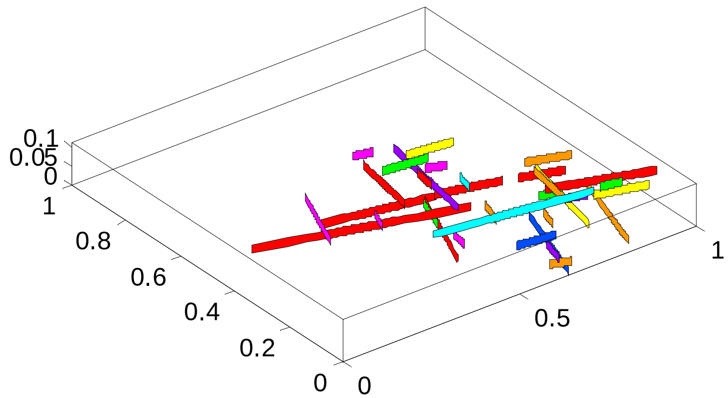


Fractures Connecting to the Injector





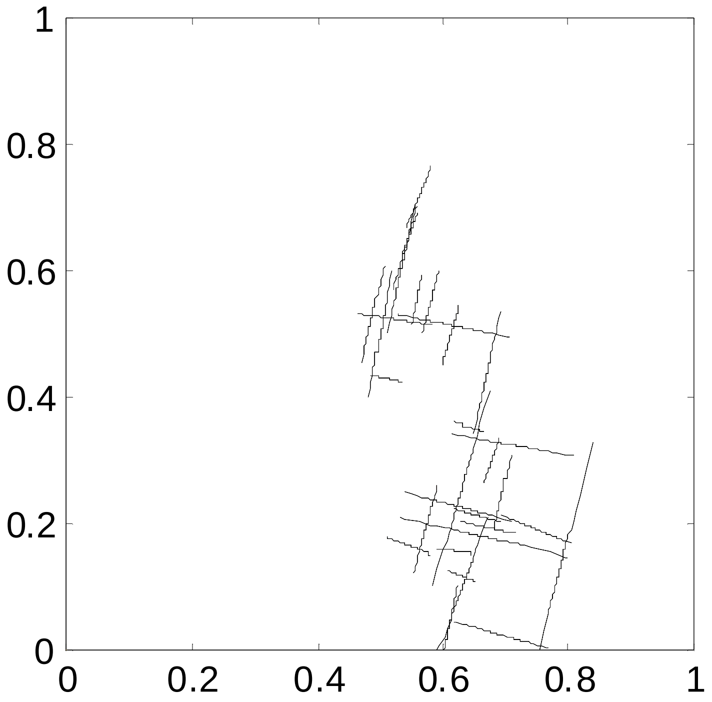


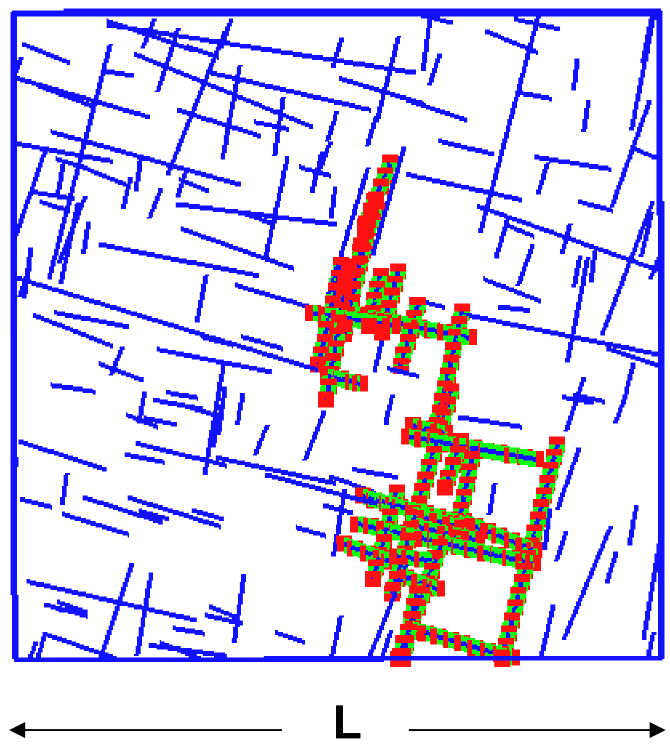


Sampling

Distance of points = coeff*L

where coeff = 0.02





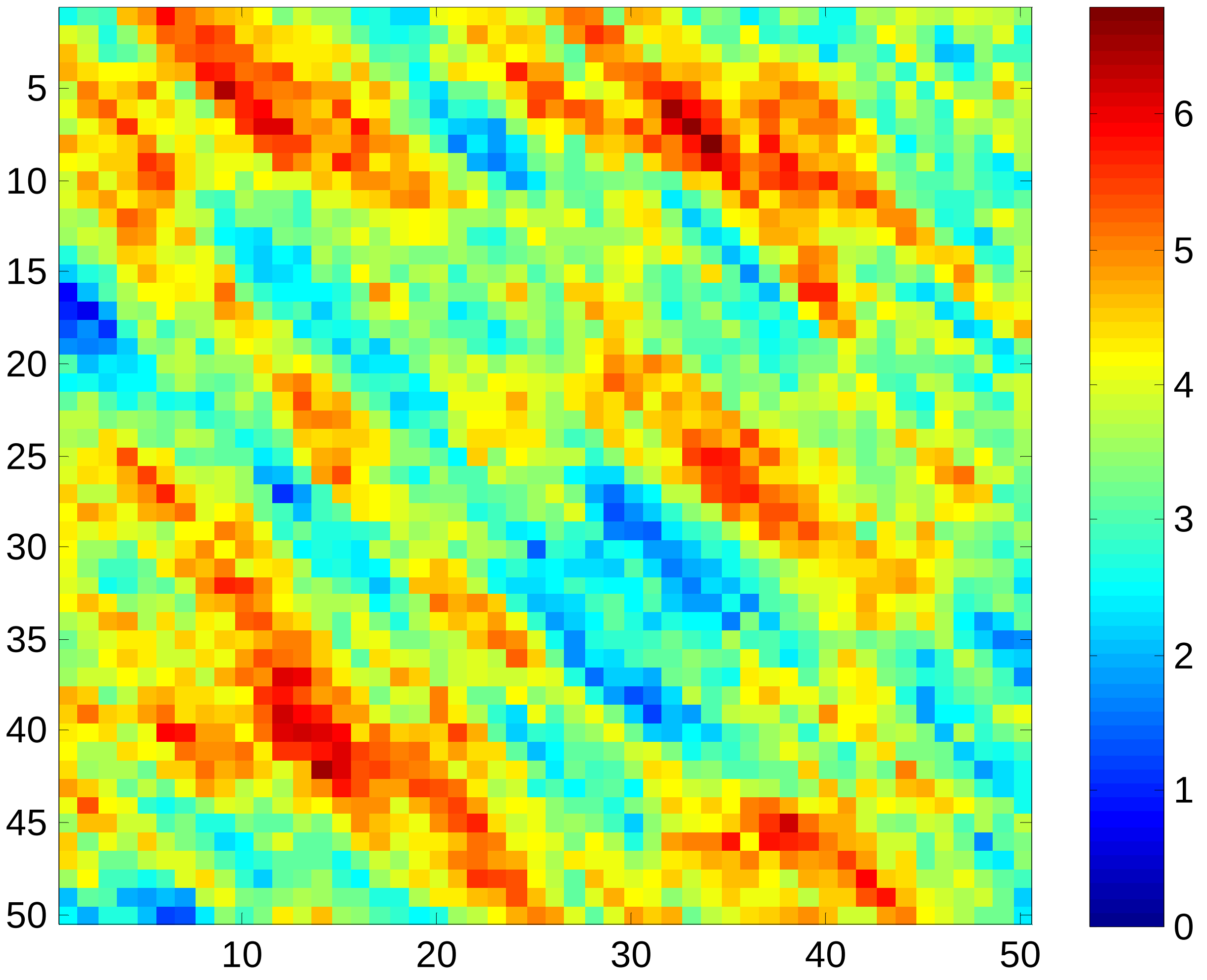
Systematically Accelerating the Fracture Simulation Workflow using Shape Matching Techniques

Content

- Motivation and Background Information
- Connectivity Analysis and Sampling
- Heterogeneous Case
- Homogeneous Case
- Instinct Methods
- Performance

