Physically Based Sound for Computer Animation and Virtual Environments

Acoustic Transfer

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Acoustic Transfer

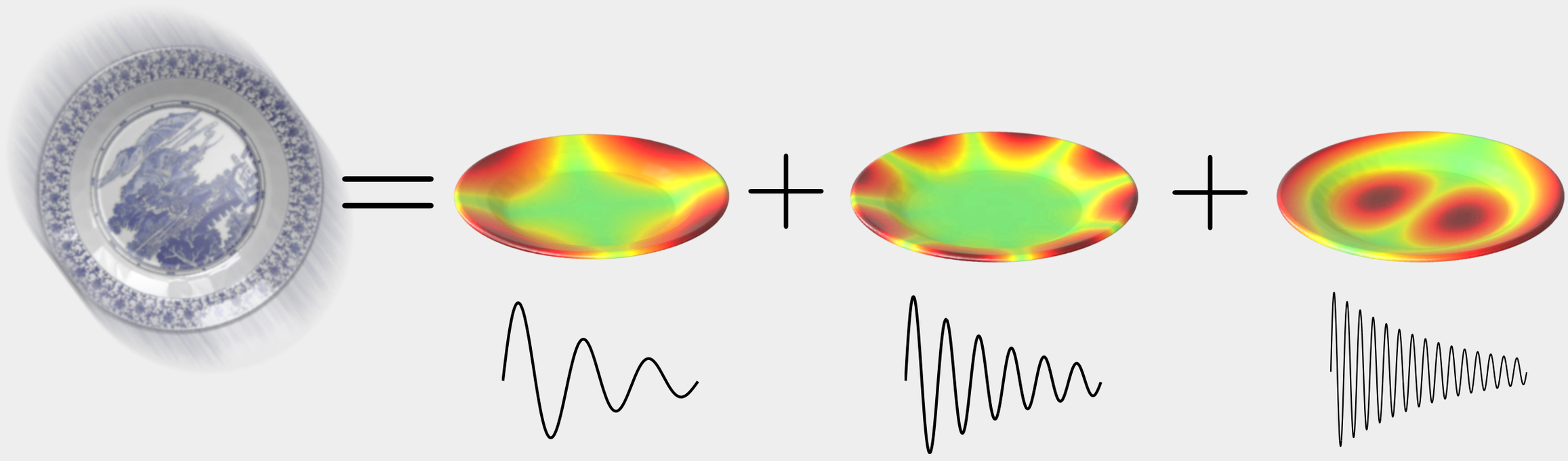
Overview

- Acoustic transfer basics
- Acoustic waves (wave equation, BCs)
- Frequency-domain wave radiation (Helmholtz eqn)
- Multipole sound sources
- Solvers and representations for AT
    - Boundary Element Method (BEM)
    - Multipole expansions
    - Equivalent source method
    - “Precomputed Acoustic Transfer”
    - FFAT Maps

Acoustic Transfer Basics

Acoustic Transfer Basics

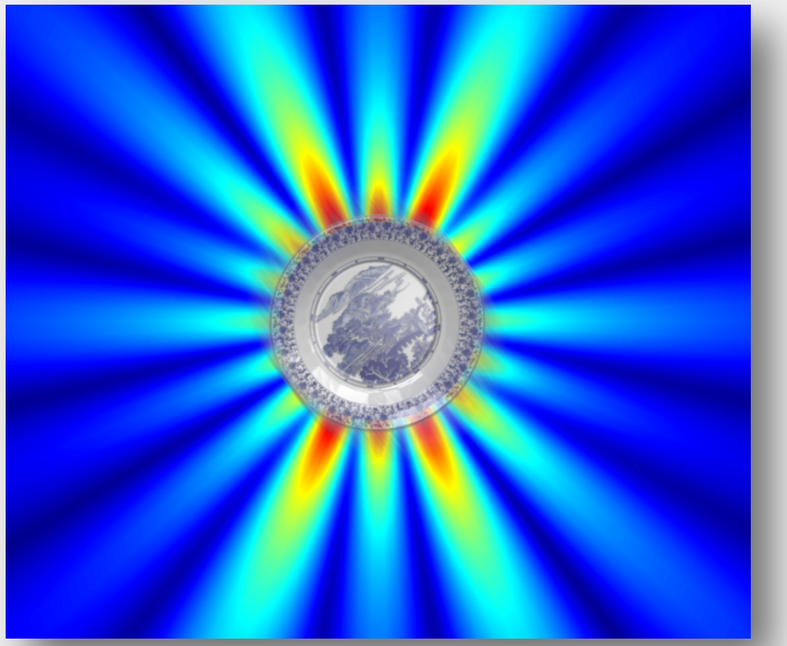
Linear modal sound

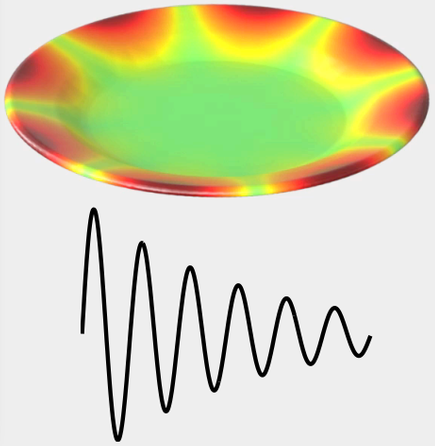


$$sound(t) = q_1(t) + q_j(t) + q_3(t)$$

Acoustic Transfer Basics

Linear modal sound





$$sound(\boldsymbol{x}, t) = \sum_j \quad a_j(\boldsymbol{x}) \quad \times \quad q_j(t)$$

Amplitude of acoustic transfer function for mode *j*

How loud is each mode?

$$sound(\boldsymbol{x}, t) = \sum_j a_j(\boldsymbol{x})\, q_j(t) = \boldsymbol{a}(\boldsymbol{x})^T \mathbf{q}(t)$$

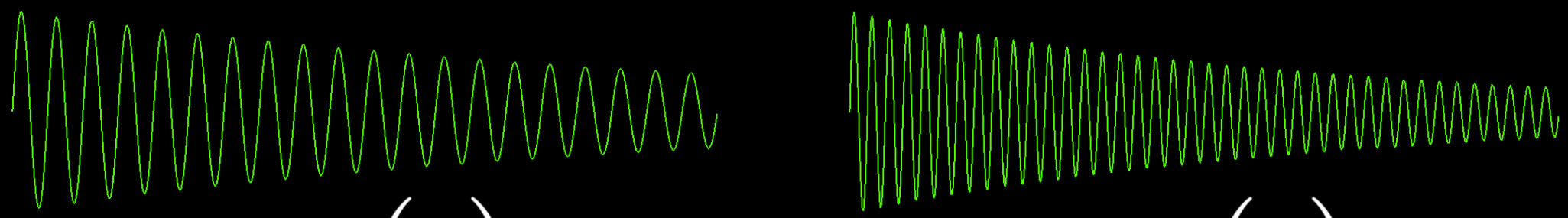
Transfer $\boldsymbol{a}(\boldsymbol{x})$ evaluated at listening position,

$\boldsymbol{x} = \boldsymbol{x}(t)$ (in body frame)

at slower rate (e.g., 250 Hz), then smoothly interpolated up to audio rate of $\mathbf{q}(t)$.

ASIDE: Point sources

Wait… what about $\frac{1}{r}$ fall off?

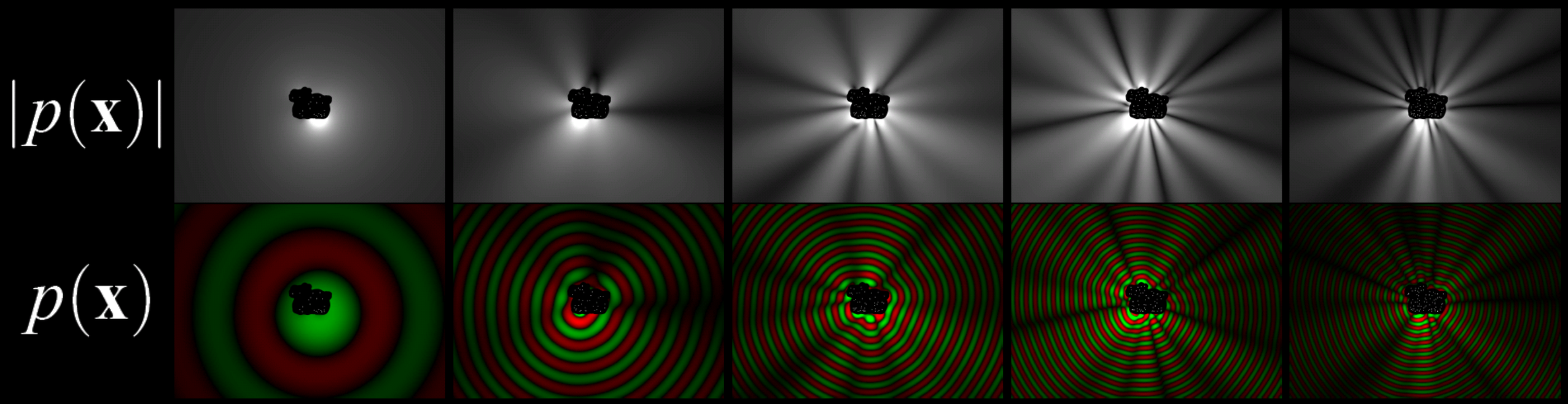


$$\frac{q_1(t) + q_2(t) + \cdots}{r}$$

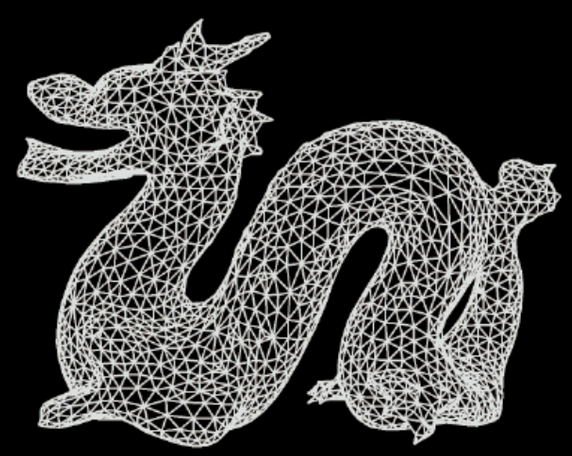
Commonly used for point sources….

ASIDE: Point sources

Not a point source!



Mode 3 (780 Hz) Mode 10 (2.9 kHz) Mode 18 (5.1 kHz) Mode 30 (7.1 kHz) Mode 40 (8.6 kHz)



Bronze Dragon (20 cm)

ASIDE: Point sources

Point-source model

$$\text{sound} \propto \frac{\sum_i C_i \, q_i(t)}{r}$$

- C=1: [van den Doel and Pai 1996; Cook 1996; …]
- $C \neq 1$: Monopole approx. (see [Cremer et al. 1990, O'Brien et al. 2002, James et al. 2006])
  - Assumes:
    - Far-field listener $r \gg R$
    - Low-frequency waves $\lambda \gg R$
    - Closed object

Acoustic Waves

Diffraction is important!

- Speed of sound, $c = 343 \text{ m/s} = \lambda f$
- Audible frequencies: 16 Hz — 20 kHz
- Audible wavelengths: 21 m — 17 mm

Diffraction Shaders [Stam 99]

ASIDE: Point sources

Comparison

[James, Barbic & Pai 2006]

Acoustic transfer renderer

Point-source renderer

Acoustic Waves

Wave equation follows from…

(see [Howe 2003], or Bridson's “fluid” course notes)

- Linearized continuity eqn (mass conservation)

$$\frac{1}{\rho}\frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot \underbrace{\mathbf{v}}_{\text{Acoustic particle velocity}} = 0$$

- Linearized momentum eqn (“ma=f”)

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla \underbrace{p}_{\text{Acoustic pressure}}$$

- Energy eqn (relates pressure and density at constant entropy)

$$p = \left(\frac{\partial p}{\partial \rho}\right)_0 \underbrace{\tilde{\rho}}_{\text{Acoustic density fluctuation}} = c^2 \, \tilde{\rho}$$

3D Wave Equation

Acoustic pressure, $p(\boldsymbol{x}, t)$, satisfies

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \qquad (\textit{Wave Equation})$$

where

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}$$

is the Laplacian.

Acoustic Waves

1D Wave Equation

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}$$

Traveling wave solutions of the form:

$$p(x,t) = f(x \pm ct)$$

Radiation boundary condition (1)

Need to impose (vibrating) solid boundaries.

