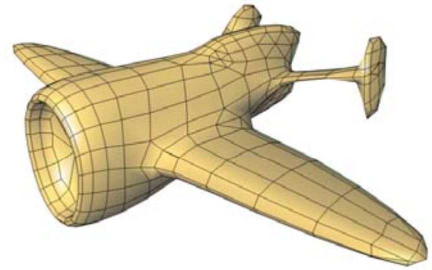
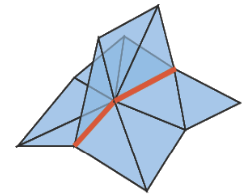
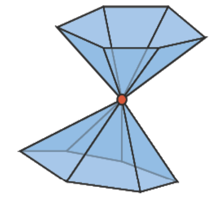
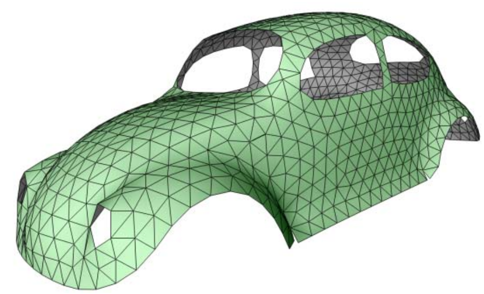
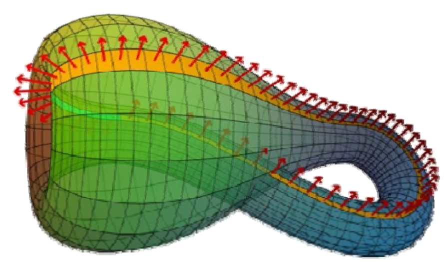






Basic Concepts





Today

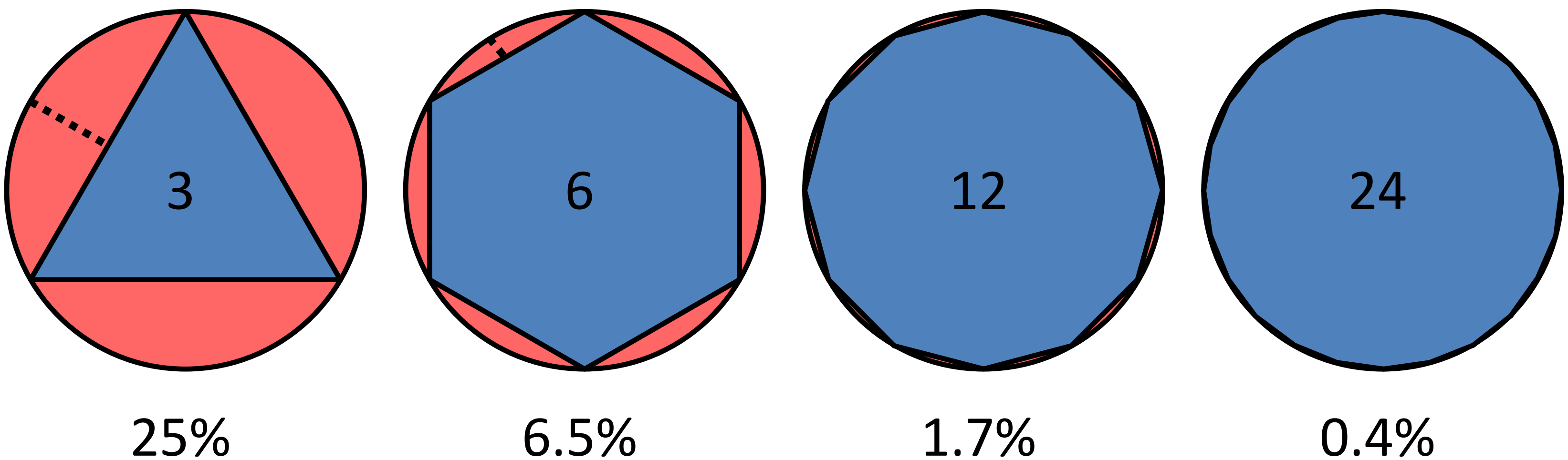
- Mesh basics
  - Zoo
  - Definitions
  - Important properties

- Mesh data structures

- HW1

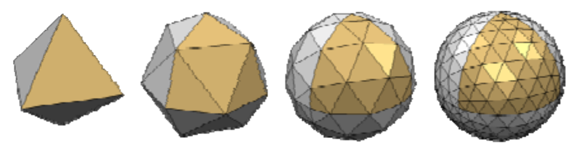
Polygonal Meshes

- Piecewise linear approximation
  - Error is $O(h^2)$



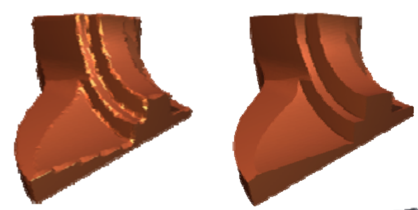
Polygonal Meshes

- Polygonal meshes are a good representation
  - approximation $O(h^2)$

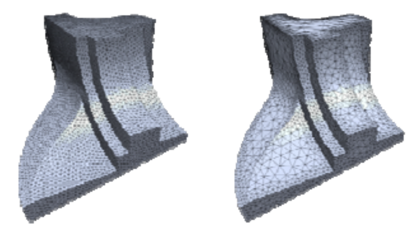


  - arbitrary topology

  - piecewise smooth surfaces



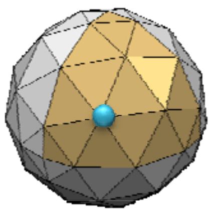
  - adaptive refinement



  - efficient rendering

Triangle Meshes

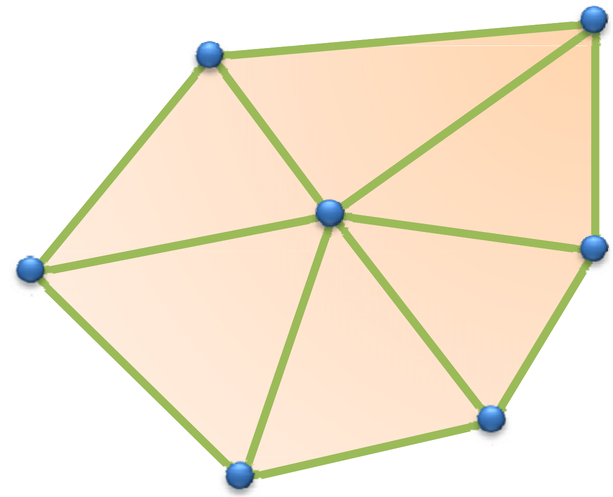
- Connectivity: vertices, edges, triangles



$$\mathcal{V} = \{v_1, \ldots, v_n\}$$

$$\mathcal{E} = \{e_1, \ldots, e_k\}\ , \quad e_i \in \mathcal{V} \times \mathcal{V}$$