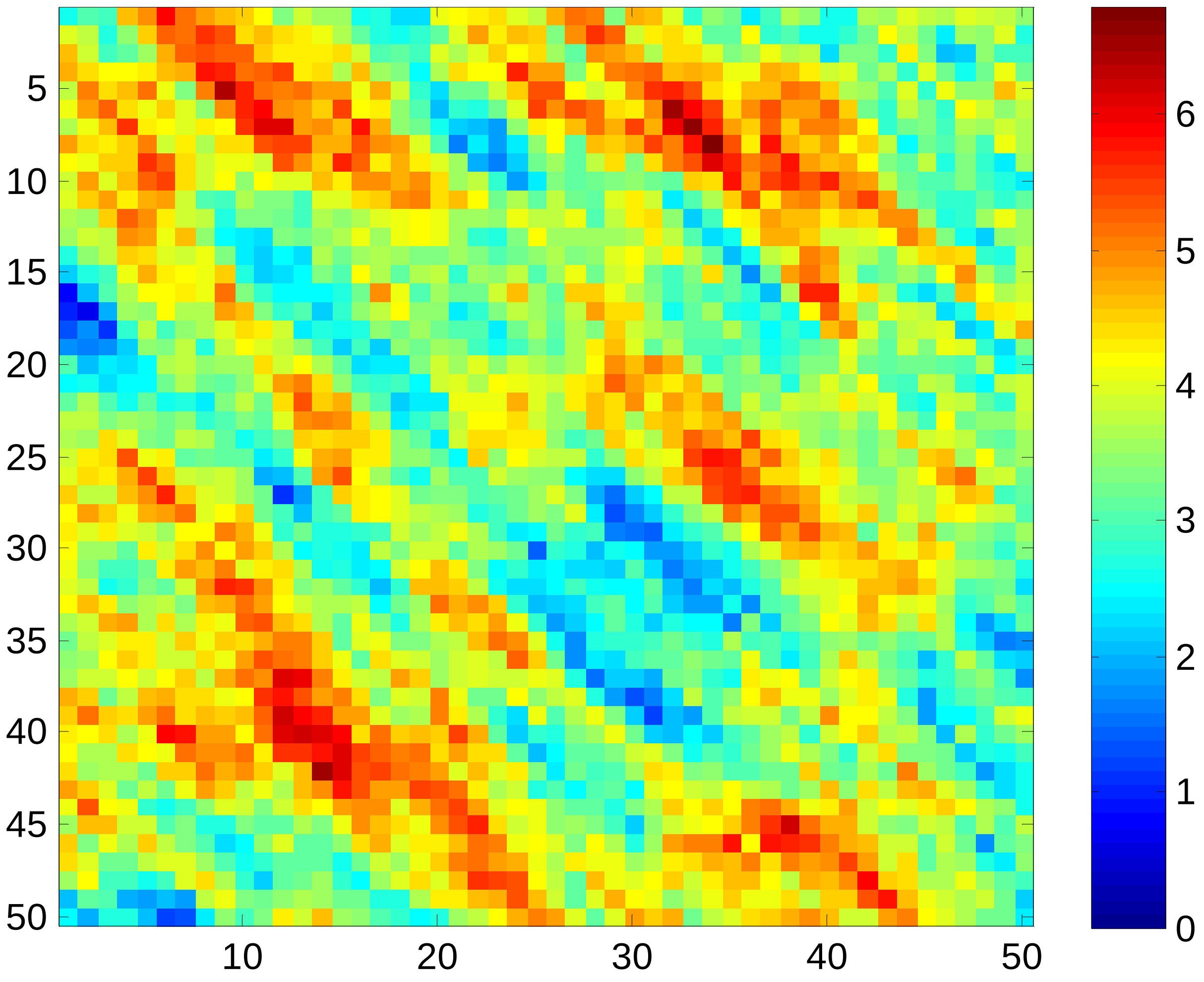
Heterogeneous Permeability Field

Log-Normal Distribution



Heterogeneous Permeability Field

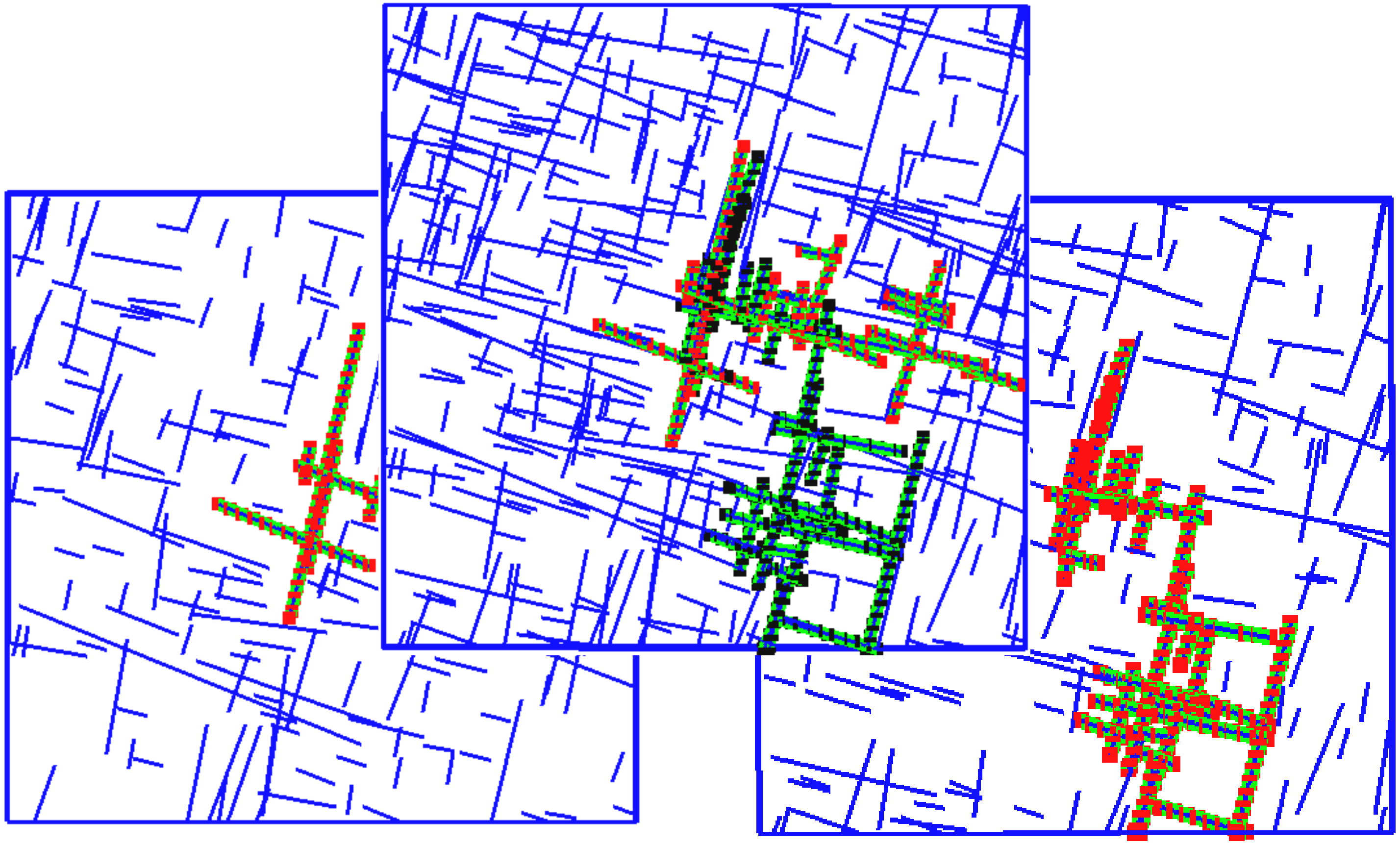
Log-Normal Distribution



Hausdorff Distance

$$d_H^Z(A,B) := \max(\sup_{a\in A}\inf_{b\in B} d(a,b), \sup_{b\in B}\inf_{a\in A} d(a,b))$$

$$d_{ab} = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

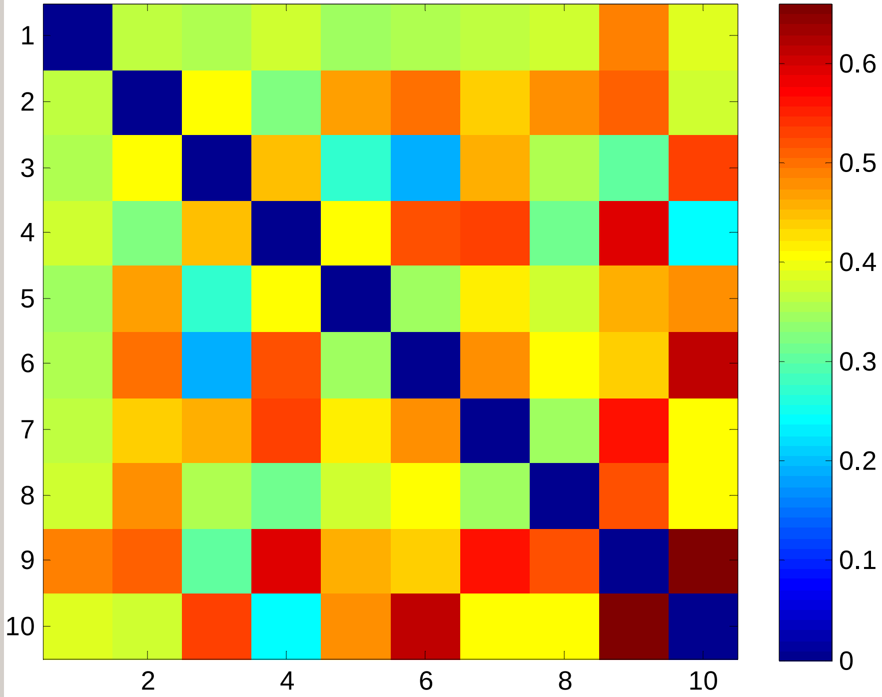


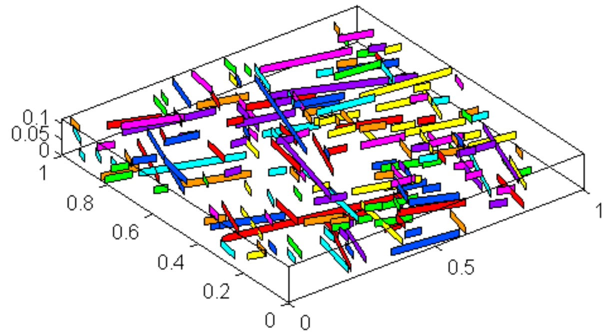
Result of Hausdorff Distance

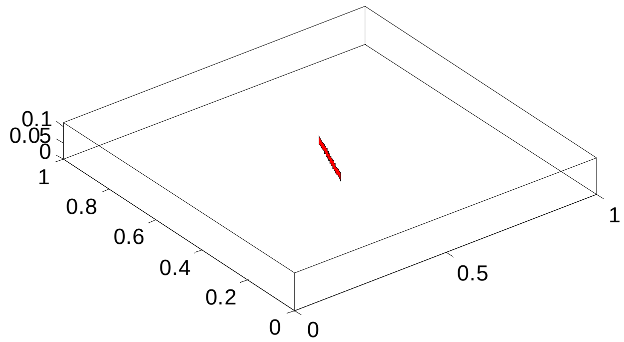
Zero diagonal

#9 is very high

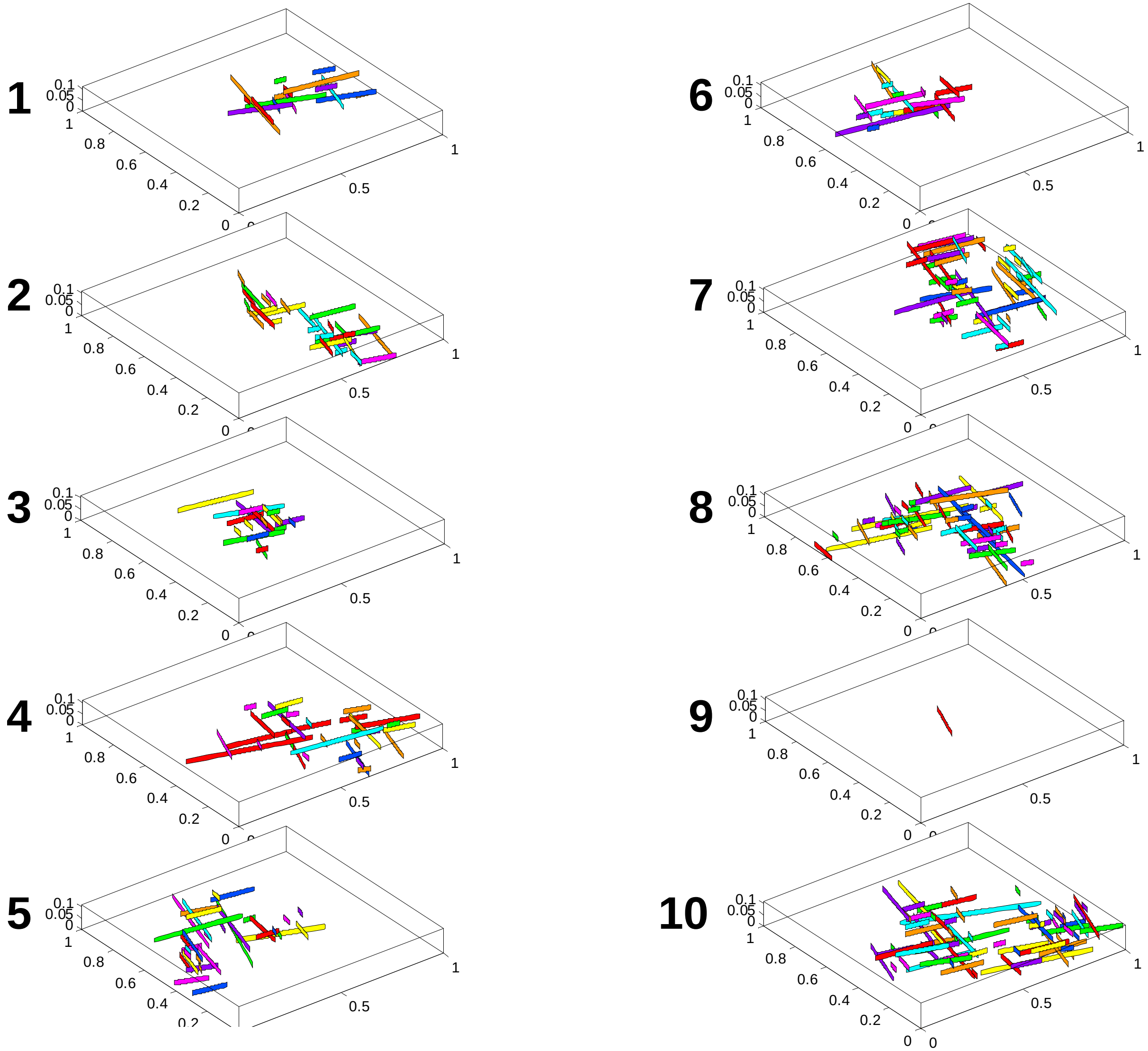
Max is (#10,#9)



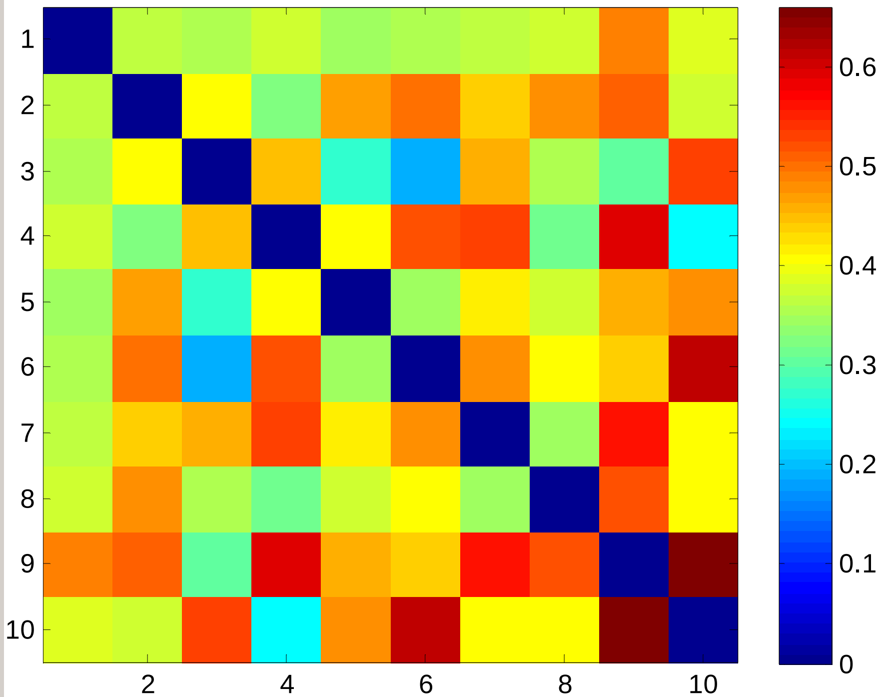




Connectivity Results



How to Choose realizations



3.4194		4.5460
3.8751		4.1098
3.3406		4.0132
3.7761		3.8751
3.5957	**sort** →	3.8576
3.8576		3.7761
4.0132		3.5957
3.5721		3.5721
4.5460		3.4194
4.1098		3.3406