From the linearized momentum equation (F=0),

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p$$

the normal component yields

$$\boxed{\frac{\partial p}{\partial n} = -\rho \; a_n(\boldsymbol{x}, t), \quad \boldsymbol{x} \in \Gamma}$$ *(Neumann BC)*

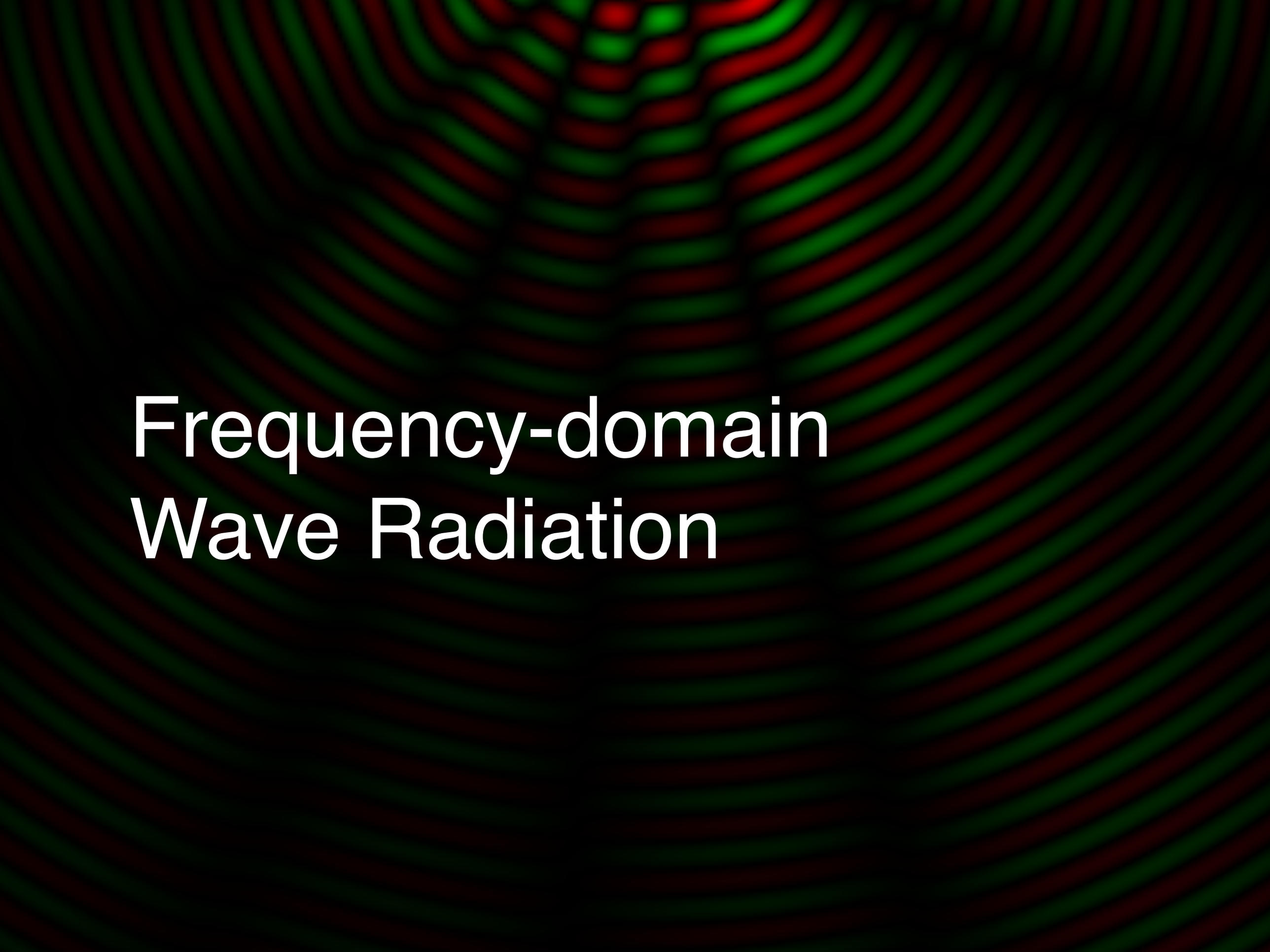
where $a_n(\boldsymbol{x}, t)$ is the surface acceleration,

and $\rho = 1.2041 \; kg/m^3$ is air density at STP.

Radiation boundary condition (2)

Need condition at infinity.

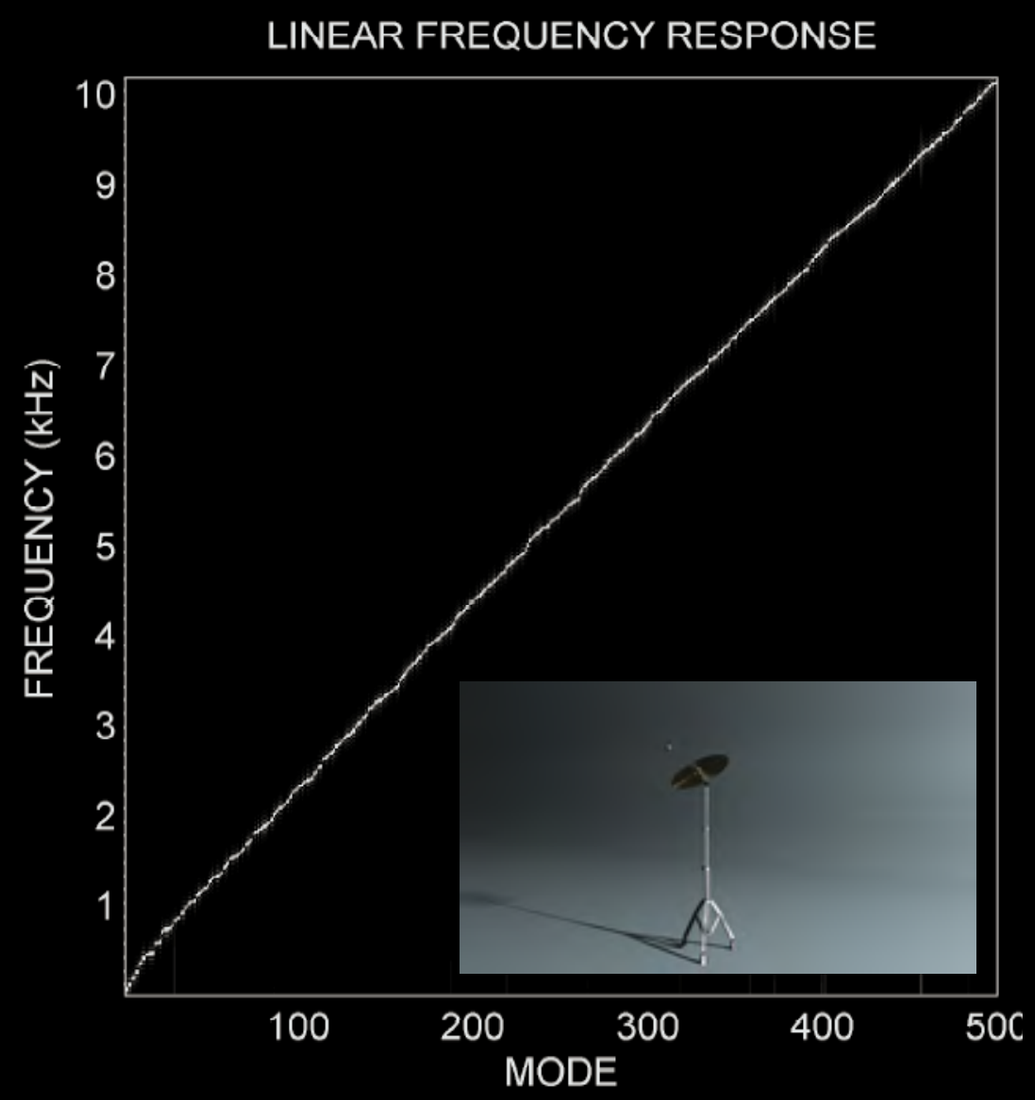
- So-called *Sommerfeld* radiation condition.
- Solutions must decay at infinity and correspond to out-going waves.
- Avoids nonphysical solutions, such as infinite sources at infinity.



Frequency-domain Wave Radiation

Frequency-domain Wave Radiation

Motivation: Individual mode vibrations are localized in frequency domain



Impulse response of a cymbal impact modeled using a linear modal vibration model.

Math Flashback

Using complex numbers to represent oscillations

By linearity of the wave equation & BCs, sufficient to consider a unit amplitude harmonic vibration at natural frequency ω,

$$e^{+i\omega t} = \cos(\omega t) + i\,\sin(\omega t)$$

Different phases introduced by $e^{+i\phi}$ factors since

$$e^{+i\omega t}e^{i\phi} = \cos(\omega t + \phi) + i\,\sin(\omega t + \phi)$$

Can think of $q(t)$ as an amplitude and a unit complex part.

Harmonic pressure oscillations

Consider harmonic pressure solutions of the form

$$p(\boldsymbol{x}, t) = p(\boldsymbol{x}) e^{+i\omega t}$$

where $p(\boldsymbol{x})$ is a complex-valued pressure field—will be the *acoustic transfer function*.

Using Euler's formula, we can write $p(\boldsymbol{x}) = |p| e^{i\phi}$ so that

$$\begin{aligned} p(\boldsymbol{x}, t) &= p(\boldsymbol{x})\, e^{+i\omega t} = |p(\boldsymbol{x})|\, e^{+i(\omega t + \phi)} \\ &= |p(\boldsymbol{x})| \cos(\omega t + \phi) + i\, |p(\boldsymbol{x})| \sin(\omega t + \phi). \end{aligned}$$

The Helmholtz equation

(the frequency-domain wave equation)

Substituting $p(\boldsymbol{x}, t) = p(\boldsymbol{x}) e^{+iwt}$ into the wave equation yields

$$0 = \nabla^2 p(\boldsymbol{x})\, e^{+i\omega t} - \frac{1}{c^2}\, p(\boldsymbol{x}) \left(\frac{\partial^2}{\partial t^2}\, e^{+i\omega t} \right)$$

$$= \left(\nabla^2 p(\boldsymbol{x}) - \frac{\omega^2}{c^2} p(\boldsymbol{x}) \right) e^{+i\omega t}$$

Never zero

and so

$$\boxed{\nabla^2 p(\boldsymbol{x}) + k^2\, p(\boldsymbol{x}) = 0} \quad (\textit{Helmholtz eqn})$$

where k is the wavenumber,

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

Radiation boundary conditions

Recall time-domain Neumann BC:

$$\boxed{\frac{\partial p}{\partial n} = -\rho\, a_n(\boldsymbol{x}, t), \quad \boldsymbol{x} \in \Gamma}$$

What is frequency-domain BC?

What is $a_n(\boldsymbol{x}, t)$?

Frequency-domain Wave Radiation

Radiation boundary conditions

Need harmonic surface acceleration $a_n(\boldsymbol{x}, t)$ in terms of eigenmode displacement, $\bar{\mathbf{u}}(\boldsymbol{x})$:

$$\mathbf{u}(\boldsymbol{x}, t) = \bar{\mathbf{u}}(\boldsymbol{x})\, e^{+i\omega t} \qquad \Longrightarrow \qquad u_n = \bar{u}_n e^{+i\omega t}$$

$$\boldsymbol{v}(\boldsymbol{x}, t) = \frac{d\mathbf{u}}{dt}(\boldsymbol{x}, t) = i\omega\, \bar{\mathbf{u}}(\boldsymbol{x})\, e^{+i\omega t} \qquad \Longrightarrow \qquad v_n = i\omega \bar{u}_n e^{+i\omega t}$$

$$\boldsymbol{a}(\boldsymbol{x}, t) = \frac{d^2\mathbf{u}}{dt^2}(\boldsymbol{x}, t) = (i\omega)^2\, \bar{\mathbf{u}}(\boldsymbol{x})\, e^{+i\omega t} \qquad \Longrightarrow \qquad a_n = -\omega^2 \bar{u}_n e^{+i\omega t}$$

$$\boxed{a_n = -\omega^2 \bar{u}_n e^{+i\omega t}}$$

Radiation boundary conditions

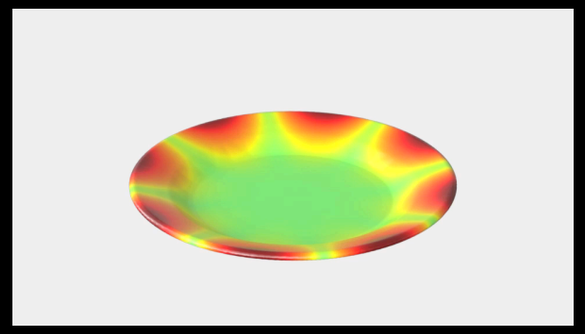
The Neumann boundary condition becomes

$$\frac{\partial p(\boldsymbol{x}, t)}{\partial n} = -\rho\, a_n(\boldsymbol{x}, t)$$

Never zero

$$\frac{\partial p(\boldsymbol{x})}{\partial n} \cancel{e^{+i\omega t}} = \rho\, \omega^2\, \bar{u}_n\, \cancel{e^{+i\omega t}}$$

$$\boxed{\frac{\partial p(\boldsymbol{x})}{\partial n} = \rho\, \omega^2\, \bar{u}_n(\boldsymbol{x})}$$



(+Sommerfeld radiation BC: Out-going decaying waves.)

