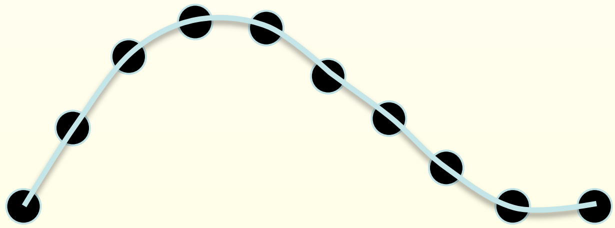
Normal Estimation for a Curve

Assume we have a clean sampling of the curve.



Our goal is to find the best approximation of the tangent direction, and thus of the normal to the curve.

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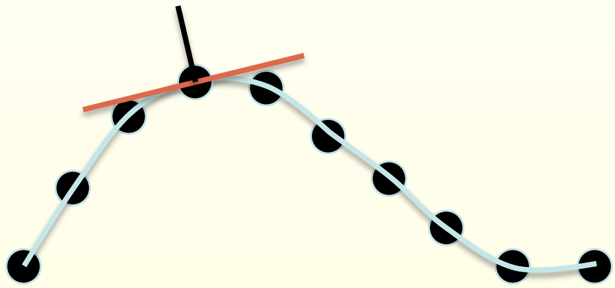

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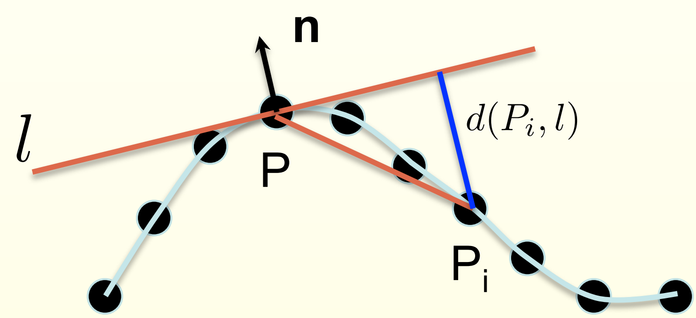

Goal: find best approximation of the normal at P.

Method: Given line l through P with normal **n**, the distance to the tangent line from another close-by point p_i:

$$d(p_i, l)^2 = \frac{((p_i - P)^T \mathbf{n})^2}{\mathbf{n}^T \mathbf{n}} = ((p_i - P)^T \mathbf{n})^2 \text{ if } \|\mathbf{n}\| = 1$$

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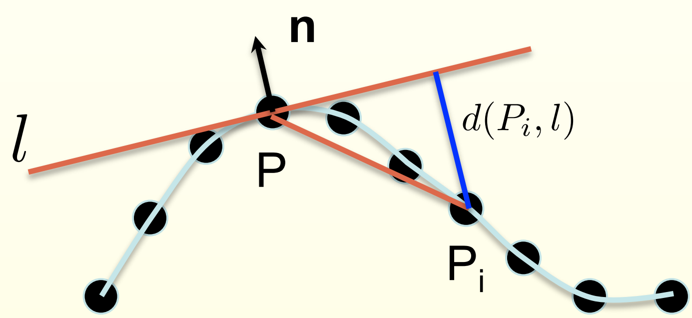

Goal: find best approximation of the normal at P.

Method: Find the line that best fits the neighborhood of P. This will approximate the tangent line.

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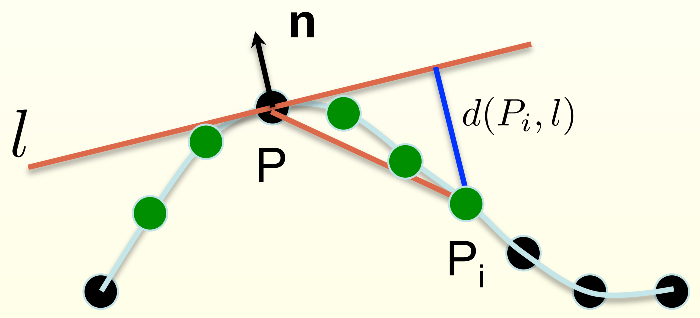

Goal: find best approximation of the normal at P.

Equivalently: Find unit-length vector **n**, minimizing $\sum_{i=1}^{k} d(p_i, l)^2$ for a set of *k* points (e.g. *k* nearest neighbors of P) .

$$\mathbf{n}_{\text{opt}} = \arg \min_{\|\mathbf{n}\|=1} \sum_{i=1}^{k} ((p_i - P)^T \mathbf{n})^2$$

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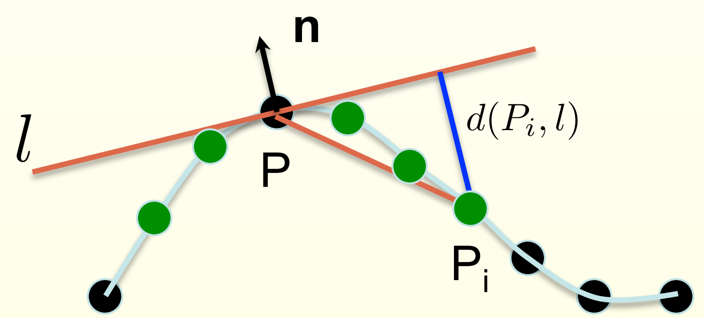