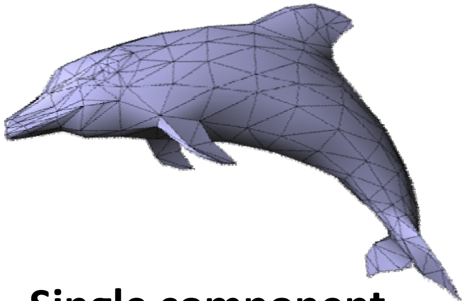
$$\mathcal{F} = \{f_1, \ldots, f_m\}\ , \quad f_i \in \mathcal{V} \times \mathcal{V} \times \mathcal{V}$$



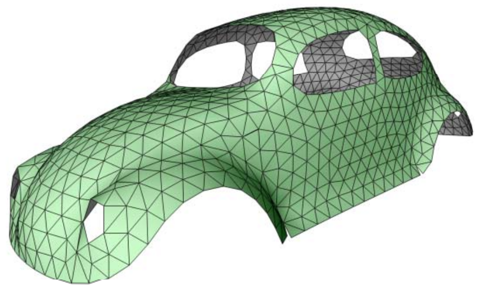
- Geometry: vertex positions

$$\mathcal{P} = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}\ , \quad \mathbf{p}_i \in \mathbb{R}^3$$

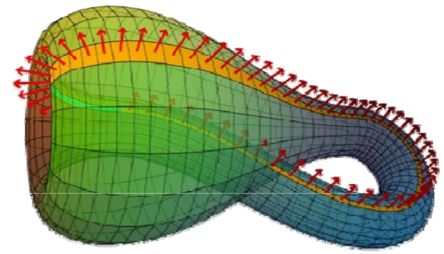
Mesh Zoo



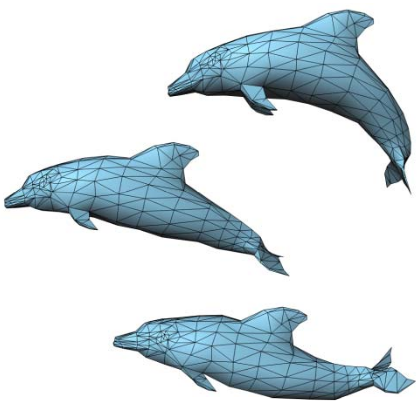
Single component, closed, triangular, orientable manifold



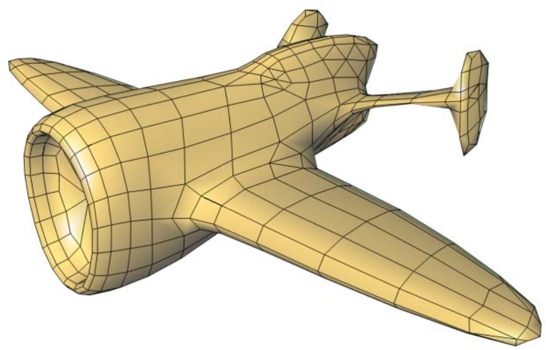
With boundaries



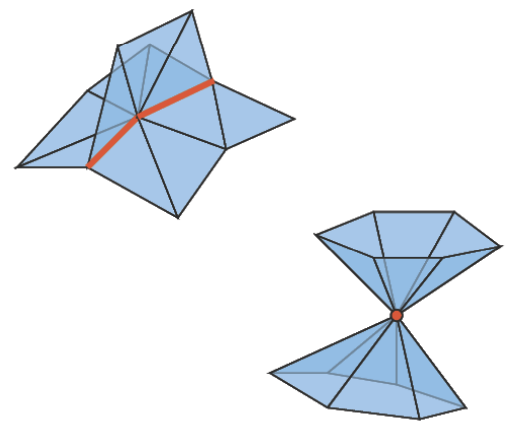
Not orientable



Multiple components



Not only triangles

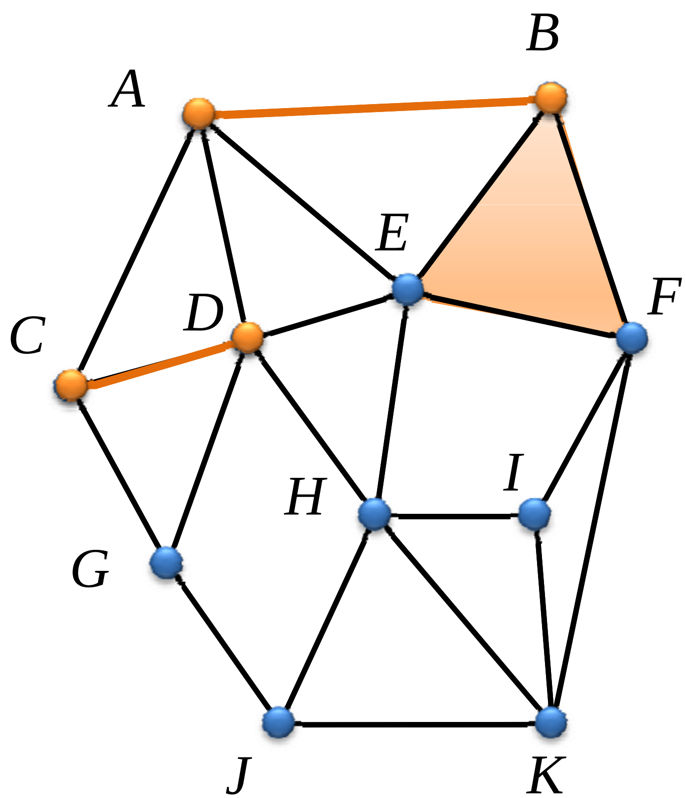


Non manifold

Mesh Definitions

- Need vocabulary to describe zoo meshes
- The connectivity of a mesh is just a graph
- We’ll start with some graph theory