Frequency-domain Wave Radiation

Acoustic Transfer BVP

Given mode data $\bar{u}_n(\boldsymbol{x})$ and frequency ω.

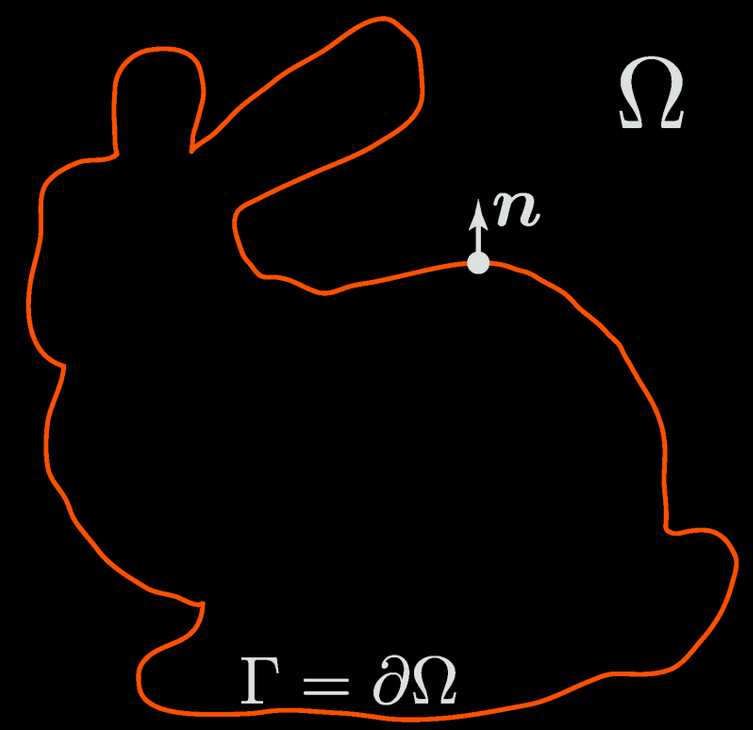
Solve exterior Helmholtz eqn

$$(\nabla^2 + k^2)p(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Omega$$

subject to BC:

$$\frac{\partial p(\boldsymbol{x})}{\partial n} = \rho\,\omega^2\,\bar{u}_n(\boldsymbol{x}), \quad \boldsymbol{x} \in \Gamma$$

(+Sommerfeld radiation BC.)



Frequency-domain Wave Radiation

Visualizing solutions

Can visualize solutions by looking at real part of

$$p(\boldsymbol{x})e^{+i\omega t}$$

or the AT amplitude

$$|p(\boldsymbol{x})|$$

MODE 01

MODE 02

MODE 03

MODE 04


MODE 05

MODE 06

MODE 07

MODE 08


MODE 09


MODE 10


MODE 11


MODE 12


MODE 13


MODE 14


MODE 15


MODE 16


MODE 17


MODE 18


MODE 19


MODE 20


MODE 21


MODE 22


MODE 23


MODE 24


MODE 25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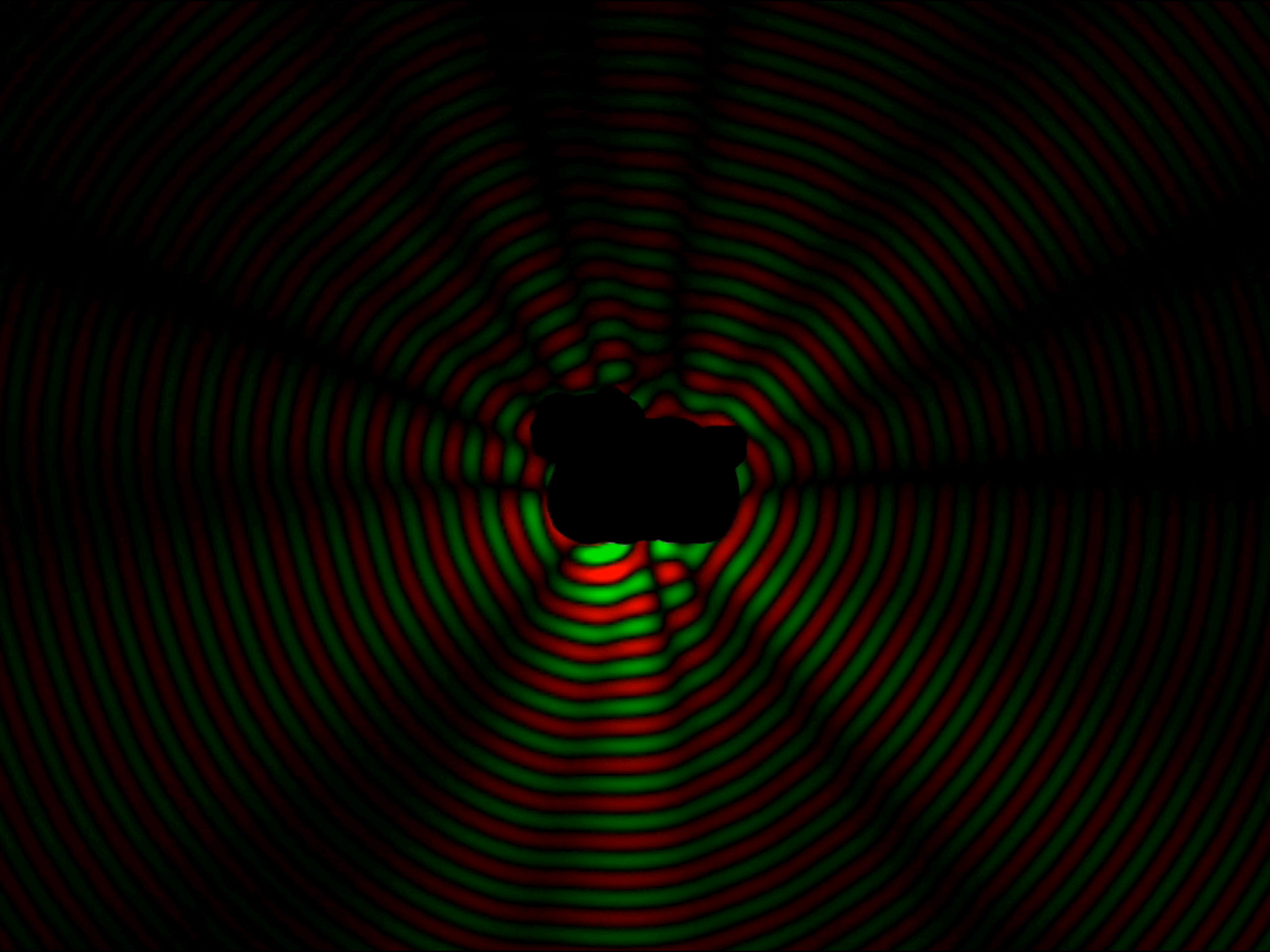
MODE 36


MODE 37


MODE 38


MODE 39


MODE 40



Multipole Sound Sources

Singular Helmholtz solutions

Satisfy Helmholtz equation

$$\nabla^2 p(\boldsymbol{x}) + k^2\, p(\boldsymbol{x}) = 0$$

almost everywhere, except where they are singular, e.g., at origin.

Very common and extremely useful.

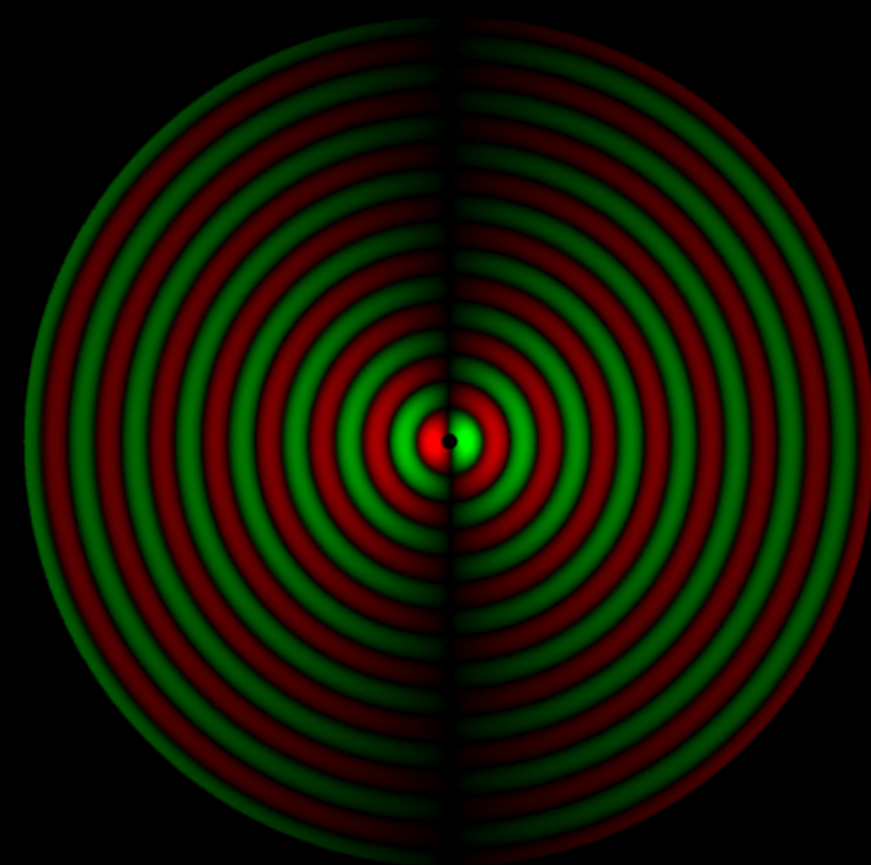
Can be summation of simpler singular solutions.

Multipole Sound Sources

Spherical Multipole sources

$S_0^0(\boldsymbol{x})$

Monopole



$S_1^m(\boldsymbol{x})$

Dipole

Multipole Sound Sources

Monopole point source

$$p(\boldsymbol{x}) = \frac{e^{-ikr}}{4\pi r}$$

where

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}, \qquad r = \|\boldsymbol{x} - \boldsymbol{x}_0\|_2$$

