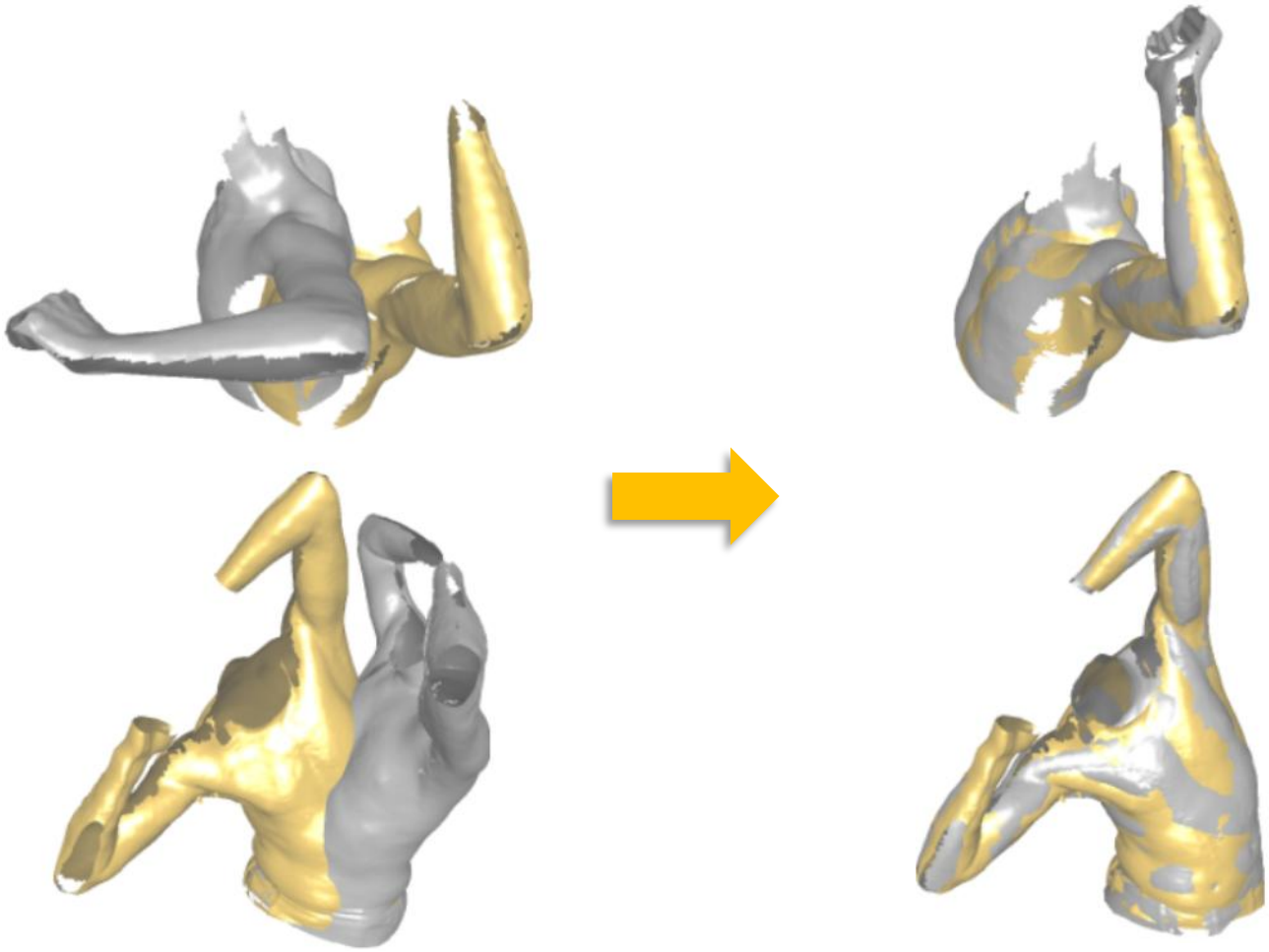
Data-Driven Shape Analysis --- Non-Rigid Registration

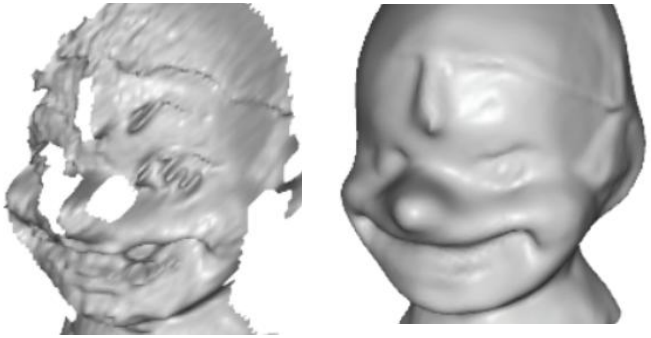
Qi-xing Huang
Stanford University

Non-rigid registration

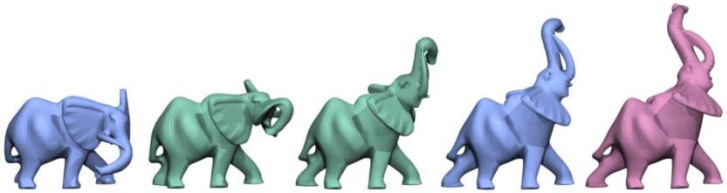


Applications

Dynamic geometry reconstruction
[Li et al. 13]



Tracking
[Li et al. 09]



Interpolation
[Kilian et al. 08]

Shape completion
[Pauly et al. 05]