Wasserstein Distance

$$d_{W,p}(A,B) := \inf_{\mu \in M(\mu_A, \mu_B)} \| d \|_{L^p(A \times B, \mu)}$$

$$d_{W,p}^{X}(A,B) := \min\left(\sum_{a,b} d(a,b)^p \mu(a,b)\right)^{1/p}$$

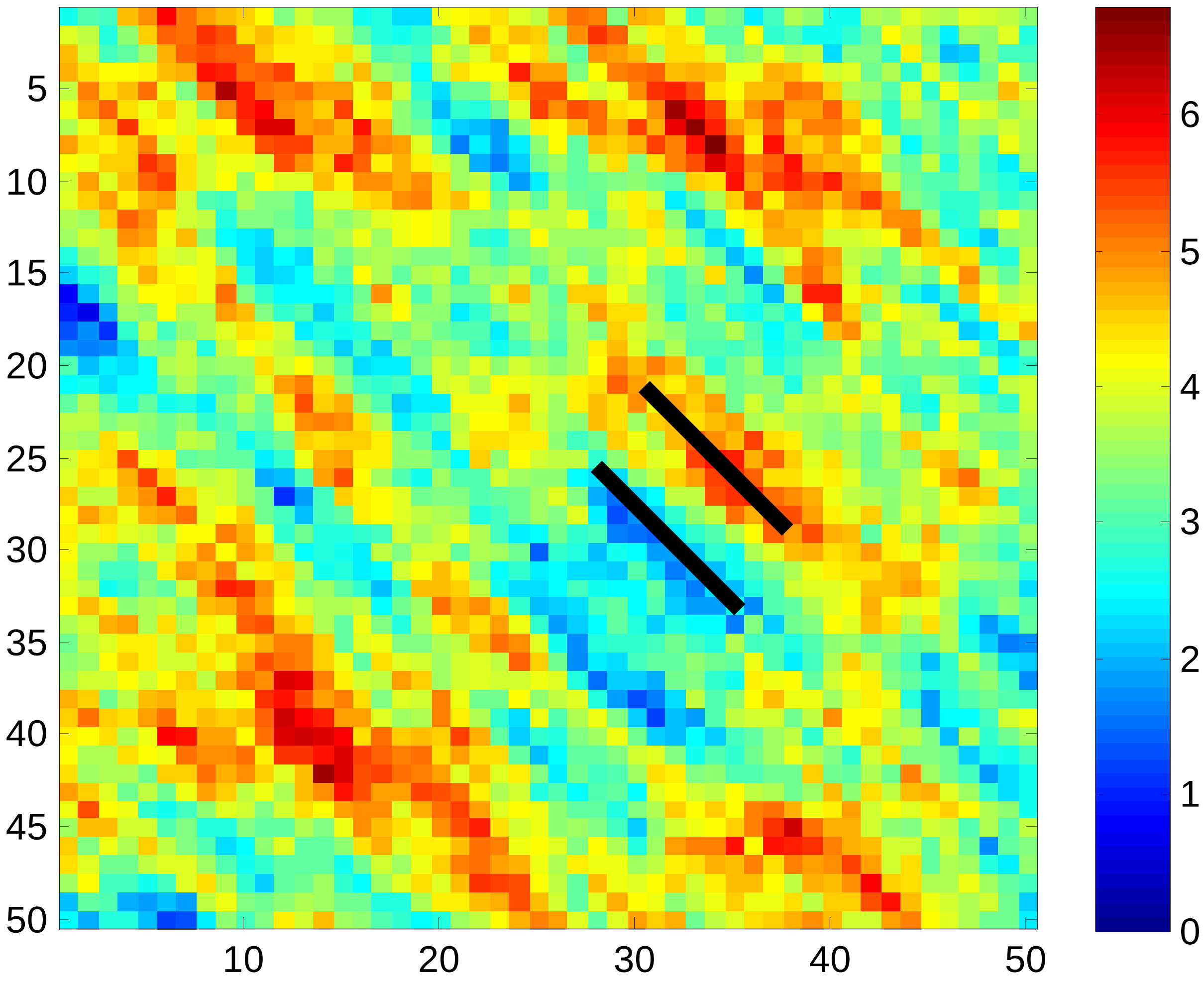
$$d_{ab} = e^{coeff \times |K_a - K_b|} \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

where *coeff* > 0, *Ka* and *Kb* are permeability value at the (x_a, y_a) and (x_b, y_b)

$p = 1$, $p = 2$ and $p = 3$

Remember that $p = 1$ is also called EMD distance.

Advantage of Wasserstein Distance



Transportation Problem

$$\mathrm{WORK}(P, Q, \mathbf{F}) = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} f_{ij}$$

$$f_{ij} \geq 0 \qquad 1 \leq i \leq m,\ 1 \leq j \leq n \qquad (1)$$

$$\sum_{j=1}^{n} f_{ij} \leq w_{\mathbf{p}_i} \qquad 1 \leq i \leq m \qquad (2)$$

$$\sum_{i=1}^{m} f_{ij} \leq w_{\mathbf{q}_j} \qquad 1 \leq j \leq n \qquad (3)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij} = \min\left(\sum_{i=1}^{m} w_{\mathbf{p}_i}, \sum_{j=1}^{n} w_{\mathbf{q}_j}\right), \qquad (4)$$

$$\mathrm{EMD}(P, Q) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} f_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}}$$

Sub sampling is needed.

where

$$w_{pi} = 1/M$$

$$w_{qi} = 1/N$$

$$\sum_{i=1}^{M}\sum_{j=1}^{N} f_{ij} = 1$$

$$d_{ab} = e^{coeff \times |K_a - K_b|}\sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

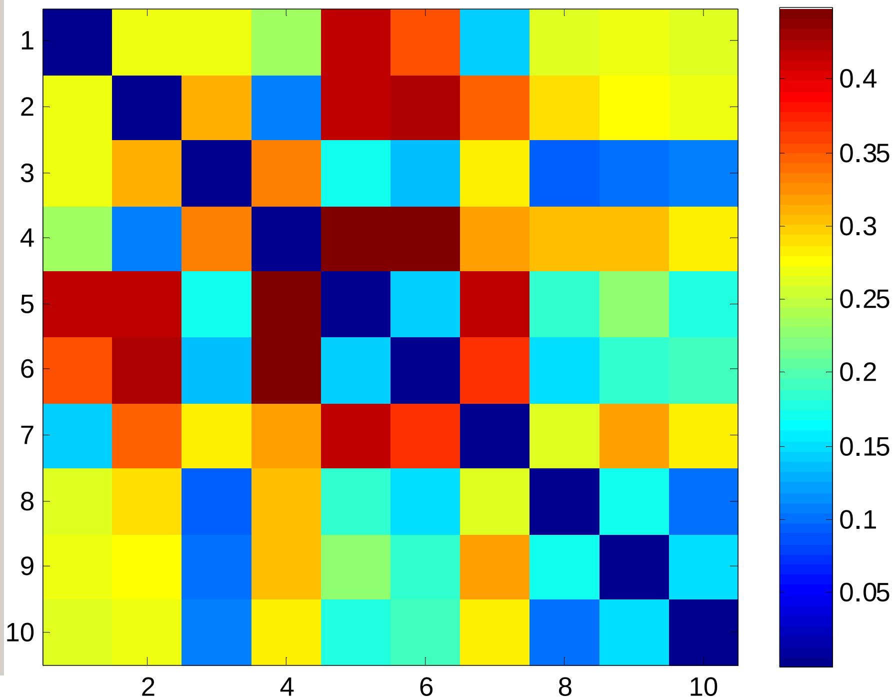
Results of Wasserstein Distance

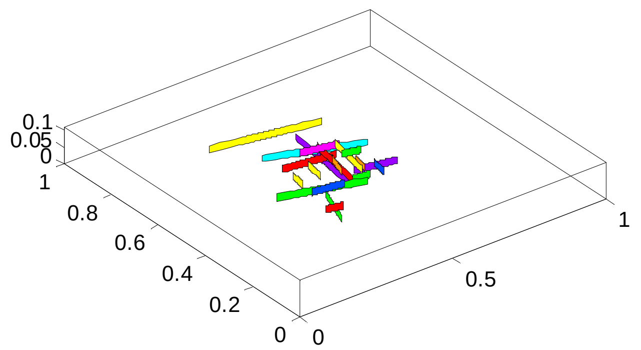
Zero diagonal

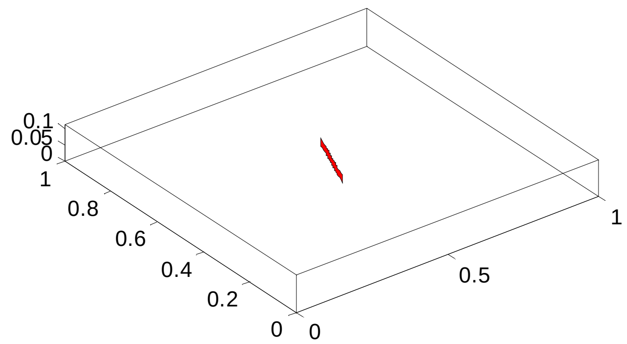
#9 is not very high, Lower than Hausdorff

Max are (#4,#5) and (#4,#6)