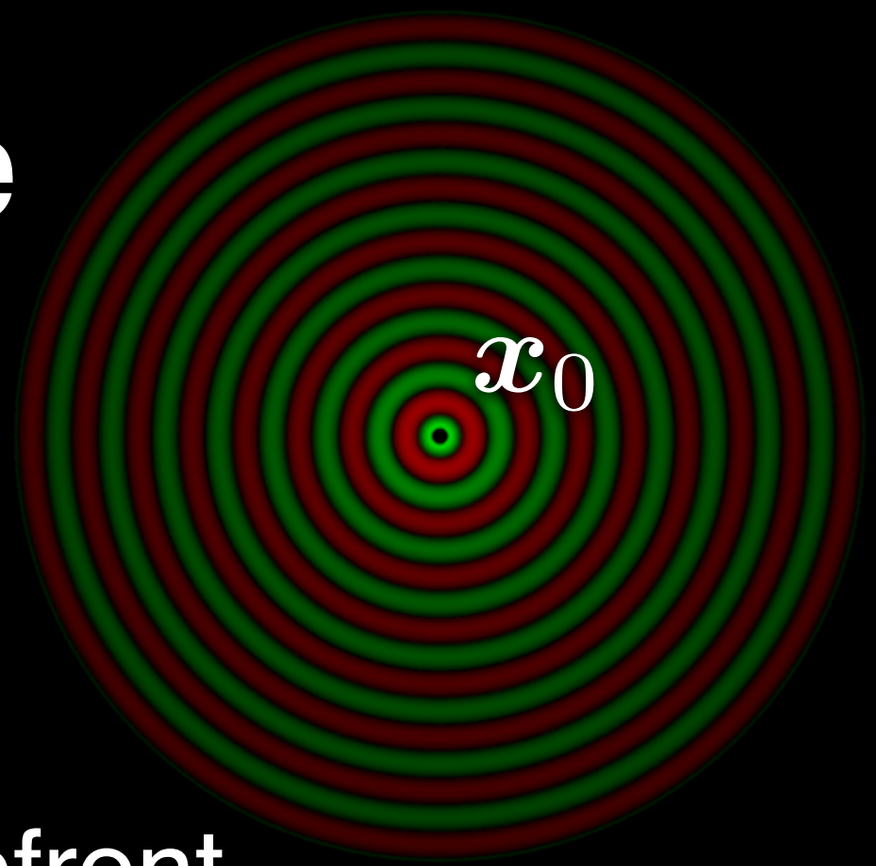
Solution to point-like volume pulsation at $\boldsymbol{x}_0$

$$\nabla^2 p(\boldsymbol{x}) + k^2 p(\boldsymbol{x}) = \delta(\boldsymbol{x} - \boldsymbol{x}_0)$$

where δ is the Kronecker delta function satisfying

$$\int_\Omega \delta(\boldsymbol{x} - \boldsymbol{x}_0)\, d\Omega_{\boldsymbol{x}} = \begin{cases} 1, & \boldsymbol{x}_0 \in \Omega \\ 0, & \boldsymbol{x}_0 \notin \Omega \end{cases}$$

Monopole point source



Corresponds to an expanding spherical wave:

$$p(x)e^{+i\omega t} = \frac{e^{i(\omega t - kr)}}{4\pi r}$$

For example, the spherical wavefront

$$\omega t - kr = B \in \mathbb{R}$$

has radius

$$r(t) = \frac{\omega}{k}t - \frac{B}{k} = ct - \frac{B}{k}$$

which is increasing at speed c.

Spherical Multipole sources

Higher-order singular sources:

$$S_n^m(\boldsymbol{x} - \boldsymbol{x}_0) = h_n^{(2)}(kr)\, Y_n^m(\theta, \phi)$$

where $n = 0, 1, 2, \ldots$ and $m = -n, \ldots, n$

$h_n^{(2)}(kr) \in \mathbb{C}$ — spherical Hankel functions of the 2nd kind

$Y_n^m(\theta, \phi) \in \mathbb{C}$ — spherical harmonics

Complex-valued. In spherical coordinates (r, θ, ϕ)

Spherical Hankel functions $h_n^{(2)}(kr)$

Radial solutions to the wave equation.

Easy—just sin/cos & increasing powers of $1/r$

$$h_0^{(2)}(kr) = \frac{i}{kr}\, e^{-ikr}$$

$$h_1^{(2)}(kr) = -\,\frac{kr - i}{(kr)^2}\, e^{-ikr}$$

$$h_2^{(2)}(kr) = -\,i\,\frac{(kr)^2 - 3ikr - 3}{(kr)^3}\, e^{-ikr}$$

$$h_3^{(2)}(kr) = \frac{(kr)^3 - 6i(kr)^2 - 15kr + 15i}{(kr)^4}\, e^{-ikr}.$$

Spherical Hankel functions $h_n^{(2)}(kr)$

Higher-orders easily evaluated using the recurrence relation:

$$h_{n+1}^{(2)}(kr) = \frac{2n+1}{kr}\, h_n^{(2)}(kr) - h_{n-1}^{(2)}(kr)$$

Derivatives also available (needed later for BCs)

$$\frac{d}{dz} h_n^{(2)}(z) = \frac{1}{2}\left[h_{n-1}^{(2)}(z) - \frac{h_n^{(2)}(z) + z\, h_{n+1}^{(2)}(z)}{z} \right]$$

Spherical Harmonics $Y_n^m(\theta, \phi)$

Complex-valued spherical harmonics:

$$Y_n^m(\theta, \phi) = \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}}\, P_n^m(\cos\theta)\, e^{im\phi}$$

where P_n^m are associated Legendre polynomials.

Widely used in physics and graphics.

Easy to compute, e.g., see Wikipedia, or "Numerical Recipes" [Press et al. 2007].

Single-point Multipole Expansion

Multipole expansion about $\boldsymbol{x}_0$

$$p(\boldsymbol{x};\boldsymbol{x}_0) = \sum_{n=0}^{\bar{n}} \sum_{m=-n}^{n} S_n^m(\boldsymbol{x}-\boldsymbol{x}_0)\, c_n^m$$

where $c_n^m \in \mathbb{C}$ are multipole expansion coefficients.

Generalized index notation:

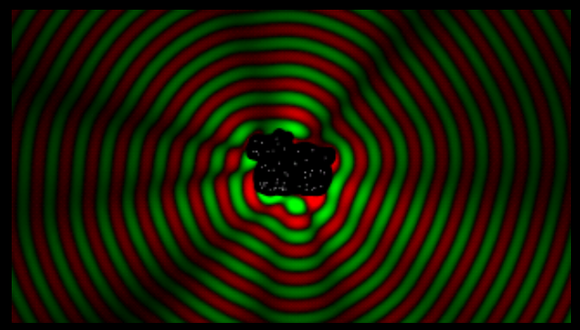
$$p(\boldsymbol{x};\boldsymbol{x}_0) = \sum_j S_j(\boldsymbol{x}-\boldsymbol{x}_0)\, c_j$$

Approximating & Representing Acoustic Transfer

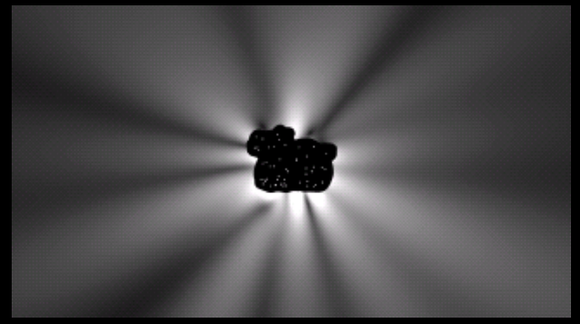
Representing Acoustic Transfer

Transfer-based Sound Rendering

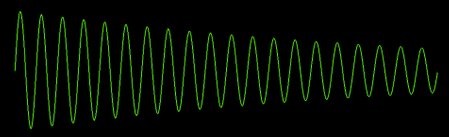
- Acoustic transfer function: $p(\mathbf{x})$



- Amplitude of unit vibration: $|p(\mathbf{x})|$



- Modal sound contribution: $|p(\mathbf{x})|\, q(t)$



Problem: Must evaluate $p(\mathbf{x})$ for each … time sample … mode … and object.

⇒ Representation is important for fast rendering.

Approximating Acoustic Transfer

Helmholtz Solvers

1. Spatial discretizations (finite difference, finite volume, finite element, etc.)
   - Need absorbing BCs (e.g., PMLs), or infinite elements
   - Memory intensive.
   - Need far-field representation.
2. Boundary element method (BEM)
   - Boundary-only formulation
   - Popular in computational acoustics
   - Fast Helmholtz BEM solvers needed for high-res geometry
3. Equivalent source methods
   - Single-point or multi-point multipole expansions
   - Can handle high-res geometry; unsuitable for thin shells.

All solvers struggle at higher frequencies, i.e., $kL \gg 1$

Approximating Acoustic Transfer

BEM Helmholtz solver

Boundary integral methods

Precomputation: Given geometry, mode & ω.

Input boundary data, $\frac{\partial p}{\partial n}$

BEM discretization:

$$H \; p = G \; \frac{\partial p}{\partial n}$$

Output boundary data, p

Software available, e.g., BEM++

Approximating Acoustic Transfer

BEM Helmholtz solver

Boundary integral methods

Runtime Evaluation: Given p and $\frac{\partial p}{\partial n}$ on boundary, evaluate $p(\boldsymbol{x})$ on Ω using *Kirchhoff integral*:

$$p(\boldsymbol{x}) = \int_\Gamma \left[G(\boldsymbol{x};\boldsymbol{y}) \frac{\partial p}{\partial \boldsymbol{n}}(\boldsymbol{y}) - \frac{\partial G}{\partial \boldsymbol{n}}(\boldsymbol{x};\boldsymbol{y})\, p(\boldsymbol{y}) \right] d\Gamma_{\boldsymbol{y}}$$

Cost: O(N) cost per mode

$\Rightarrow$ O(MN) transfer evaluation cost for M modes.

(Expensive.)

Note: Special formulations needed for thin shells.

BEM $\Rightarrow$ Multipole expansion

Precomputation:

Given p and $\dfrac{\partial p}{\partial n}$ on a closed boundary Γ.

Coefficients of single-point multipole expansion are

$$c_n^m = ik \int_\Gamma \left[R_n^{-m}(\boldsymbol{y} - \boldsymbol{x}_0) \, \frac{\partial p}{\partial \boldsymbol{n}}(\boldsymbol{y}) - p(\boldsymbol{y}) \, \frac{\partial R_n^{-m}}{\partial \boldsymbol{n}}(\boldsymbol{y} - \boldsymbol{x}_0) \right] d\Gamma_{\boldsymbol{y}}$$

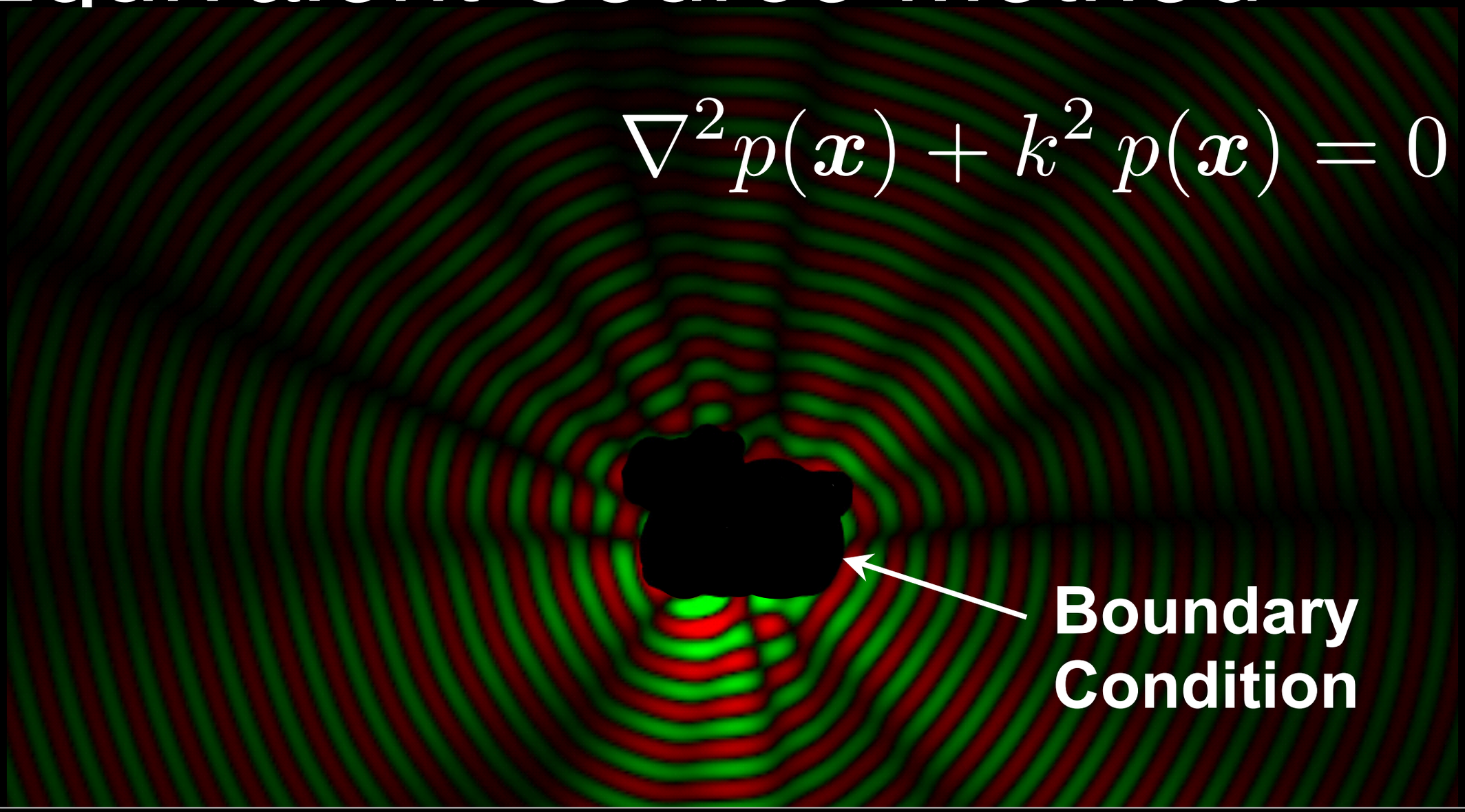
where $R_n^m(\boldsymbol{r}) = j_n(kr) \, Y_n^m(\theta, \phi)$ are *regular* basis functions, and $j_n(kr) \in \mathbb{R}$ are spherical Bessel functions (easy to compute), e.g., $j_0(kr) = \mathrm{sinc}(kr)$

