Physically Based Sound for Computer Animation and Virtual Environments

Acoustic Transfer

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Acoustic Transfer

Overview

• Acoustic transfer basics
• Acoustic waves (wave equation, BCs)
• Frequency-domain wave radiation (Helmholtz eqn)
• Multipole sound sources
• Solvers and representations for AT
  • Boundary Element Method (BEM)
  • Multipole expansions
  • Equivalent source method
  • “Precomputed Acoustic Transfer”
  • FFAT Maps
Acoustic Transfer Basics
Acoustic Transfer Basics

Linear modal sound

\[ \text{sound}(t) = q_1(t) + q_j(t) + q_3(t) \]
Acoustic Transfer Basics

Linear modal sound

\[ \text{sound}(x, t) = \sum_j a_j(x) \times q_j(t) \]

Amplitude of acoustic transfer function for mode \( j \)
Acoustic Transfer Basics

How loud is each mode?

\[ \text{sound}(x, t) = \sum_j a_j(x) q_j(t) = a(x)^T q(t) \]

Transfer \( a(x) \) evaluated at listening position, \( x = x(t) \) (in body frame)

at slower rate (e.g., 250 Hz), then smoothly interpolated up to audio rate of \( q(t) \).
Wait… what about \( \frac{1}{r} \) fall off?

\[
q_1(t) + q_2(t) + \cdots
\]

Commonly used for point sources…. 

ASIDE: Point sources
Not a point source!

Bronze Dragon (20 cm)
Point-source model

\[
\text{sound} \propto \sum_i C_i q_i(t) \frac{1}{r}
\]

- \(C=1\): [van den Doel and Pai 1996; Cook 1996; ...]
- \(C \neq 1\): Monopole approx. (see [Cremer et al. 1990, O’Brien et al. 2002, James et al. 2006])

- Assumes:
  - Far-field listener \(r \gg R\)
  - Low-frequency waves \(\lambda \gg R\)
  - Closed object
Acoustic Waves

Diffraction is important!

- Speed of sound, \( c = 343 \text{ m/s} = \lambda f \)
- Audible frequencies: 16 Hz — 20 kHz
- Audible wavelengths: 21 m — 17 mm

Diffraction Shaders [Stam 99]
ASIDE: Point sources

Comparison

[James, Barbic & Pai 2006]

Acoustic transfer renderer

Point-source renderer
Acoustic Waves
Wave equation follows from...
(see [Howe 2003], or Bridson's “fluid” course notes)

- **Linearized continuity eqn** (mass conservation)
  \[
  \frac{1}{\rho} \frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot \mathbf{v} = 0
  \]

- **Linearized momentum eqn** (“ma=t”)
  \[
  \rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p
  \]

- **Energy eqn** (relates pressure and density at constant entropy)
  \[
  p = \left( \frac{\partial p}{\partial \tilde{\rho}} \right)_0 \quad \tilde{\rho} = c^2 \tilde{\rho}
  \]
Acoustic Waves

3D Wave Equation

Acoustic pressure, \( p(x, t) \), satisfies

\[
\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad (\text{Wave Equation})
\]

where

\[
\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}
\]

is the Laplacian.
Acoustic Waves

1D Wave Equation

\[
\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}
\]

Traveling wave solutions of the form:

\[
p(x, t) = f(x \pm ct)
\]
Acoustic Waves

Radiation boundary condition (1)

Need to impose (vibrating) solid boundaries. From the linearized momentum equation \((F=0)\),
\[
\rho \frac{\partial v}{\partial t} = -\nabla p
\]
the normal component yields
\[
\frac{\partial p}{\partial n} = -\rho \ a_n(x, t), \quad x \in \Gamma \quad \text{(Neumann BC)}
\]
where \(a_n(x, t)\) is the surface acceleration, and \(\rho = 1.2041 \text{ kg/m}^3\) is air density at STP.
Acoustic Waves

Radiation boundary condition (2)

Need condition at infinity.

- So-called *Sommerfeld radiation condition*.
- Solutions must decay at infinity and correspond to out-going waves.
- Avoids nonphysical solutions, such as infinite sources at infinity.
Frequency-domain Wave Radiation

Motivation: Individual mode vibrations are localized in frequency domain

Impulse response of a cymbal impact modeled using a linear modal vibration model.
Using complex numbers to represent oscillations

By linearity of the wave equation & BCs, sufficient to consider a unit amplitude harmonic vibration at natural frequency $\omega$,

$$e^{+i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

Different phases introduced by $e^{+i\phi}$ factors since

$$e^{+i\omega t} e^{i\phi} = \cos(\omega t + \phi) + i \sin(\omega t + \phi)$$

Can think of $q(t)$ as an amplitude and a unit complex part.
Frequency-domain Wave Radiation

**Harmonic pressure oscillations**

Consider harmonic pressure solutions of the form

\[ p(x, t) = p(x) e^{+i\omega t} \]

where \( p(x) \) is a complex-valued pressure field—will be the *acoustic transfer function*.