Graph Definitions



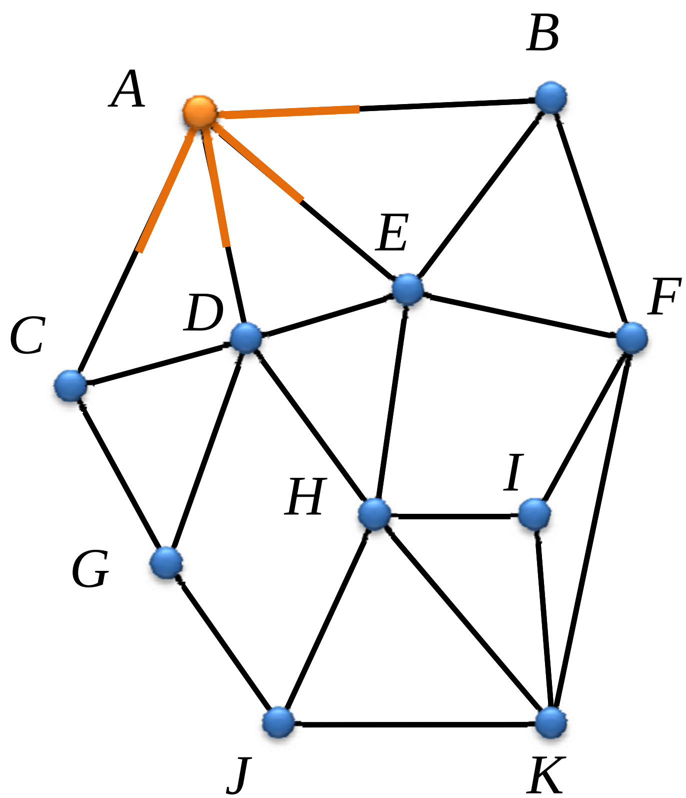
G = graph = <V, E>

V = vertices = {A, B, C, ..., K}

E = edges = {(AB), (AE), (CD), ...}

F = faces = {(ABE), ($DHJG$), ...}

Graph Definitions



Vertex degree or **valence** =
number of incident edges

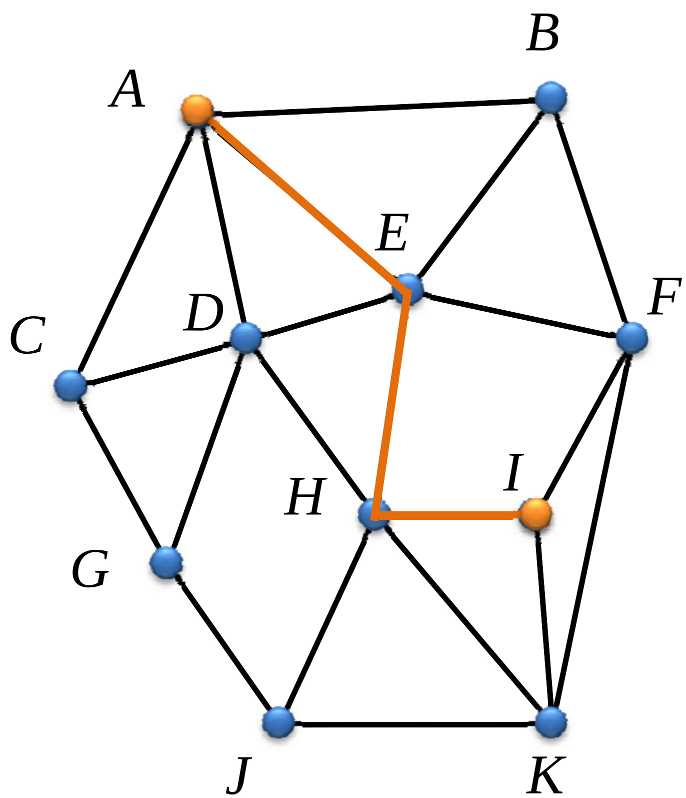
deg(A) = 4

deg(E) = 5

Regular mesh =

all vertex degrees are equal

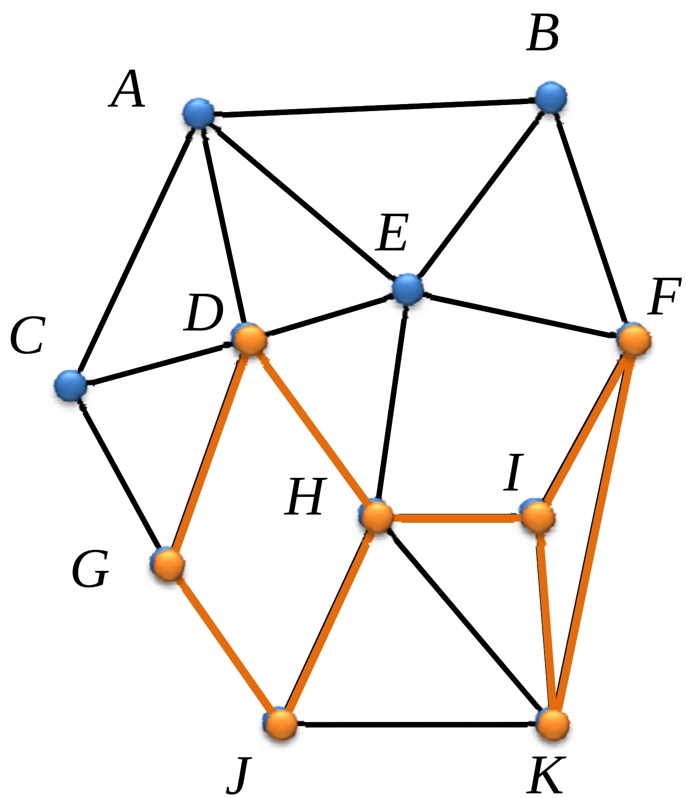
Connectivity



Connected =

path of edges connecting every two vertices

Connectivity



Connected =

path of edges connecting every two vertices

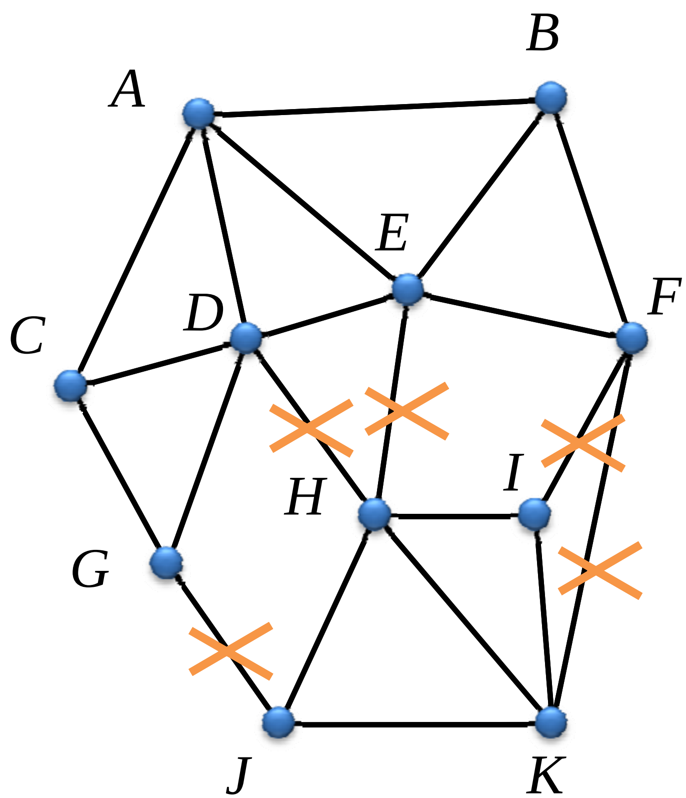
Subgraph =

G'=<V',E'> is a subgraph of G=<V,E> if

V' is a subset of V and

E' is the subset of E incident on V

Connectivity



Connected =

path of edges connecting every two vertices

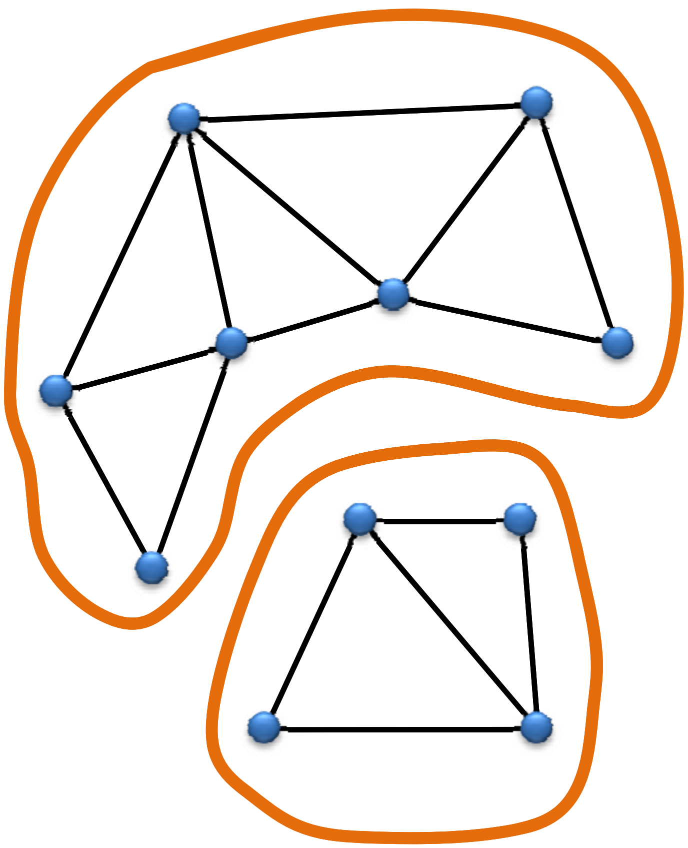
Subgraph =

