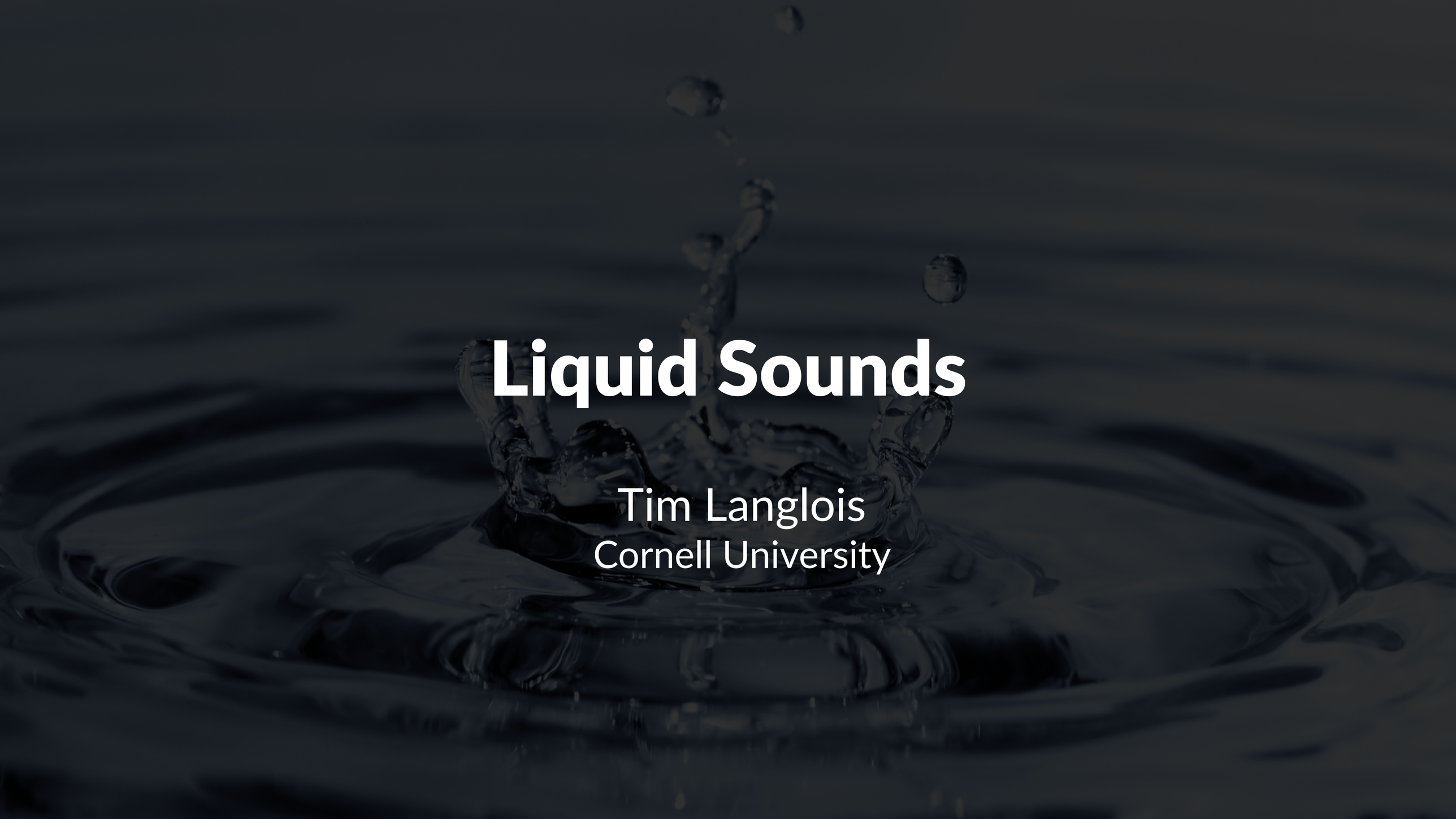
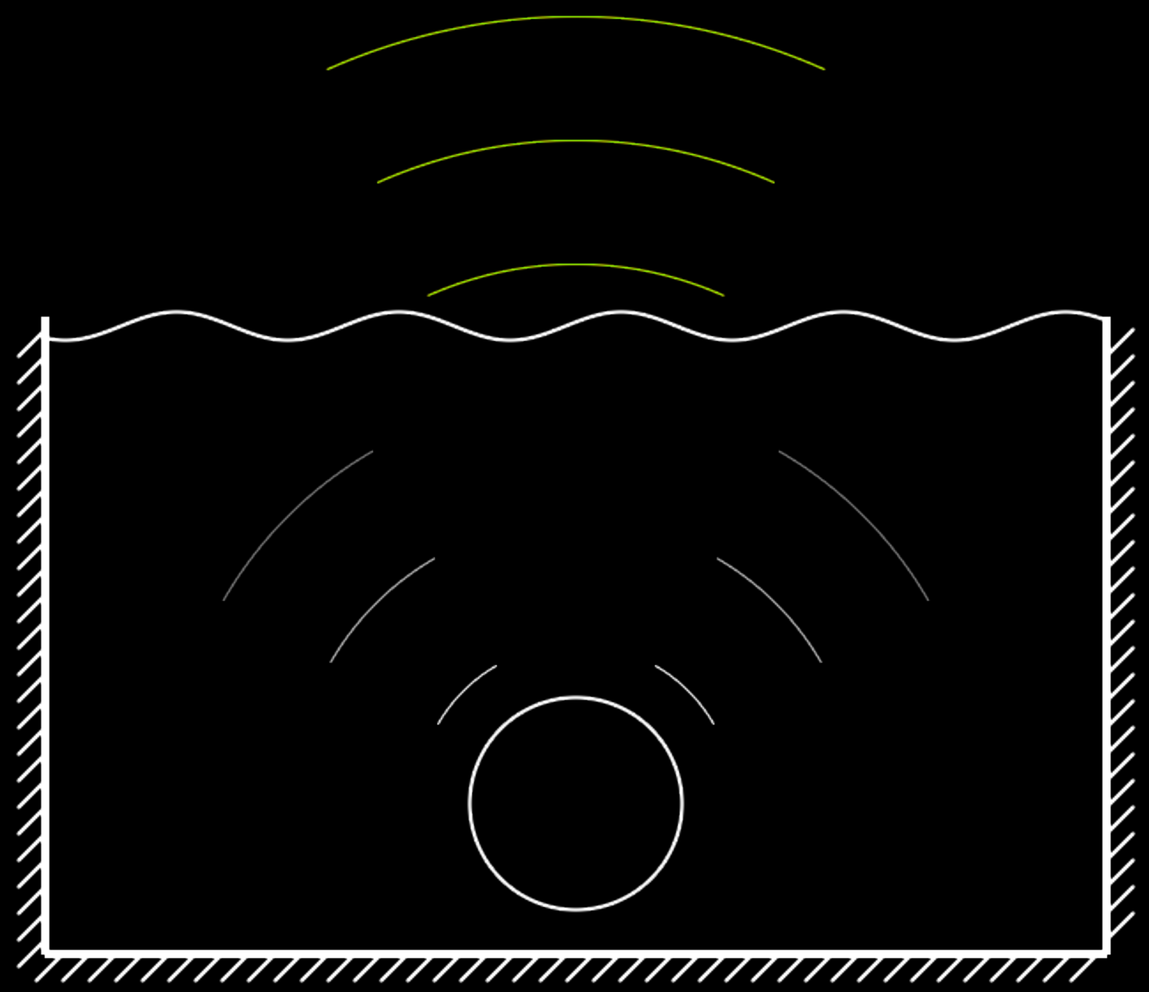


Liquid Sounds

Tim Langlois
Cornell University

Overview



How does water make sound?

- Mainly from bubbles
- When bubbles form, they are forced into volume oscillations
- Behave like a damped oscillator
  - Frequency depends on size, shape, and position

