Edgebreaker
Connectivity Compression for Triangle Meshes

Jarek Rossignac, TVCG 1999
Contribution

- Lossless compression for a triangle mesh, using $\approx 2$ bits per triangle for simple meshes
  - Only a slight increase for meshes with holes and handles
- Linear encoding size $O(|T|)$
  - Improves upon $O(|T| \log |T|)$ for many previous approaches
  - The constant is better than for previous approaches
Basic Idea

- Destroy triangles of the mesh one-by-one, starting from the boundary and spiralling inwards
- For each destruction operation, store an opcode indicating the type of the operation
  - Sequence of opcodes is called “history”
  - Length of history == number of triangles, hence linear size encoding
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Edgebreaker in action
Terminology

- **Simple triangle mesh**
  - Edge-connected collection of triangles with one or zero bounding loops
    - Piecewise linear surface homeomorphic to a disk or a sphere
- **Hole**: Interior triangles missing
  - Like holes in a sheet of paper
- **Handle**: The surface has genus $> 0$
  - Doughnuts, teacups etc.
Basics

- Simple triangle mesh T
- “Gate” $g := \text{some half-edge on bounding loop}$
- $X := \text{triangle incident on } g, \ v := \text{third vertex of } X$
Types of Triangles

- The g-X-v combo can be one of the 5 types C L E R S

- Compression scheme: remove X, store the operation (C, L, E, R or S), update the bounding loop and advance the gate
Data Structures

- Mesh: (pseudo) winged-edge structure
- History H: stores CLERS opcodes
- List P: stores vertex positions
- Stack: stores “deferred” loops
Operation C (central)

- v == internal vertex before the op
  == boundary vertex after the op
- Position of v is appended to list P
Operation L (left)
Operation R (right)
Operation S (split)

- Two loops created.
- Push gate of one loop onto the stack and continue with the other loop.
Operation E (end)

- Loop shrinks to zero
- Pop the gate of the next loop to be processed from the stack
Edgebreaker in action
Edgebreaker in action

Stack (before op)

History = C
Edgebreaker in action

History = CC
Edgebreaker in action

History = CCR
Edgebreaker in action

Stack (before op)

History = CCRR
Edgebreaker in action

Stack (before op)

History = CCRRRR
Edgebreaker in action
Edgebreaker in action

Stack (before op)

History = CCRRRSL
Edgebreaker in action

History = CCRRRSLE
Edgebreaker in action

History = CCRRRSLELC
Edgebreaker in action

History = CCRRRSLELCLCR
Edgebreaker in action

History = CCRRRSLELCRRR
Edgebreaker in action

History = CCRRRSLELELCRRRC
Edgebreaker in action

History = CCRRRSLELCRR CR
History = CCRRRSLELCRRCRR
Edgebreaker in action

History = CCRRRSLELCRRCRRR
Edgebreaker in action

Stack (before op)

History = CCRRRSLELCRRCRRRE
Crux Observations

- Different graphs have different histories.
  - Allows reconstruction of topology.

- There is a bijection between C operations and interior vertices of the triangulation.
  - Allows reconstruction of embedding.
Compression

- The history H can be encoded as follows:
  - Write 0 for C
  - Write 100 for S
  - Write 101 for R
  - Write 110 for L
  - Write 111 for E

- (The resulting binary string may be further compressed using any good compression algorithm.)
Compression

- Number of bits required = \( b \)
  \[ = |C| + 3(|S| + |L| + |R| + |E|) \]
- Denote boundary vertices \( V_E \) and interior vertices \( V_I \).
- \( |T| = |C| + |S| + |L| + |R| + |E| \) (each op destroys one triangle)
- \( |C| = |V_I| \) (observe from algorithm)
- \( |T| = 2|V_I| + |V_E| - 2 \) (property of triangulations)
- Hence \( b = 2|T| + |V_E| - 2 \leq 3|T| \)
Compression

• Assume mesh has small boundary.
  – e.g., a closed genus 0 surface can be “opened up” by “cutting along” one of its edges; the resulting surface has a boundary of length 2.

• Then $b \approx 2|T|$, i.e. 2 bits per triangle.
  – Compact repr. of any planar triangulated graph!

• Since CL and CE sequences are impossible, encoding can be made even shorter.

• In other situations, different coding schemes may be used, all of which guarantee $b \approx 2|T|$. 
Improvement in guarantees:

[King-Rossignac '99] show coding schemes which guarantee $1.84|T|$ length for the encoded history of closed genus 0 meshes.

In practice:

[Rossignac-Szymczak '99] show that entropy codes “usually” give $0.91|T|$ to $1.26|T|$ lengths.
Decompression

- [Rossignac '99] proposes (somewhat convoluted) $O(|T|^2)$ algorithm.
  - Traverses history in compression order, uses preprocessing pass to compute constants + offsets and generation pass to actually recreate the graph.
- [Isenburg-Snoeyink '01] propose single-pass $O(|T|)$ algorithm.
  - Traverses history in reverse order (why did it take two years to devise this “obvious” scheme???)
Spirale Reversi in action

Stack (after op)

History = CCRRRSLELCRRCRRRE
Spirale Reversi in action

History = CCRRRSLELCRRCRRR

Stack (after op)
Spirale Reversi in action

History = CCRRRSLELCRRCCR

Stack (after op)

History = CCRRRSLELCRRCCR
Spirale Reversi in action

Stack (after op)

History = CCRRRSLELCRRCR
Spirale Reversi in action

Stack (after op)

History = CCRRRSLELCRRRC
Spirale Reversi in action

History = CCRRRSLELCRR

Stack (after op)
Spirale Reversi in action

Stack (after op)

History = CCRRRSLELCR
Spirale Reversi in action

History = CCRRRSLELC

Stack (after op)
Spirale Reversi in action

Stack (after op)

History = CCRRRSLEL
Spirale Reversi in action

Stack (after op)

History = CCRRRSLE
Spirale Reversi in action

Stack (after op)

History = CCRRRS\textsubscript{L}
Spirale Reversi in action

History = CCRRRS
Spirale Reversi in action

Stack
(after op)

History = CCRRRR
Spirale Reversi in action

Stack (after op)

History = CCRRR
Spirale Reversi in action

Stack
(after op)

History = CCR
Spirale Reversi in action

Stack (after op)

History = CC
Spirale Reversi in action

Stack (after op)

History = C
Spirale Reversi in action

Stack (final)

History = (null)
Patching holes

• While processing a loop, the third vertex $v$ might lie on the boundary of a hole.

• **Solution:** Introduce M operation which merges the current loop with the hole.
Handling handles

- While processing one loop, we might run into a situation where the third vertex $v$ is on some other loop created by a previous split operation.

- **Solution:**
  - Modify split operation $S$ to mark and push additional information about the deferred loop.
  - When we reach a vertex marked as above, execute an $M'$ operation, which merges the two loops.
Discussion

- What about huge meshes?
  - [Isenburg-Gumhold '03] “Out-of-Core Compression for Gigantic Polygon Meshes”, SIGGRAPH '03.

- Can a different decimation order yield progressively decodable meshes?