Runtime Evaluation: $O(\bar{n}^2)$ cost per mode. (Cheaper)

(see [Zheng and James 2010] for details)

Approximating Acoustic Transfer

Equivalent Source Method



- Method of Trefftz [Trefftz 1926]
- Source simulation methods [Ochmann 1995; 1999]
- Precomputed Acoustic Transfer [James, Barbic & Pai 2006]

Approximating Acoustic Transfer

Equivalent Source Method

Solution represented as a single- or multi-point multipole expansion of the form:

$$p(\boldsymbol{x}) = \sum_{j=1}^{N} \psi_j(\boldsymbol{x})\, c_j = \boldsymbol{\psi}(\boldsymbol{x}) \cdot \mathbf{c}$$

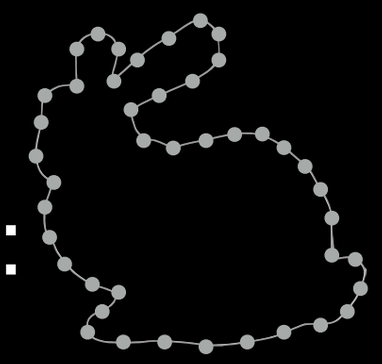
where $\psi_j(\boldsymbol{x})$ are basis functions like $S_n^m(\boldsymbol{x} - \boldsymbol{x}_0)$.
Since $p(\boldsymbol{x})$ satisfies Helmholtz equation outside Γ,

$\Rightarrow$ only need to ***find*** $\mathbf{c}$ ***that minimizes BC error!***

$$\frac{\partial p(\boldsymbol{x})}{\partial n} = \rho\, \omega^2\, \bar{u}_n(\boldsymbol{x}) \quad \Longrightarrow \quad \frac{\partial \boldsymbol{\psi}(\boldsymbol{x})}{\partial n} \cdot \mathbf{c} = \rho\, \omega^2\, \bar{u}_n(\boldsymbol{x})$$

Approximating Acoustic Transfer

Equivalent Source Method



BC least-squares problem (collocation scheme):

$$\frac{\partial \boldsymbol{\psi}}{\partial n}(\boldsymbol{x}_1) \cdot \mathbf{c} = \rho\,\omega^2\,\bar{u}_n(\boldsymbol{x}_1)$$

$$\frac{\partial \boldsymbol{\psi}}{\partial n}(\boldsymbol{x}_2) \cdot \mathbf{c} = \rho\,\omega^2\,\bar{u}_n(\boldsymbol{x}_2)$$

$$\vdots$$

$$\frac{\partial \boldsymbol{\psi}}{\partial n}(\boldsymbol{x}_M) \cdot \mathbf{c} = \rho\,\omega^2\,\bar{u}_n(\boldsymbol{x}_M)$$

$$\mathbf{A}\,\mathbf{c} = \mathbf{b}$$

Ill-conditioned system: use TSVD or regularized QR.

Other weighted-residual schemes used (see Ochmann).

Approximating Acoustic Transfer

“Precomputed Acoustic Transfer”

[James et al. 2006]

Equiv. source limitation: Volumetric objects only, since singular sources must be *inside* object ⇒ Not suitable for thin shells.

PAT paper first solves Helmholtz using BEM, then fits sources to p(x) sampled on an offset surface.

- Optimized placement of sources using multi-level randomized method.

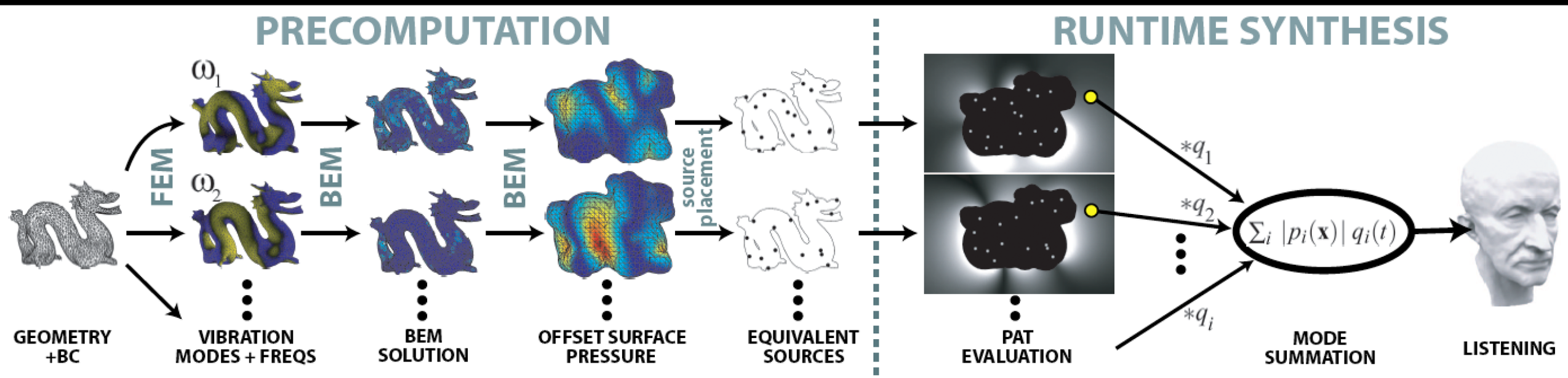
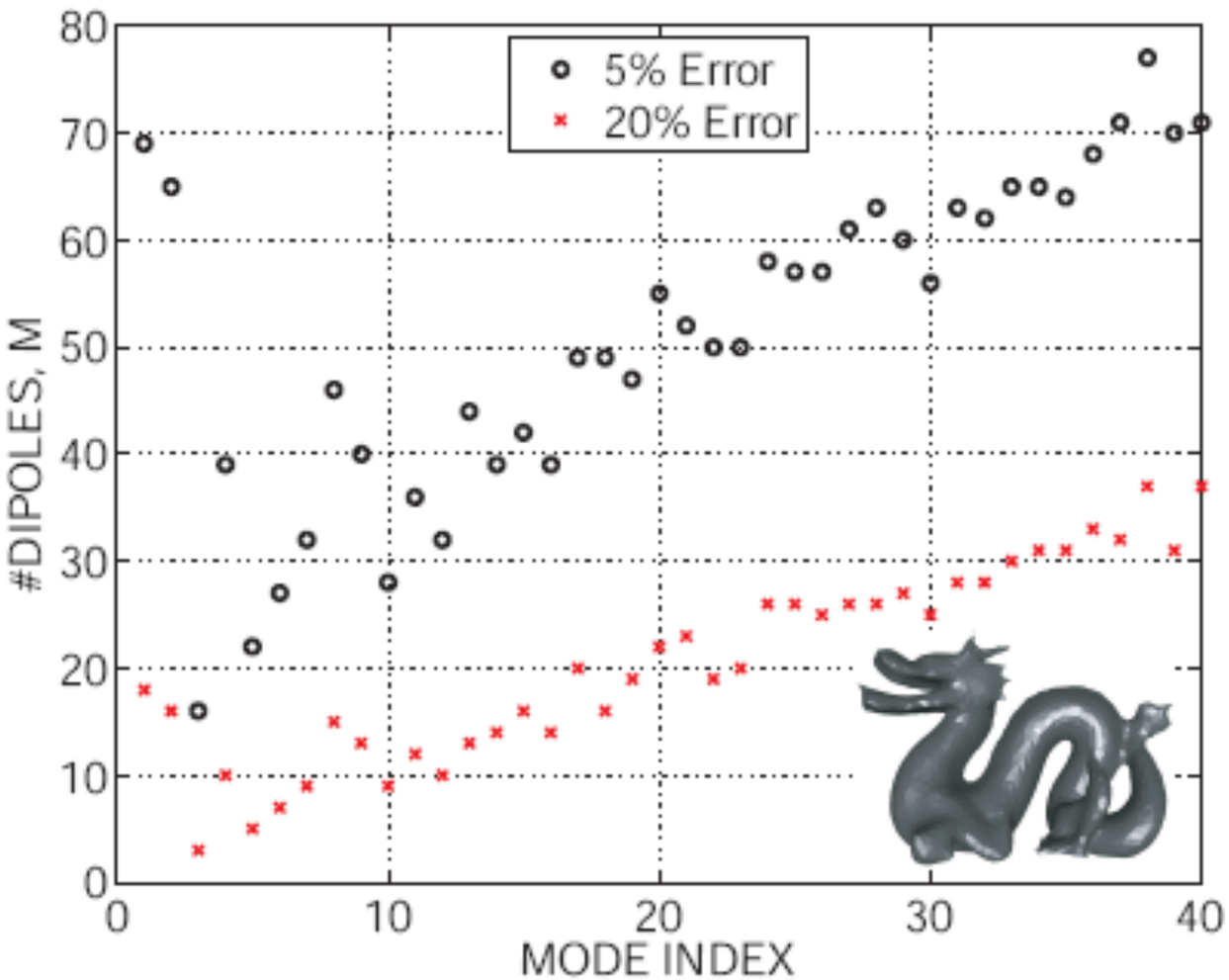
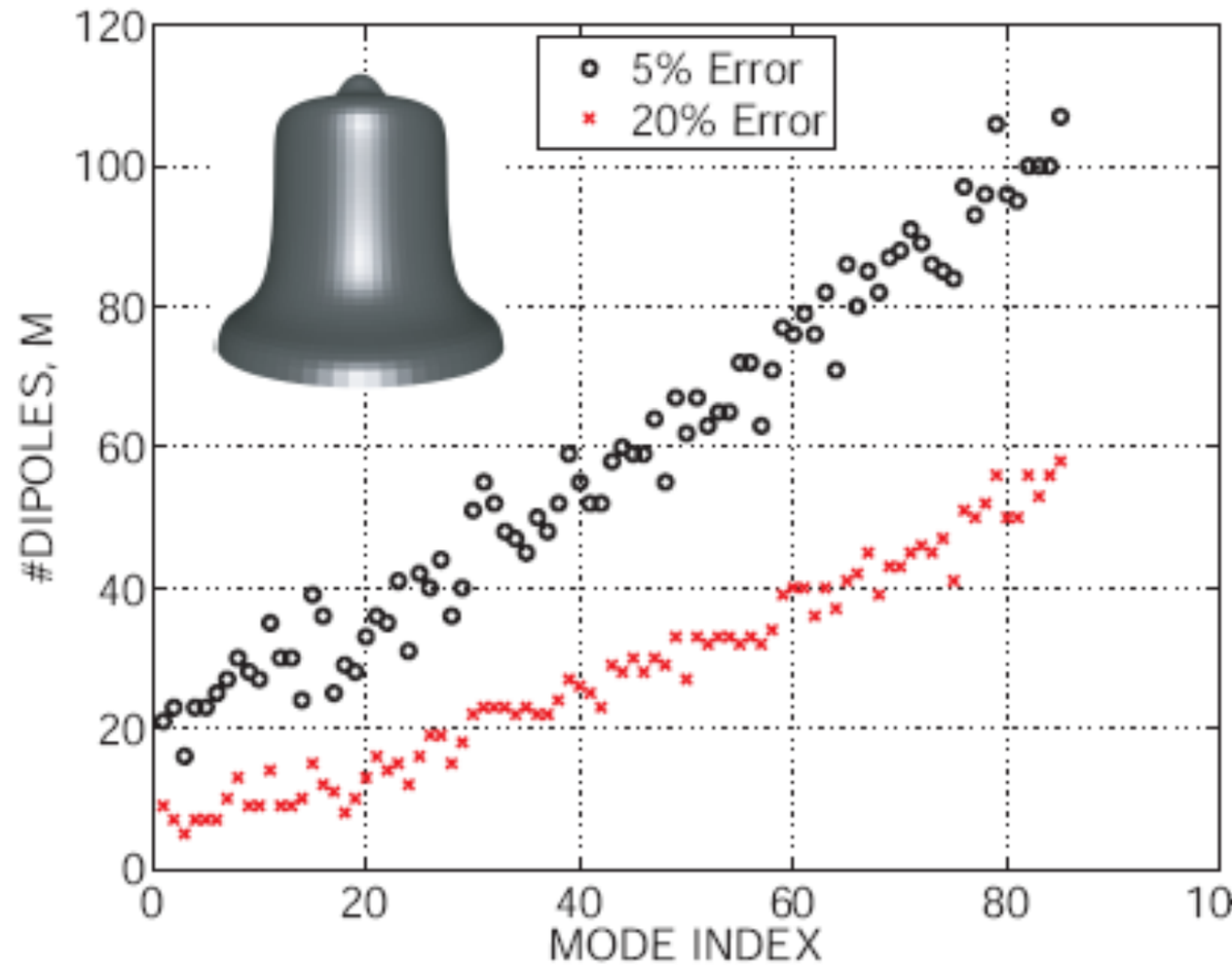


Figure 2: **Overview of Precomputed Acoustic Transfer (PAT)**

Precomputed Acoustic Transfer [James et al. 2006]

How many dipoles to fit?