Application --- distance learning

Input

Rigid

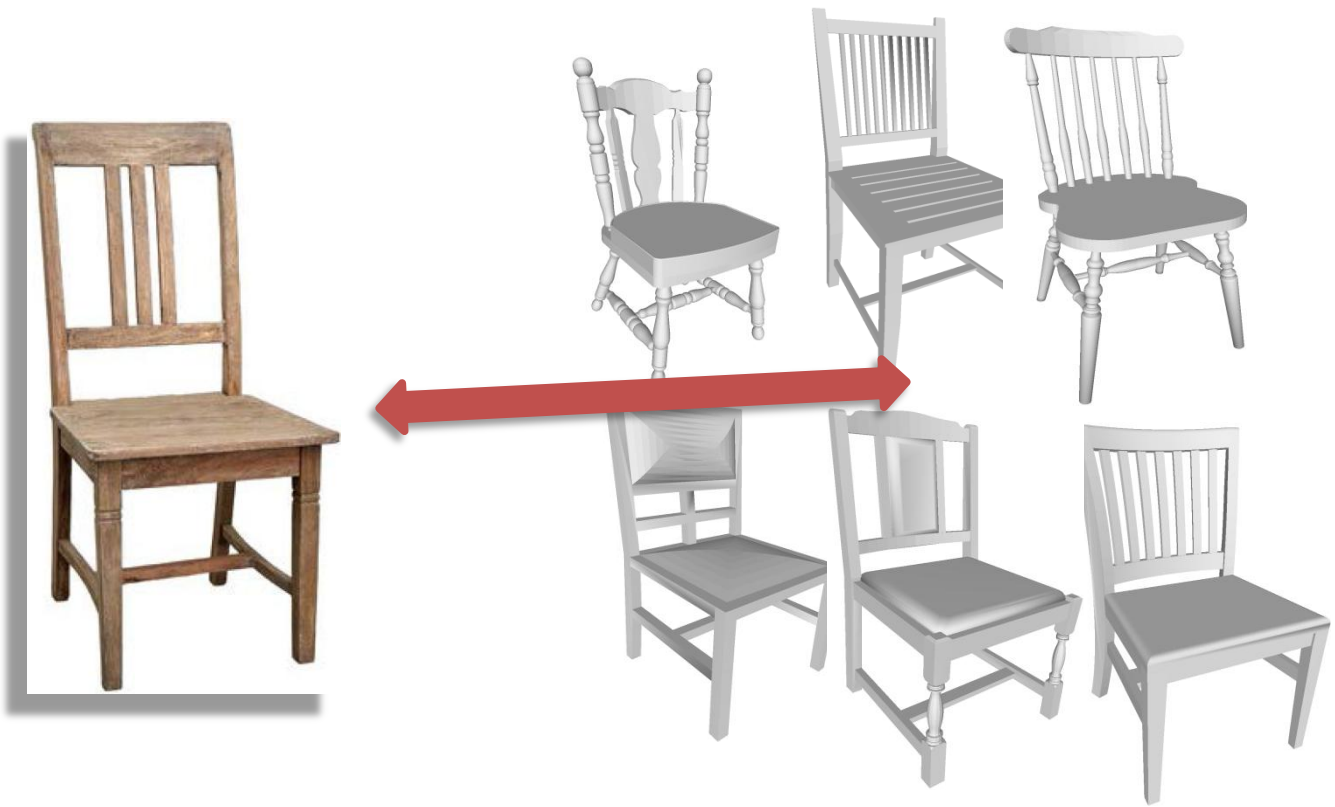
Non-rigid

with-arms side windsor rex

Distance field Spin images

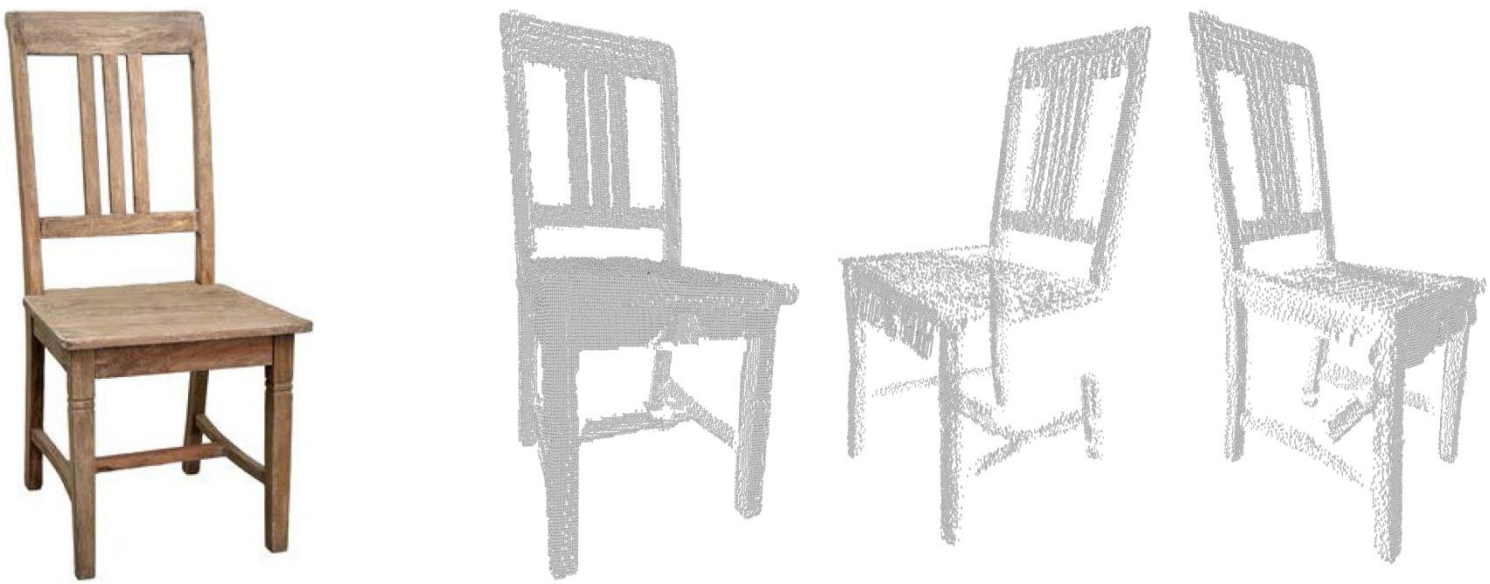
Fine-Grained Semi-Supervised Labeling of Large Shape Collections, Q. Huang, H. Su, L. Guibas, SIGGRAPH ASIA' 13

Application --- depth inference



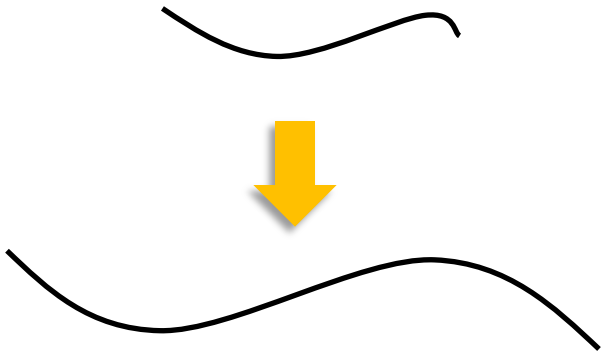
Estimating 3D Attributes of Images from Shape Collections, with H. Su, N. Mitra, Y. Li, and L. Guibas, SIGGRAPH' 14

Application --- depth inference

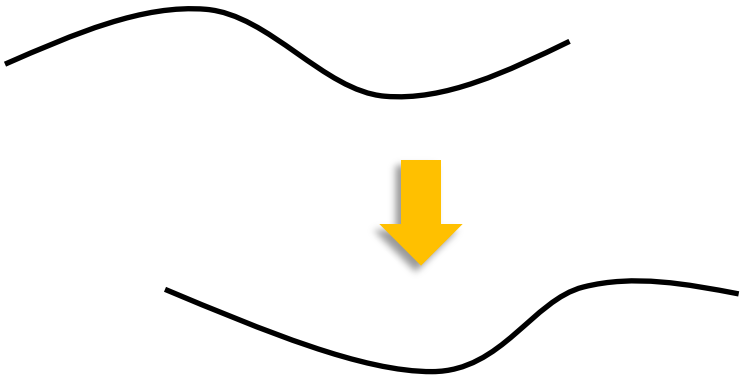


Estimating 3D Attributes of Images from Shape Collections, with H. Su, N. Mitra, Y. Li, and L. Guibas, SIGGRAPH' 14

Two important cases



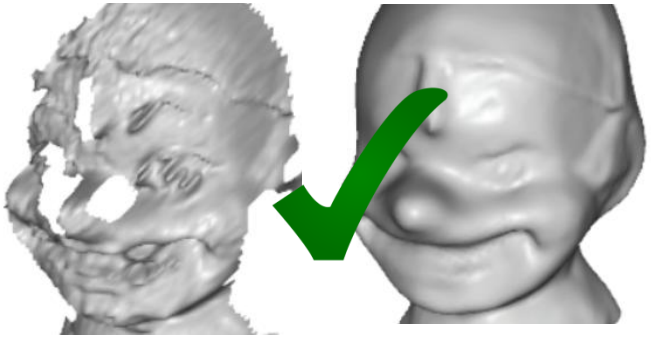
“Embedding”



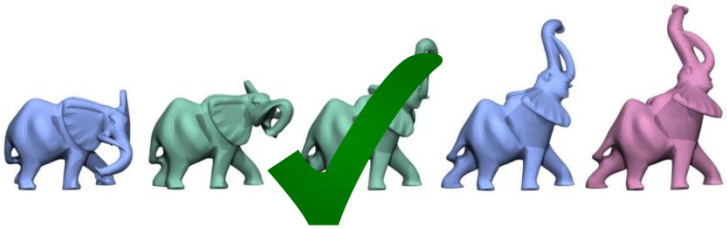
Partially overlap

“Embedding”?