$G'=\langle V',E'\rangle$ is a subgraph of $G=\langle V,E\rangle$ if

V' is a subset of V and

E' is a subset of E incident on V'

Connectivity



Connected =

path of edges connecting every two vertices

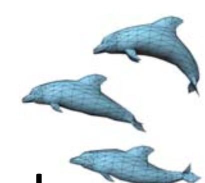
Subgraph =

$G'=<V',E'>$ is a subgraph of $G=<V,E>$ if

V' is a subset of V and

E' is the subset of E incident on V

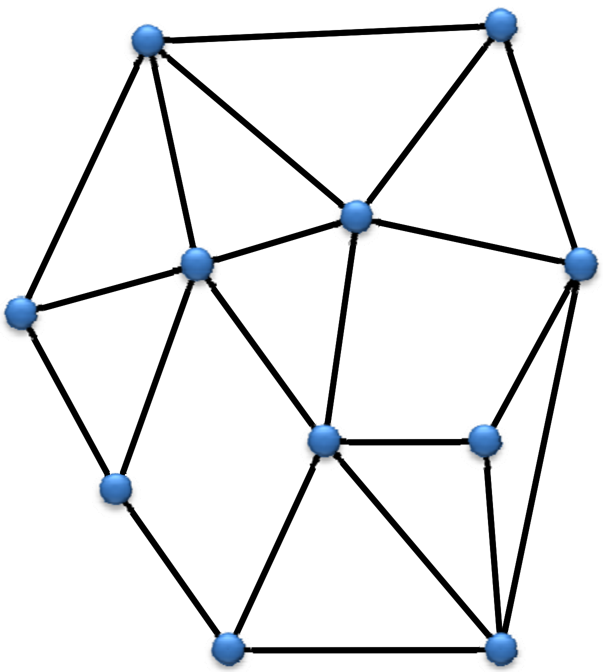
Connected Component =



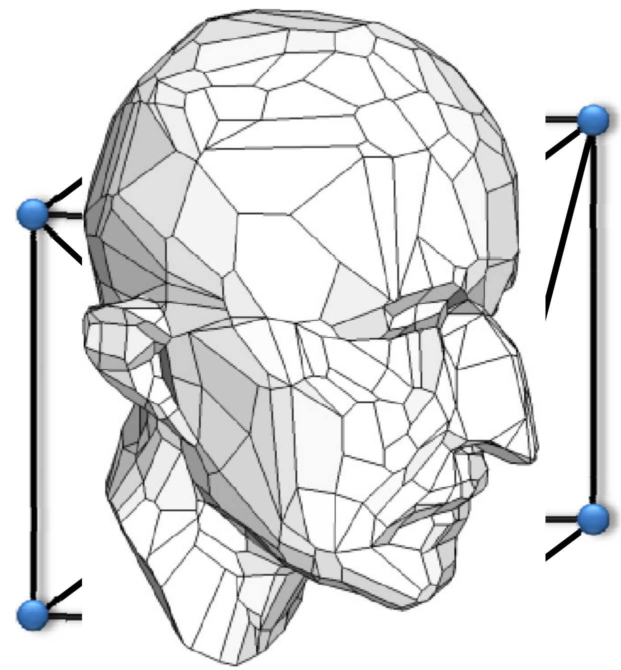
maximally connected subgraph

Graph Embedding

Embedding: G is embedded in $\mathbb{R}^d$, if each vertex is assigned a position in $\mathbb{R}^d$



Embedded in $\mathbb{R}^2$

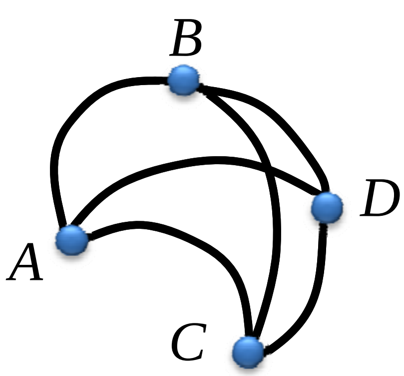


Embedded in $\mathbb{R}^3$

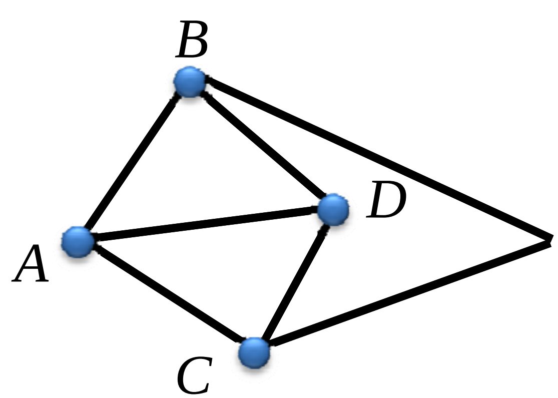
Planar Graphs

Planar Graph =
Graph whose vertices and edges
can be embedded in R^2 such that
its edges do not intersect