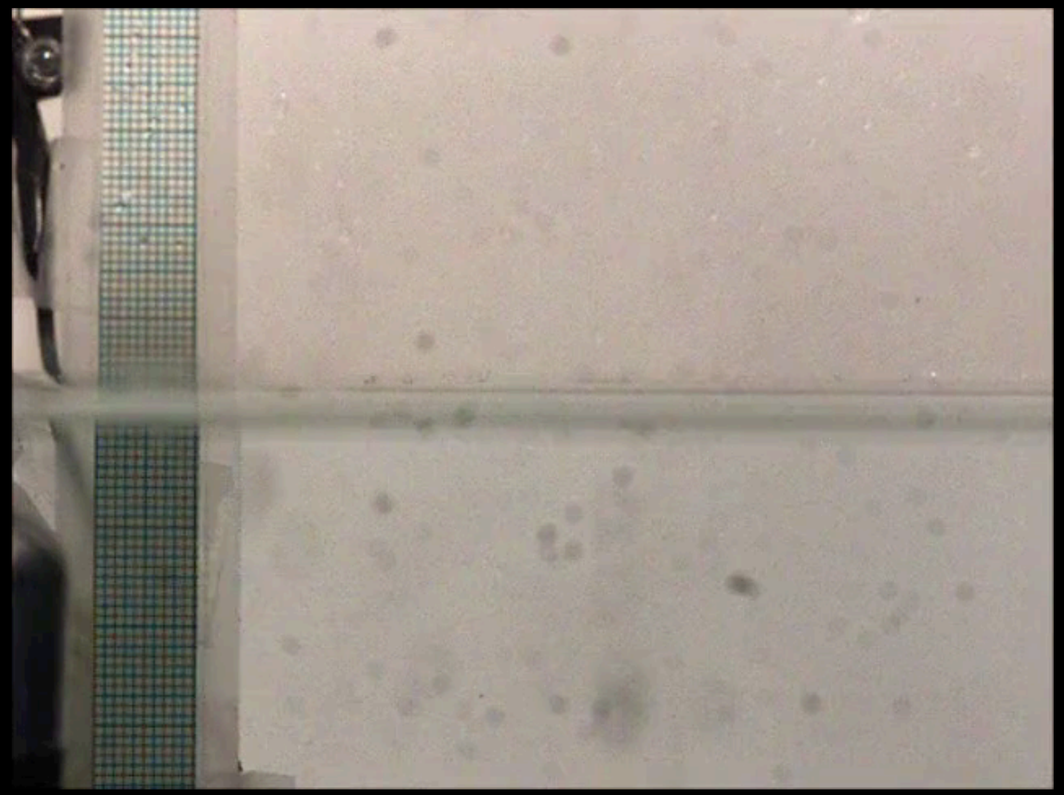
5.0 mm (1.3 kHz) ⟶

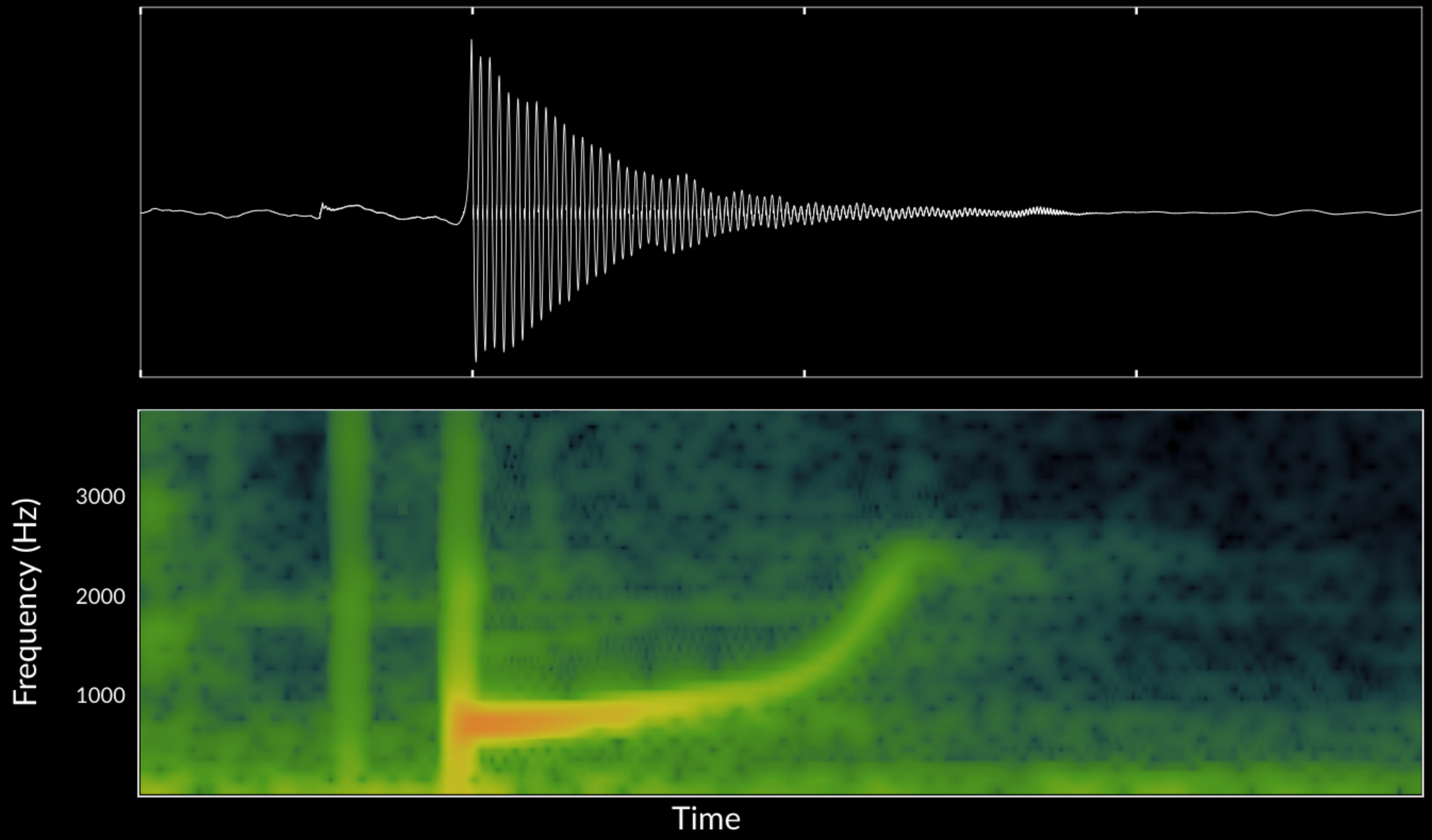
2.0 mm (3.3 kHz) ⟶

1.0 mm (6.6 kHz) ⟶

0.5 mm (13. kHz) ⟶

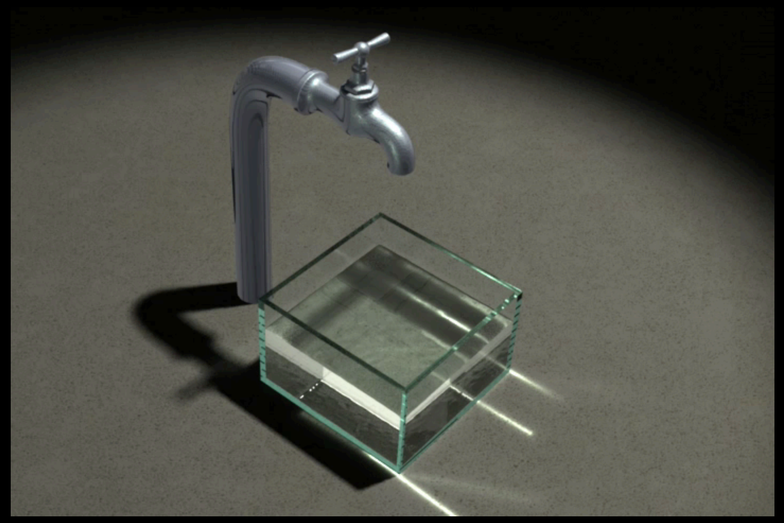
[Zheng and James 2009]



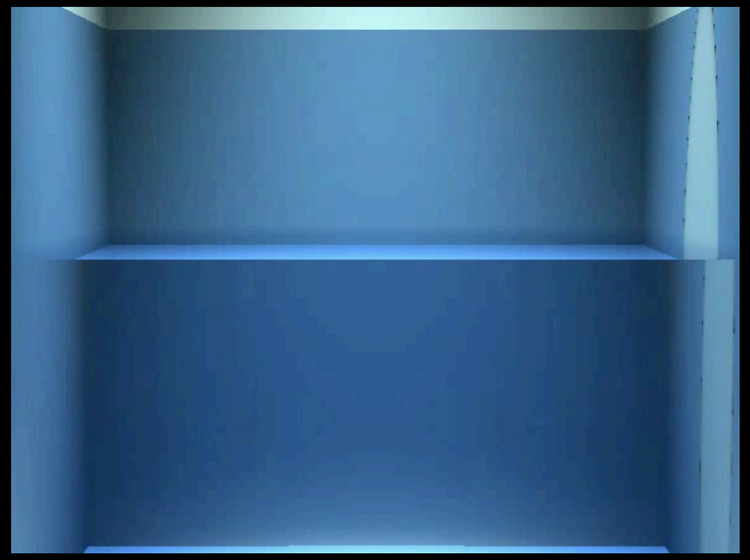

Frequency (Hz)
3000
2000
1000
Time

Fluid Sound Simulation

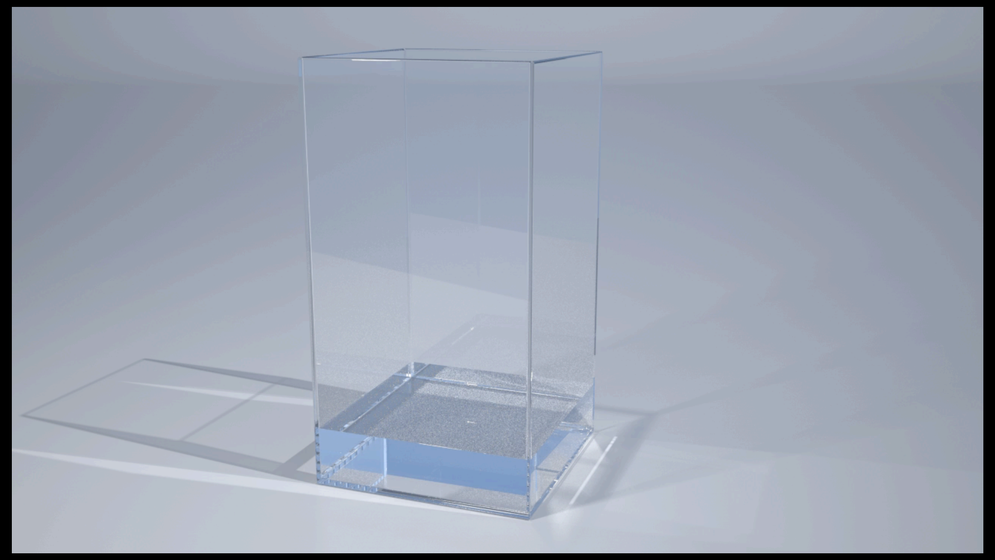
Harmonic Fluids
[Zheng & James 2009]



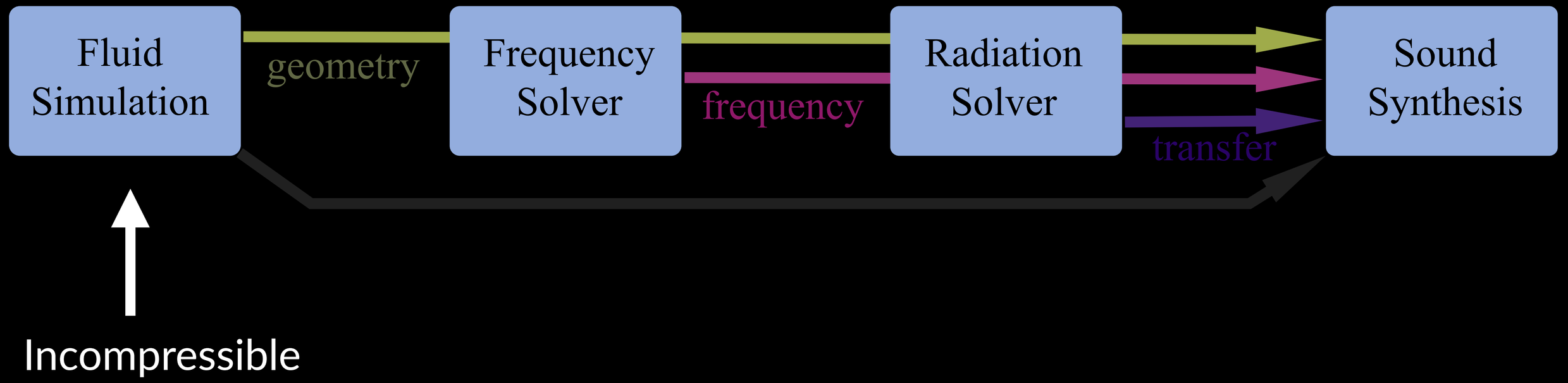
Sounding Liquids
[Moss et al. 2010]