Dynamic geometry reconstruction
[Li et al. 13]



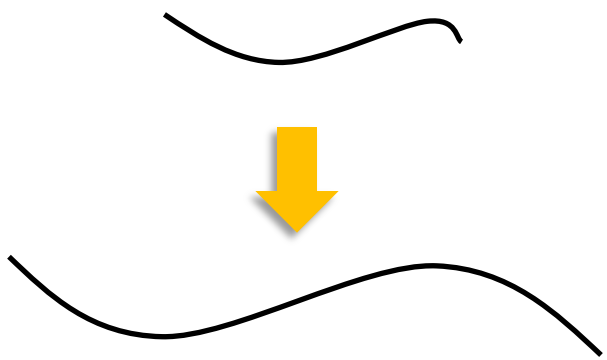
Tracking
[Li et al. 09]



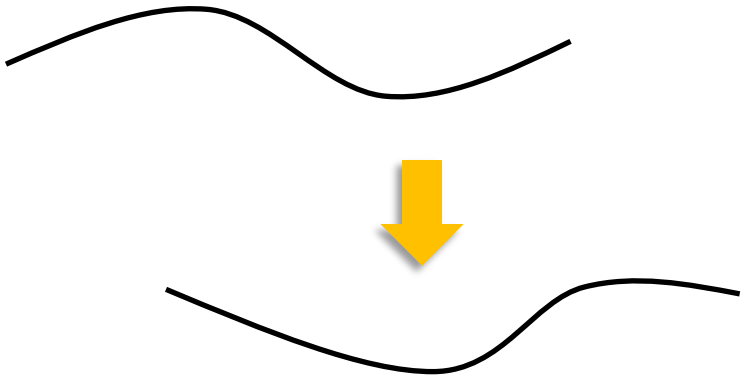
Interpolation
[Kilian et al. 08]

Shape completion
[Pauly et al. 05]

Two important cases

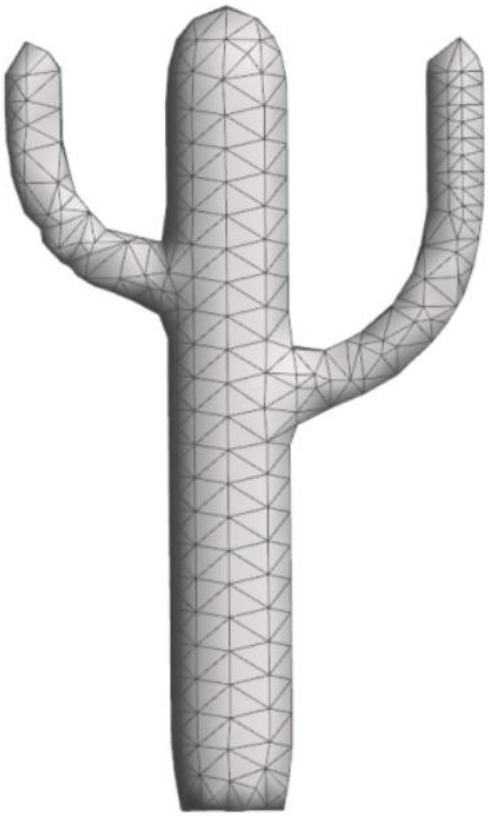


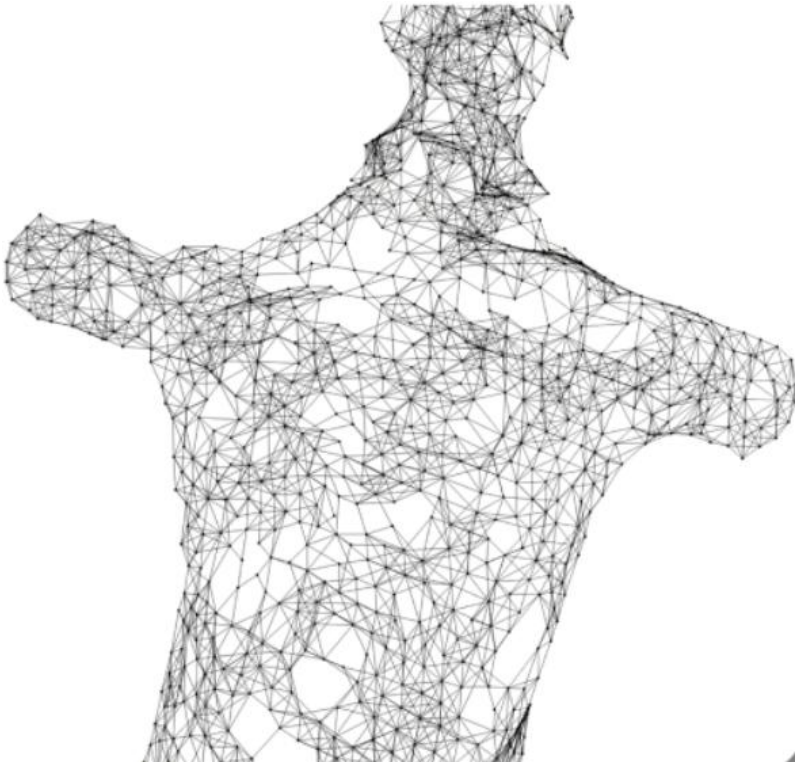
“Embedding”



Partially overlap

Shape representation



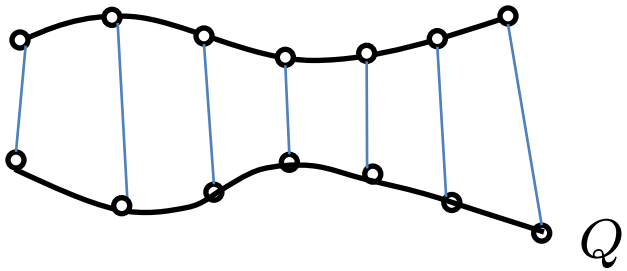


Graph representation

Non-Rigid ICP

- Compute closest point pairs

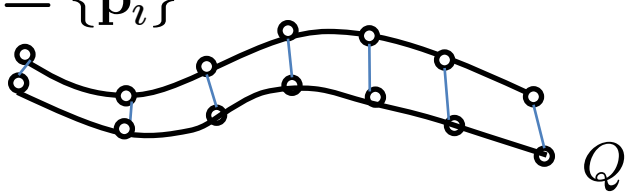
$P = \{\mathbf{p}_i\}$



$\{(\mathbf{p}_i, \mathbf{q}_i)\}$

- Deform the source shape P

$P = \{\mathbf{p}_i\}$

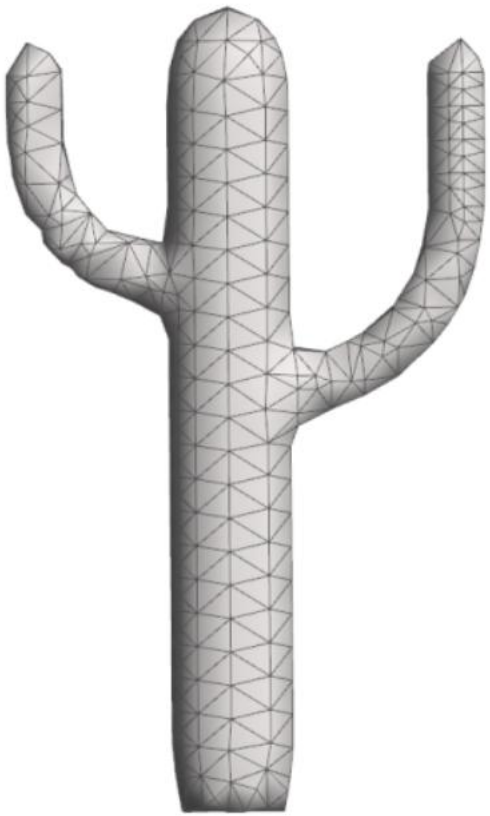


$$\sum_{i=1}^{N} \|\mathbf{p}_i - \mathbf{q}_i\|^2 + \lambda d^2(\{\mathbf{p}_i\}, \{\mathbf{p}_i^{rest}\})$$