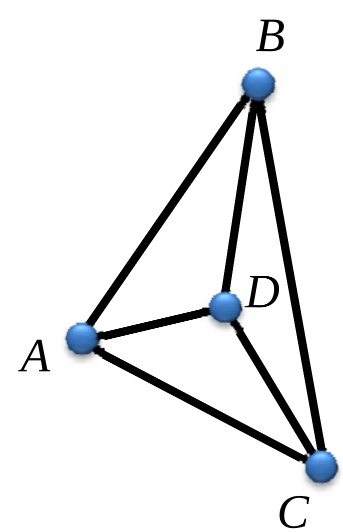
Planar Graph



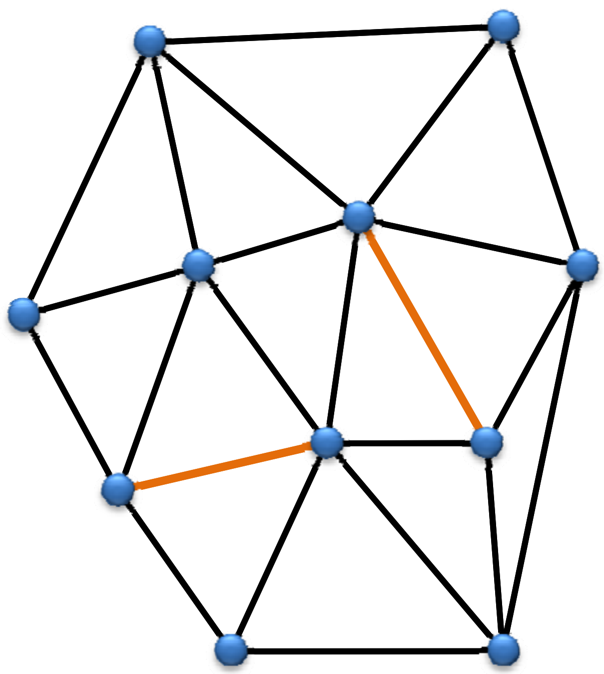
Plane Graph



Straight Line Plane Graph



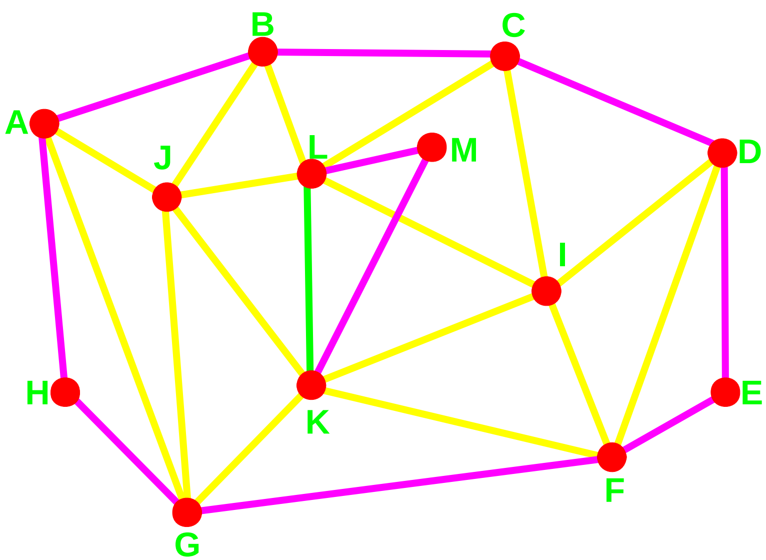
Triangulation



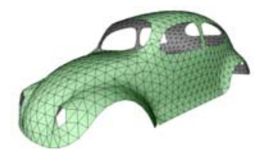
Triangulation:

Straight line plane graph where every face is a *triangle*.

Mesh

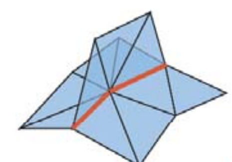


Mesh:
straight-line graph embedded in R^3

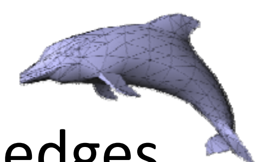


Boundary edge:
adjacent to exactly *one* face

Regular edge:
adjacent to exactly *two* faces



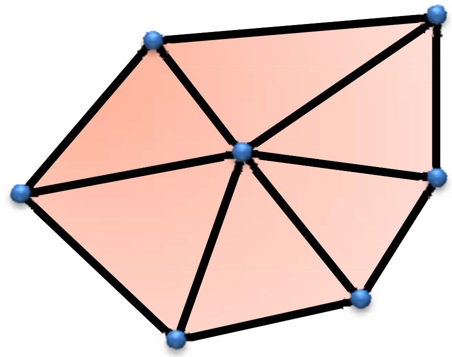
Singular edge:
adjacent to more than two faces

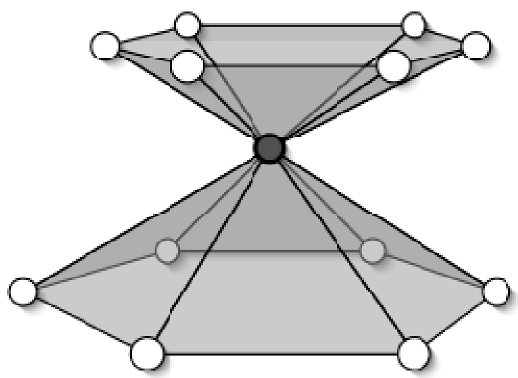


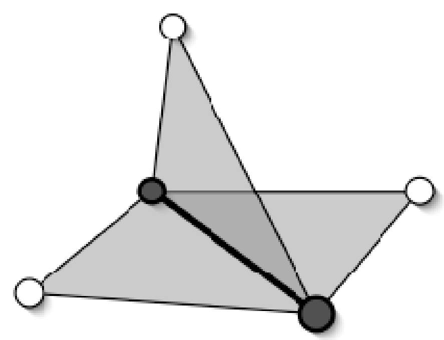
Closed mesh:
mesh with no boundary edges

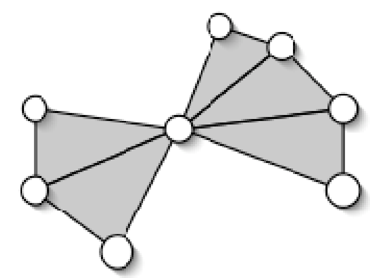
2-Manifolds Meshes

Disk-shaped neighborhoods







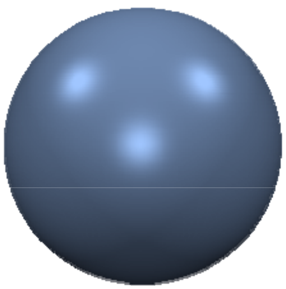


non-manifolds

Global Topology: Genus

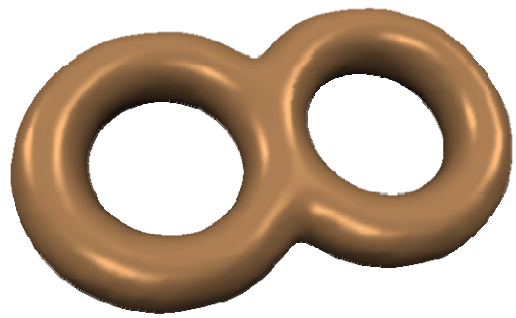
Genus:

Half the maximal number of closed paths that do not disconnect the mesh (= the number of holes)

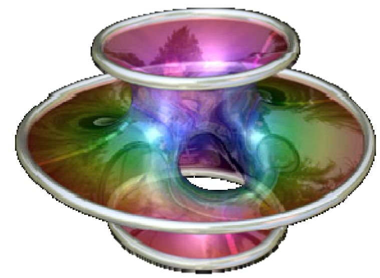


Genus 0

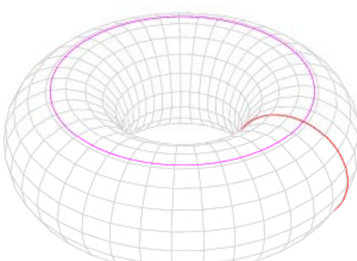
Genus 1

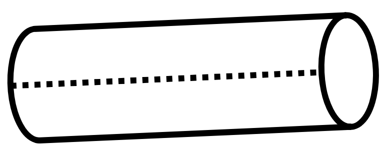


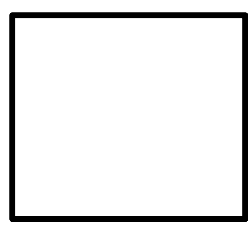
Genus 2



Genus ?