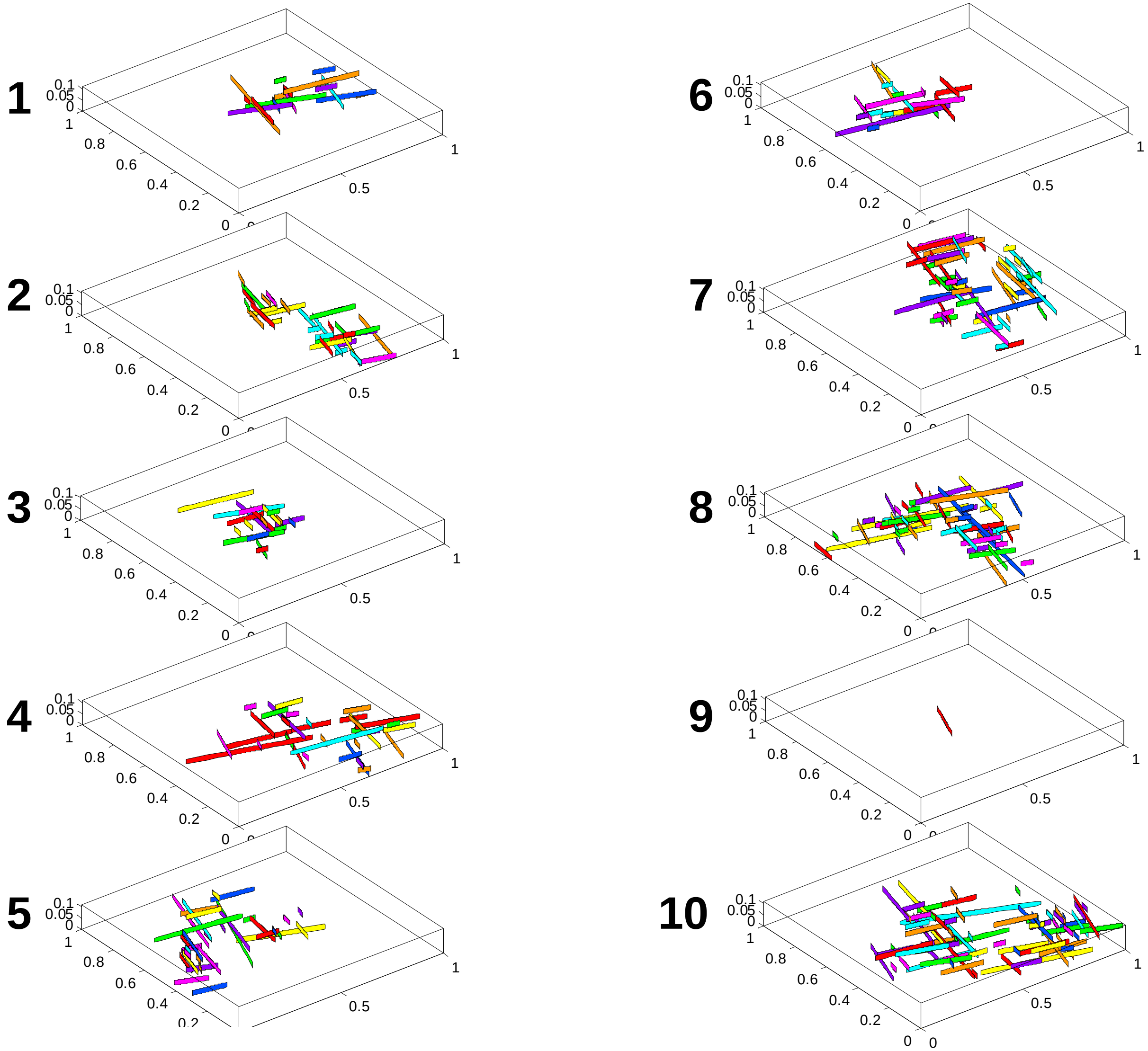
Values ↑ when p ↑



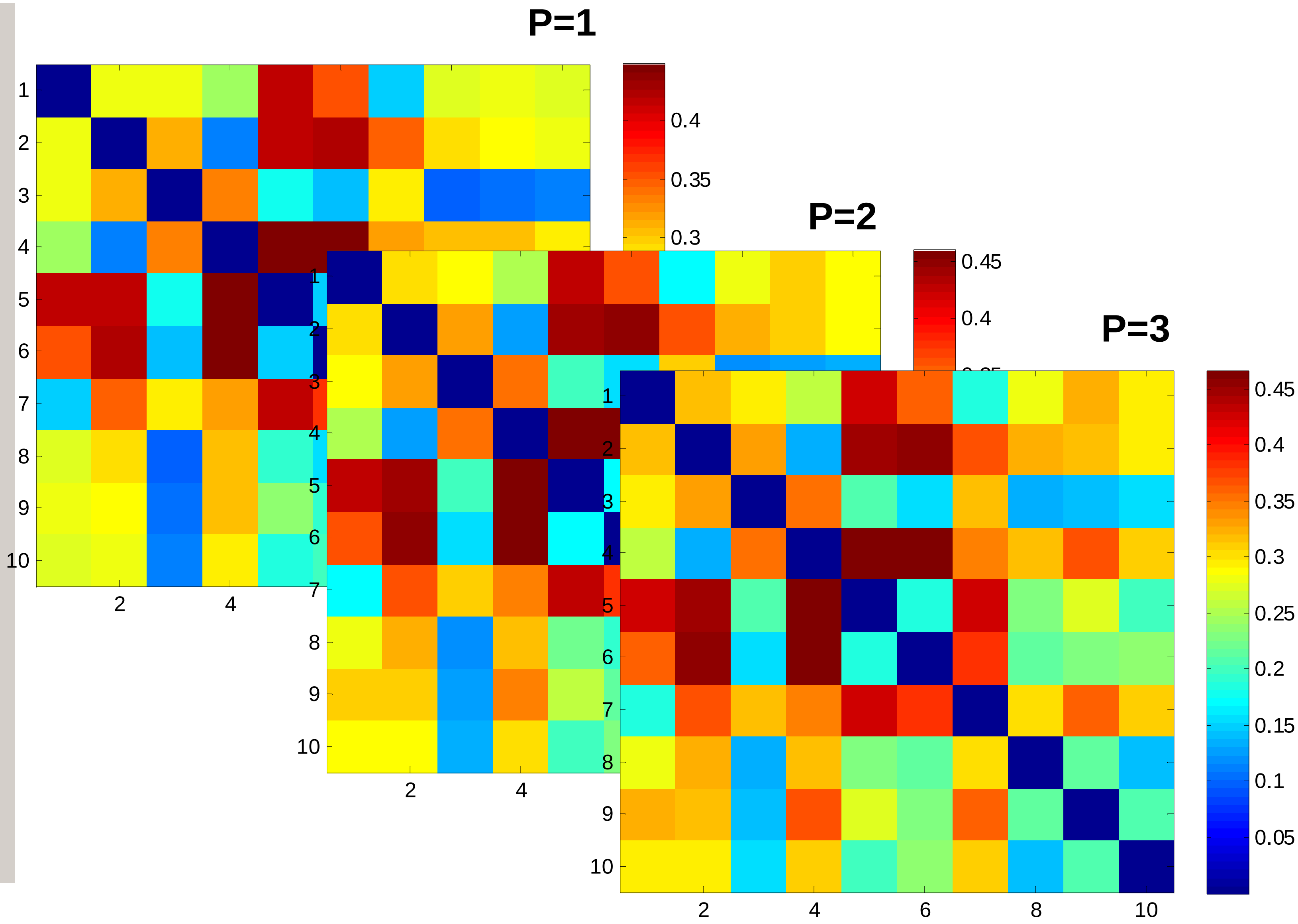




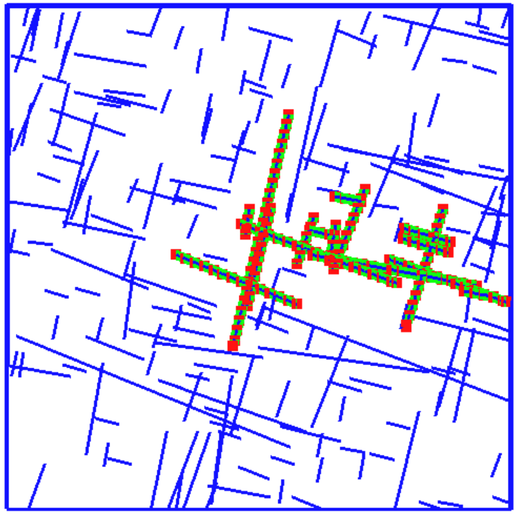
Connectivity Results

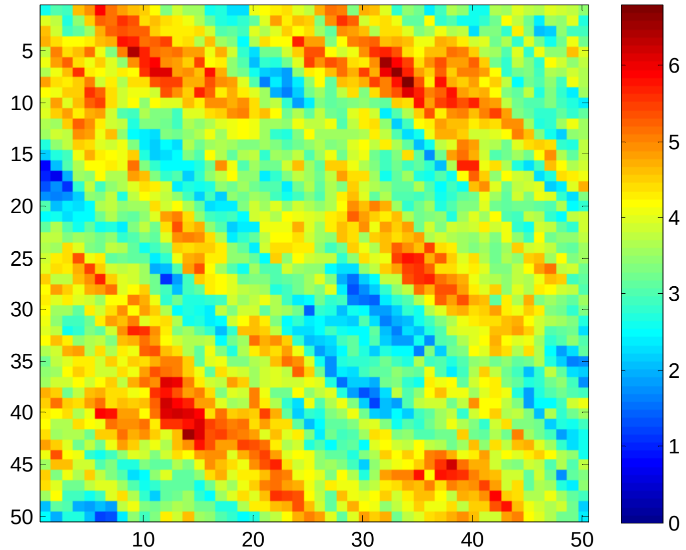


Different p of Wasserstein Distance



Wasserstein Distance using Diff Weights





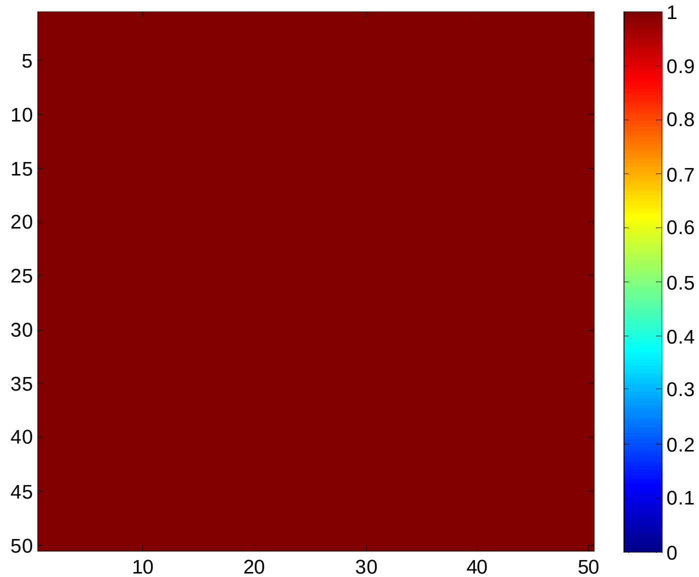
where

$$w_{pi} = \mathrm{Perm}_p(i)/\mathrm{sum}(\mathrm{Perm}_p)$$

$$w_{qi} = \mathrm{Perm}_q(i)/\mathrm{sum}(\mathrm{Perm}_q)$$

Using different Perm

EMD = 0.0242



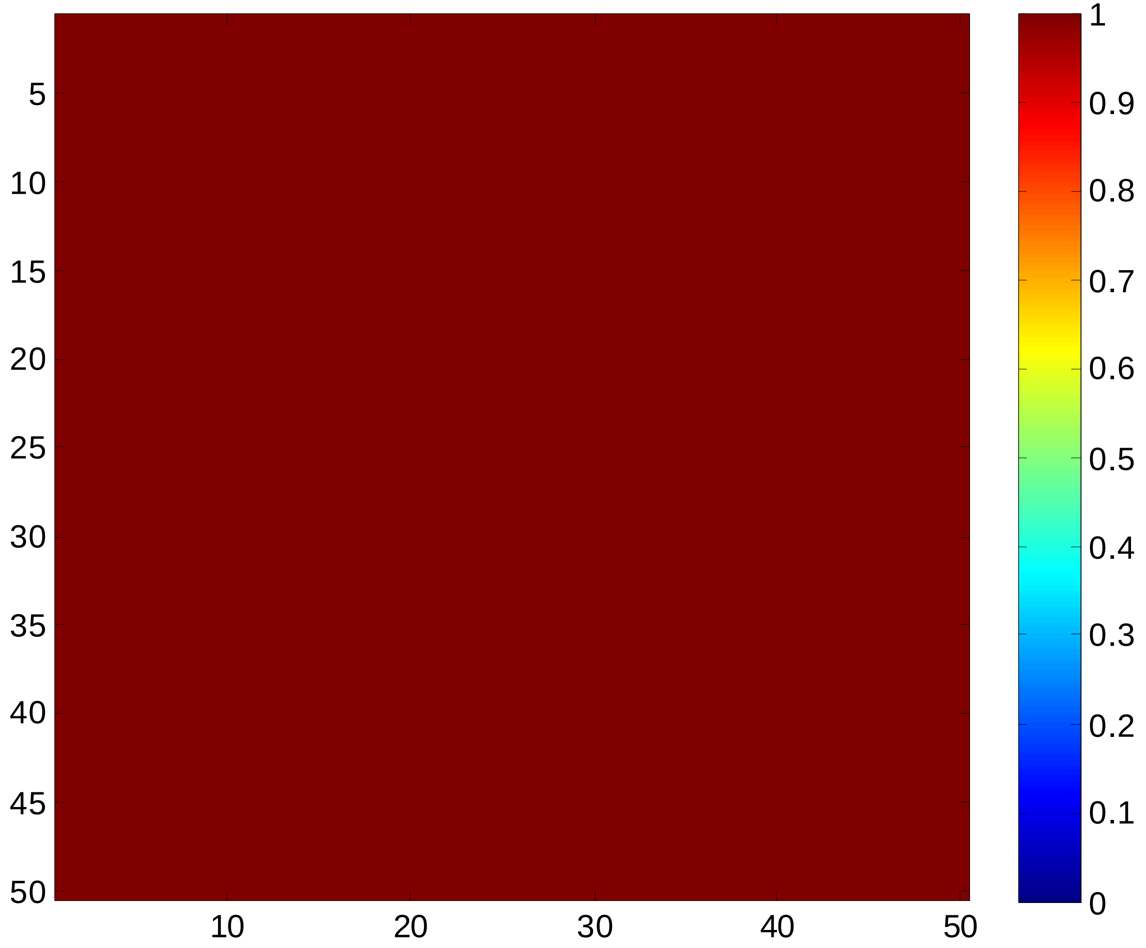
Systematically Accelerating the Fracture Simulation Workflow using Shape Matching Techniques

Content

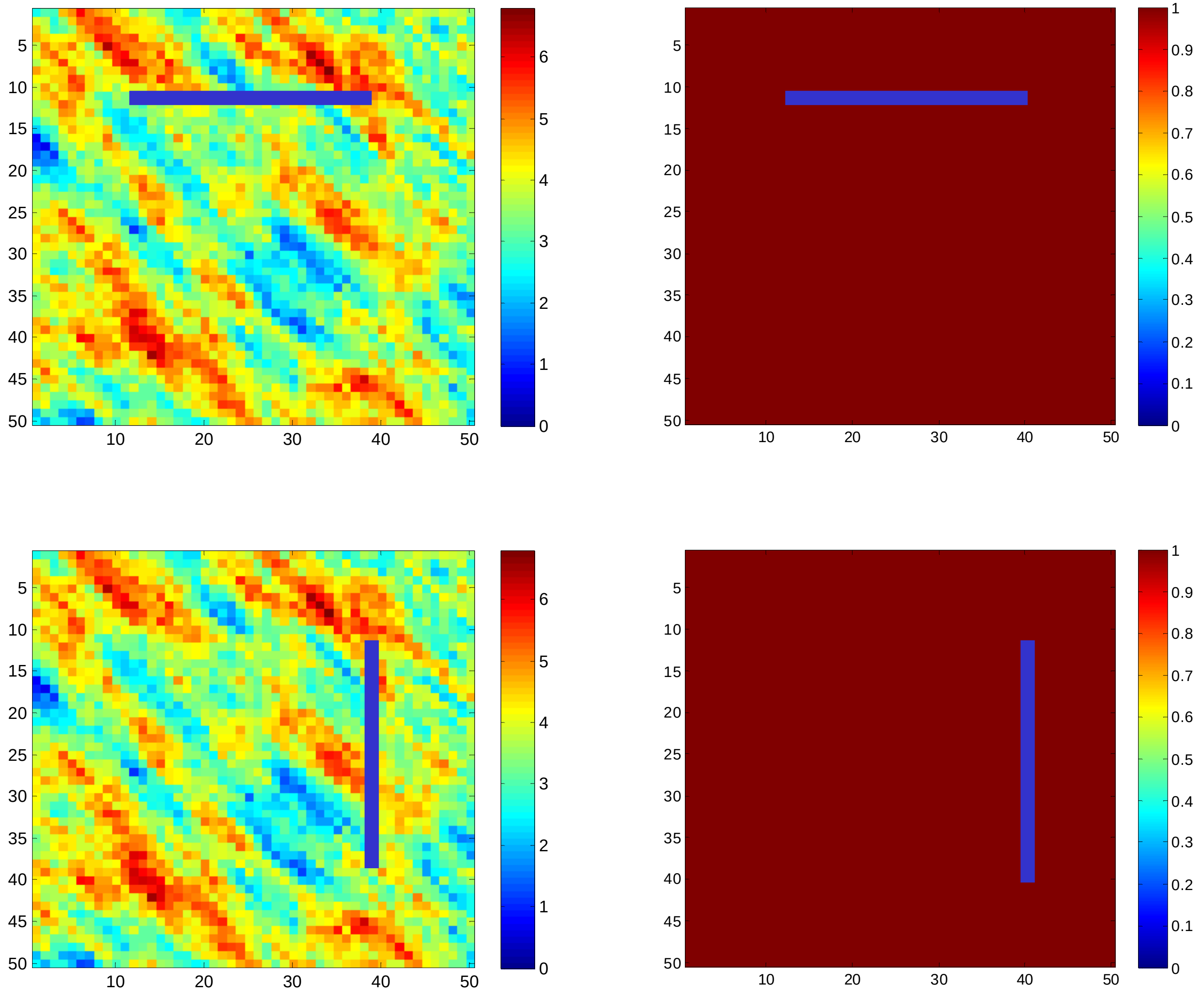
- Motivation and Background Information
- Connectivity Analysis and Sampling
- Heterogeneous Case
- Homogeneous Case
- Instinct Methods
- Performance