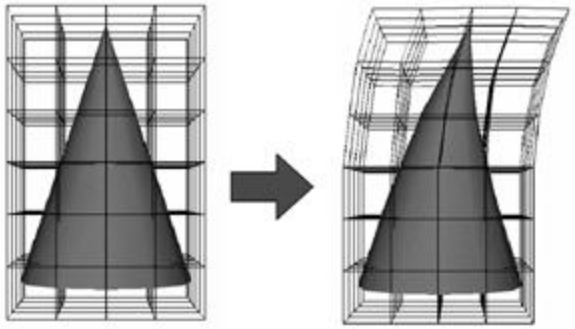
Distance term *Deformation term*

Deformation Formulation

Deformation term



Intrinsic version



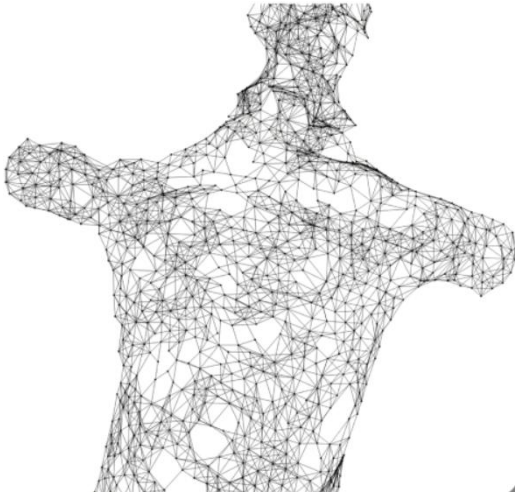
Extrinsic version

Intrinsic formulation

- Distance-preserving

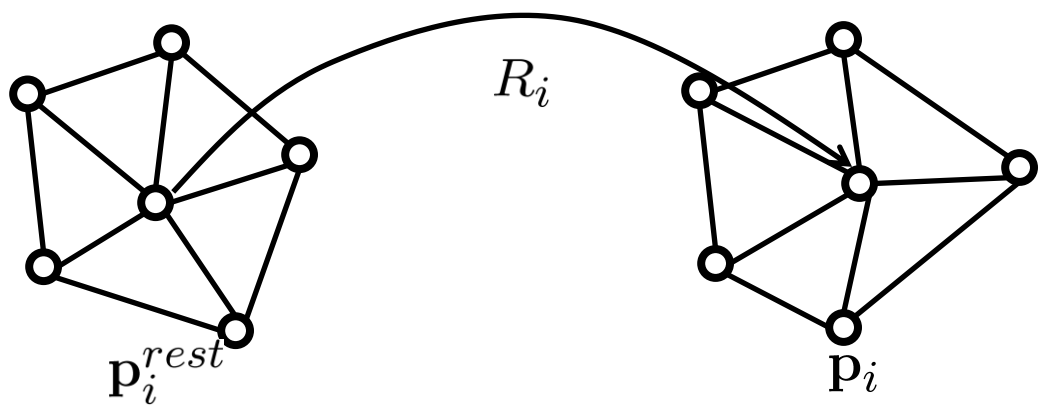
$$d^2(\{\mathbf{p}_i\}, \{\mathbf{p}_i^{rest}\}) = \sum_{(i,j)\in\mathcal{E}} (\|\mathbf{p}_i - \mathbf{p}_j\| - \|\mathbf{p}_i^{rest} - \mathbf{p}_j^{rest}\|)^2$$

- Issues
  - Hard to optimize
  - Sensitive to graph quality



Intrinsic formulation

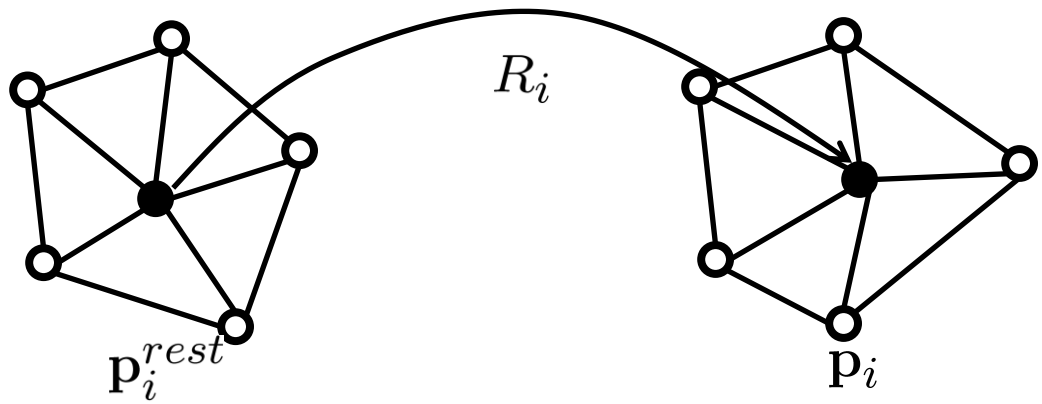
- As-rigid-as possible formulation
  - Associate each node with a rotation matrix
    - Deformation gradient [Summer 04]
  - Controls the transformation of the neighborhood



$$\sum_{i=1}^{N} \sum_{j \in N(i)} \|R_i(\mathbf{p}_i^{rest} - \mathbf{p}_j^{rest}) - (\mathbf{p}_i - \mathbf{p}_j)\|^2$$

Intrinsic formulation

- A variant [summer et al. 07]



$$\sum_{i=1}^{N} \sum_{j \in N(i)} \|R_i(\mathbf{p}_i^{rest} - \mathbf{p}_j^{rest}) - (\mathbf{p}_i - \mathbf{p}_j)\|^2$$

Soft constraints:

$$\mathrm{Rot}(\mathbf{R}) = (\mathbf{c}_1 \cdot \mathbf{c}_2)^2 + (\mathbf{c}_1 \cdot \mathbf{c}_3)^2 + (\mathbf{c}_2 \cdot \mathbf{c}_3)^2 + (\mathbf{c}_1 \cdot \mathbf{c}_1 - 1)^2 + (\mathbf{c}_2 \cdot \mathbf{c}_2 - 1)^2 + (\mathbf{c}_3 \cdot \mathbf{c}_3 - 1)^2$$