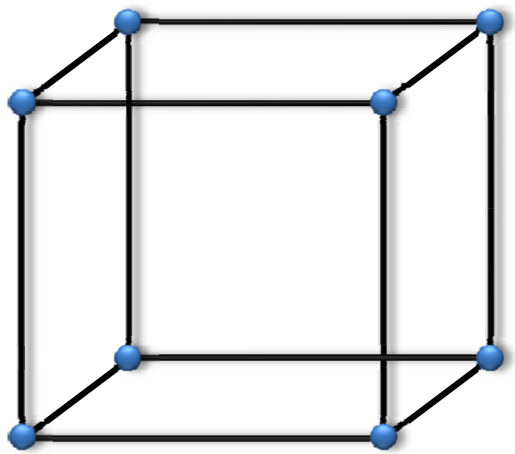


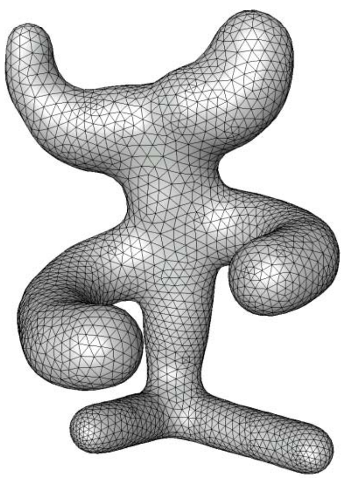


Closed 2-Manifold Polygonal Meshes

Euler-Poincaré formula

$$V + F - E = \boxed{\chi}$$ **Euler characteristic**





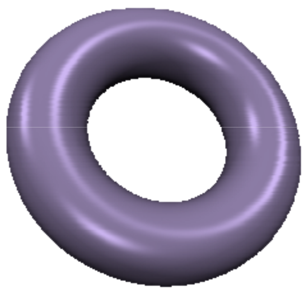
$V = 8$
$E = 12$
$F = 6$
$\chi = 8 + 6 - 12 = \mathbf{2}$

$V = 3890$
$E = 11664$
$F = 7776$
$\chi = \mathbf{2}$

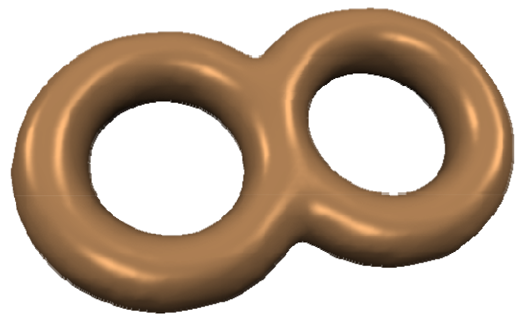
Closed 2-Manifold Polygonal Meshes

Euler-Poincaré formula

$$V + F - E = \chi = 2$$



$V = 1500$, $E = 4500$
$F = 3000$, $\mathbf{g = 1}$
$\chi = \mathbf{0}$



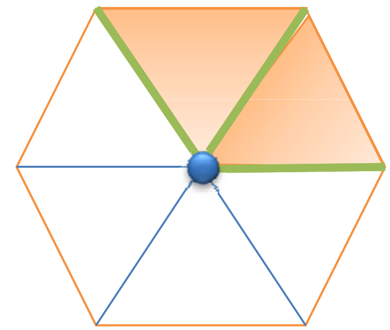
$\mathbf{g = 2}$
$\chi = \mathbf{-2}$

Closed 2-Manifold Triangle Meshes

- *Triangle* mesh statistics

  $E \approx 3V$

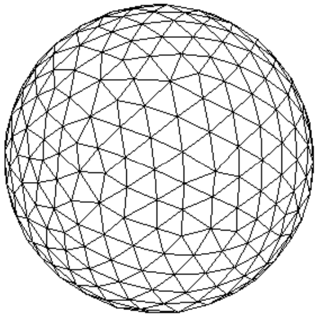
  $F \approx 2V$



- Avg. valence ≈ 6
  Show using Euler Formula

- When can a closed triangle mesh be 6-regular?

Exercise



Theorem: Average vertex degree in closed manifold triangle mesh is ~6

Proof: In such a mesh, 3F = 2E by counting edges of faces.

By Euler's formula: V+F-E = V+2E/3-E = 2-2g.
Thus E = 3(V-2+2g)

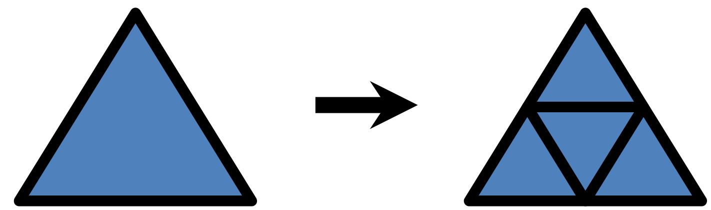
So Average(deg) = 2E/V = 6(V-2+2g)/V ~ 6 for large V

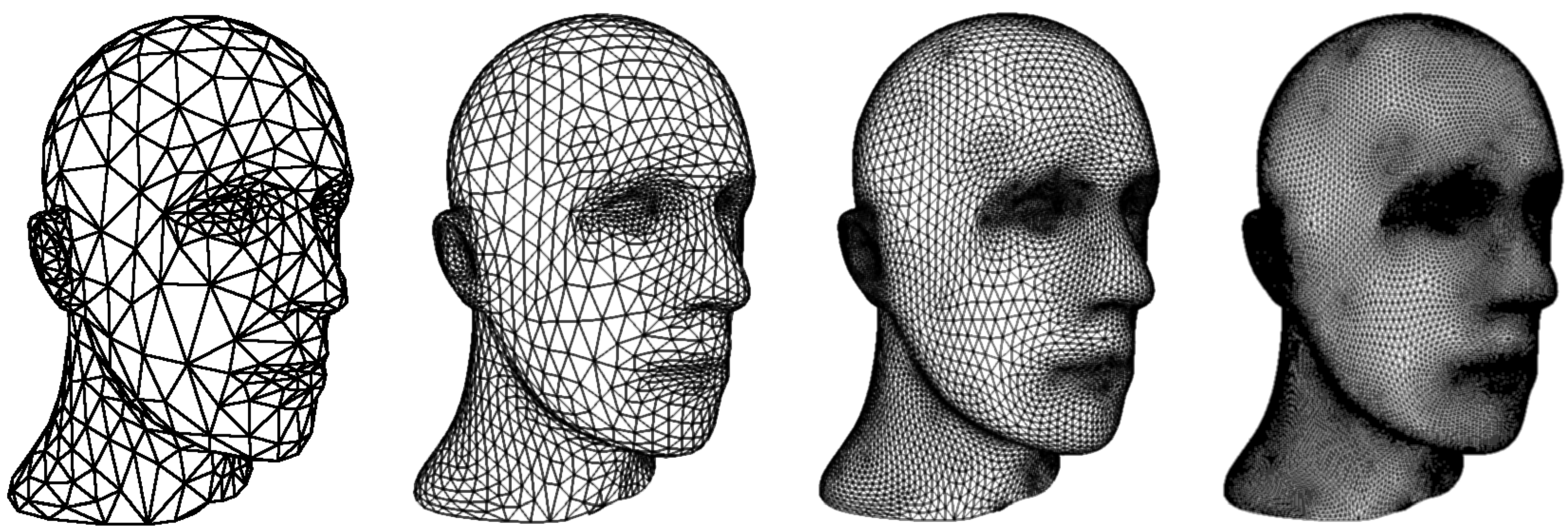
Corollary: Only toroidal (g=1) closed manifold triangle mesh can be regular (all vertex degrees are 6)