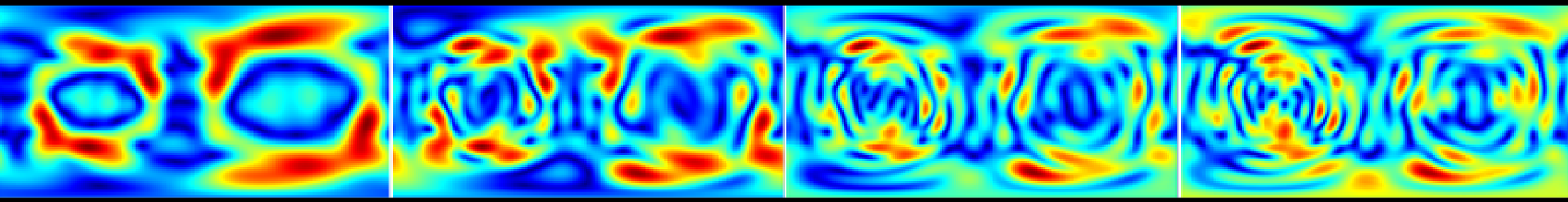
Evaluation cost ≈ 0.3 μsec/dipole (in 2006)

More coefficients for higher frequency modes.

Representing Acoustic Transfer

FFAT Maps

[Chadwick et al. 2009]



Goal:

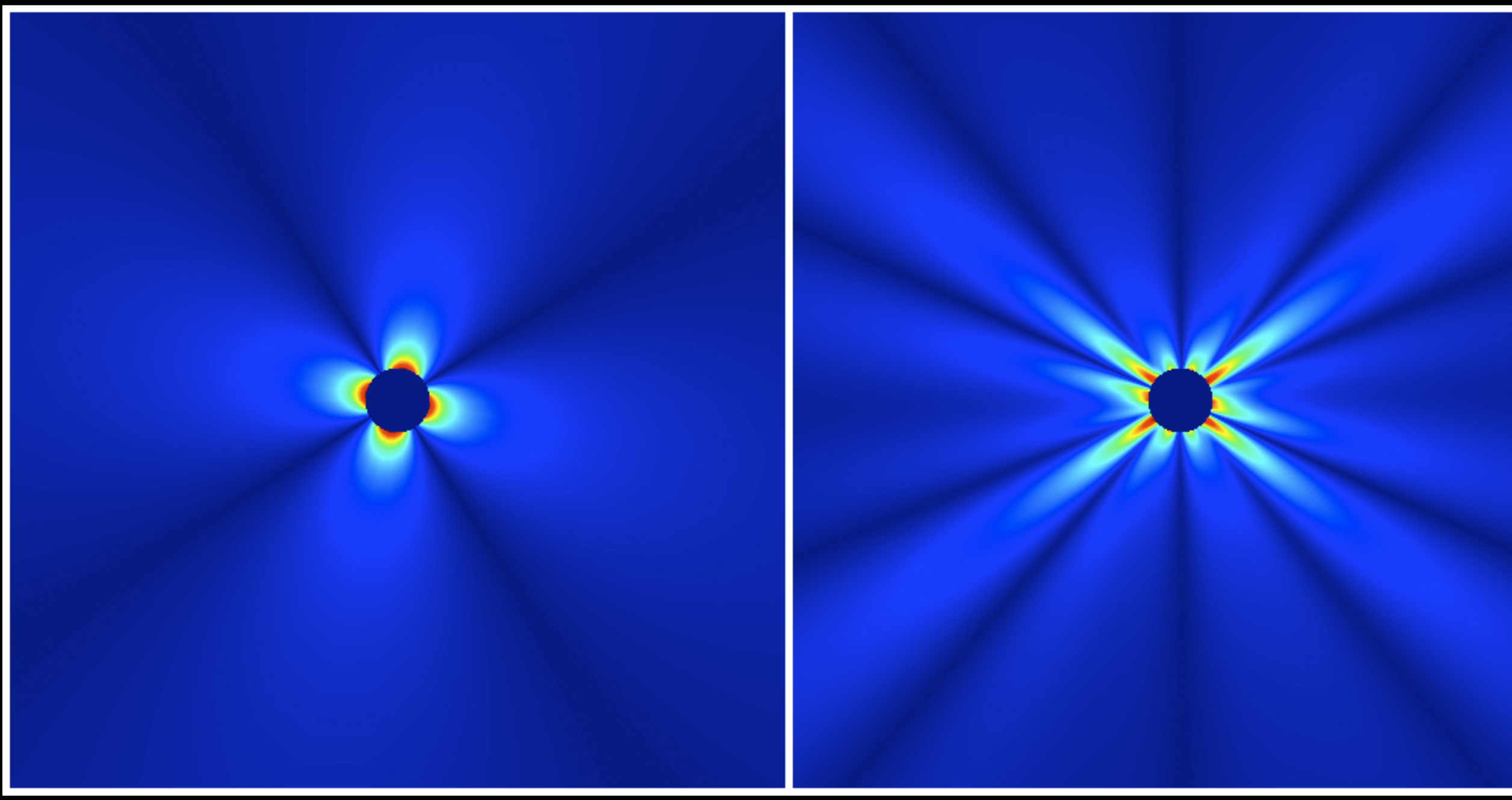
- O(1) transfer evaluation cost per mode.
- O(M) transfer evaluation cost per object (M modes)
- Avoid increasing cost for high-frequency modes

Approach:

- Exploit radial structure & speed-memory trade-off
- Texture map out-going radiation

Representing Acoustic Transfer

Exploiting radial structure



Far-Field Acoustic Transfer (FFAT)

- Consider an M-term asymptotic expansion

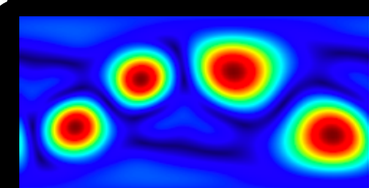
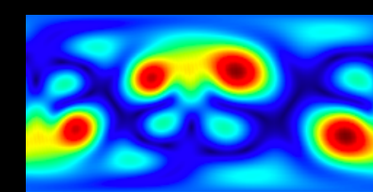
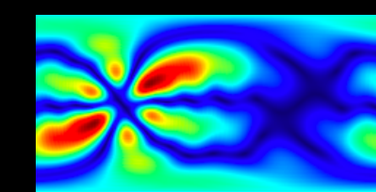
$$p(x) \sim h_0(kR)\left\{\Psi_1(\theta,\phi) + \frac{\Psi_2(\theta,\phi)}{kR} + \cdots + \frac{\Psi_M(\theta,\phi)}{(kR)^{M-1}}\right\}$$

$$-\frac{ie^{-ikR}}{kR}$$

Representing Acoustic Transfer

Far-Field Acoustic Transfer (FFAT)

- Consider an M-term asymptotic expansion

$$p(x) \sim$$  $$\left\{ \right.$$  $$+ \frac{}{kR} +$$  $$+ \cdots + \frac{}{(kR)^{M-1}} \left. \right\}$$ 

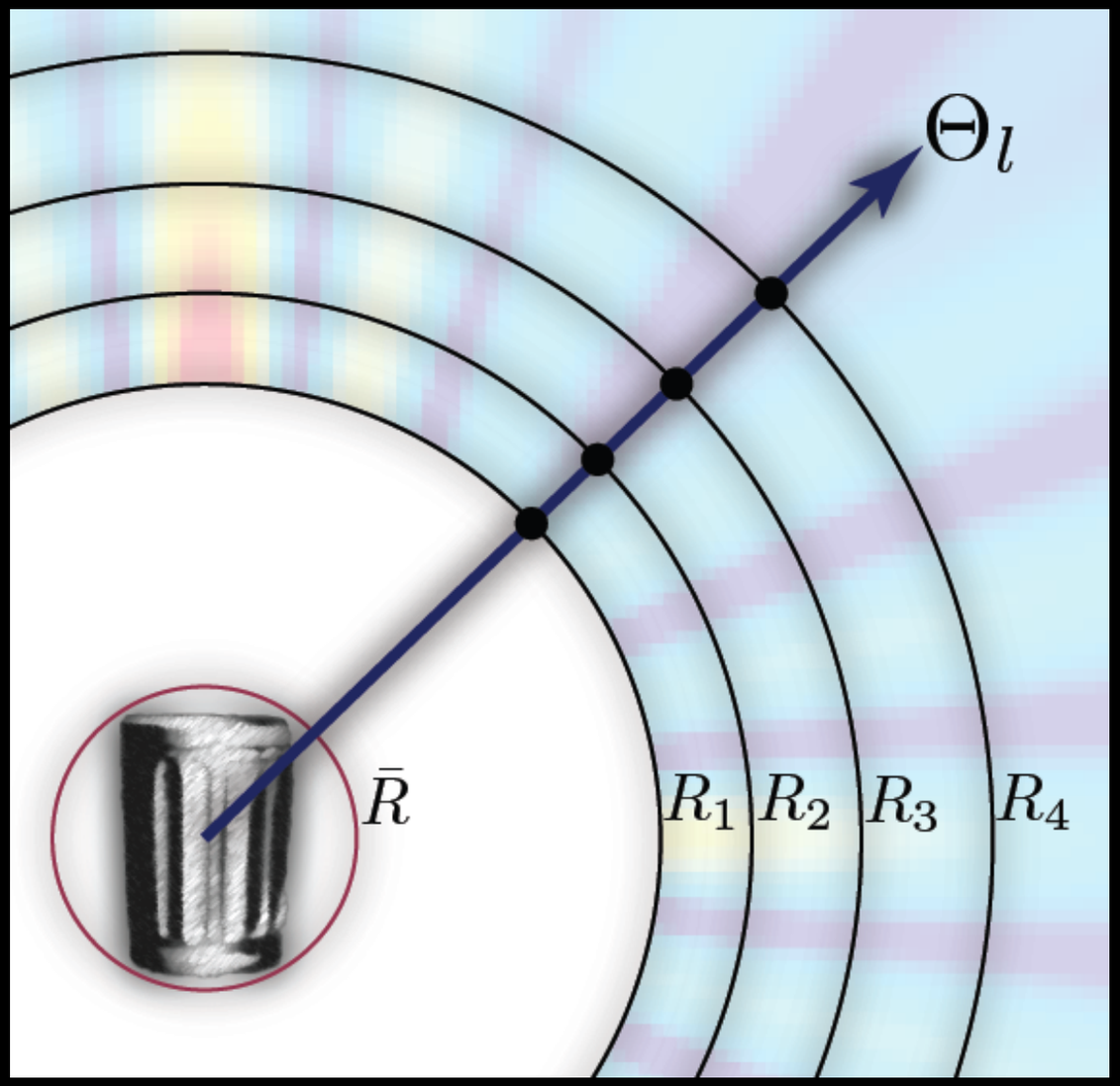
$$-\frac{ie^{-ikR}}{kR}$$

FFAT Maps [Chadwick et al. 2009]

- Low-error transfer, e.g., M=4
- O(1) transfer evaluation cost
  - ~75 µsec / 100 modes

Representing Acoustic Transfer

Precomputing FFAT Maps



Precompute pressure samples on concentric spherical shells using fast multipole BEM [Greengard and Rokhlin 1987; Gumerov and Duraiswami 2005]

- FastBEM implementation [Liu 2009]