Homogeneous Permeability Field

Everywhere is 1



Heterogeneity vs Homogeneity



Gromov-Hausdorff Distance

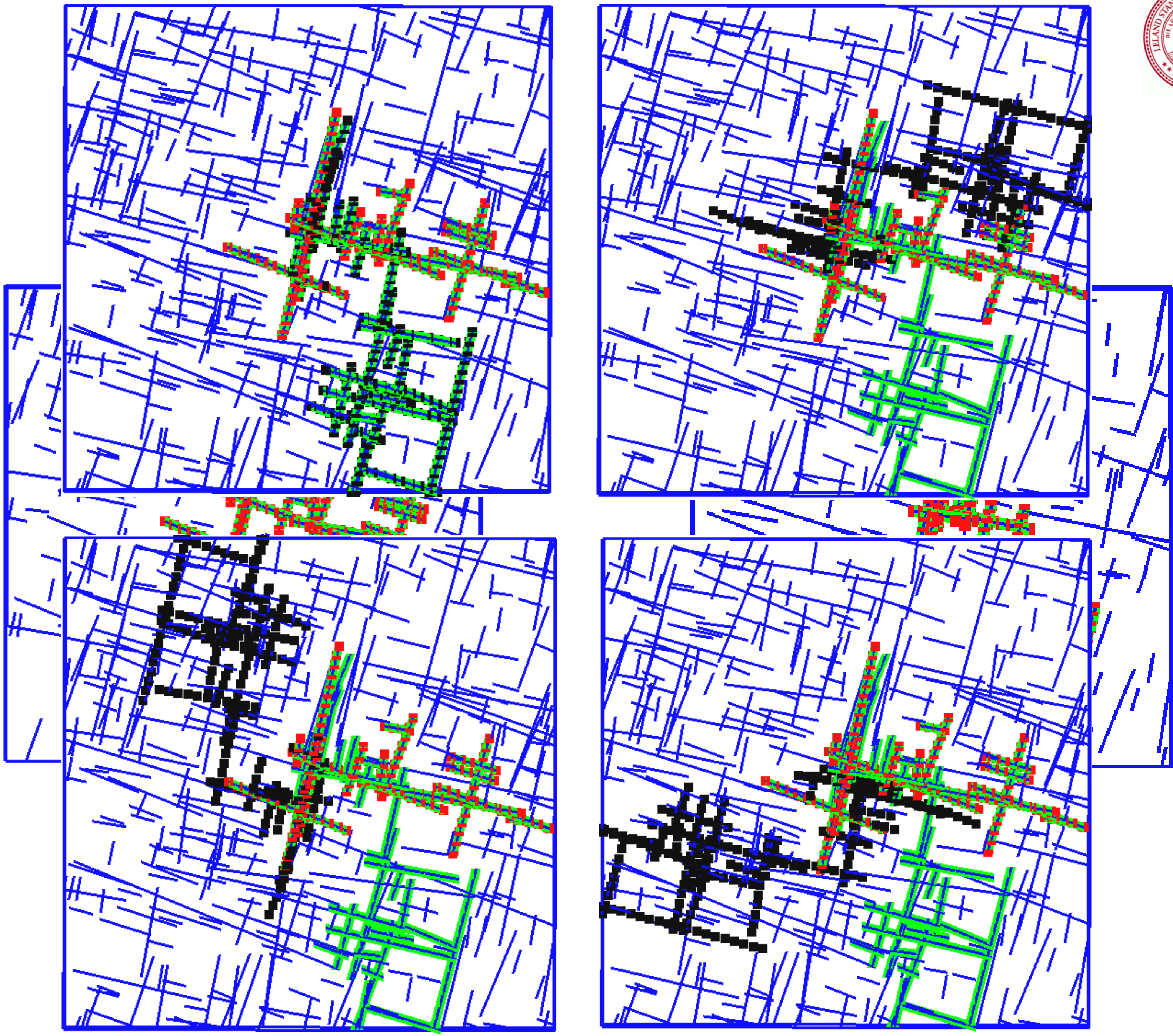
$$d_{GH}(X,Y) := \inf_{Z,f,g} d_H^Z(f(X), g(Y))$$

where

$f(X) = X$

$g(Y) =$

- Rotate(Y, 0),
- Rotate(Y, 90),
- Rotate(Y, 180),
- Rotate(Y, 270)

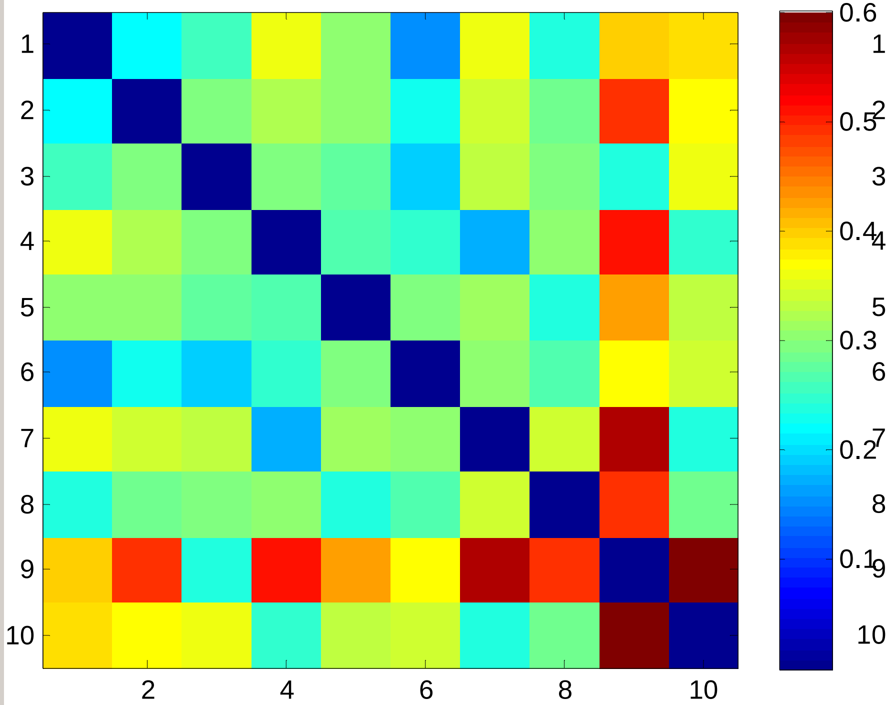


Results of Gromov-Hausdorff Distance

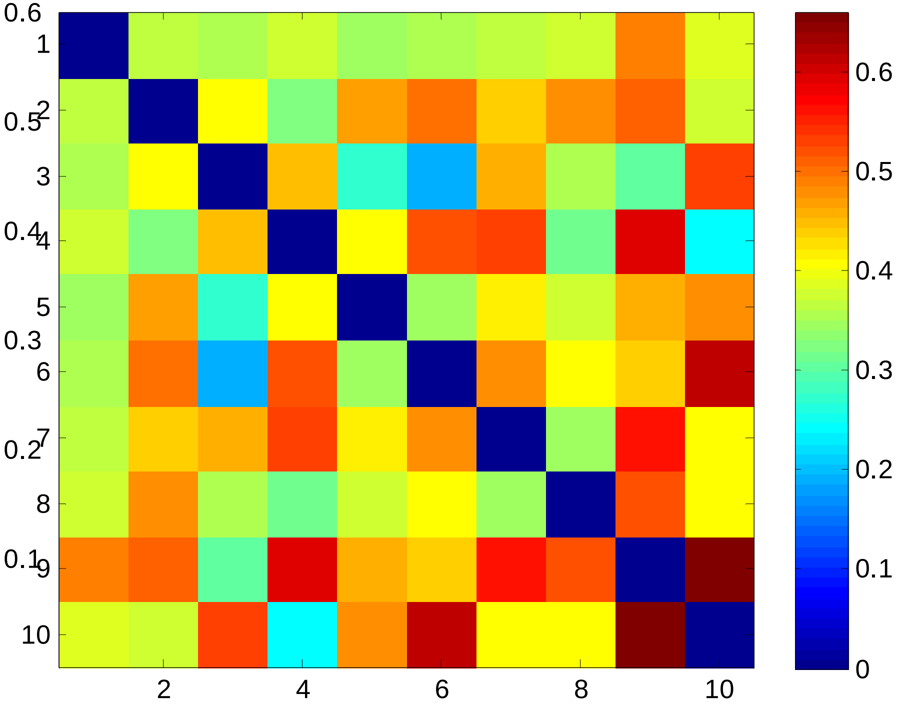
Zero diagonal

#9 is very high

Much lower than Hausdorff



Hausdorff



Gromov-Wasserstein Distance

$$d_{GW,p}(X,Y) := \inf_{Z,f,g} d_{W,p}^{Z}(f(X), g(Y))$$

where

p = 1, p = 2, p = 3

$f(X) = X$

$g(Y) =$

- Rotate(Y, 0),
- Rotate(Y, 90),
- Rotate(Y, 180),
- Rotate(Y, 270)

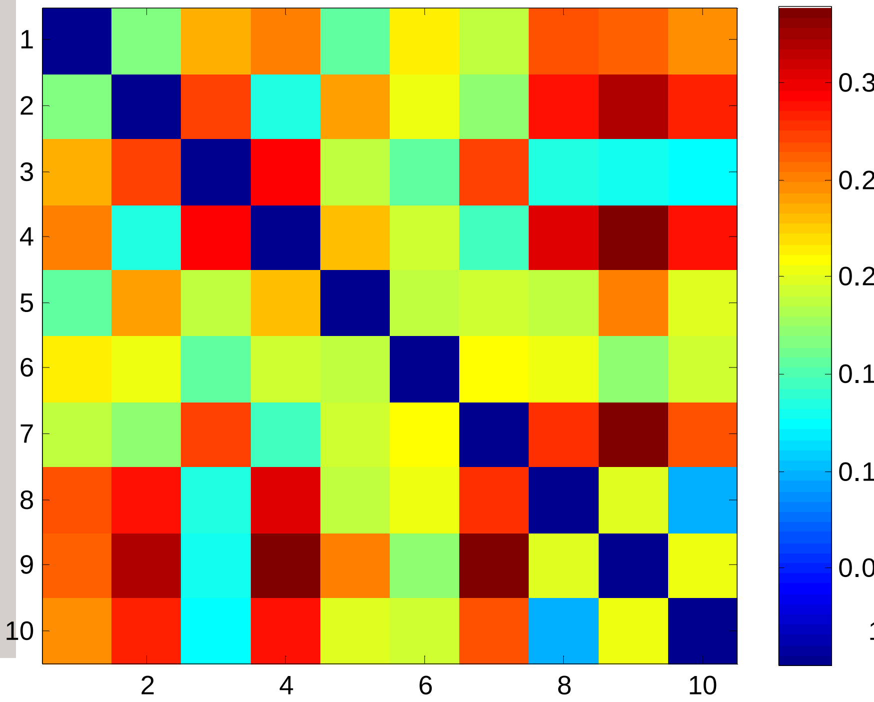
Results of Gromov-Wasserstein Distance

Zero diagonal

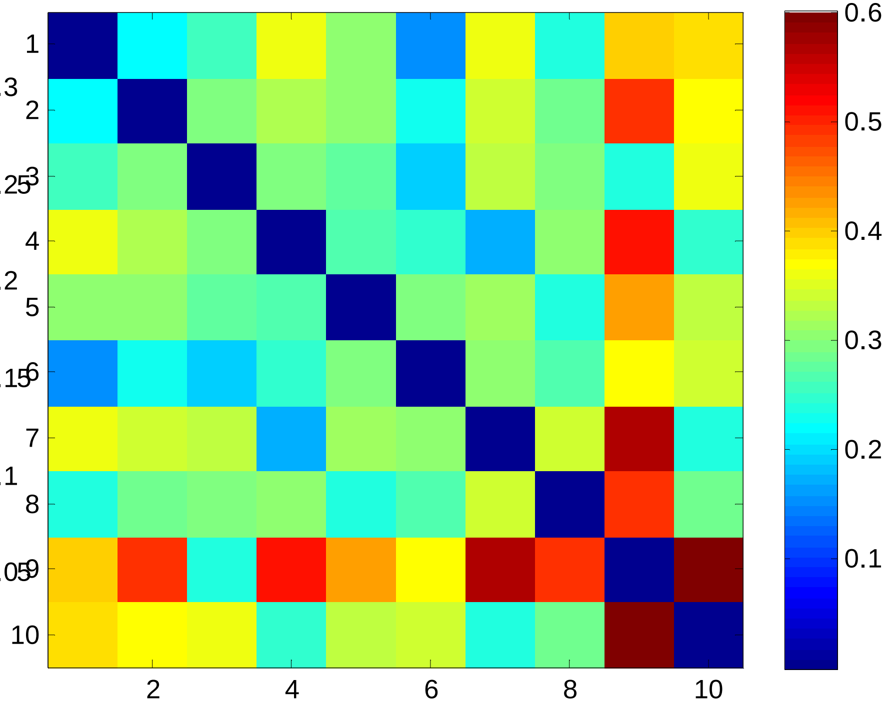
Lower than Gromov-Hausdorff and Wasserstein