Proof: In regular mesh average degree is *exactly* 6.
Can happen only if g=1

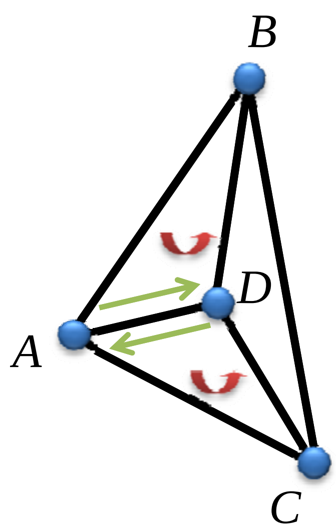
Regularity

- semi-regular





Orientability



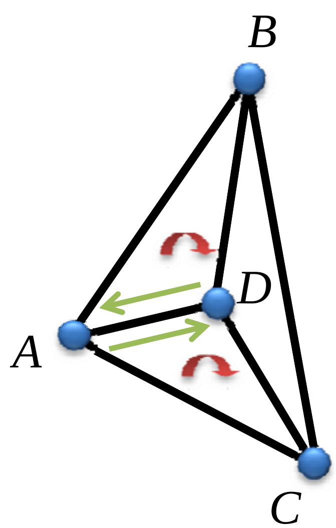
Face Orientation =
clockwise or anticlockwise order in which the vertices listed

defines direction of face **normal**

Oriented **CCW**:
{(C,**D,A**), (**A,D**,B), (C,B,D)}

Oriented **CW**:
{(C,A,D), (D,A,B), (B,C,D)}

Orientability



Face Orientation =
clockwise or anticlockwise order in which the vertices listed

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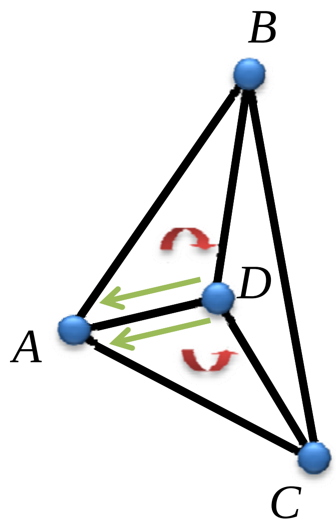
Oriented **CCW**:
{(C,D,A), (A,D,B), (C,B,D)}

Oriented **CW**:
{(C,**A,D**), (**D,A**,B), (B,C,D)}

Not oriented:
{(C,D,A), (D,A,B), (C,B,D)}

Orientability



Face Orientation =
clockwise or anticlockwise order in which the vertices listed

defines direction of face **normal**

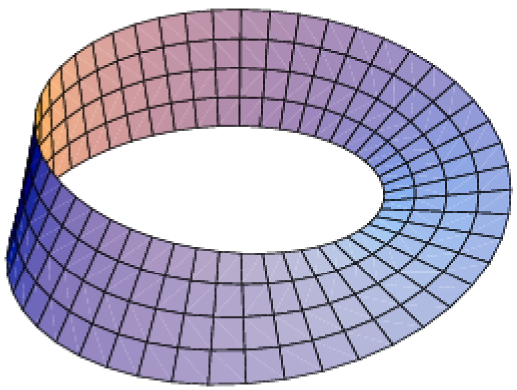
Oriented **CCW**:
{(C,D,A), (A,D,B), (C,B,D)}

Oriented **CW**:
{(C,A,D), (D,A,B), (B,C,D)}

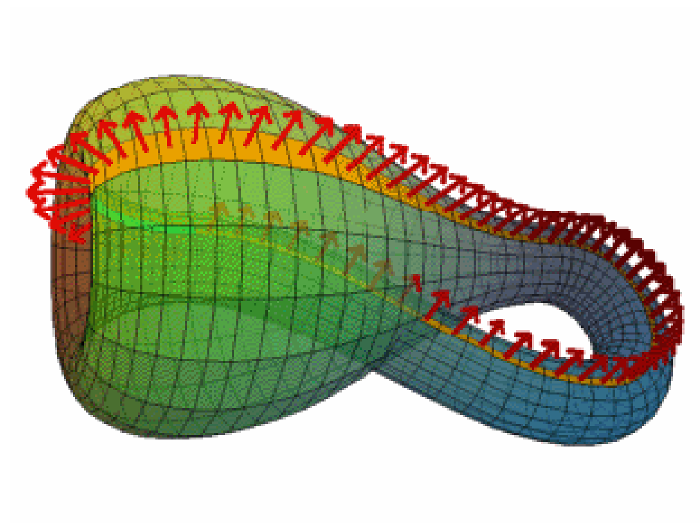
Not oriented:
{(C,**D,A**), (**D,A**,B), (C,B,D)}

Orientable Plane Graph =
orientations of faces can be chosen so that each non-boundary edge is oriented in *both* directions