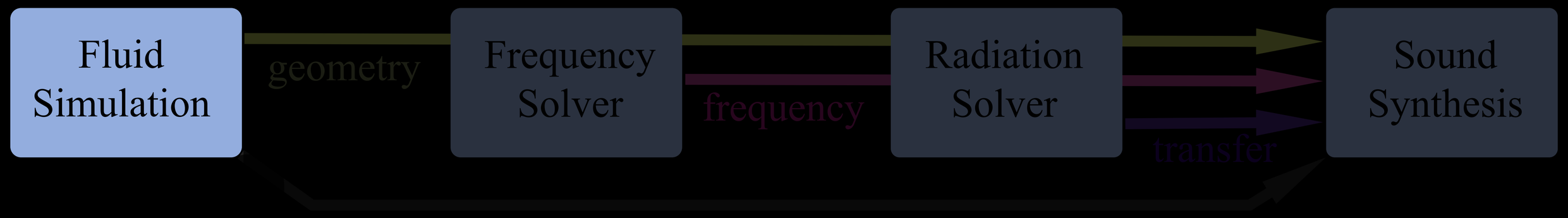
Toward Animating Water
with Complex Acoustic Bubbles
[Langlois et al. 2016]



Overview


Fluid Simulation
geometry
Frequency Solver
frequency
Radiation Solver
transfer
Sound Synthesis
Incompressible

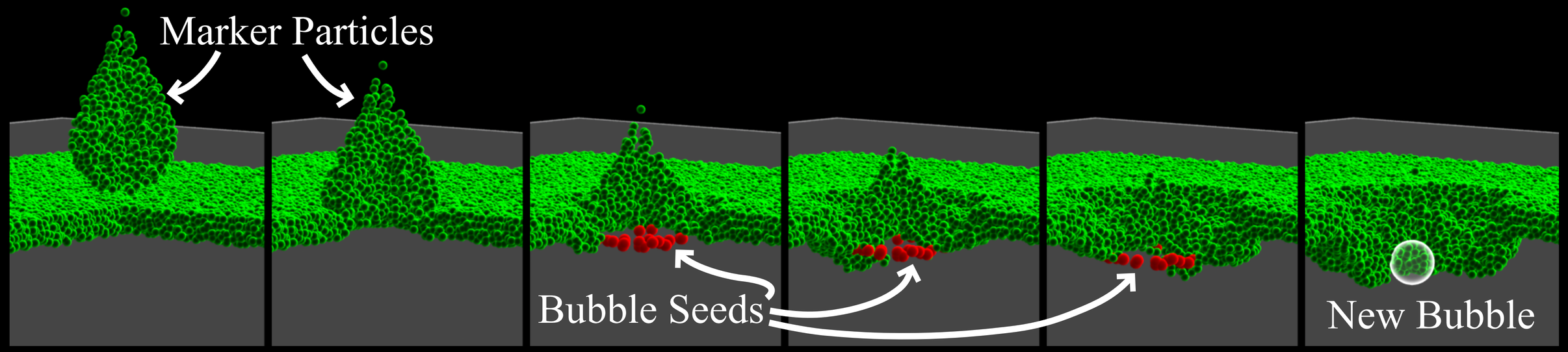
Overview


Fluid Simulation
geometry
Frequency Solver
frequency
Radiation Solver
transfer
Sound Synthesis

Fluid Simulation Choices

Particles

- FLIP/PIC [Harmonic Fluids]
  - Bubbles are particles - easy to track
  - Stochastic entrainment model, requires hand tuning
  - No splitting/merging of bubbles



[Zheng and James 2009]

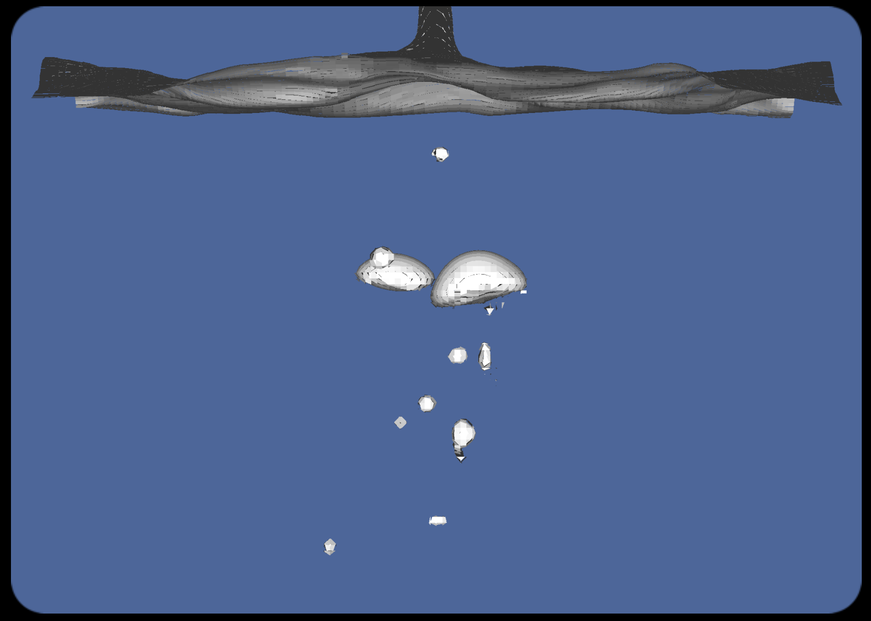
Fluid Simulation Choices

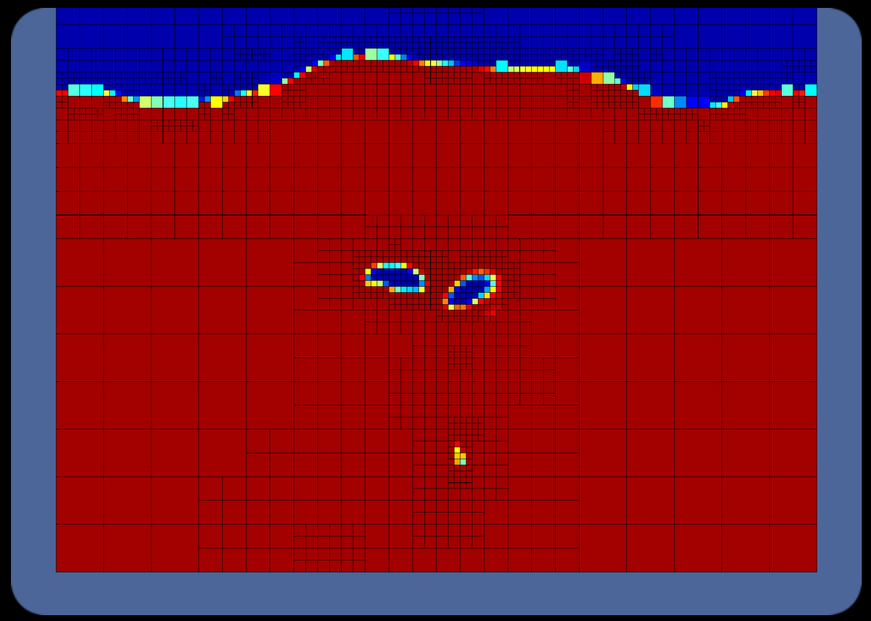
(Two phase) Eulerian Simulation

- Define a Volume-of-Fluid field
  - 0 in air, 1 in water
- Other variables are interpolated based on this field

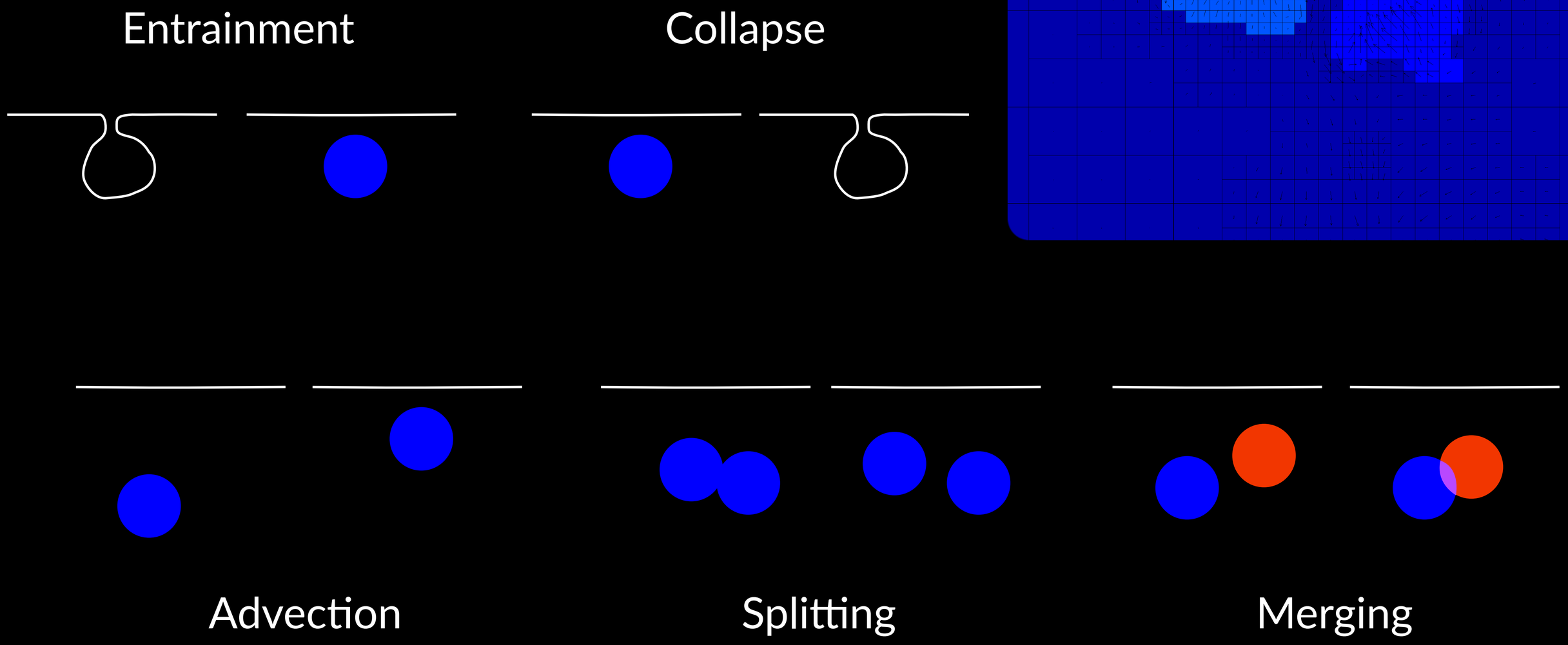
$$\rho = c\,\rho_{water} + (1 - c)\,\rho_{air}$$