Using Euler’s formula, we can write

\[ p(x) = |p| e^{i\phi} \]

so that

\[
\begin{align*}
p(x, t) &= p(x) e^{+i\omega t} = |p(x)| e^{+i(\omega t + \phi)} \\
&= |p(x)| \cos(\omega t + \phi) + i |p(x)| \sin(\omega t + \phi).
\end{align*}
\]
Frequency-domain Wave Radiation

The Helmholtz equation
(the frequency-domain wave equation)

Substituting \( p(x, t) = p(x)e^{i\omega t} \) into the wave equation yields

\[
0 = \nabla^2 p(x) e^{i\omega t} - \frac{1}{c^2} p(x) \left( \frac{\partial^2}{\partial t^2} e^{i\omega t} \right)
\]

\[
= \left( \nabla^2 p(x) - \frac{\omega^2}{c^2} p(x) \right) e^{i\omega t}
\]

and so

\[
\nabla^2 p(x) + k^2 p(x) = 0
\]

\( (\text{Helmholtz eqn}) \)

where \( k \) is the wavenumber,

\[
k = \frac{\omega}{c} = \frac{2\pi}{\lambda}
\]
Frequency-domain Wave Radiation

Radiation boundary conditions

Recall time-domain Neumann BC:

\[ \frac{\partial p}{\partial n} = -\rho \, a_n(x, t), \quad x \in \Gamma \]

What is frequency-domain BC?
What is \( a_n(x, t) \)?
Frequency-domain Wave Radiation

Radiation boundary conditions

Need harmonic surface acceleration $a_n(x, t)$ in terms of eigenmode displacement, $\bar{u}(x)$:

\[
\begin{align*}
\mathbf{u}(x, t) &= \bar{u}(x) e^{+i\omega t} \quad \Rightarrow \quad \mathbf{u}_n = \bar{u}_n e^{+i\omega t} \\
\mathbf{v}(x, t) &= \frac{d\mathbf{u}}{dt}(x, t) = i\omega \bar{u}(x) e^{+i\omega t} \quad \Rightarrow \quad \mathbf{v}_n = i\omega \bar{u}_n e^{+i\omega t} \\
\mathbf{a}(x, t) &= \frac{d^2\mathbf{u}}{dt^2}(x, t) = (i\omega)^2 \bar{u}(x) e^{+i\omega t} \quad \Rightarrow \quad \mathbf{a}_n = -\omega^2 \bar{u}_n e^{+i\omega t}
\end{align*}
\]

\[a_n = -\omega^2 \bar{u}_n e^{+i\omega t}\]
Frequency-domain Wave Radiation

Radiation boundary conditions

The Neumann boundary condition becomes

$$\frac{\partial p(x, t)}{\partial n} = -\rho \ a_n(x, t)$$

$$\frac{\partial p(x)}{\partial n} e^{+i\omega t} = \rho \omega^2 \ u_n e^{+i\omega t}$$

Never zero

$$\frac{\partial p(x)}{\partial n} = \rho \omega^2 \ u_n(x)$$

(+Sommerfeld radiation BC: Out-going decaying waves.)
Solve exterior Helmholtz eqn

\[(\nabla^2 + k^2)p(x) = 0, \quad x \in \Omega\]

subject to BC:

\[\frac{\partial p(x)}{\partial n} = \rho \omega^2 \bar{u}_n(x), \quad x \in \Gamma\]

(+Sommerfeld radiation BC.)

Given mode data \(\bar{u}_n(x)\) and frequency \(\omega\).
Frequency-domain Wave Radiation

Visualizing solutions

Can visualize solutions by looking at real part of

$$p(\mathbf{x})e^{+i\omega t}$$

or the AT amplitude

$$|p(\mathbf{x})|$$
MODE 10
MODE 12
MODE 15
MODE 23
MODE 25
MODE 28
MODE 31
MODE 38
Multipole Sound Sources
Multipole Sound Sources

Singular Helmholtz solutions

Satisfy Helmholtz equation

\[ \nabla^2 p(x) + k^2 p(x) = 0 \]

almost everywhere, except where they are singular, e.g., at origin.