Using Lagrange multiplier:

$$\frac{\partial}{\partial \mathbf{n}}\left(\sum_{i=1}^{k}((p_i - P)^T\mathbf{n})^2\right) - \lambda\frac{\partial}{\partial \mathbf{n}}\left(\mathbf{n}^T\mathbf{n}\right) = 0$$

$$\sum_{i=1}^{k} 2(p_i - P)(p_i - P)^T\mathbf{n} = 2\lambda\mathbf{n}$$

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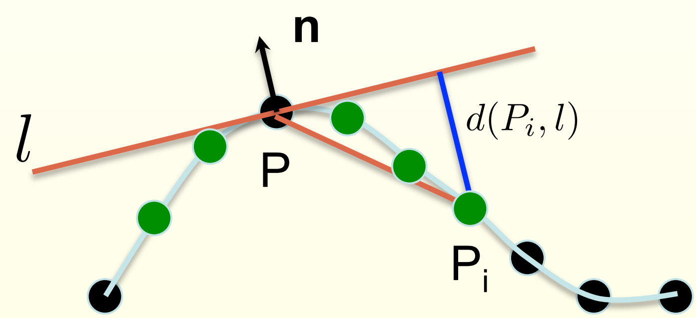

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$$\left(\sum_{i=1}^{k}(p_i - P)(p_i - P)^T\right)\mathbf{n} = \lambda\mathbf{n} \quad \Rightarrow \quad C\mathbf{n} = \lambda\mathbf{n}$$

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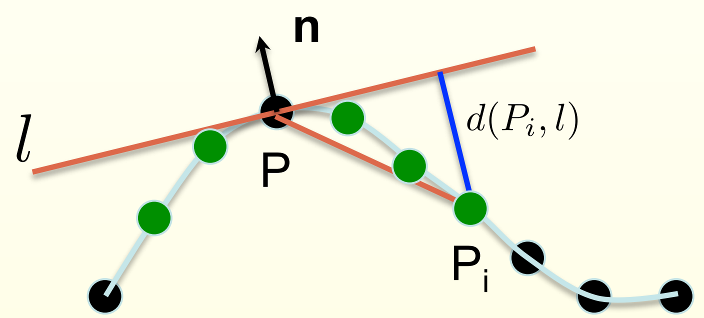

The normal **n** must be an eigenvector of the matrix:

$$C\mathbf{n} = \lambda\mathbf{n} \qquad C = \sum_{i=1}^{k} (p_i - P)(p_i - P)^T$$

Moreover, since:

$$\mathbf{n}_{\text{opt}} = \arg \min_{\|\mathbf{n}\|=1} \sum_{i=1}^{k} ((p_i - P)^T \mathbf{n})^2 = \arg \min_{\|\mathbf{n}\|=1} \mathbf{n}^T C \mathbf{n}$$

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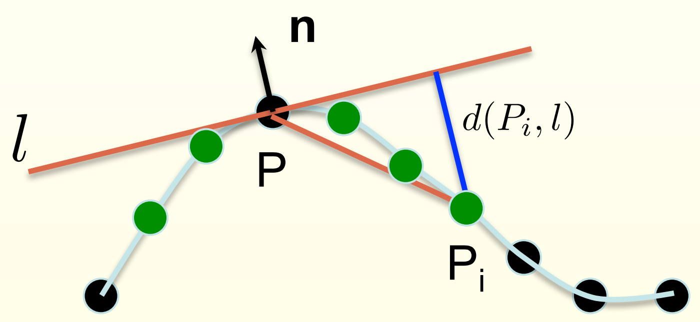


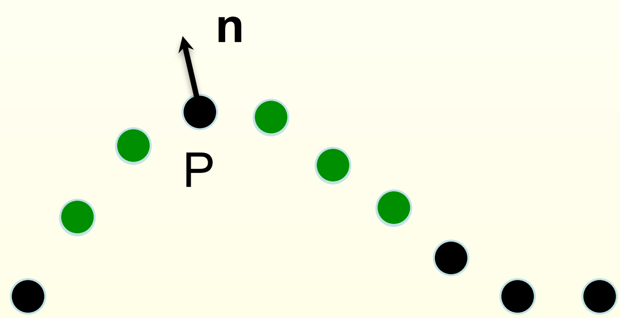
The normal **n** must be an eigenvector of the matrix:

$$C\mathbf{n} = \lambda\mathbf{n} \qquad C = \sum_{i=1}^{k}(p_i - P)(p_i - P)^T$$

Moreover, $\mathbf{n}_{\text{opt}}$ must be the eigenvector corresponding to the **smallest eigenvalue** of C.

Normal Estimation for a Curve

Method Outline (PCA):



1. Given a point P in the point cloud, find its *k* nearest neighbors.
2. Compute $C = \sum_{i=1}^{k} (p_i - P)(p_i - P)^T$
3. **n:** eigenvector corresponding to the smallest eigenvalue of *C*.

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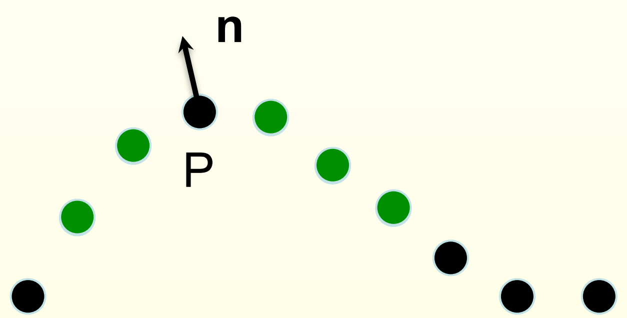

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Same principle for surfaces!