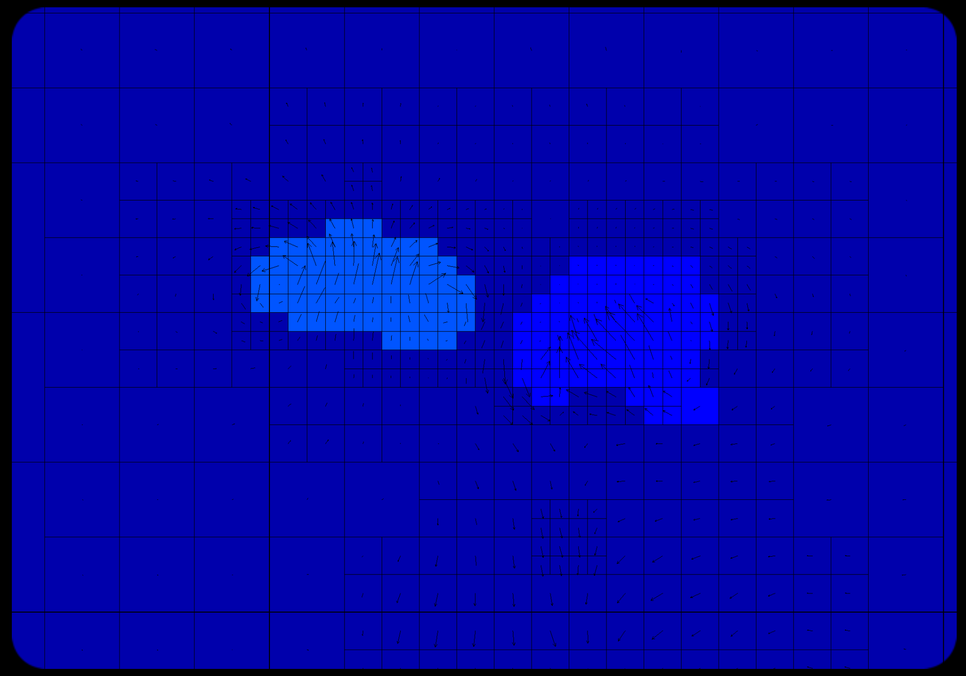
- Entrainment handled accurately
- Tracking is more difficult
- Sounding Liquids - two options
  - Level sets with particles for tiny bubbles
  - Shallow water equations



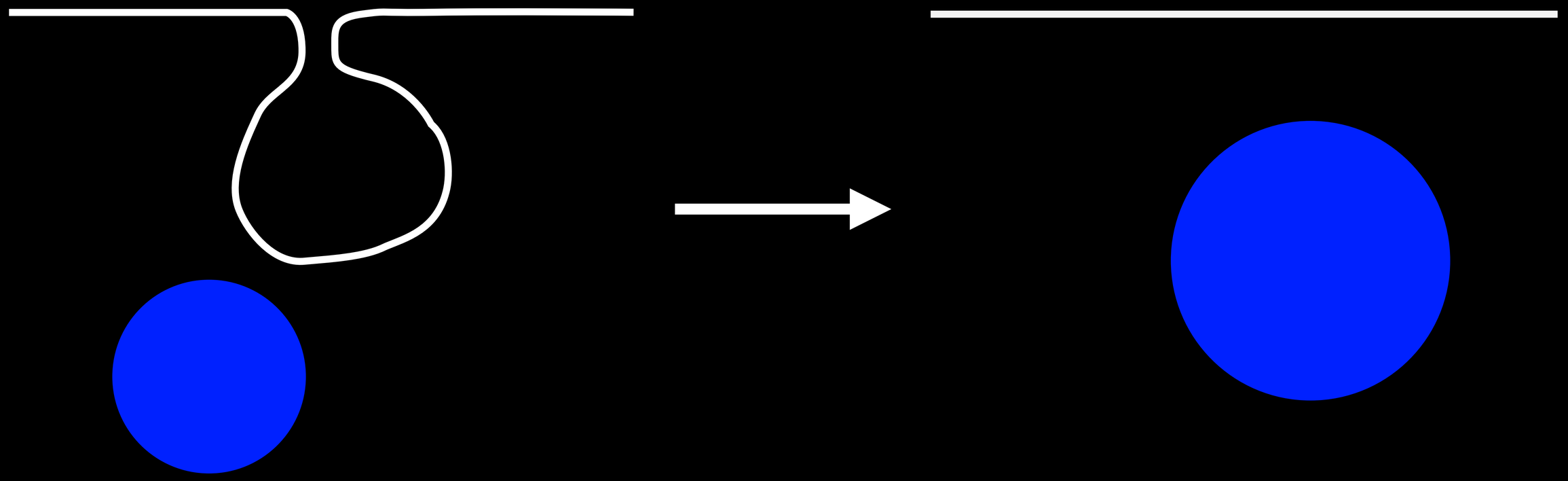


Bubble Tracking



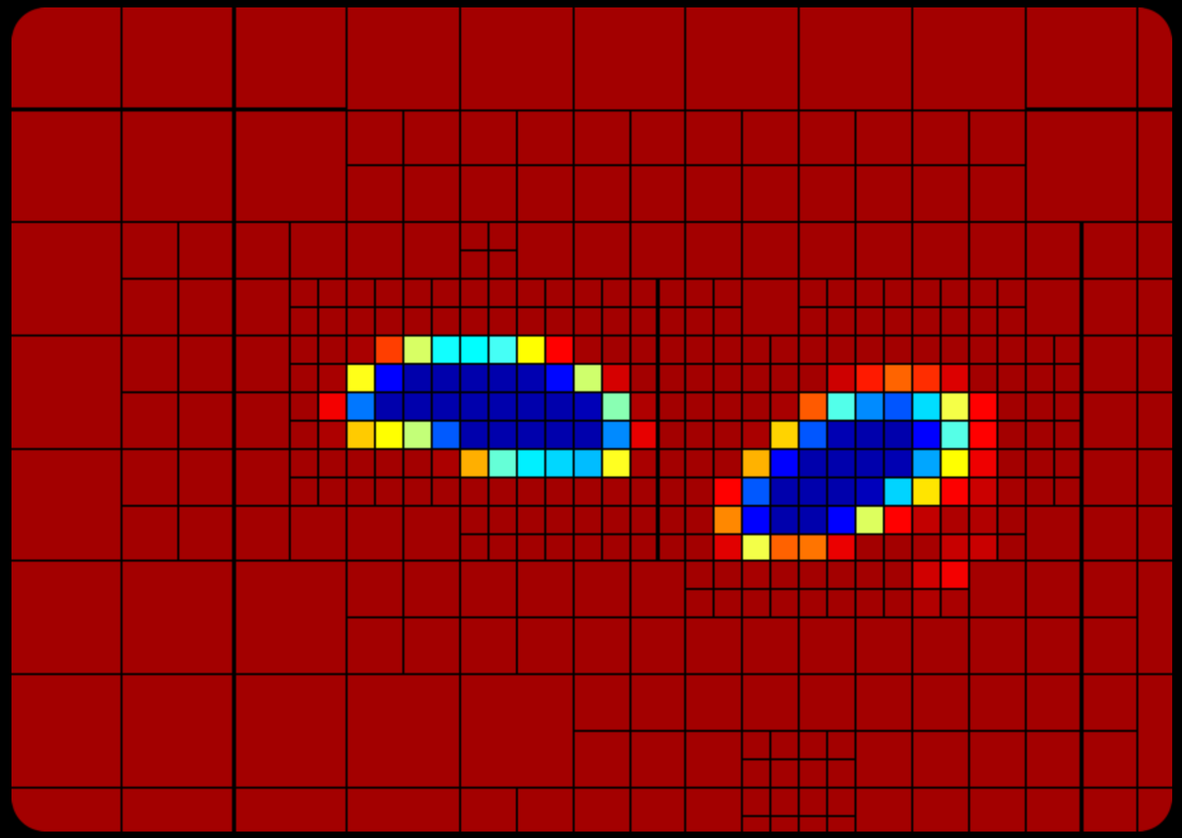


Implementation: Simultaneous events



Implementation: Resolution limits

- Resolving all bubble sizes can be challenging
- HF - not an issue
- SL - add bubble particles that are coupled with the flow for tiny bubbles
- CAB - add tiny bubbles in the audio domain



Overview


Fluid Simulation
geometry
Frequency Solver
frequency
Radiation Solver
transfer
Sound Synthesis