Very common and extremely useful.
Can be summation of simpler singular solutions.
Multipole Sound Sources

Spherical Multipole sources

$S_0^0(x)$
Monopole

$S_1^m(x)$
Dipole
Multipole Sound Sources

Monopole point source

\[ p(x) = \frac{e^{-ikr}}{4\pi r} \]

where

\[ k = \frac{\omega}{c} = \frac{2\pi}{\lambda}, \quad r = \|x - x_0\|_2 \]

Solution to point-like volume pulsation at \( x_0 \)

\[ \nabla^2 p(x) + k^2 p(x) = \delta(x - x_0) \]

where \( \delta \) is the Kronecker delta function satisfying

\[ \int_{\Omega} \delta(x - x_0) \, d\Omega_x = \begin{cases} 1, & x_0 \in \Omega \\ 0, & x_0 \notin \Omega \end{cases} \]
Multipole Sound Sources

Monopole point source

Corresponds to an expanding spherical wave:

\[ p(x) e^{+i \omega t} = \frac{e^{i(\omega t - k r)}}{4\pi r} \]

For example, the spherical wavefront

\[ \omega t - kr = B \in \mathbb{R} \]

has radius

\[ r(t) = \frac{\omega}{k} t - \frac{B}{k} = ct - \frac{B}{k} \]

which is increasing at speed \( c \).
Multipole Sound Sources

Spherical Multipole sources

Higher-order singular sources:

\[ S^m_n (\bm{x} - \bm{x}_0) = h_n^{(2)}(kr) \ Y^m_n (\theta, \phi) \]

where \( n = 0, 1, 2, \ldots \) and \( m = -n, \ldots, n \)

\( h_n^{(2)}(kr) \in \mathbb{C} \) — spherical Hankel functions of the 2\textsuperscript{nd} kind

\( Y^m_n (\theta, \phi) \in \mathbb{C} \) — spherical harmonics

Complex-valued. In spherical coordinates \((r, \theta, \phi)\)
Multipole Sound Sources

Spherical Hankel functions $h^{(2)}_n(kr)$

Radial solutions to the wave equation.

Easy—just sin/cos & increasing powers of $1/r$

\[
\begin{align*}
    h^{(2)}_0(kr) &= \frac{i}{kr} e^{-ikr} \\
    h^{(2)}_1(kr) &= -\frac{kr - i}{(kr)^2} e^{-ikr} \\
    h^{(2)}_2(kr) &= -i\frac{(kr)^2 - 3ikr - 3}{(kr)^3} e^{-ikr} \\
    h^{(2)}_3(kr) &= \frac{(kr)^3 - 6i(kr)^2 - 15kr + 15i}{(kr)^4} e^{-ikr}.
\end{align*}
\]
Multipole Sound Sources

**Spherical Hankel functions** $h_n^{(2)}(kr)$

Higher-orders easily evaluated using the recurrence relation:

$$h_{n+1}^{(2)}(kr) = \frac{2n + 1}{kr} h_n^{(2)}(kr) - h_{n-1}^{(2)}(kr)$$

Derivatives also available (needed later for BCs)

$$\frac{d}{dz} h_n^{(2)}(z) = \frac{1}{2} \left[ h_{n-1}^{(2)}(z) - \frac{h_n^{(2)}(z) + \frac{z}{z} h_{n+1}^{(2)}(z)}{z} \right]$$
Multipole Sound Sources

Spherical Harmonics $Y_n^m(\theta, \phi)$

Complex-valued spherical harmonics:

$$Y_n^m(\theta, \phi) = \sqrt{\frac{2n + 1}{4\pi} \frac{(n - m)!}{(n + m)!}} P_n^m(\cos \theta) e^{im\phi}$$

where $P_n^m$ are associated Legendre polynomials.

Widely used in physics and graphics.

Easy to compute, e.g., see Wikipedia, or “Numerical Recipes” [Press et al. 2007].
Multipole Sound Sources

Single-point Multipole Expansion

Multipole expansion about \( \mathbf{x}_0 \)

\[
p(\mathbf{x}; \mathbf{x}_0) = \sum_{n=0}^{\bar{n}} \sum_{m=-n}^{n} S_n^m (\mathbf{x} - \mathbf{x}_0) c_n^m
\]

where \( c_n^m \in \mathbb{C} \) are multipole expansion coefficients.

