Data-Driven Shape Analysis
--- Non-Rigid Registration

Qi-xing Huang
Stanford University
Non-rigid registration
Applications

Dynamic geometry reconstruction
[Li et al. 13]

Tracking
[Li et al. 09]

Interpolation
[Kilian et al. 08]

Shape completion
[Pauly et al. 05]
Application --- distance learning

Input

Rigid

Non-rigid

with-arms  side  windsor  rex

Distance field  Spin images

Fine-Grained Semi-Supervised Labeling of Large Shape Collections, Q. Huang, H. Su, L. Guibas, SIGGRAPH ASIA’ 13
Application --- depth inference

Estimating 3D Attributes of Images from Shape Collections, with H. Su, N. Mitra, Y. Li, and L. Guibas, SIGGRAPH’ 14
Application --- depth inference

Estimating 3D Attributes of Images from Shape Collections, with H. Su, N. Mitra, Y. Li, and L. Guibas, SIGGRAPH’ 14
Two important cases

“Embedding”  Partially overlap
“Embedding”?

Dynamic geometry reconstruction
[Li et al. 13]

Tracking
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Shape completion
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Two important cases

“Embedding”

Partially overlap
Shape representation

Graph representation
Non-Rigid ICP

• Compute closest point pairs

\[ P = \{ p_i \} \]

\[ \{(p_i, q_i)\} \]

• Deform the source shape P

\[ P = \{ p_i \} \]

\[ \sum_{i=1}^{N} \|p_i - q_i\|^2 + \lambda d^2(\{p_i\}, \{p_i^{rest}\}) \]

Distance term  Deformation term
Deformation Formulation
Deformation term

Intrinsic version

Extrinsic version
Intrinsic formulation

- Distance-preserving

\[ d^2(\{p_i\}, \{p_i^{rest}\}) = \sum_{(i,j) \in \mathcal{E}} \left( \|p_i - p_j\| - \|p_i^{rest} - p_j^{rest}\| \right)^2 \]

- Issues
  - Hard to optimize
  - Sensitive to graph quality
Intrinsic formulation

• As-rigid-as possible formulation
  – Associate each node with a rotation matrix
    • Deformation gradient [Summer 04]
  – Controls the transformation of the neighborhood

\[
\sum_{i=1}^{N} \sum_{j \in N(i)} \| R_i (p_i^{rest} - p_j^{rest}) - (p_i - p_j) \|^2
\]
Intrinsic formulation

- A variant [summer et al. 07]

\[
\sum_{i=1}^{N} \sum_{j \in N(i)} \| R_i (p_i^{rest} - p_j^{rest}) - (p_i - p_j) \|^2
\]

\[
\text{Rot}(R) = (c_1 \cdot c_2)^2 + (c_1 \cdot c_3)^2 + (c_2 \cdot c_3)^2 + (c_1 \cdot c_1 - 1)^2 + (c_2 \cdot c_2 - 1)^2 + (c_3 \cdot c_3 - 1)^2
\]
Deformation field

- Deform neighboring points
- Partition-of-unity

\[ \hat{x} = \sum_{i=1}^{N} w_i(x)[R_i(x - p_i^{rest}) + (p_i - p_i^{rest})] \]

The transformation associated with each node

Blending weight [Summer et al. 07]
Weight function

- Compact support
- Smooth

\[ w_i(t) = \begin{cases} 
  (1 - t)^2 & \text{if } t < 1 \\
  0 & \text{otherwise}
\end{cases} \]

\[ t = \frac{||x - p_i^{rest}||}{r_{max}} \]
Extrinsic formulation

• Free-form deformation [Sederberg and Parry 86]

\[ \phi(x, y, z) = \sum_{I=1}^{n} b_I(x, y, z) X_I \]
Extrinsic formulation

• Free-form deformation [Sederberg and Parry 86]

Free-Form Deformation of Solid Geometric Models, T. Sederberg, S. Parry, SIGGRAPH’ 86
Extrinsic formation

Skeleton-based

Cage-based

Green coordinates, Y. Lipman, D. Levin, D. Cohen-Or, SIGGRAPH’08
Optimization
Optimization

The objective function is also used for shape manipulation

\[ \sum_{i=1}^{N} \| p_i - q_i \|^2 + \lambda \sum_{(i,j) \in \mathcal{E}} \| R_i(p_i^{\text{rest}} - p_j^{\text{rest}}) - (p_i - p_j) \|^2 \]

Variables

- \( p_i \)
- \( q_i \)
- \( p_i^{\text{rest}} \)
- \( p_j \)
- \( R_i \)