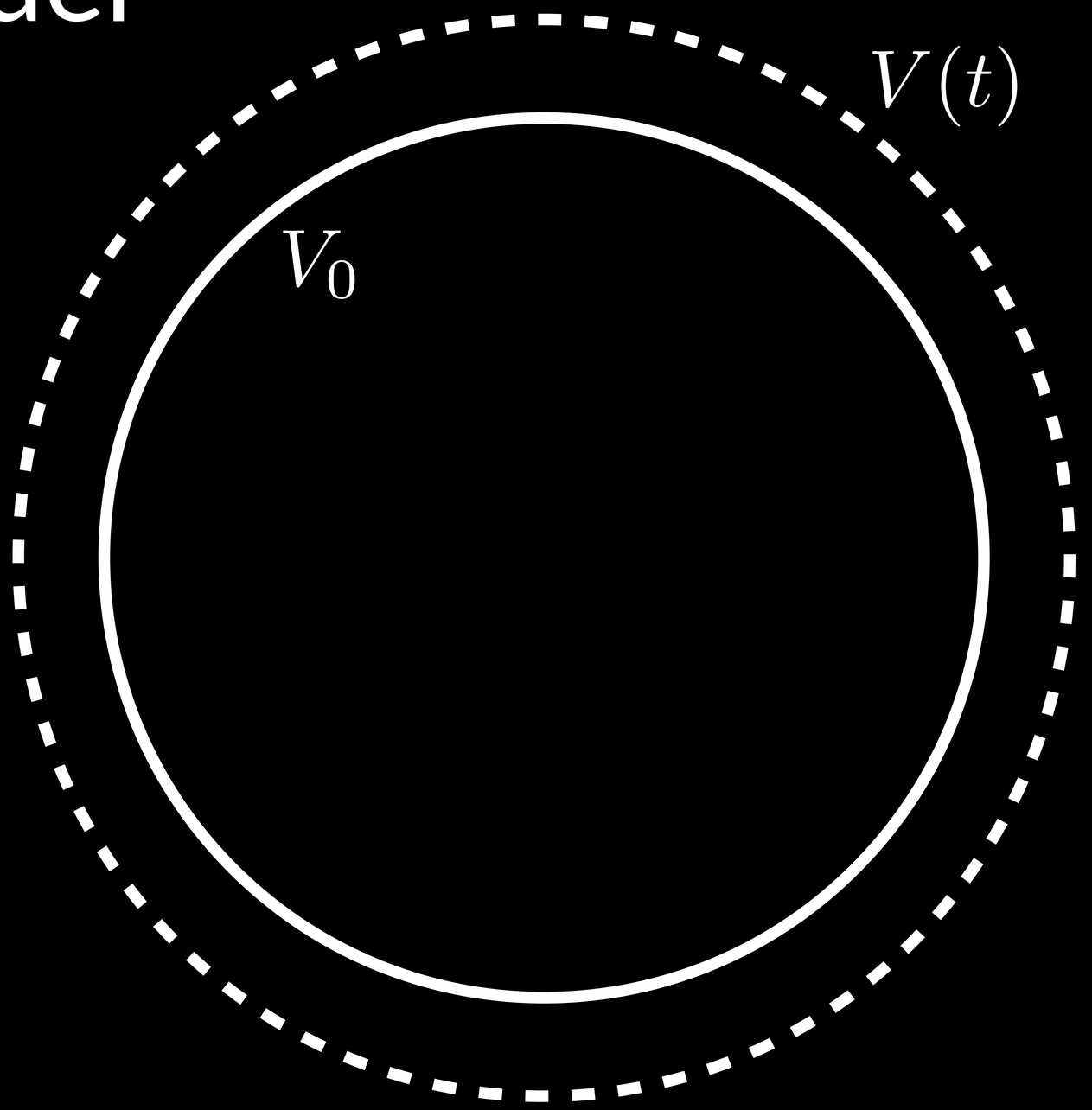
Bubble Vibration Model

- Analogy: Spring
  - Bubble is spring force
  - Liquid is mass

$$v = V(t) - V_0$$

$$\ddot{v} + 2\beta\dot{v} + \omega^2 v = \frac{p(t)}{m}$$

$$\omega^2 = \frac{\kappa}{m}$$



Frequency Modeling Choices

- Minnaert Frequency - HF, SL
  - Developed almost a century ago
  - Isolated, spherical bubbles

$$\omega_m = \sqrt{\frac{3\gamma p_0}{r_0^2 \rho} - \frac{2\sigma}{r_0^3 \rho}}$$

- Pitch rise
  - HF - ad-hoc formula based on distance to fluid surface
  - SL - ad-hoc formula based on age of bubble
- CAB - solve for frequency at each time step (computationally expensive)

Effective Mass [Strasberg 1953]

Kinetic energy: $$E = \frac{1}{2} m \dot{v}^2 \implies m = \frac{2E}{\dot{v}^2}$$

Bubbles are acoustically compact sources

3.3 kHz
1482 m/s in water
449 mm wavelength

Effective Mass [Strasberg 1953]

Kinetic energy: $$E = \frac{1}{2} m \dot{v}^2 \implies m = \frac{2E}{\dot{v}^2}$$

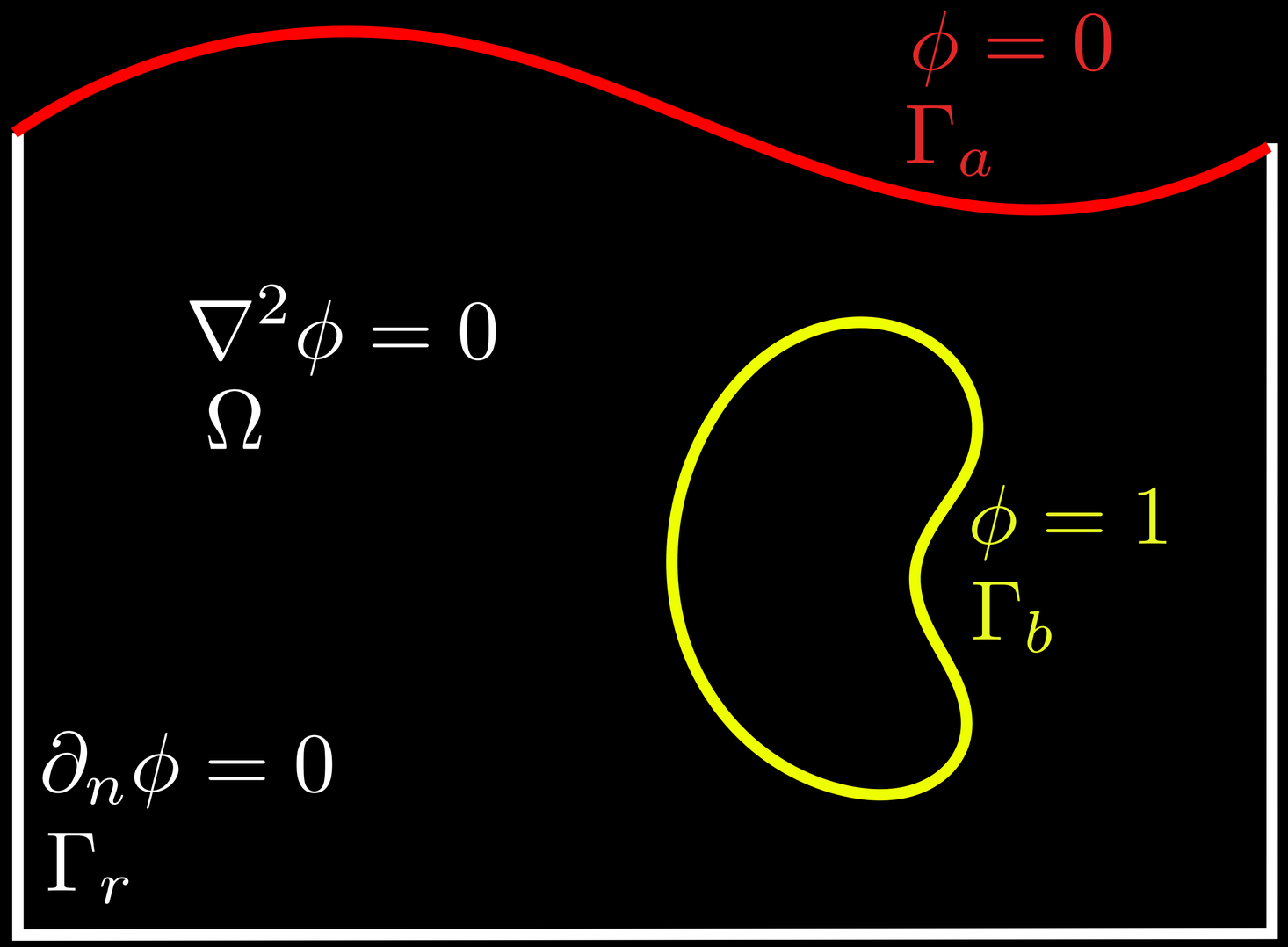
Bubbles are acoustically compact sources

Acoustic particle velocity $= \nabla \phi \qquad \nabla^2 \phi = 0$

$$E = \frac{\rho}{2} \int_\Omega (\nabla \phi)^2 \, d\Omega$$

$$\int_\Omega (\nabla \phi)^2 \, d\Omega = \boxed{-\int_{\partial\Omega} \phi \, \partial_n \phi \, dS} - \cancelto{0}{\int_\Omega \phi \nabla^2 \phi \, d\Omega}$$

Bubble BVP


$\phi = 0$
Γ_a
$\nabla^2 \phi = 0$
Ω
$\phi = 1$
Γ_b
$\partial_n \phi = 0$
Γ_r

Effective Mass

$$m = -\frac{\rho}{\dot{v}^2} \int_{\Gamma_b} \phi \, \partial_n \phi \, dS$$

Bubble Capacitance Analogy

- Effective bubble capacitance C:

$$m = \frac{\rho\phi_b}{\dot{v}} = \frac{\rho}{4\pi C}$$

- Equivalent to electrostatic capacitance
  - ϕ is electrostatic potential
  - $\dot{v}$ is flux