Generalized index notation:

\[
p(\mathbf{x}; \mathbf{x}_0) = \sum_j S_j (\mathbf{x} - \mathbf{x}_0) c_j
\]
Approximating & Representing Acoustic Transfer
Representing Acoustic Transfer

Transfer-based Sound Rendering

- Acoustic transfer function: \( p(x) \)
- Amplitude of unit vibration: \( |p(x)| \)
- Modal sound contribution: \( |p(x)| q(t) \)

**Problem**: Must evaluate \( p(x) \) for each … time sample … mode … and object.
⇒ Representation is important for fast rendering.
Approximating Acoustic Transfer

Helmholtz Solvers

1. **Spatial discretizations** (finite difference, finite volume, finite element, etc.)
   - Need absorbing BCs (e.g., PMLs), or infinite elements
   - Memory intensive.
   - Need far-field representation.

2. **Boundary element method (BEM)**
   - Boundary-only formulation
   - Popular in computational acoustics
   - Fast Helmholtz BEM solvers needed for high-res geometry

3. **Equivalent source methods**
   - Single-point or multi-point multipole expansions
   - Can handle high-res geometry; unsuitable for thin shells.

All solvers struggle at higher frequencies, i.e., $kL \gg 1$
Approximating Acoustic Transfer

BEM Helmholtz solver

Boundary integral methods

**Precomputation:** Given geometry, mode & $\omega$.

Input boundary data, $\frac{\partial p}{\partial n}$

BEM discretization:

\[
H \quad p = G \quad \frac{\partial p}{\partial n}
\]

Output boundary data, $p$

Software available, e.g., BEM++
Approximating Acoustic Transfer

**BEM Helmholtz solver**
Boundary integral methods

**Runtime Evaluation:** Given \( p \) and \( \frac{\partial p}{\partial n} \) on boundary, evaluate \( p(x) \) on \( \Omega \) using *Kirchhoff integral*:

\[
p(x) = \int_{\Gamma} \left[ G(x; y) \frac{\partial p}{\partial n}(y) - \frac{\partial G}{\partial n}(x; y) p(y) \right] d\Gamma_y
\]

**Cost:** \( O(N) \) cost per mode

\[ \Rightarrow O(MN) \] transfer evaluation cost for \( M \) modes.

(Expensive.)

Note: Special formulations needed for thin shells.
Approximating Acoustic Transfer

BEM $\Rightarrow$ Multipole expansion

Precomputation:

Given $p$ and $\frac{\partial p}{\partial n}$ on a closed boundary $\Gamma$. Coefficients of single-point multipole expansion are

$$c^m_n = i k \int_{\Gamma} \left[ R^{-m}_n(y - x_0) \frac{\partial p}{\partial n}(y) - p(y) \frac{\partial R^{-m}_n}{\partial n}(y - x_0) \right] d\Gamma_y$$

where $R^m_n(r) = j_n(kr) Y^m_n(\theta, \phi)$ are regular basis functions, and $j_n(kr) \in \mathbb{R}$ are spherical Bessel functions (easy to compute), e.g., $j_0(kr) = \text{sinc}(kr)$

Runtime Evaluation: $O(\bar{n}^2)$ cost per mode. (Cheaper) (see [Zheng and James 2010] for details)
Approximating Acoustic Transfer

**Equivalent Source Method**

\[ \nabla^2 p(x) + k^2 p(x) = 0 \]

- Method of Trefftz [Trefftz 1926]
- Source simulation methods [Ochmann 1995; 1999]
- Precomputed Acoustic Transfer [James, Barbic & Pai 2006]
Approximating Acoustic Transfer

**Equivalent Source Method**

Solution represented as a single- or multi-point multipole expansion of the form:

\[ p(x) = \sum_{j=1}^{N} \psi_j(x) c_j = \psi(x) \cdot c \]

where \( \psi_j(x) \) are basis functions like \( S_m^n(x - x_0) \).