\( S(\beta) = \sum_{i=1}^{m} r_i(\beta)^2. \)

Gauss-Newton optimization

Alternating optimization

\{A\} \leftrightarrow \{B\}
Gauss-Newton optimization
Gaussian-Newton optimization

Non-linear least squares:

\[ f(x) = \sum_{i=1}^{N} r_i^2(x) \rightarrow \sum_{i=1}^{N} (r_i(x_k) + \nabla r_i(x_k)d_k)^2 \]

Search direction:

\[ d_k = -\left( \sum_{i=1}^{N} \nabla r_i(x)\nabla r_i(x)^T \right)^{-1} \left( \sum_{i=1}^{N} \nabla r_i(x)r_i(x) \right) \]

Update:

\[ x_{k+1} = x_k + \alpha d_k \]

Convergence rate:

\[ \epsilon_{k+1} \approx C(\epsilon_k^2 + O(f(x^*))\epsilon_k) \]
Gauss-Newton optimization

\[ \sum_{i=1}^{N} \| p_i - q_i \|^2 + \lambda \sum_{(i,j) \in \mathcal{E}} \| R_i(p_i^{\text{rest}} - p_j^{\text{rest}}) - (p_i - p_j) \|^2 \]

First order approximation:

\[ p_i = p_i^k + d_i \]
\[ R_i = (I_3 + c_i \times) R_i^k \]

Exponential map:

\[ R_i^{k+1} = \exp(c_i \times) R_i^k \]

where

\[ \exp(c) = I_3 + \frac{c}{\|c\|}(c \times) + \frac{1-\cos(\|c\|)}{\|c\|^2}(c \times)^2 \]

Rodrigues’ formula
Alternating optimization
Alternating optimization

\[ \sum_{i=1}^{N} \| p_i - q_i \|^2 + \lambda \sum_{(i,j) \in \mathcal{E}} \| R_i (p_i^{rest} - p_j^{rest}) - (p_i - p_j) \|^2 \]

- When rotations are fixed

\[ \sum_{i=1}^{N} \| p_i - q_i \|^2 + \lambda \sum_{(i,j) \in \mathcal{E}} \| e_{ij} - (p_i - p_j) \|^2 \]

Linear system, which allows pre-factorization

- When positions are fixed

\[ R_i = \arg \min_{R_i} \sum_{j \in N(i)} \| R_i e_{ij}^{rest} - e_{ij} \|^2 \]

Registration with known correspondences --- closed form solution
Alternate optimization

• Shape editing

As-rigid-as possible shape manipulation, O. Sorkine and M. Alexa, SGP’07
Two important cases

“Embedding”

Partially overlap
Partial similarity

- Find weighted close-point pairs

\[ P = \{p_i\} \]

\[ \{(w_i, p_i, q_i)\} \]

- Optimize deformed shape

\[
\sum_{i=1}^{N} w_i \|p_i - q_i\|^2 + \lambda d^2(\{p_i\}, \{q_i\})
\]

\[
\sum_{i=1}^{N} w_i ((p_i - q_i)^T n_i)^2 + \lambda d^2(\{p_i\}, \{q_i\})
\]

*Optimize strategy is the same*
Partial similarity

• Weight function

\[ P = \{p_i\} \quad \{w_i, p_i, q_i\} \]

\[ w_i = \exp\left(-\frac{||p_i - q_i||^2}{2\sigma^2}\right) \]

• Partial case
  – Do not optimize a global energy function
  – Easily get stuck into local minimum

Lecture 6: Avoid aligning partially overlap scans
Implementation details

- Use hierarchical deformation structures

Robust Single-View Geometry and Motion Reconstruction, H. Li, B. Adams, L. Guibas, M. Pauly, SIGGRAPH ASIA'09
Implementation details

• Use feature descriptors to compute point correspondences

Closest point
Feature descriptors
Spectral matching
Propagation
Further reading

- Articulated Object Reconstruction and Markerless Motion Capture from Depth Video, Y. Pekelny and C. Gotsman, Eurographics'08
- Global correspondence optimization for non-rigid registration of depth scans, H. Li, R. Sumner, M. Pauly, SGP’08
- Non-Rigid Registration Under Isometric Deformations, Q. Huang, B. Adams, M. Wiche, and L. Guibas, SGP’08
- Robust single-View geometry and motion reconstruction, H. Li, B. Adams, L. Guibas, M. Pauly, SIGGRAPH ASIA'09
- Animation Cartography - Intrinsic Reconstruction of Shape and Motion, A. Tevs, A. Berner, M. Wand, I. Ihrke, M. Bokeloh, J. Kerber, H. Seidel, TOG'13