Max are (#4,#9) and (#7,#9)

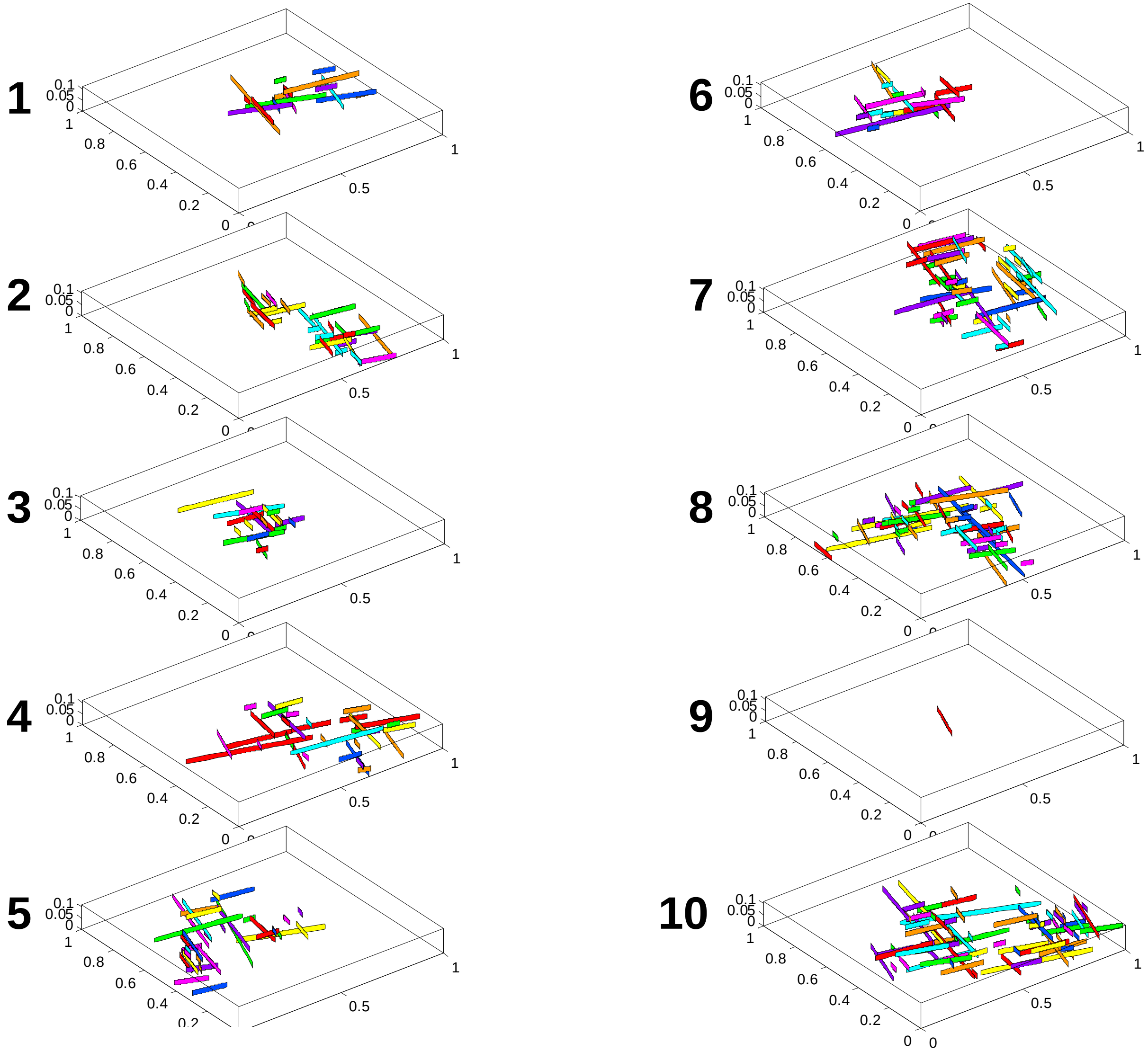
Values ↑ when p ↑



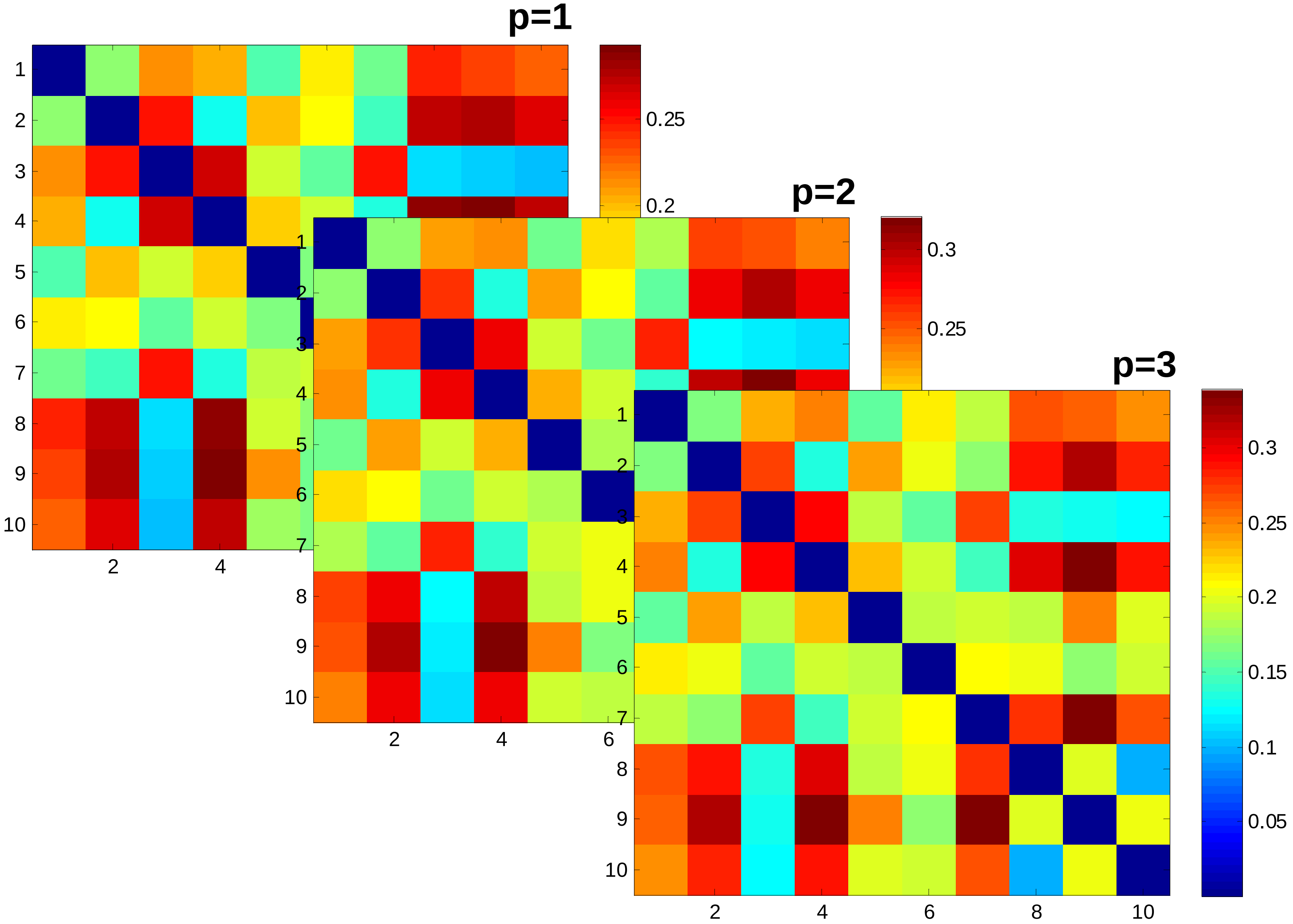
Gromov-Hausdorff



Connectivity Results



Different p of Gromov-Wasserstein Distance



Systematically Accelerating the Fracture Simulation Workflow using Shape Matching Techniques

Content

- Motivation and Background Information
- Connectivity Analysis and Sampling
- Heterogeneous Case
- Homogeneous Case
- Intrinsic Methods
- Performance

Shape Distribution

Investigate the intrinsic method

D2: Measures the distance between two random points on the surface.

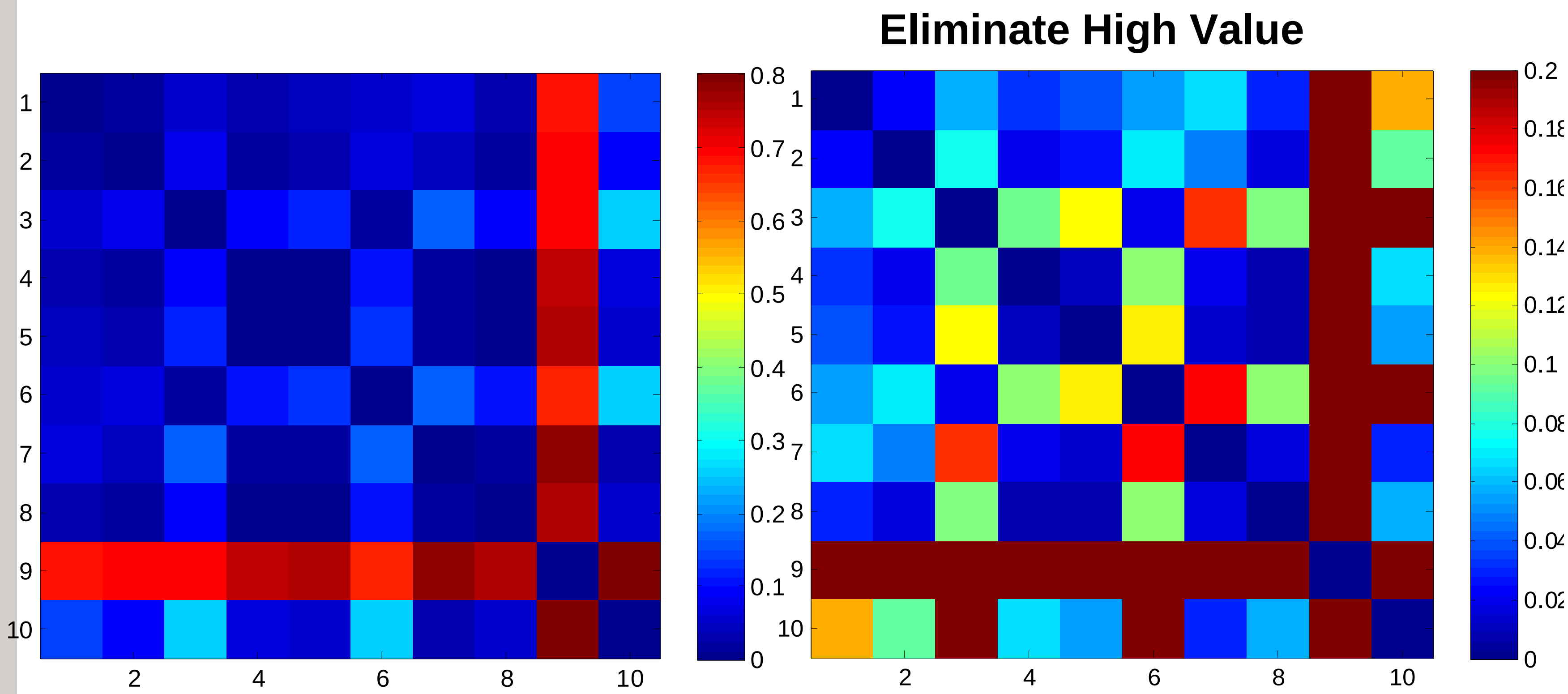
Bhattacharyya: $D(f,g) = 1 - \int \sqrt{fg}$