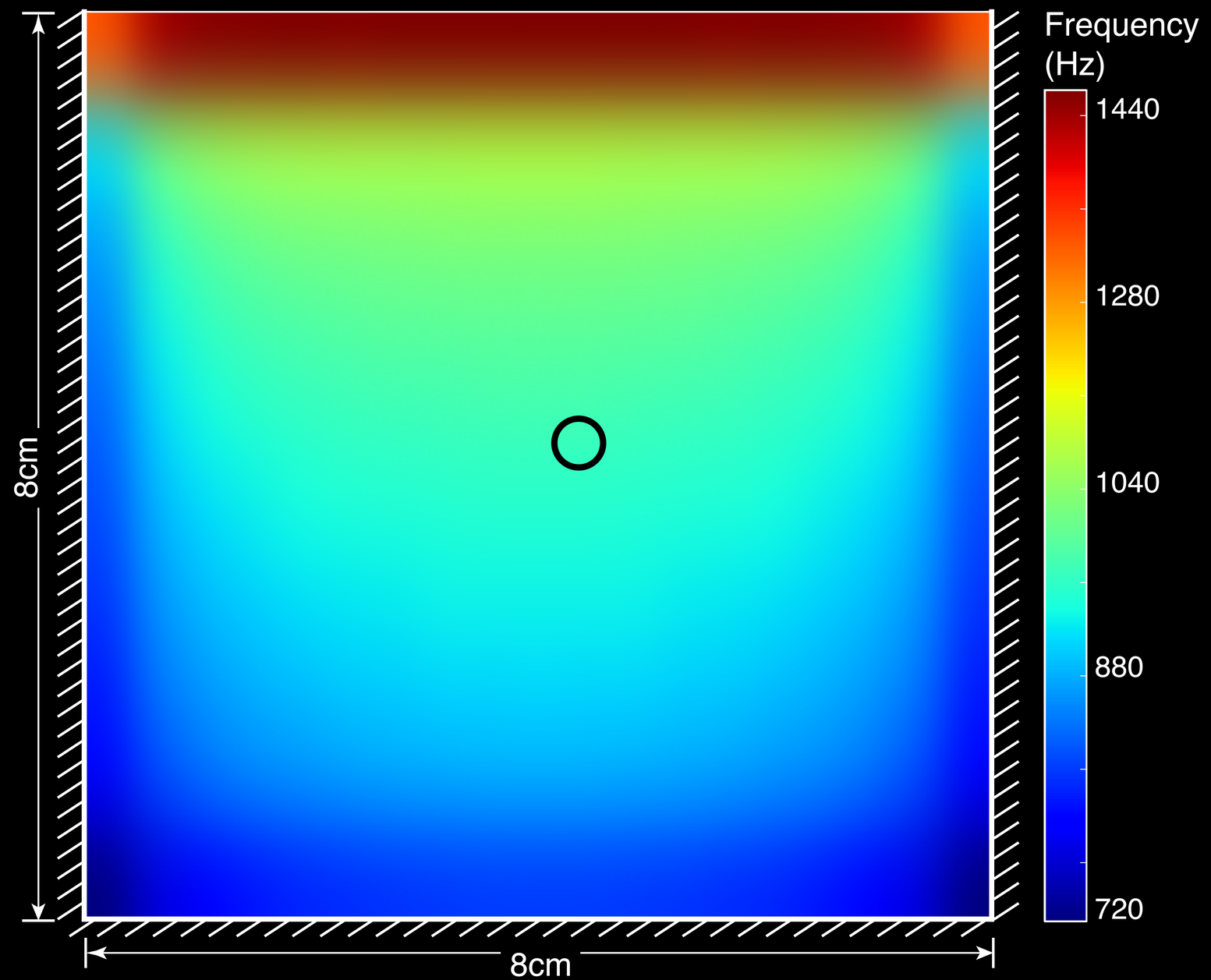
$$4\pi C = \frac{\dot{v}}{\phi_b}$$

- Generalized frequency model:

$$\boxed{\omega^2 = \frac{\kappa}{m} = \frac{4\pi\gamma P_0}{\rho V_0} C}$$

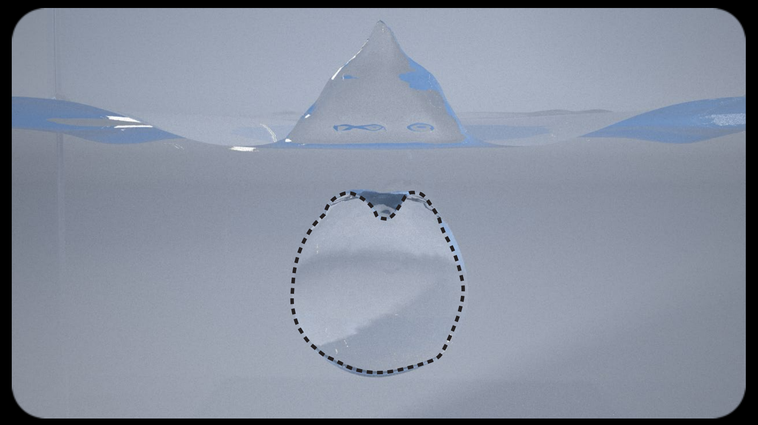
- Reproduces classical Minnaert frequency for spherical bubbles

Frequency vs Bubble Position

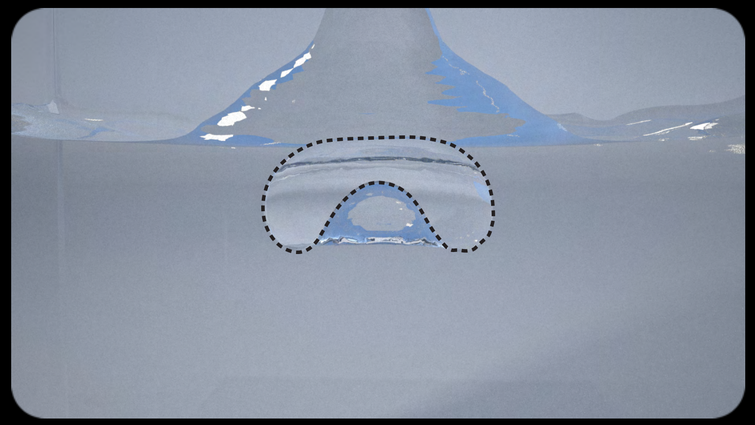


Frequency Rise

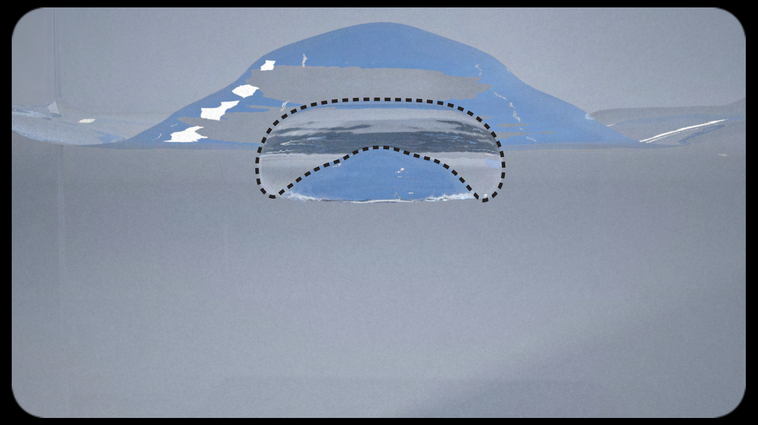
Radius: 5.8mm
Minneart Frequency: 567 Hz
Cross-section view