Non-Orientable Surfaces

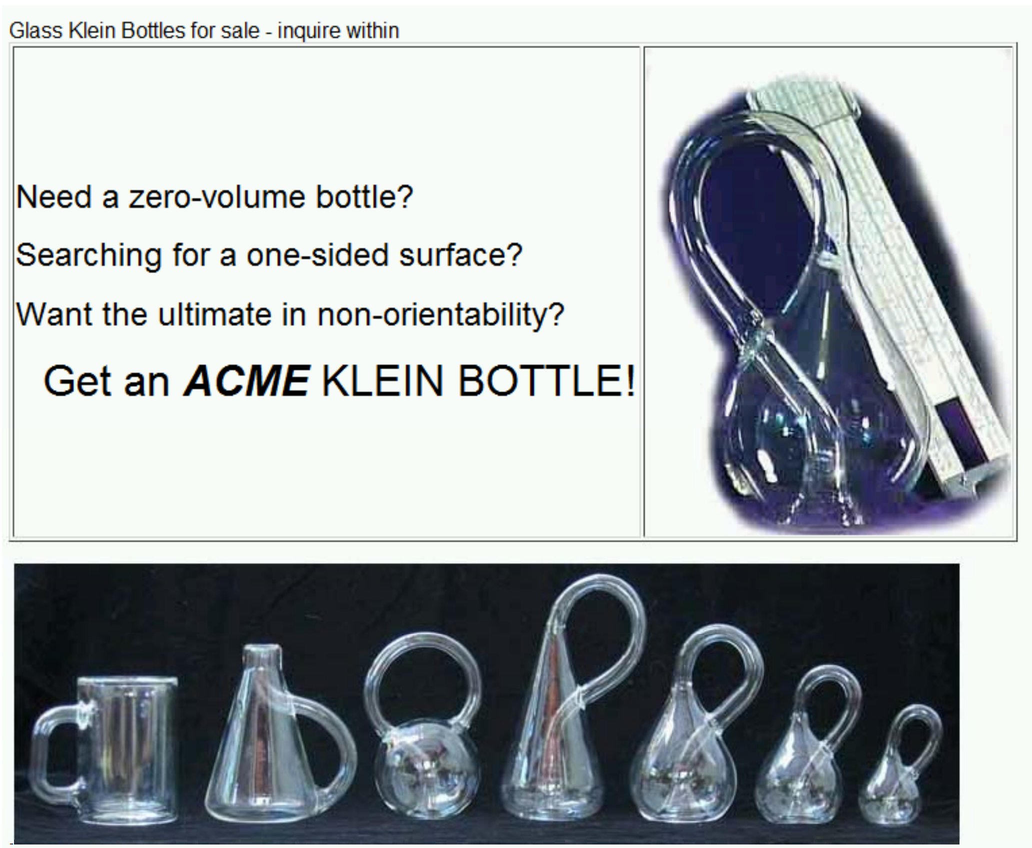


Mobius Strip



Klein Bottle

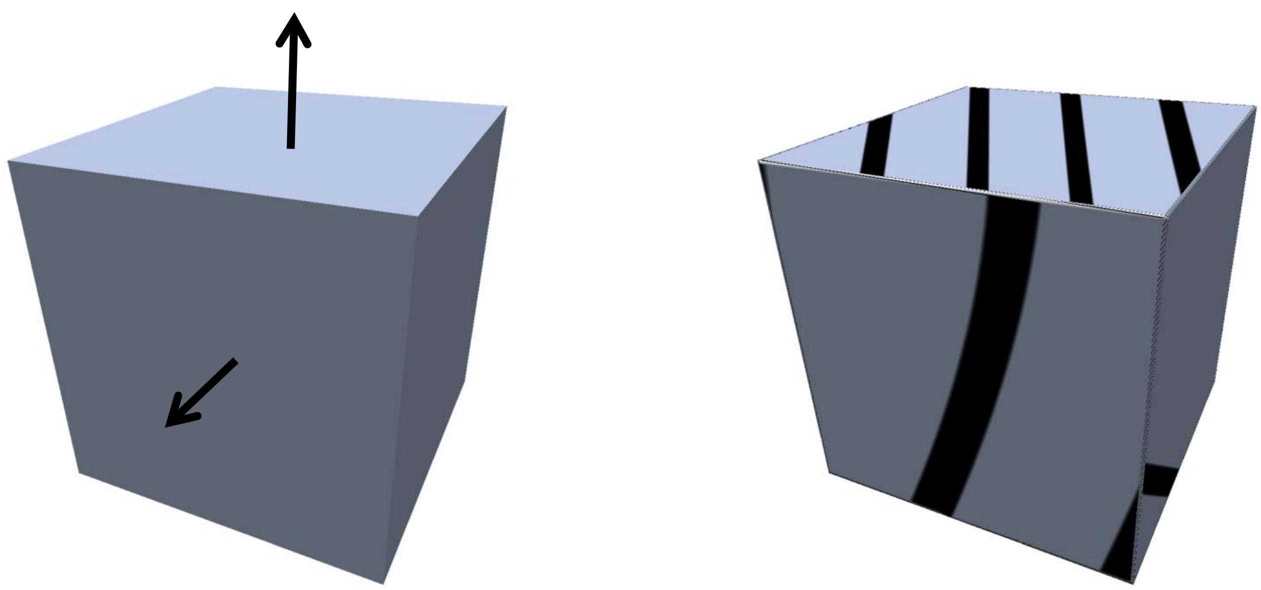
Garden Variety Klein Bottles



http://www.kleinbottle.com/

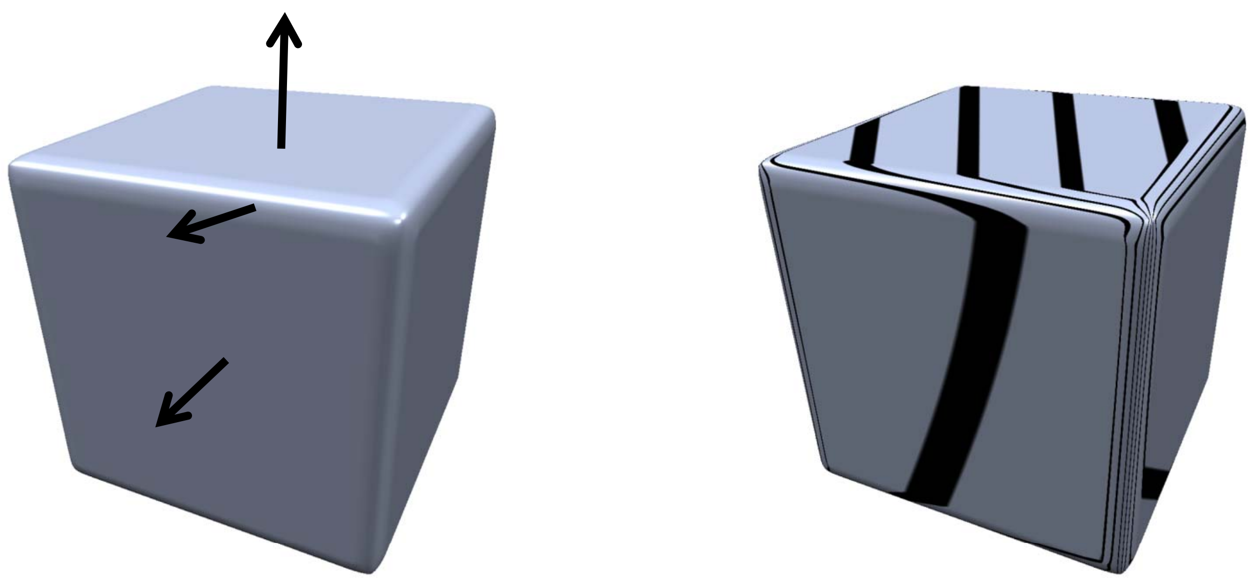
Smoothness

- Position continuity = C^0



Smoothness

- Position continuity = C^0
- Tangent continuity ≈ C^1



Smoothness

- Position continuity = C^0
- Tangent continuity ≈ C^1
- Curvature continuity ≈ C^2