Results of Shape Distribution

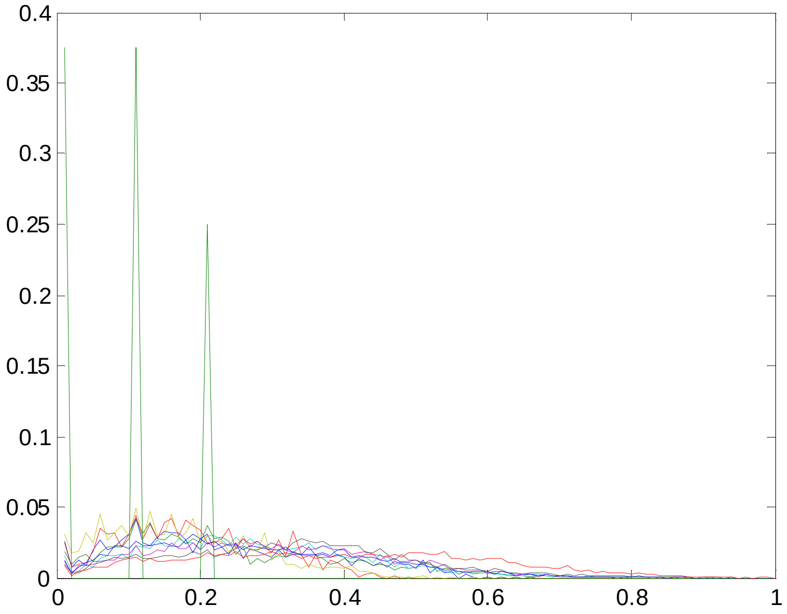
Zero diagonal

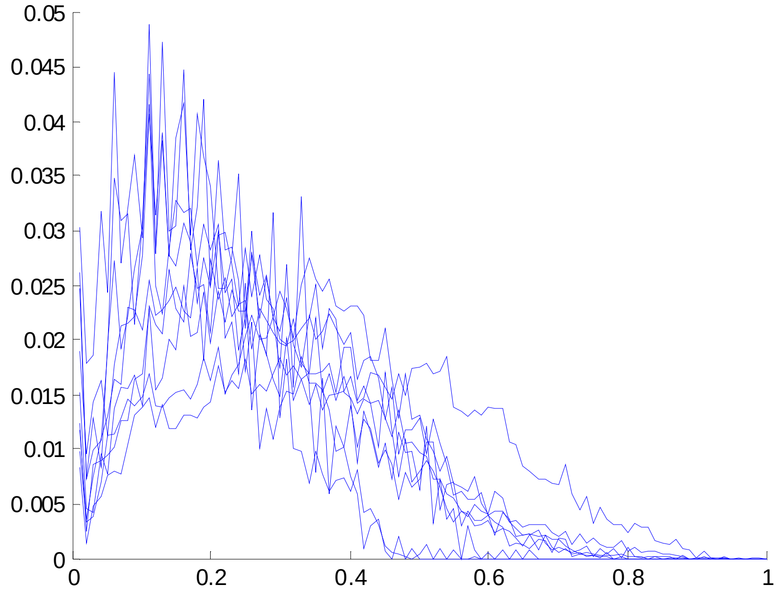
#9 is extremely high

Hard to discriminate values



Histogram of Shape Distribution





EMD using Shape Context

$$\mathrm{WORK}(P, Q, \mathbf{F}) = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} f_{ij}$$

$$f_{ij} \geq 0 \qquad 1 \leq i \leq m,\ 1 \leq j \leq n \qquad (1)$$

$$\sum_{j=1}^{n} f_{ij} \leq w_{\mathbf{p}_i} \qquad 1 \leq i \leq m \qquad (2)$$

$$\sum_{i=1}^{m} f_{ij} \leq w_{\mathbf{q}_j} \qquad 1 \leq j \leq n \qquad (3)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij} = \min\left(\sum_{i=1}^{m} w_{\mathbf{p}_i}, \sum_{j=1}^{n} w_{\mathbf{q}_j} \right), \qquad (4)$$

$$\mathrm{EMD}(P, Q) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} f_{ij}}{\sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}}$$

where

$$w_{pi} = 1/M$$

$$w_{qi} = 1/N$$

$$\sum_{i=1}^{M} \sum_{j=1}^{N} f_{ij} = 1$$

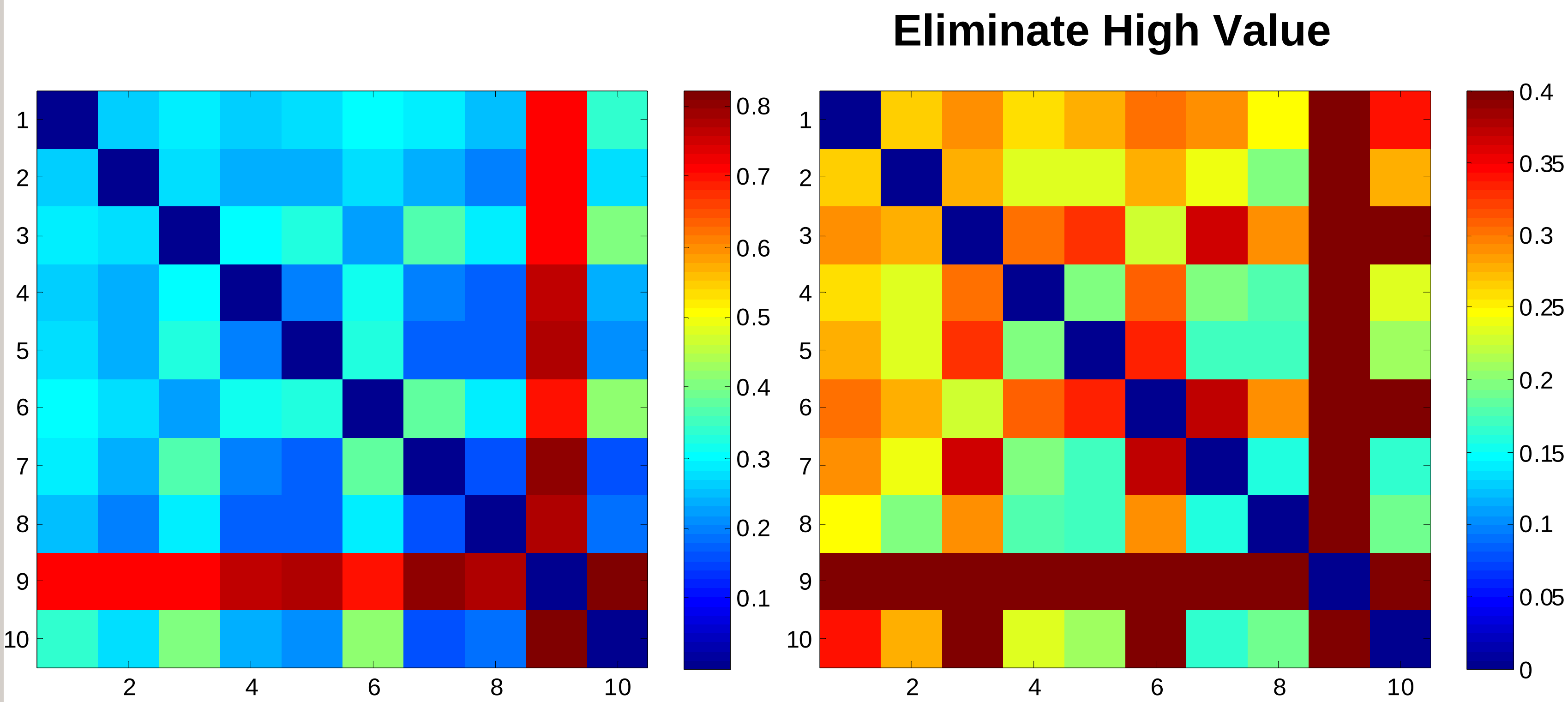
$$d_{ij}(a_i, b_j) = 1 - \int \sqrt{a_i b_j}$$

Results of EMD using Shape Context

Zero diagonal

#9 is extremely high

Better than Shape Distribution



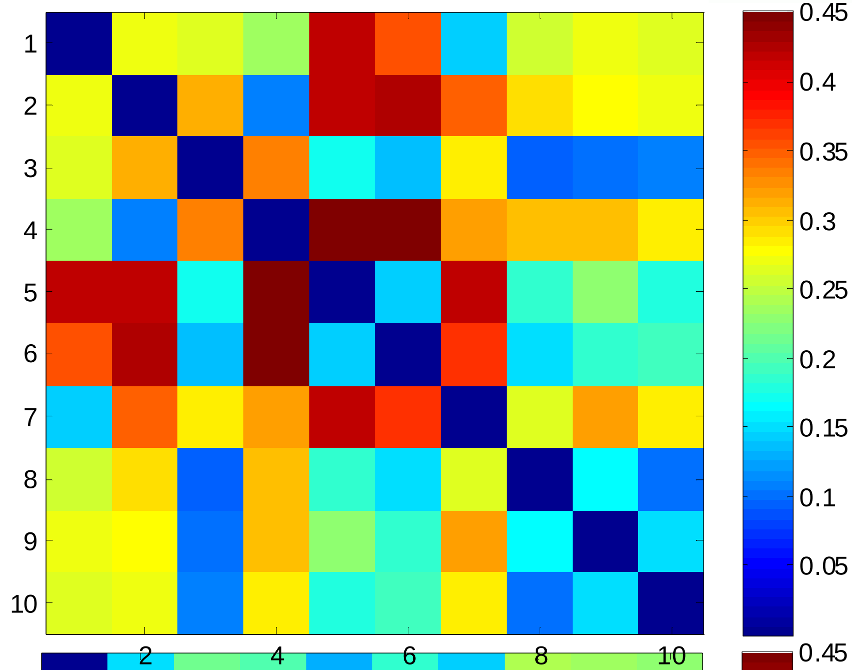
EMD using Shape Context vs EMD using Euclidean Distance