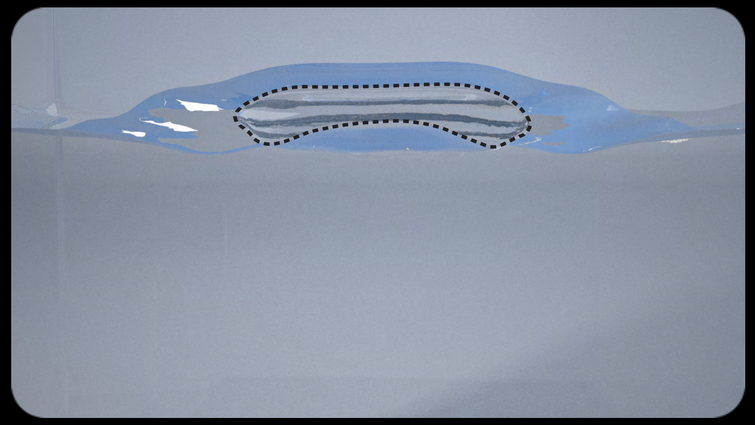
$f = 651\ (1)$



$f = 745\ (1.14)$

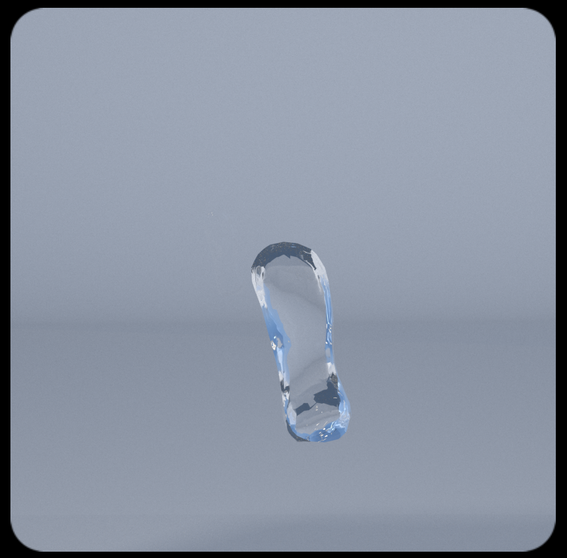


$f = 825\ (1.27)$



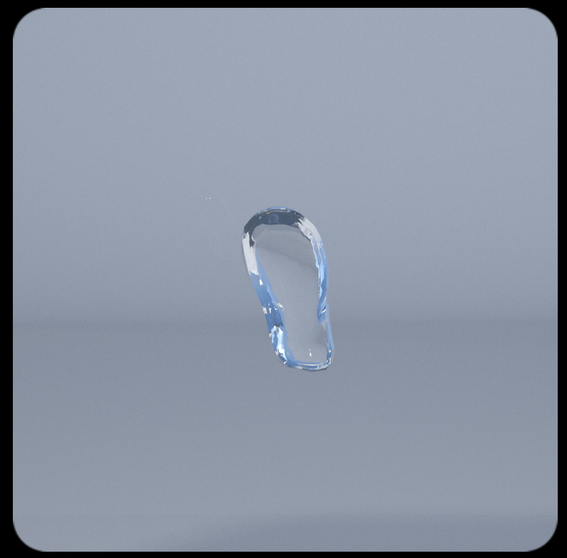
$f = 1117\ (1.72)$

Shape Dependence



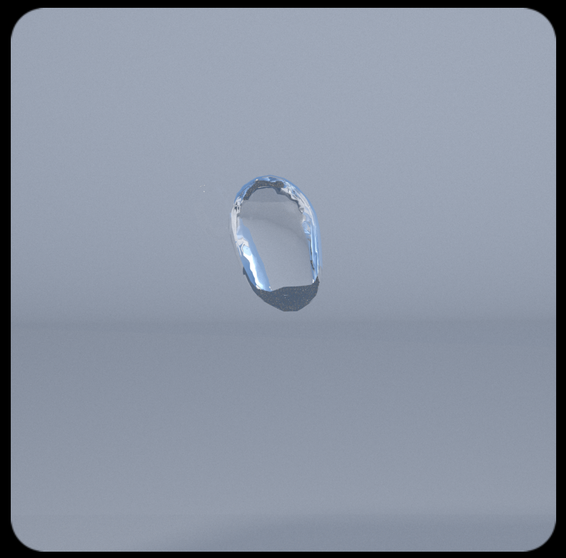
$f = 1209\ (1)$

$t = 0\ (\mathrm{ms})$



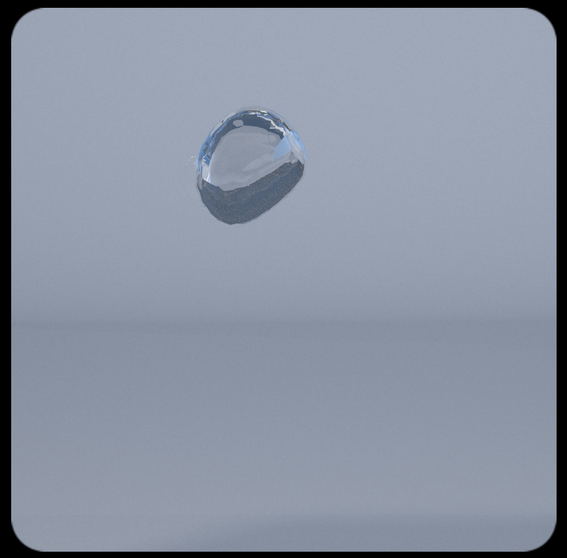
$f = 1158\ (0.96)$

$t = 3$



$f = 1103\ (0.91)$

$t = 6$

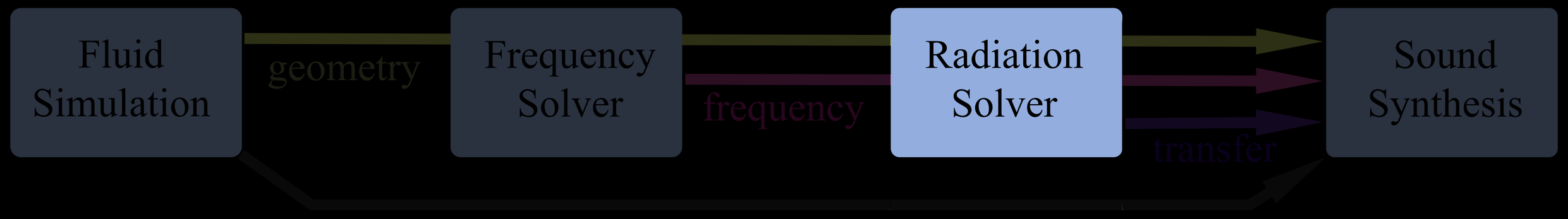


$f = 1034\ (0.85)$

$t = 13$

Radius: 2.4mm Minneart Frequency: 1347 Hz

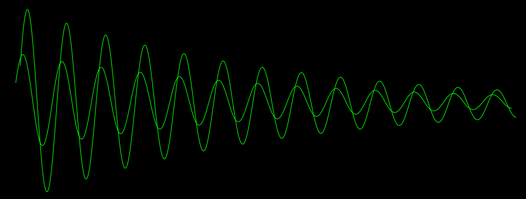
Overview


Fluid Simulation
geometry
Frequency Solver
frequency
Radiation Solver
transfer
Sound Synthesis

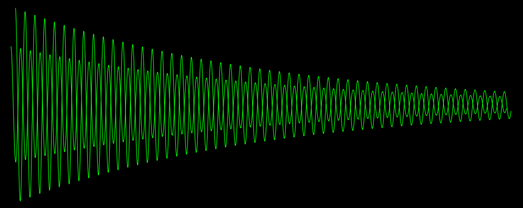
Transfer Function

=

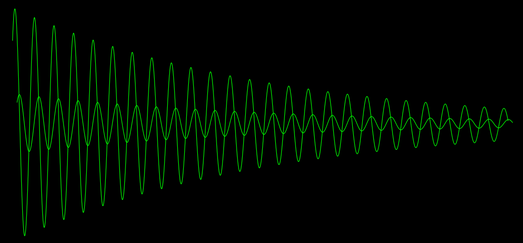
Bubble 1



Bubble 2



Bubble 3



$$p_1 q_1(t) \; + \; p_2 q_2(t) \; + \; p_3 q_3(t) \; + \; \cdots$$

Final Sound:

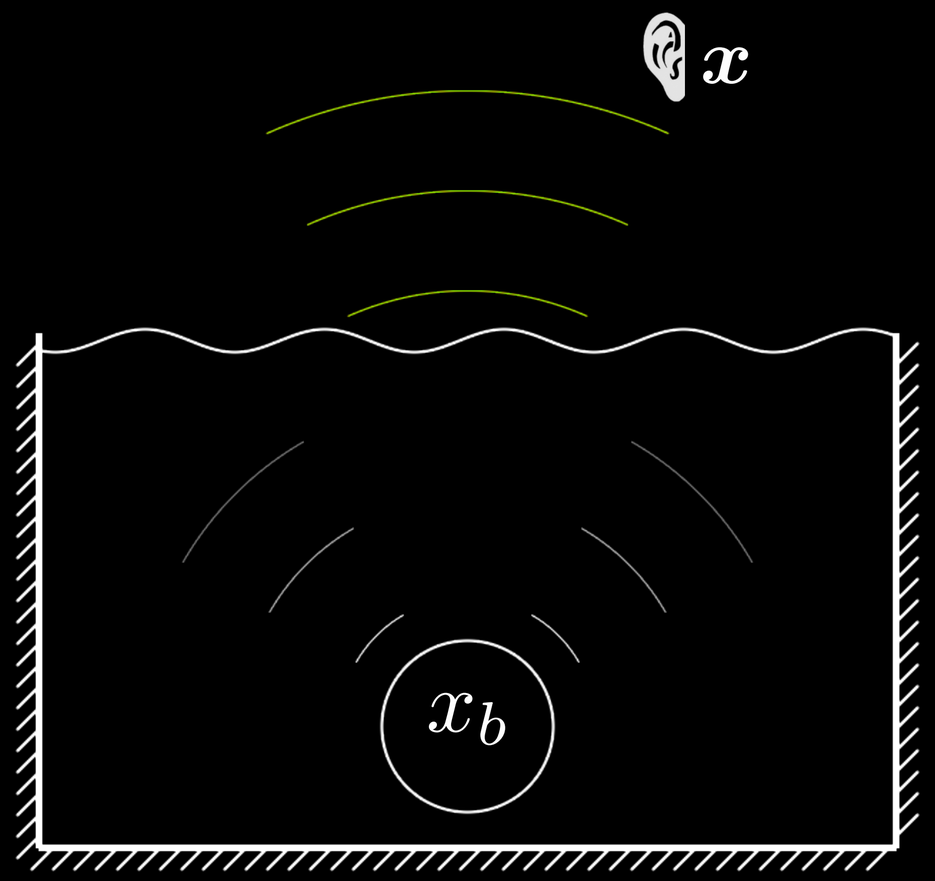
$$\sum_{\substack{\text{for each} \\ \text{bubble}}} p_i q_i(t)$$

Transfer Function

Helmholtz Problems

$$(\nabla^2 + k^2(\boldsymbol{x}))P(\boldsymbol{x};\boldsymbol{x}_b) = S_b\delta(\boldsymbol{x} - \boldsymbol{x}_b)$$

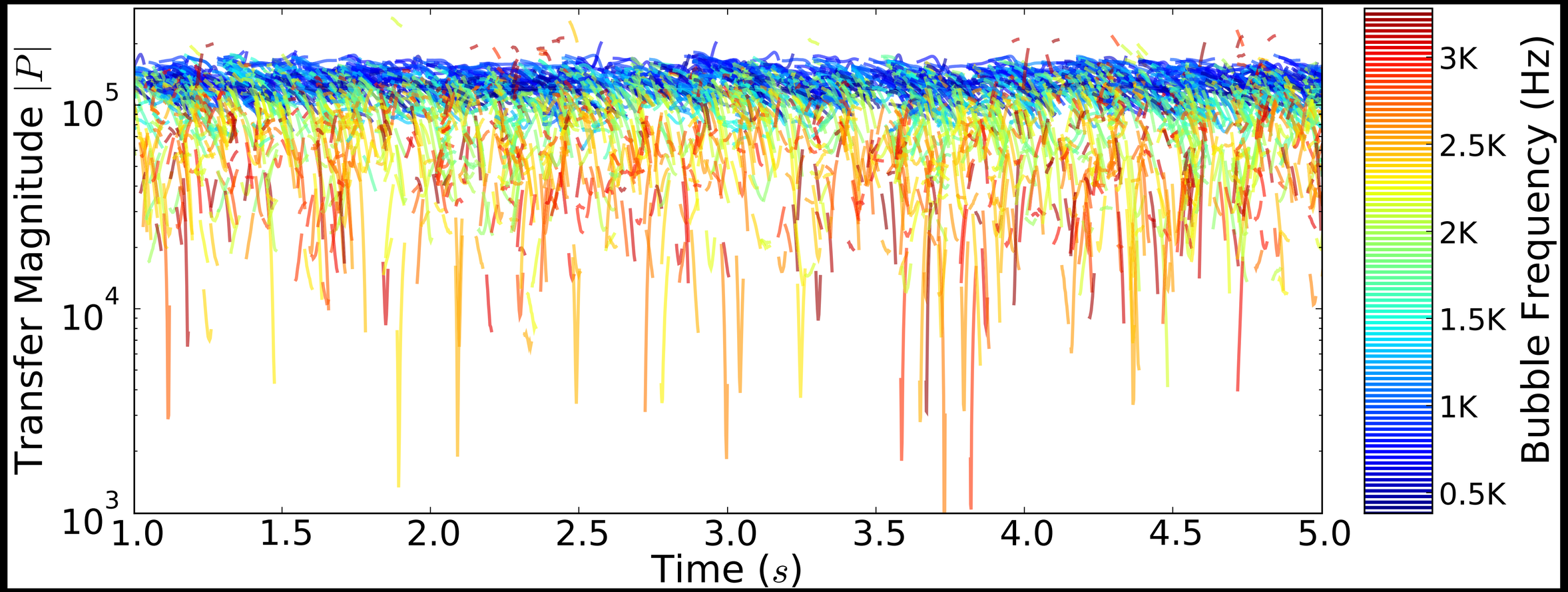
wave number transfer function



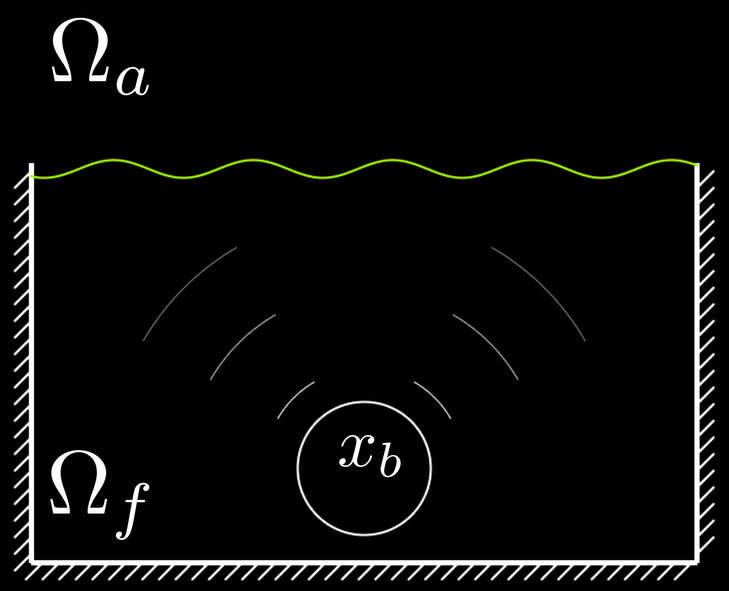
Acoustic Radiation Importance

- Frequency dependent
- Spatially and temporally varying

[Zheng & James 2009]