Since \( p(x) \) satisfies Helmholtz equation outside \( \Gamma \),
\[ \Rightarrow \text{ only need to find } c \text{ that minimizes BC error!} \]

\[ \frac{\partial p(x)}{\partial n} = \rho \omega^2 \bar{u}_n(x) \quad \Rightarrow \quad \frac{\partial \psi(x)}{\partial n} \cdot c = \rho \omega^2 \bar{u}_n(x) \]
Approximating Acoustic Transfer

Equivalent Source Method

BC least-squares problem (collocation scheme):

\[
\frac{\partial \psi}{\partial n}(x_1) \cdot c = \rho \omega^2 \bar{u}_n(x_1)
\]

\[
\frac{\partial \psi}{\partial n}(x_2) \cdot c = \rho \omega^2 \bar{u}_n(x_2)
\]

\[\vdots\]

\[
\frac{\partial \psi}{\partial n}(x_M) \cdot c = \rho \omega^2 \bar{u}_n(x_M)
\]

Ill-conditioned system: use TSVD or regularized QR.
Other weighted-residual schemes used (see Ochmann).
Approximating Acoustic Transfer

“Precomputed Acoustic Transfer”
[James et al. 2006]

Equiv. source limitation: Volumetric objects only, since singular sources must be inside object ⇒ Not suitable for thin shells.

PAT paper first solves Helmholtz using BEM, then fits sources to p(x) sampled on an offset surface.
- Optimized placement of sources using multi-level randomized method.

Figure 2: Overview of Precomputed Acoustic Transfer (PAT)
Precomputed Acoustic Transfer [James et al. 2006]

How many dipoles to fit?

Evaluation cost ≈ 0.3 µsec/dipole (in 2006)

More coefficients for higher frequency modes.
Representing Acoustic Transfer

FFAT Maps
[Chadwick et al. 2009]

Goal:

• O(1) transfer evaluation cost per mode.
• O(M) transfer evaluation cost per object (M modes)
• Avoid increasing cost for high-frequency modes

Approach:

• Exploit radial structure & speed-memory trade-off
• Texture map out-going radiation
Representing Acoustic Transfer

Exploiting radial structure
Representing Acoustic Transfer

Far-Field Acoustic Transfer (FFAT)

• Consider an M-term asymptotic expansion

\[ p(x) \sim h_0(kR) \left\{ \Psi_1(\theta, \phi) + \frac{\Psi_2(\theta, \phi)}{kR} + \cdots + \frac{\Psi_M(\theta, \phi)}{(kR)^{M-1}} \right\} \]
Representing Acoustic Transfer

Far-Field Acoustic Transfer (FFAT)

• Consider an M-term asymptotic expansion

\[ p(x) \sim \frac{ie^{-ikR}}{kR} + \frac{kR}{(kR)^{M-1}} \]

**FFAT Maps** [Chadwick et al. 2009]

• Low-error transfer, e.g., M=4
• O(1) transfer evaluation cost
• \(~75 \mu\text{sec} / 100 \text{ modes}\)
Representing Acoustic Transfer

Precomputing FFAT Maps

Precompute pressure samples on concentric spherical shells using fast multipole BEM
[Greengard and Rokhlin 1987; Gumerov and Duraiswami 2005]
- FastBEM implementation [Liu 2009]

Estimate FFAT Maps using least squares

\[ \sum_{j=1}^{M} \frac{h_0(kR_i)}{(kR_i)^{j-1}} \Psi_j(\Theta_l) = p(R_i, \Theta_l) \quad \iff \quad \sum_{j=1}^{M} A_{ij} \Psi_{jl} = p_{il} \]
Representing Acoustic Transfer

FFAT Maps (Trash Can)

Mode 140

\[ |\Psi_1(\theta, \phi)| \quad |\Psi_2(\theta, \phi)| \]
Representing Acoustic Transfer

FFAT Maps (Trash Can)

Mode 160

$|\Psi_1(\theta, \phi)|$

$|\Psi_2(\theta, \phi)|$
Representing Acoustic Transfer

FFAT Map Accuracy

Trash Can (M=4)

Mode 199 (44333 Hz)
# Representing Acoustic Transfer

## FFAT Map Accuracy

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Comparison of Acoustic Transfer Methods

“Ground Truth”
Fast multipole method
(FastBEM)
Evaluation time: 13.2 hours

Far-Field Acoustic Transfer Map
FFAT Map Approximation
(M=4 textures/mode)
Evaluation time: <1 second
Frequency-domain Wave Radiation

Concept: Radiation Efficiency
(see [Cremer et al. 1990])

Ratio comparing radiated acoustic power $\Pi_{rad}$ to surface vibrational power

$$\sigma = \frac{\Pi_{rad}}{\rho c S \langle v_{n}^{2} \rangle_{S}} = \frac{\Pi_{rad}}{\rho c S \omega^{2} \langle \bar{u}_{n}^{2} \rangle_{S}}$$

Strongly depends on shape, mode & frequency.
Frequency-domain Wave Radiation

Concept: Radiation Efficiency

(see [Cremer et al. 1990])