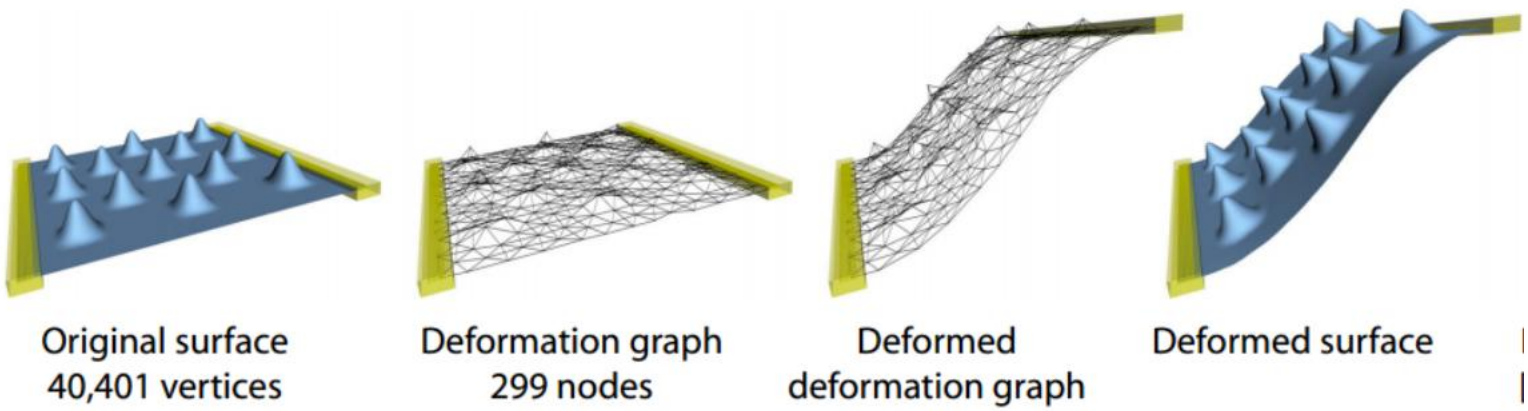
Deformation field

- Deform neighboring points
- Partition-of-unity

The transformation associated with each node

$$\hat{\mathbf{x}} = \sum_{i=1}^{N} w_i(\mathbf{x})[R_i(\mathbf{x} - \mathbf{p}_i^{rest}) + (\mathbf{p}_i - \mathbf{p}_i^{rest})]$$

Blending weight [Summer et al. 07]

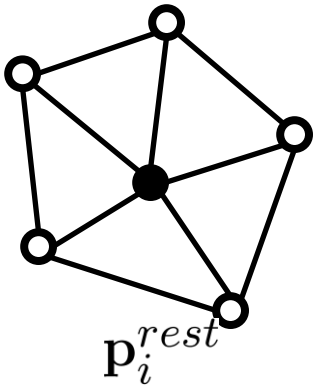


Weight function

- Compact support
- Smooth

$$w_i(t) = \begin{cases} (1-t)^2 & t < 1 \\ 0 & \textit{otherwise} \end{cases}$$

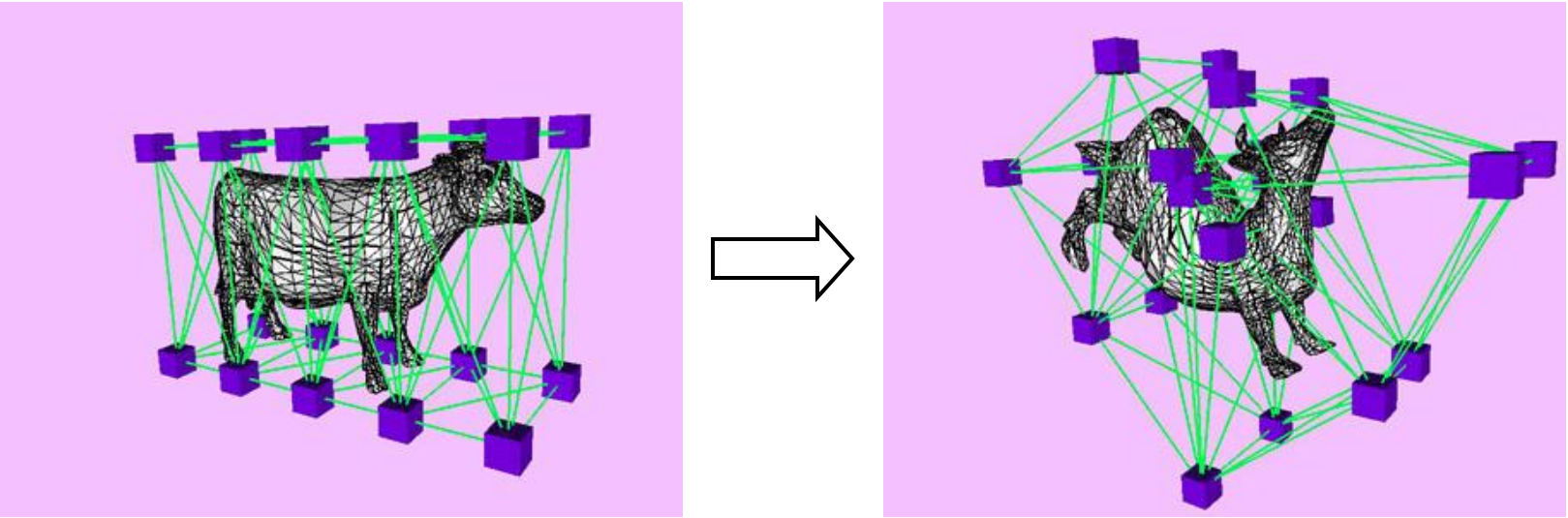
$$t = \frac{\|\mathbf{x} - \mathbf{p}_i^{rest}\|}{r_{\text{max}}}$$



Extrinsic formulation

- Free-form deformation [Sederberg and Parry 86]

$$\varphi(x,y,z) = \sum_{I=1}^{n} b_I(x,y,z)\boldsymbol{X}_I$$



Free-Form Deformation of Solid Geometric Models, T. Sederberg, S. Parry , SIGGRAPH' 86

Extrinsic formulation

- Free-form deformation [Sederberg and Parry 86]

