Homework 1

Due: Friday, October 16, 2009

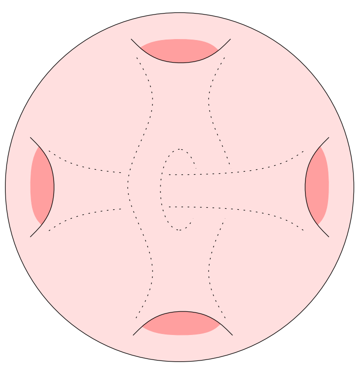
Problem 1

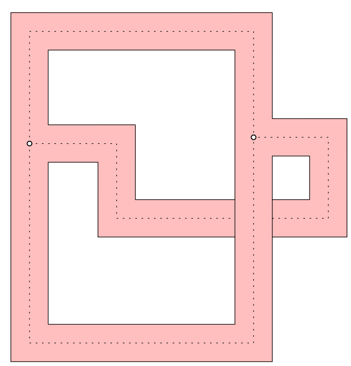
Homeomorphisms. (10 points.) Give explicit homeomorphisms to show that the following spaces with topologies inherited from the respective containing Euclidean spaces are homeomorphic:

- $\mathbb{R}$, the real line;
- $(0, 1)$, the open interval;
- $\mathbb{S}^1 - \{(0, 1)\}$, the circle with one point removed.

Problem 2

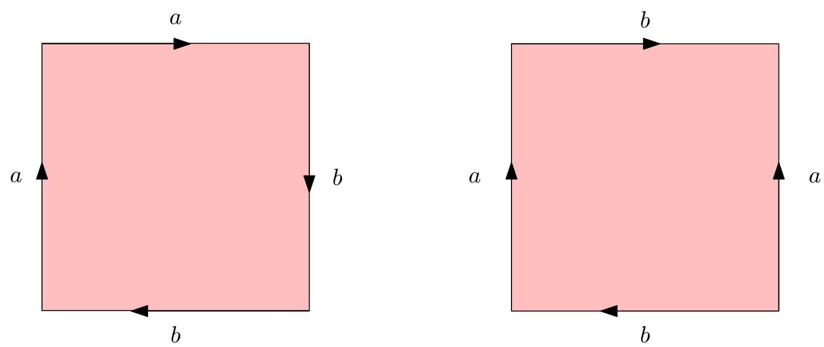
Classifying 2-manifolds. (20 points.) Characterize the two surfaces below in terms of genus, boundary, and orientability.





Problem 3

Klein bottle. (20 points.) Cut and paste the standard polygonal schema for the Klein bottle (a, a, b, b) to obtain the polygonal schema in which opposite edges of a square are identified (a, b, a^{-1}, b).



Problem 4

Order complex. (20 points.) A *flag* in a simplicial complex K in $\mathbb{R}^d$ is a nested sequence of proper faces, $\sigma_0 < \sigma_1 < \ldots < \sigma_k$. The collection of flags form an abstract simplicial complex A sometimes referred to as the *order complex* of K. Prove that A has a geometric realization in $\mathbb{R}^d$.

Problem 5

Alpha complexes. (10 points.) Let $S \subseteq \mathbb{R}^d$ be a finite set of points in general position. Recall that Čech(r) and Alpha(r) are the Čech and alpha complexes for radius $r \geq 0$, $\text{Čech}(r) = \text{Nrv}\{B_x(r)\}_{x \in S}$, and $\text{Alpha}(r) = \text{Nrv}\{B_x(r) \cap \text{Vor}_x\}_{x \in S}$. Is it true that $\text{Alpha}(r) = \text{Čech}(r) \cap \text{Delaunay}$? If yes, prove the following two subcomplex relations. If no, give examples to show which subcomplex relations are not valid.

1. $\text{Alpha}(r) \subseteq \text{Čech}(r) \cap \text{Delaunay}$
2. $\text{Čech}(r) \cap \text{Delaunay} \subseteq \text{Alpha}(r)$

Extra credit

Deciding Isomorphism. (30 points.) What is the computational complexity of recognizing isomorphic abstract simplicial complexes?