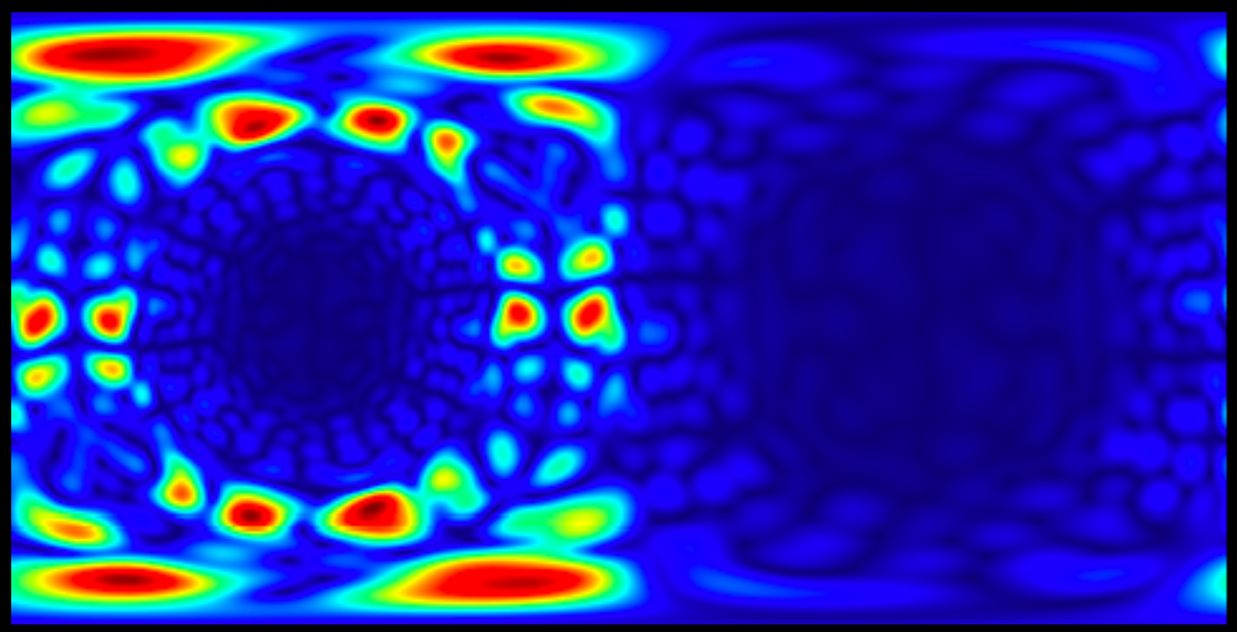
Estimate FFAT Maps using least squares

$$\sum_{j=1}^{M} \frac{h_0(kR_i)}{(kR_i)^{j-1}} \Psi_j(\Theta_l) = p(R_i, \Theta_l) \quad \Leftrightarrow \quad \sum_{j=1}^{M} A_{ij} \Psi_{jl} = p_{il}$$

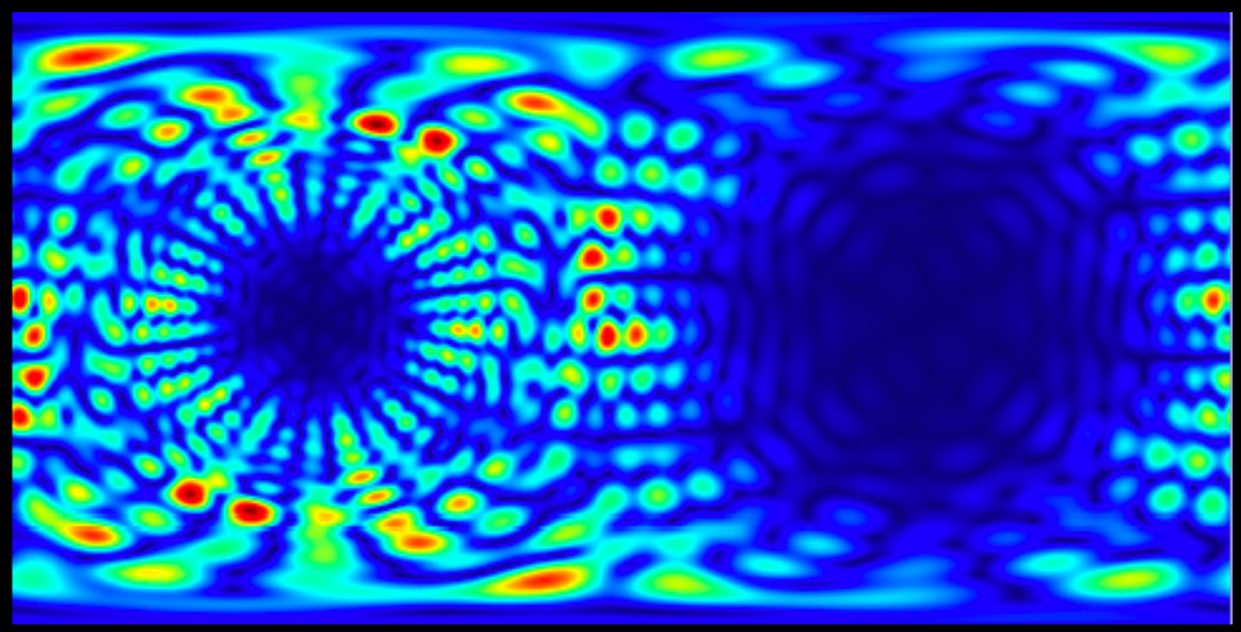
Representing Acoustic Transfer

FFAT Maps (Trash Can)

Mode 140



$|\Psi_1(\theta, \phi)|$

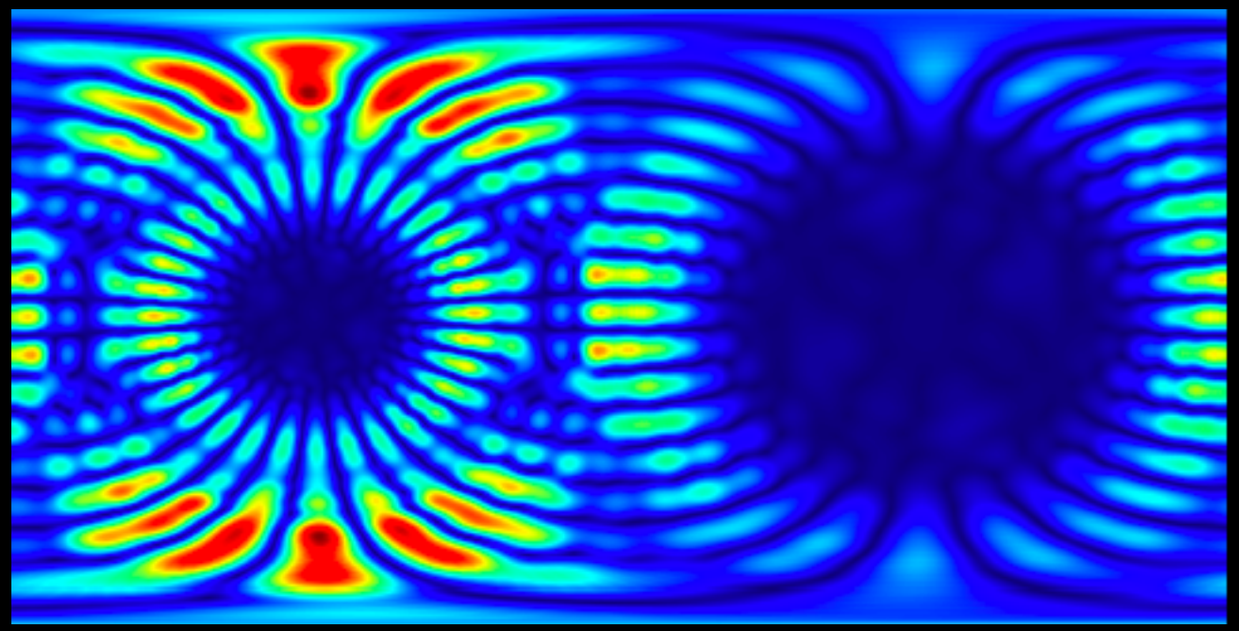


$|\Psi_2(\theta, \phi)|$

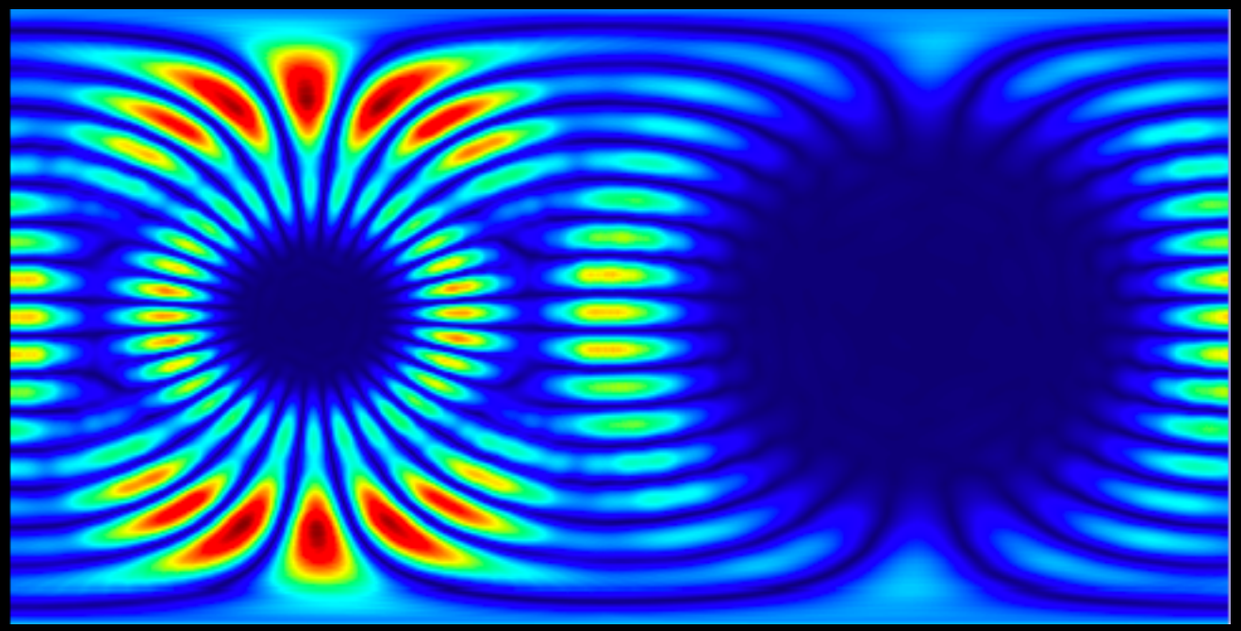
Representing Acoustic Transfer

FFAT Maps (Trash Can)