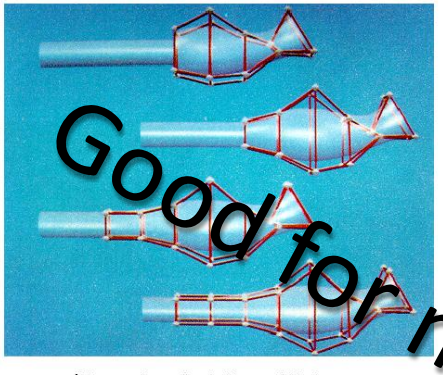


Fig. 11 Local C^1 Control Points

Fig. 12 C^{-1}, C^0, C^1 and C^2 Local Deformations

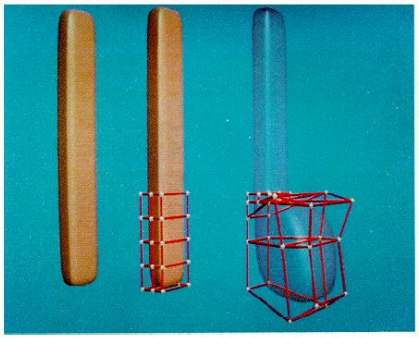


Fig. 14 Local FFD

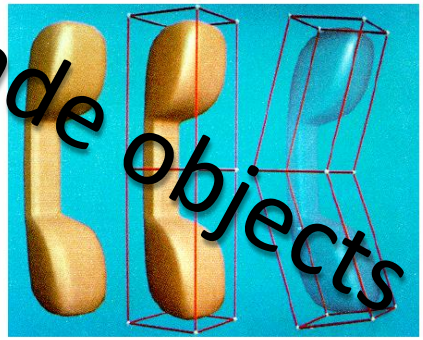
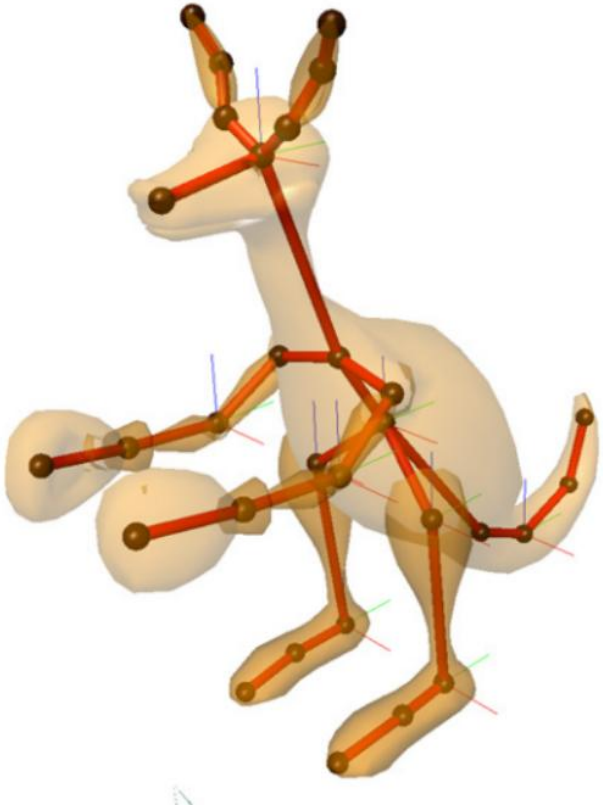


Fig. 15 Global FFD

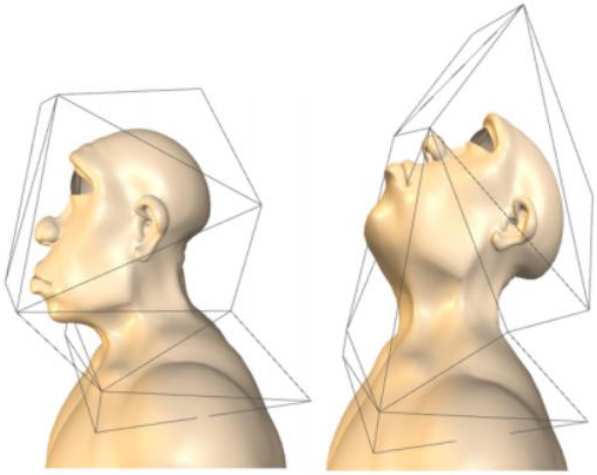
Good for man-made objects

Free-Form Deformation of Solid Geometric Models, T. Sederberg, S. Parry , SIGGRAPH' 86

Extrinsic formation



Skeleton-based

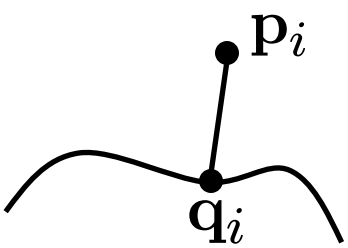


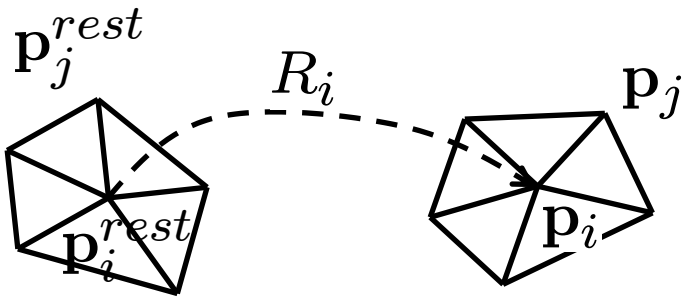
Cage-based

Green coordinates, Y. Lipman, D. Levin, D. Cohen-Or, SIGGRAPH’08

Optimization

Optimization





$$\sum_{i=1}^{N} \|\boxed{\mathbf{p}_i} - \mathbf{q}_i\|^2 + \lambda \sum_{(i,j)\in\mathcal{E}} \|\boxed{R_i}(\mathbf{p}_i^{rest} - \mathbf{p}_j^{rest}) - (\boxed{\mathbf{p}_i} - \boxed{\mathbf{p}_j})\|^2$$

Variables

$$S(\boldsymbol{\beta}) = \sum_{i=1}^{m} r_i(\boldsymbol{\beta})^2.$$

Gauss-Newton optimization

{A} ⇌ {B}

Alternating optimization

The objective function is also used for shape manipulation

Gauss-Newton optimization

Gaussian-Newton optimization

Non-linear least squares:

$$f(\mathbf{x}) = \sum_{i=1}^{N} r_i^2(\mathbf{x}) \Rightarrow \sum_{i=1}^{N} (r_i(\mathbf{x}_k) + \nabla r_i(\mathbf{x}_k)\mathbf{d}_k)^2$$

Search direction:

Approximated Hessian

Gradient

$$\mathbf{d}_k = -(\sum_{i=1}^{N} \nabla r_i(\mathbf{x}) \nabla r_i(\mathbf{x})^T)^{-1} (\sum_{i=1}^{N} \nabla r_i(\mathbf{x}) r_i(\mathbf{x}))$$

Update:

Line search

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha \mathbf{d}_k$$

Convergence rate:

Residual

$$\epsilon_{k+1} \approx C(\epsilon_k^2 + O(f(\mathbf{x}^*))\epsilon_k)$$

Gauss-Newton optimization

$$\sum_{i=1}^{N} \|\mathbf{p}_i - \mathbf{q}_i\|^2 + \lambda \sum_{(i,j)\in\mathcal{E}} \|R_i(\mathbf{p}_i^{rest} - \mathbf{p}_j^{rest}) - (\mathbf{p}_i - \mathbf{p}_j)\|^2$$

First order approximation:

$$\mathbf{p}_i = \mathbf{p}_i^k + \mathbf{d}_i$$

$$R_i = (I_3 + \mathbf{c}_i\times)R_i^k$$

$$\mathbf{c}\times = \begin{pmatrix} 0 & -c_z & c_y \\ c_z & 0 & -c_x \\ -c_y & c_x & 0 \end{pmatrix}$$

Exponential map:

$$R_i^{k+1} = \exp(\mathbf{c}_i\times)R_i^k \quad \text{where}$$

$$\exp(\mathbf{c}) = I_3 + \frac{\mathbf{c}}{\|\mathbf{c}\|}(\mathbf{c}\times) + \frac{1-\cos(\|\mathbf{c}\|)}{\|\mathbf{c}\|^2}\mathbf{c}\times)^2$$

Rodrigues' formula

Alternating optimization

Alternating optimization

$$\sum_{i=1}^{N} \|\mathbf{p}_i - \mathbf{q}_i\|^2 + \lambda \sum_{(i,j)\in\mathcal{E}} \|R_i(\mathbf{p}_i^{rest} - \mathbf{p}_j^{rest}) - (\mathbf{p}_i - \mathbf{p}_j)\|^2$$

- When rotations are fixed

$$\sum_{i=1}^{N} \|\mathbf{p}_i - \mathbf{q}_i\|^2 + \lambda \sum_{(i,j)\in\mathcal{E}} \|\mathbf{e}_{ij} - (\mathbf{p}_i - \mathbf{p}_j)\|^2$$