#9 is higher

(#3,#7/#9/#10) is higher

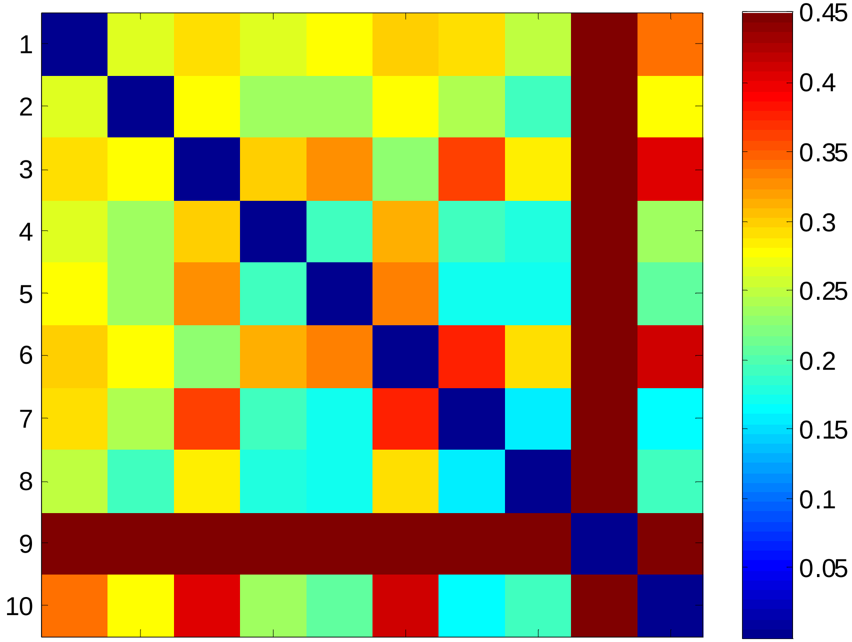
Values are higher

W1

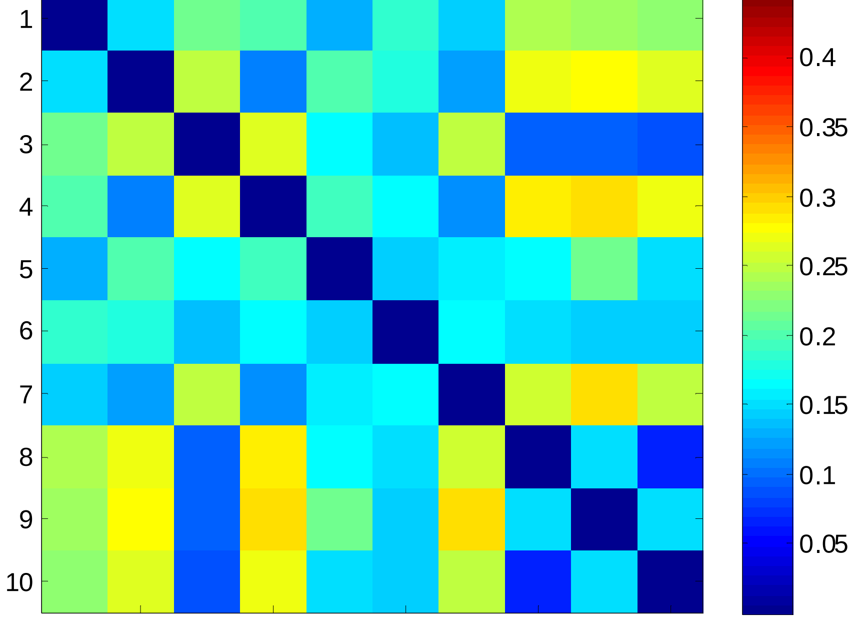


EMD using

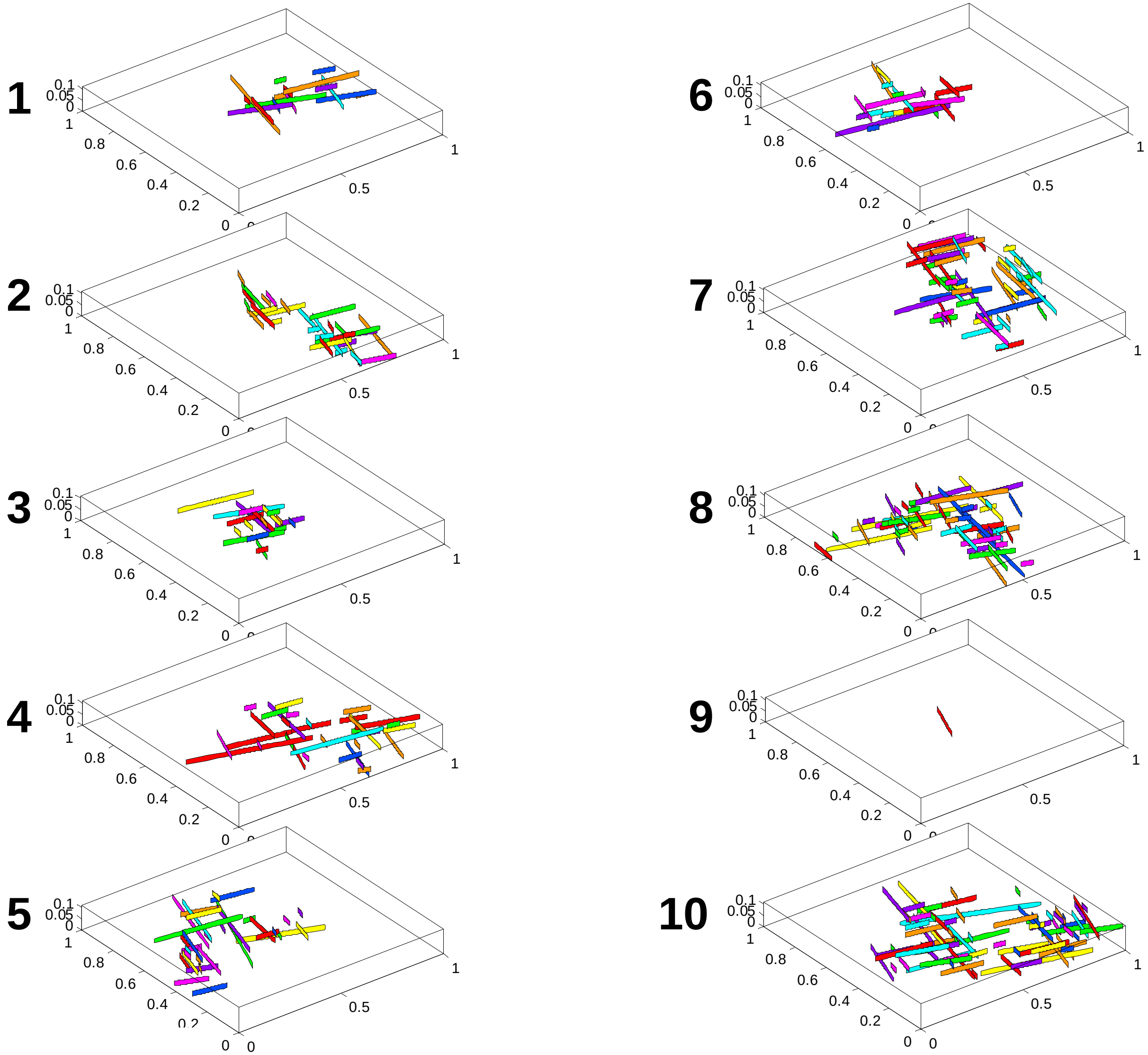
Shape Context



Gromov-W1



Connectivity Results



Systematically Accelerating the Fracture Simulation Workflow using Shape Matching Techniques

Content

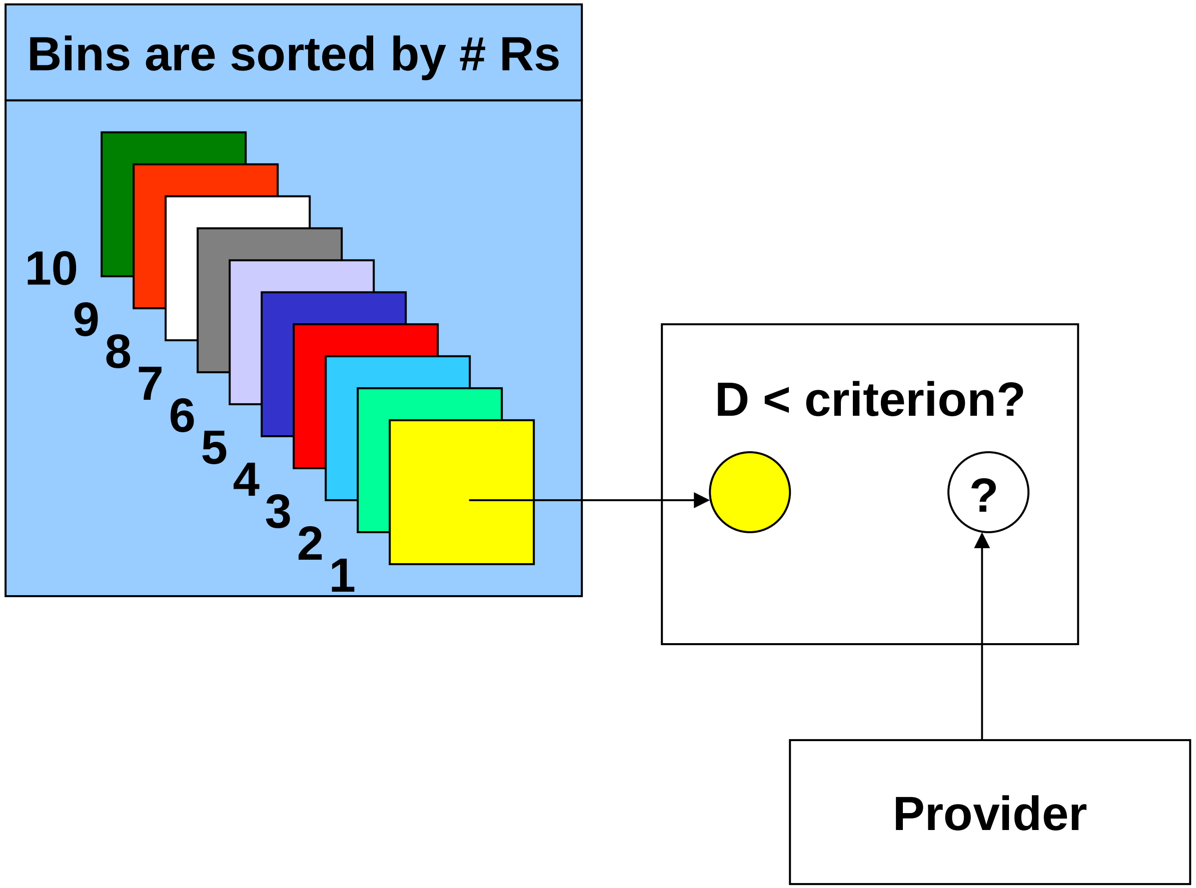
- Motivation and Background Information
- Connectivity Analysis and Sampling
- Heterogeneous Case
- Homogeneous Case
- Instinct Methods
- Performance

Performance

Method	Ave Time (s)	#
GROMOV-Wasserstein	26.62083929	1
EMD using Shape Context	6.75475	2
Wasserstein	6.270928571	3
GROMOV-HAUSDORFF	1.486035714	4
HAUSDORFF	0.321196429	5
Shape Distribution	0	6

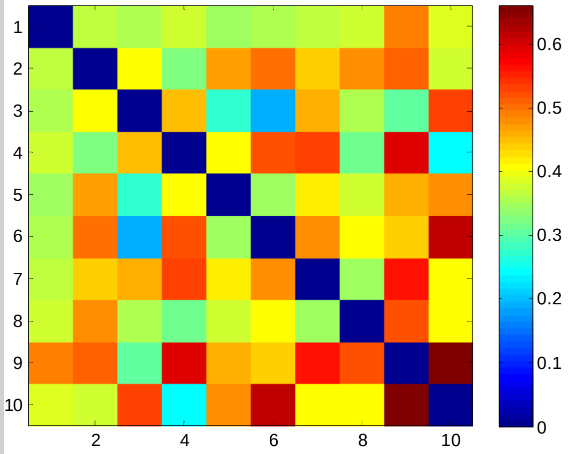
Reduce from $O(N^2)$ to $O(MN)$

M = 10, N = 1000

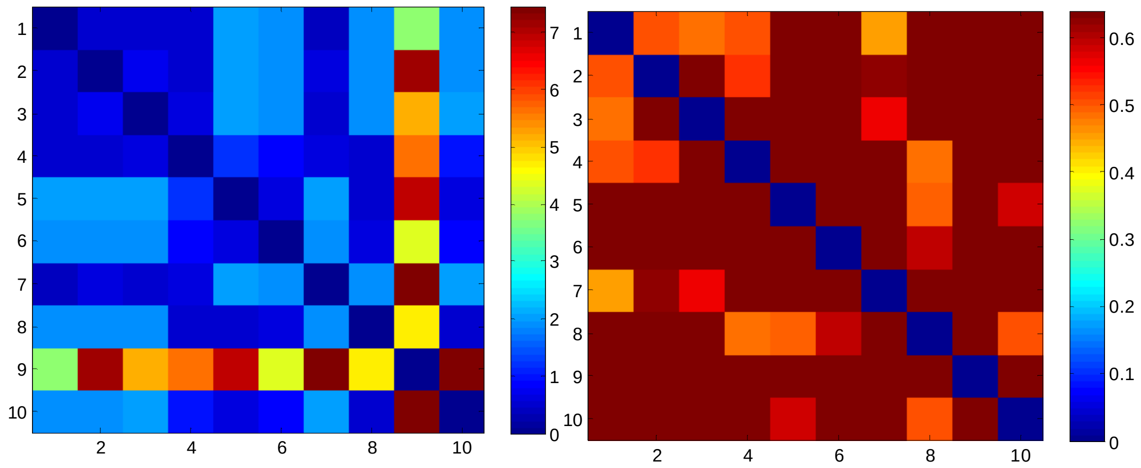


Hausdorff Distance with Heterogeneity

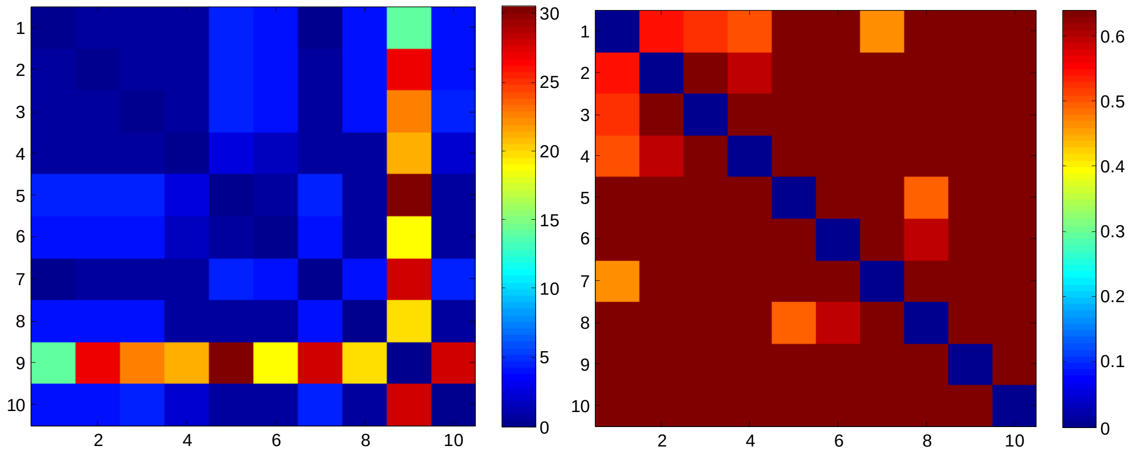
Hetero Coeff = 0



Hetero Coeff = 2



Hetero Coeff = 3



$$d_{ab} = e^{coeff \times |K_a - K_b|} \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

Summary and Conclusions

In the cases I studied, observations are below:

- In heterogeneous cases, use Wasserstein distance or Hausdorff distance using permeability information.
- In homogeneous cases, Gromov-Hausdorff distance is better than Gromov-Wasserstein distance because its values are higher and much faster.
- In both heterogeneous cases and homogeneous cases, EMD using Shape Context is better than Shape Distribution using "D2" because its values are much higher and even better than Wasserstein distance (p=1).
- The performance of Gromov-Wasserstein is low, practically, we had better avoid calculating Gromov- distance directly.

Provided a systematic approach to accelerate the fracture simulation process.