Mode 160



$|\Psi_1(\theta, \phi)|$



$|\Psi_2(\theta, \phi)|$

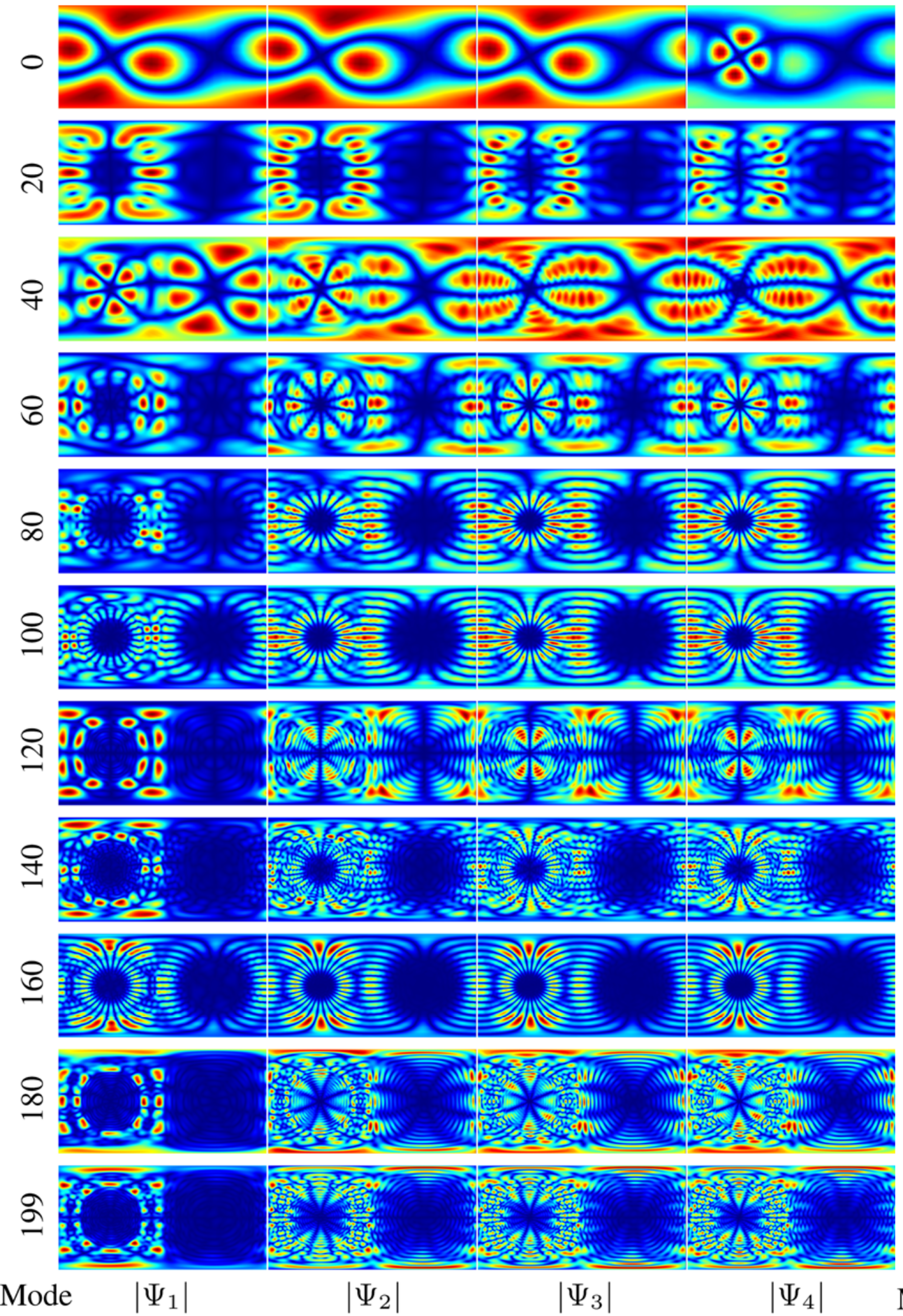


Figure 13: FFAT Maps (Trash Can)

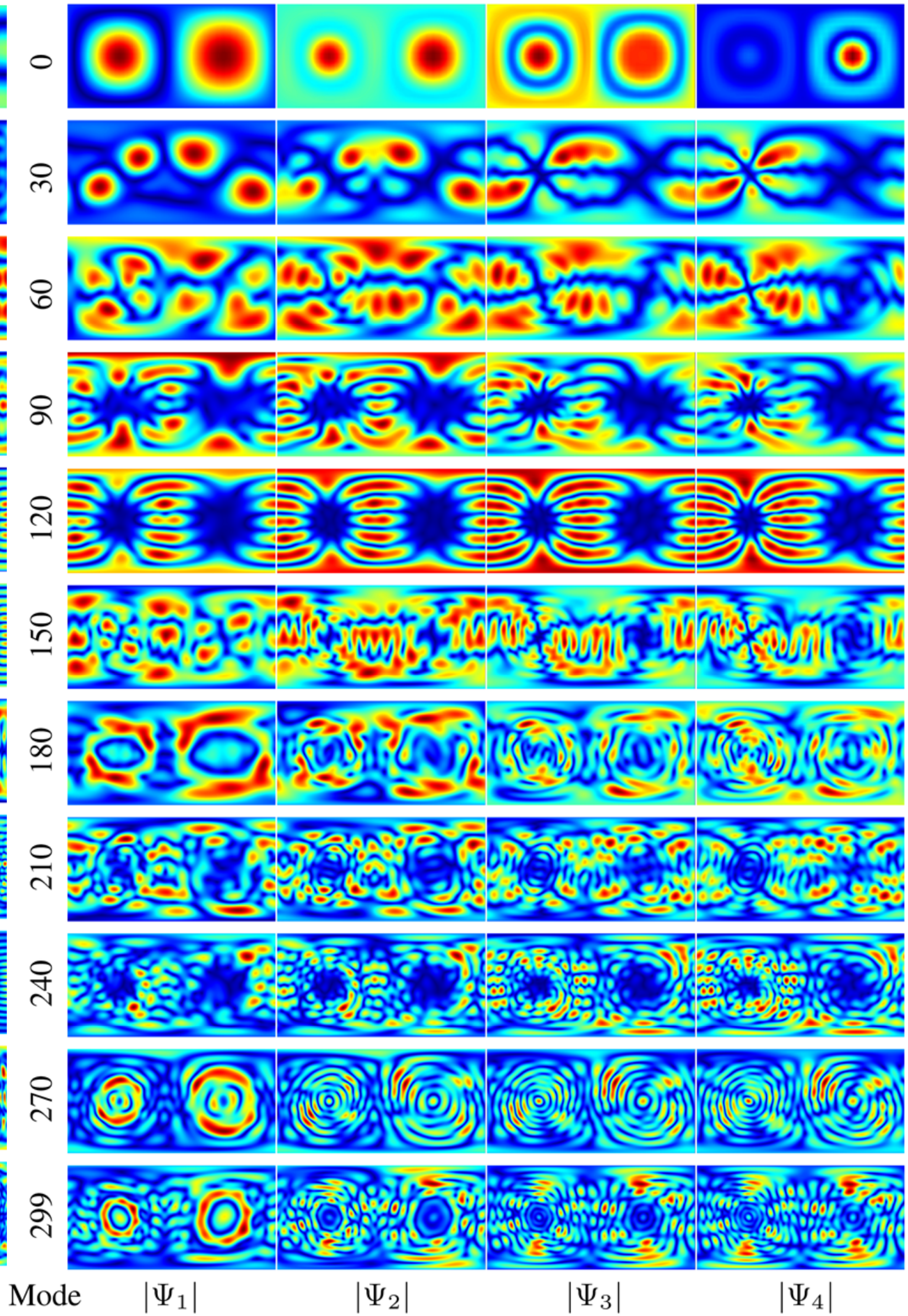
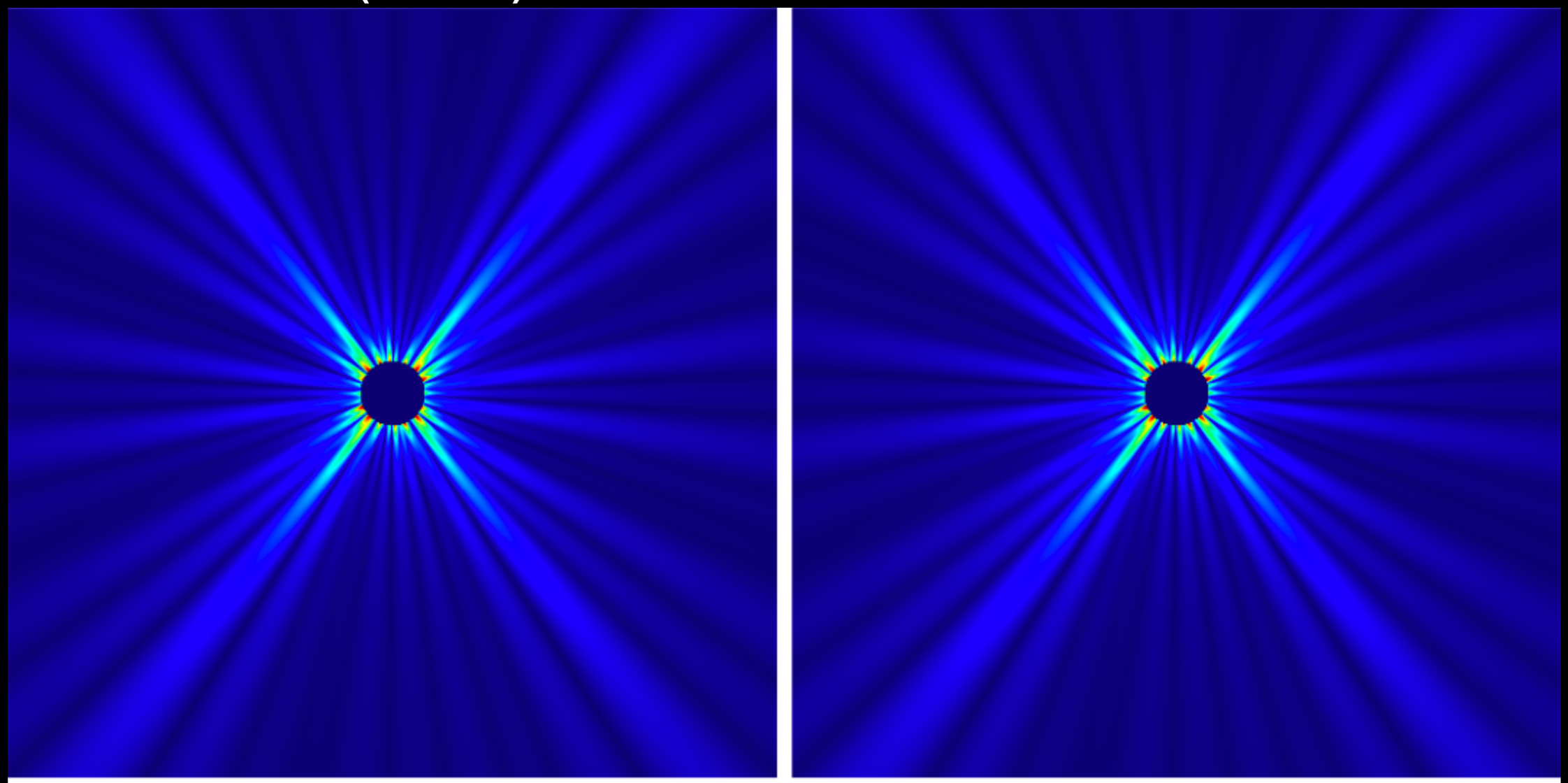


Figure 12: FFAT Maps (Water Bottle)

Representing Acoustic Transfer

FFAT Map Accuracy

Trash Can (M=4)



Mode 199 (4433 Hz)

Representing Acoustic Transfer

FFAT Map Accuracy

Model	Mode	Freq (Hz)	kL	θ Resolution (T)	Average Relative Error (%)			
					$M=1$	$M=2$	$M=3$	$M=4$
Trash Can	0	71	0.90	18	1.47	6.73	0.95	1.04
	50	1880	25.82	93	19.51	7.11	1.28	0.69
	100	2823	38.79	132	44.44	28.36	20.00	5.62
	150	3698	50.81	168	67.21	35.20	8.27	1.76
	199	4433	60.80	198	53.37	42.64	17.05	3.14
Trash Lid	0	112	1.13	20	3.12	8.47	1.36	0.78
	50	2296	23.13	110	36.12	21.91	6.41	1.74
	100	3997	40.27	180	51.54	24.36	18.07	2.36
	150	5736	57.79	252	14.09	3.96	1.32	0.47
	199	6791	68.22	295	60.40	23.39	31.04	2.94
Water Bottle	0	116	0.98	20	251.35	16.00	1.90	0.032
	75	1321	11.13	70	26.86	11.80	2.98	0.54
	150	2197	18.51	106	11.14	6.69	0.86	0.16
	225	2906	24.49	135	14.03	8.41	1.33	0.31
	299	3593	30.27	164	32.26	15.84	3.33	0.45
Recycling Bin	0	62	0.70	18	10.85	3.53	0.40	0.43
	75	820	9.17	49	13.66	3.79	1.05	0.65
	150	1329	14.85	70	8.54	3.72	0.87	0.65
	225	1791	20.01	89	7.36	2.89	0.70	0.35
	299	2209	24.68	107	12.67	5.03	2.07	0.54

Comparison of Acoustic Transfer Methods

"Ground Truth"

Fast multipole method (FastBEM)

Evaluation time: 13.2 hours

Far-Field Acoustic Transfer Map

FFAT Map Approximation (M=4 textures/mode)

Evaluation time: <1 second

Concept: Radiation Efficiency

(see [Cremer et al. 1990])

Ratio comparing radiated acoustic power Π_{rad} to surface vibrational power

$$\sigma = \frac{\Pi_{rad}}{\rho\, c\, S\, \langle v_n^2 \rangle_S} = \frac{\Pi_{rad}}{\rho\, c\, S\, \omega^2\, \langle \bar{u}_n^2 \rangle_S}$$

Strongly depends on shape, mode & frequency.

Frequency-domain Wave Radiation

Concept: Radiation Efficiency

(see [Cremer et al. 1990])