Linear system, which allows pre-factorization

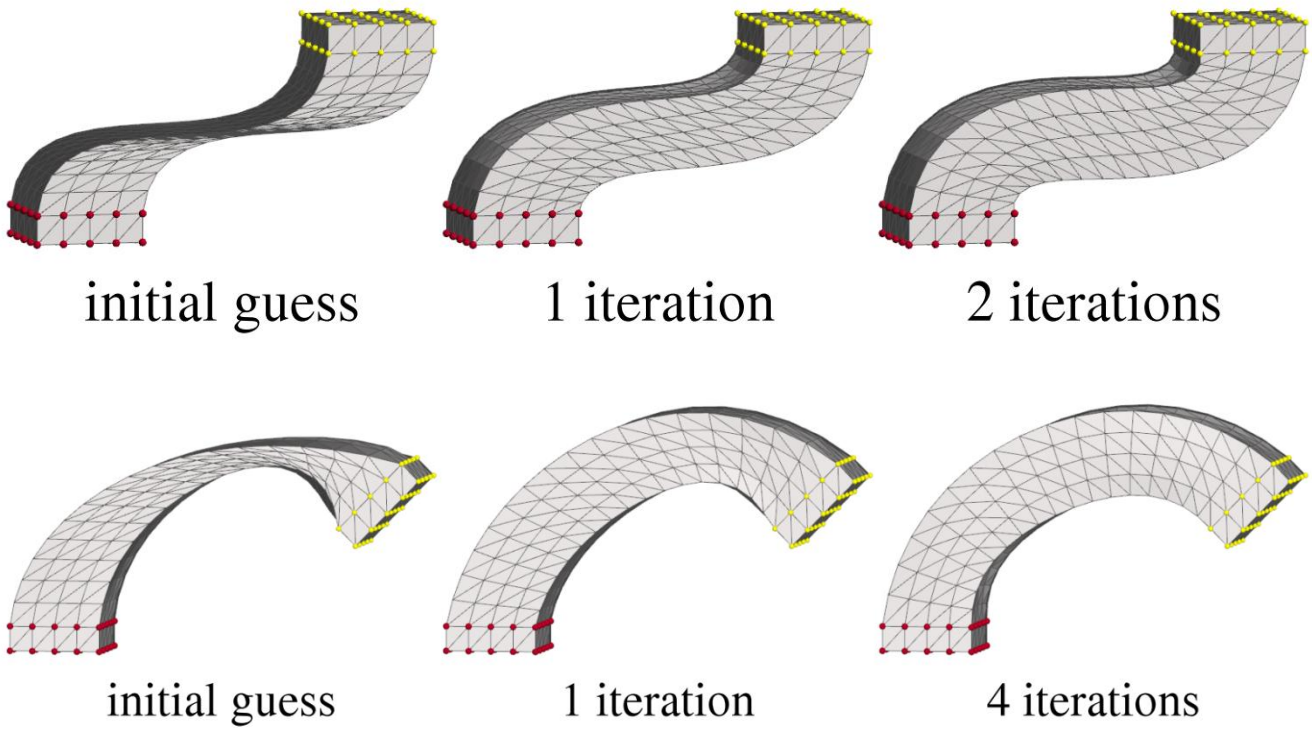
- When positions are fixed

$$R_i = \arg\min_{R_i} \sum_{j\in N(i)} \|R_i \mathbf{e}_{ij}^{rest} - \mathbf{e}_{ij}\|^2$$

Registration with known correspondences --- closed form solution

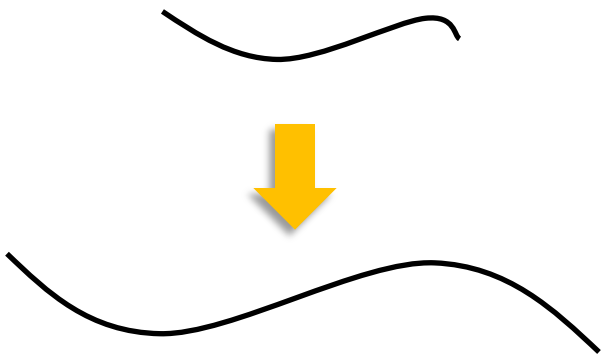
Alternate optimization

- Shape editing

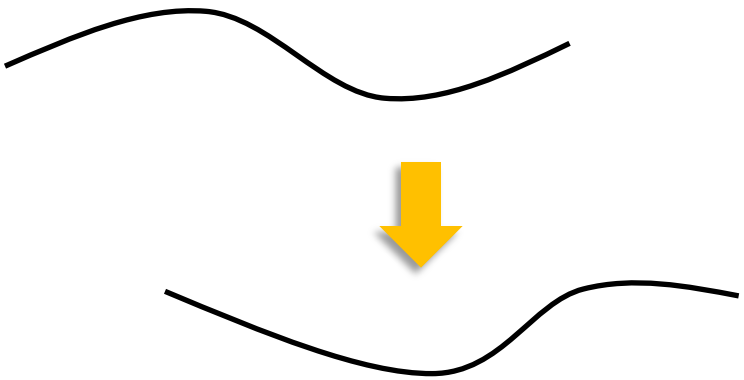


As-rigid-as possible shape manipulation, O. Sorkine and M. Alexa, SGP'07

Two important cases



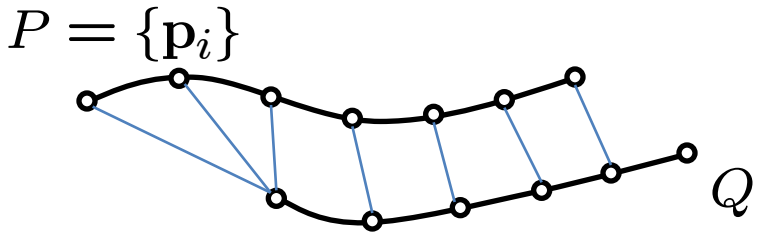
"Embedding"



Partially overlap

Partial similarity

- Find weighted close-point pairs



$$\{(w_i, \mathbf{p}_i, \mathbf{q}_i)\}$$

- Optimize deformed shape

$$\sum_{i=1}^{N} w_i \|\mathbf{p}_i - \mathbf{q}_i\|^2 + \lambda d^2(\{\mathbf{p}_i\}, \{\mathbf{q}_i\})$$

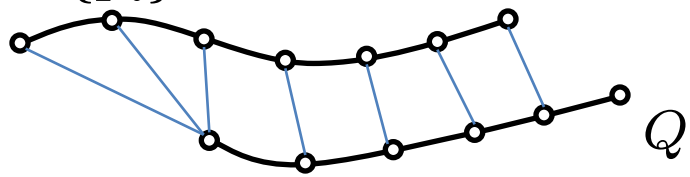
$$\sum_{i=1}^{N} w_i ((\mathbf{p}_i - \mathbf{q}_i)^T \mathbf{n}_i)^2 + \lambda d^2(\{\mathbf{p}_i\}, \{\mathbf{q}_i\})$$

