Shape Matching: A Metric Geometry Approach
CS468

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Description

This seminar will touch upon the use of ideas from metric geometry for tackling the problem of Matching/Comparison of Shapes.

We will cover different ideas of M. Gromov that have proven extremely useful in the context of this applied problem.

The central idea is to consider shapes as metric spaces (or measure metric spaces) and, then, use one of various notions of distance between metric spaces to obtain a measure of dissimilarity between shapes. The choice of the metric with which one augments the shapes encodes the degree of invariance one obtains from the dissimilarity measure.

We will discuss the necessary theoretical background and emphasize issues arising from the numerical implementation of these ideas.

Connections with several standard approaches to the problem of shape comparison will be discussed.

General Information

time: Mondays, 2.15-4.05
room: Building 380, room 380D
class website: `http://graphics.stanford.edu/courses/cs468-08-fall/`

Syllabus

1. Introduction. Review of various Shape Matching algorithms
2. Mathematical background:
   - Correspondences, Hausdorff distance
   - Transportation Measures
   - Köenig's Lemma
   - Earth Mover's Distance (Monge-Kantorovich, Wasserstein)
3. The Gromov-Hausdorff framework

- Introducing invariances: Rigid Isometries.
- Gromov-Hausdorff distance. Sampling and simple lower bounds
- Connection between Hausdorff distance under rigid isometries and GH distance.
- Features of GH distance. Some comments on approximate implementation algorithms.
- Critique of the GH distance.

4. L^p-Gromov-Hausdorff distances (GH distances based on Mass Transportation ideas)

   - L^p-Gromov-Hausdorff distances. Motivation.
   - The L^p-Gromov-Hausdorff distances explain many pre-existing methods in the literature.
   - L^p-Gromov-Hausdorff distances for partial shape matching.
   - Lower bounds for the GH and L^∞-GH distance based on Topological Persistence constructions. Connections with the works of Frosini et al. and Carlsson et al.
   - Implementation details.

5. The unifying global picture. Relating the GH framework with other approaches in the literature.