Optimize strategy is the same

Partial similarity

- Weight function

$P = \{\mathbf{p}_i\}$



$$\{(w_i, \mathbf{p}_i, \mathbf{q}_i)\}$$

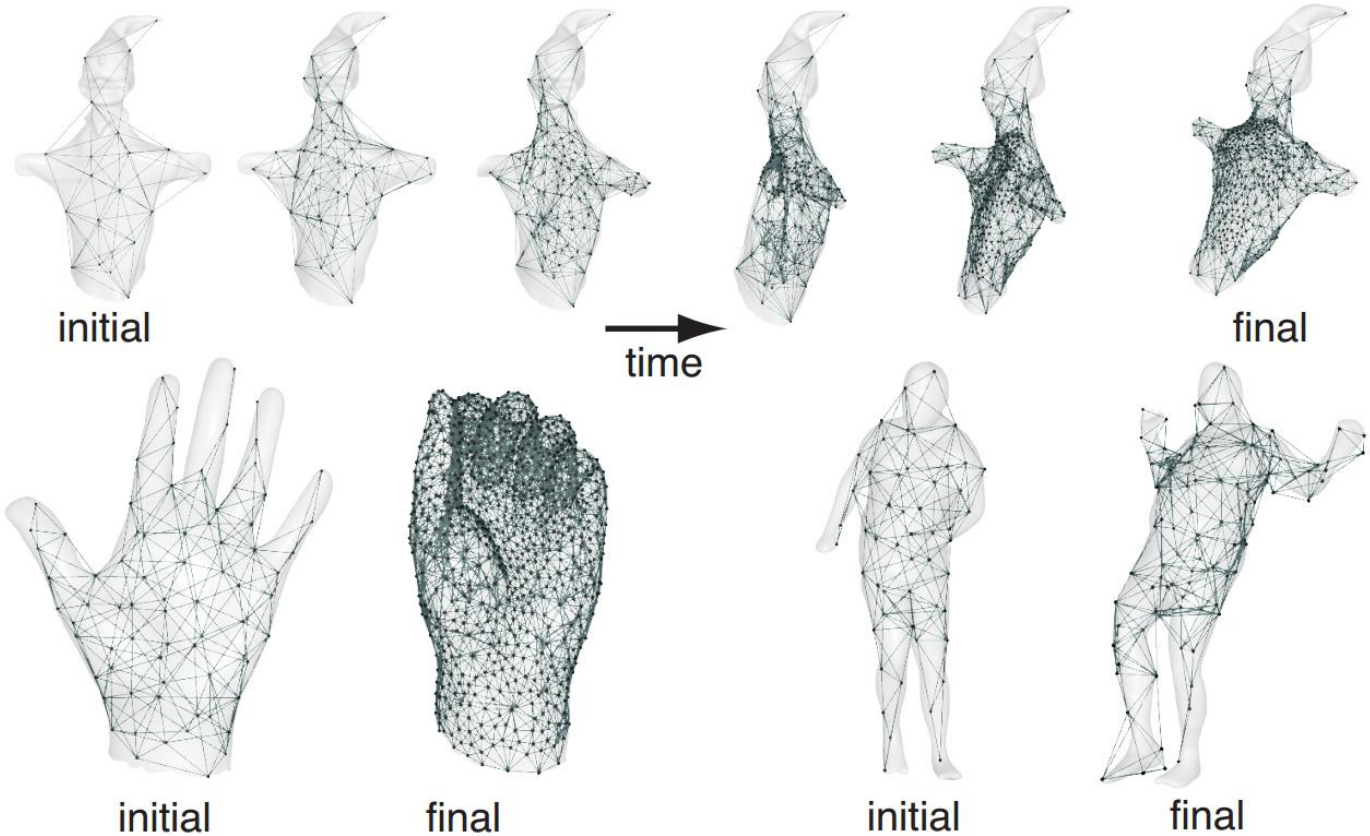
$$w_i = \exp(-\frac{\|\mathbf{p}_i - \mathbf{q}_i\|^2}{2\sigma^2})$$

- Partial case
  - Do not optimize a global energy function
  - Easily get stuck into local minimum

Lecture 6: Avoid aligning partially overlap scans

Implementation details

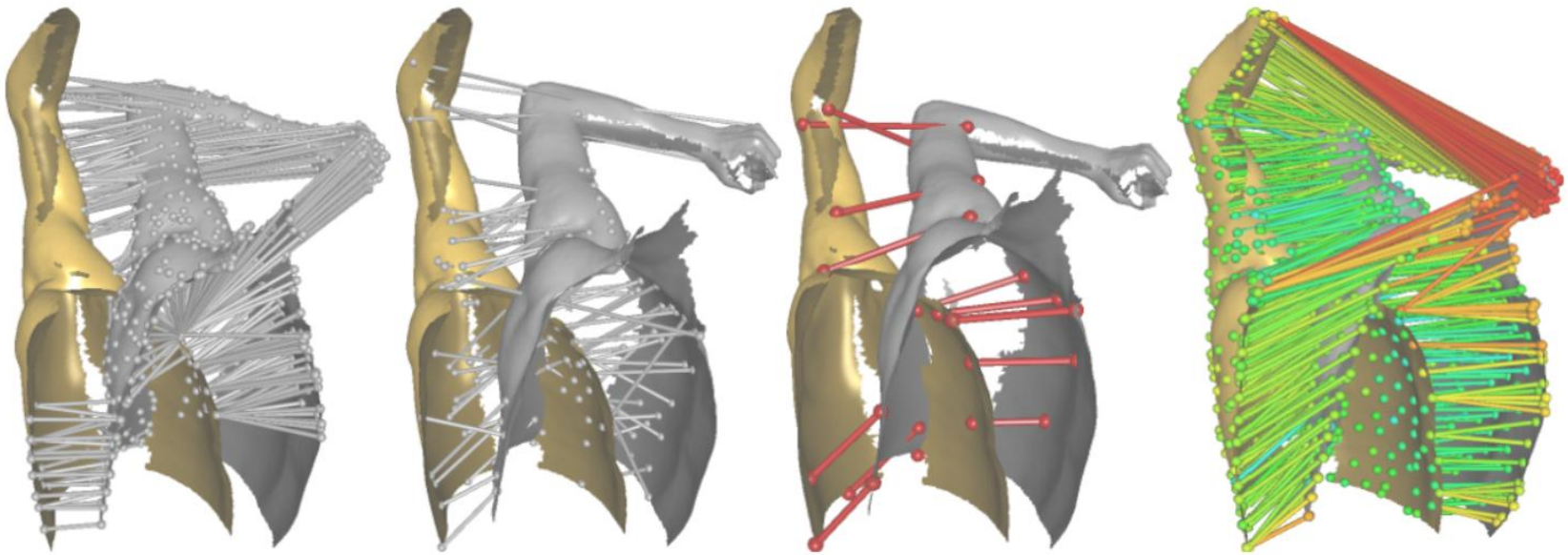
- Use hierarchical deformation structures



Robust Single-View Geometry and Motion Reconstruction, H. Li, B. Adams, L. Guibas, M. Pauly, SIGGRAPH ASIA'09

Implementation details

- Use feature descriptors to compute point correspondences



Closest point | Feature descriptors | Spectral matching | Propagation

Further reading

- Articulated Object Reconstruction and Markerless Motion Capture from Depth Video, Y. Pekelny and C. Gotsman, Eurographics' 08
- Global correspondence optimization for non-rigid registration of depth scans, H. Li, R. Sumner, M. Pauly, SGP’08
- Non-Rigid Registration Under Isometric Deformations, Q. Huang, B. Adams, M. Wiche, and L. Guibas, SGP’08
- Robust single-View geometry and motion reconstruction, H. Li, B. Adams, L. Guibas, M. Pauly, SIGGRAPH ASIA'09
- 3D self-portraits, H. Li, E. Vouga, A. Gudym, J. Barron, L. Luo, G. Gusev, SIGGRAPH ASIA'13
- Animation Cartography - Intrinsic Reconstruction of Shape and Motion, A. Tevs, A. Berner, M. Wand, I. Ihrke, M. Bokeloh, J. Kerber, H. Seidel, TOG'13
- Dynamic Geometry Processing, W. Chang, H. Li, N. Mitra, M. Pauly, M. Wand: *Eurographics 2012 tutorial*