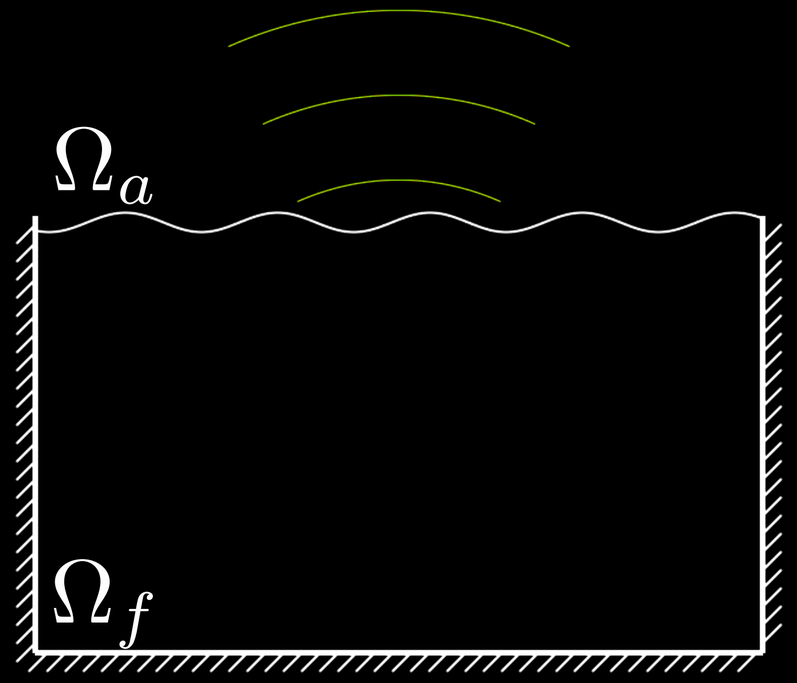


Harmonic Fluids: Dual-domain approximation



Estimate surface vibration

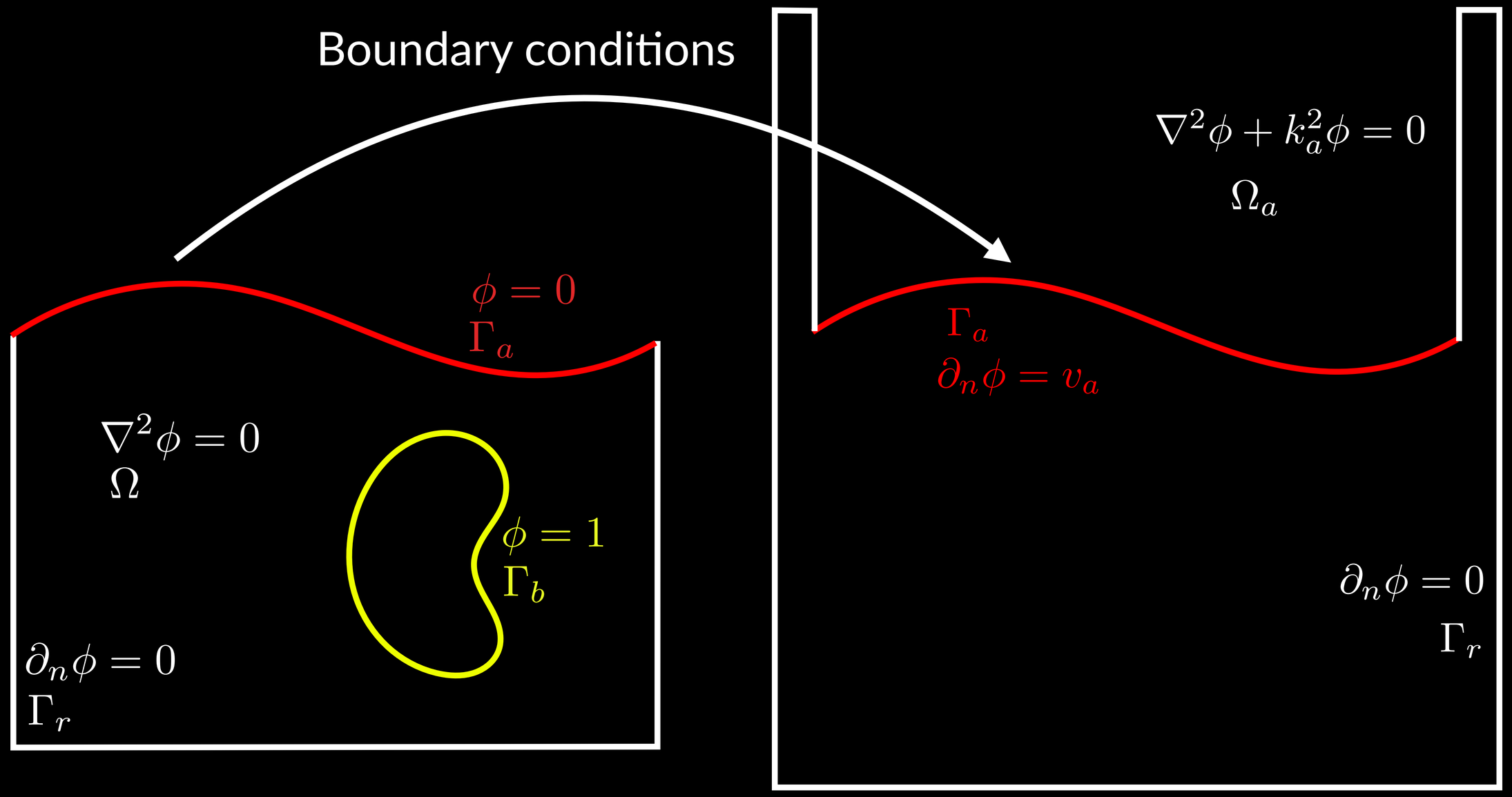
$$(\nabla^2 + k_f^2)P_f = S_b\delta(\boldsymbol{x} - \boldsymbol{x}_b),\ \boldsymbol{x} \in \Omega_f$$



Estimate sound radiation

$$(\nabla^2 + k_a^2)P_a = 0,\ \boldsymbol{x} \in \Omega_a$$

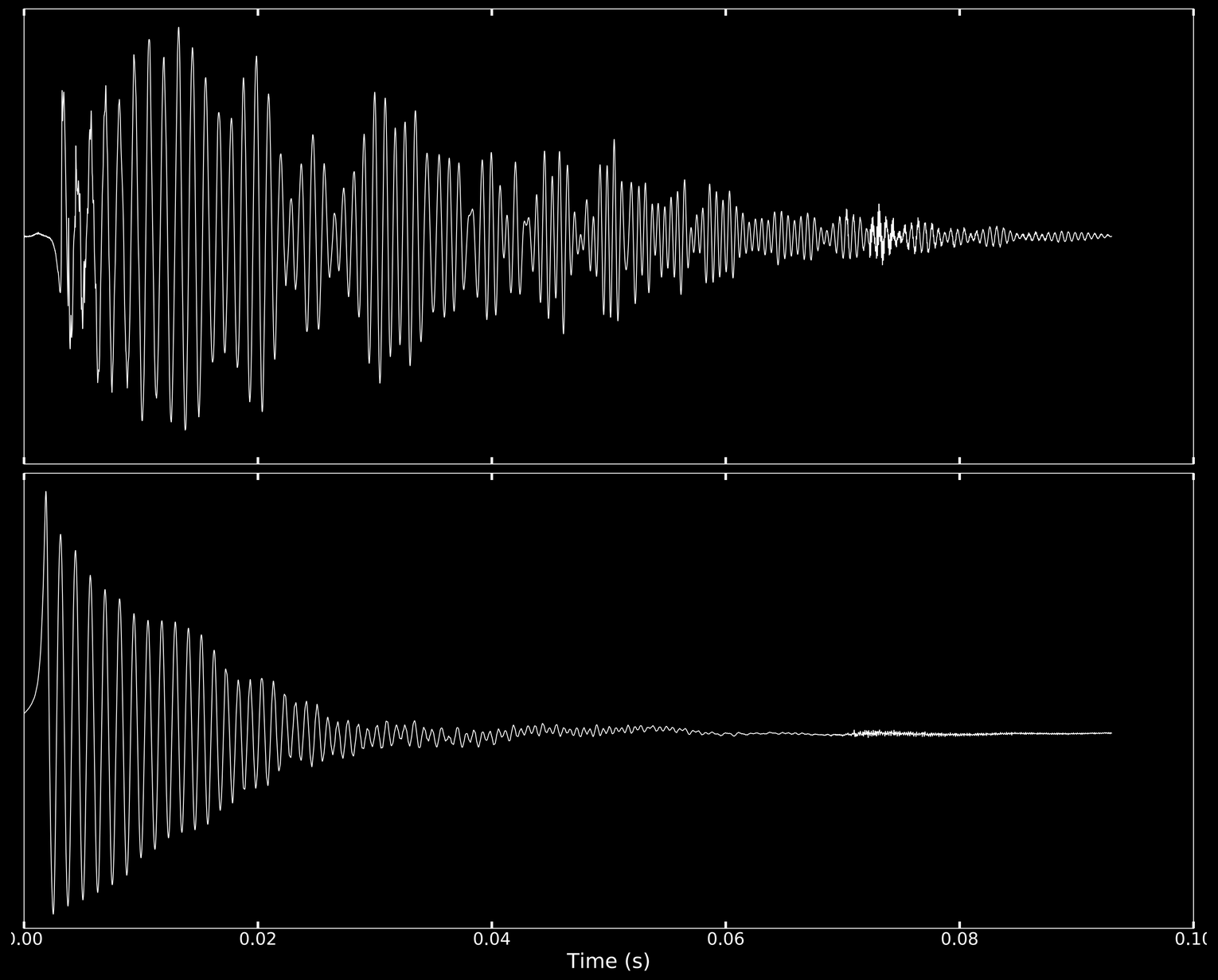
Complex Acoustic Bubbles



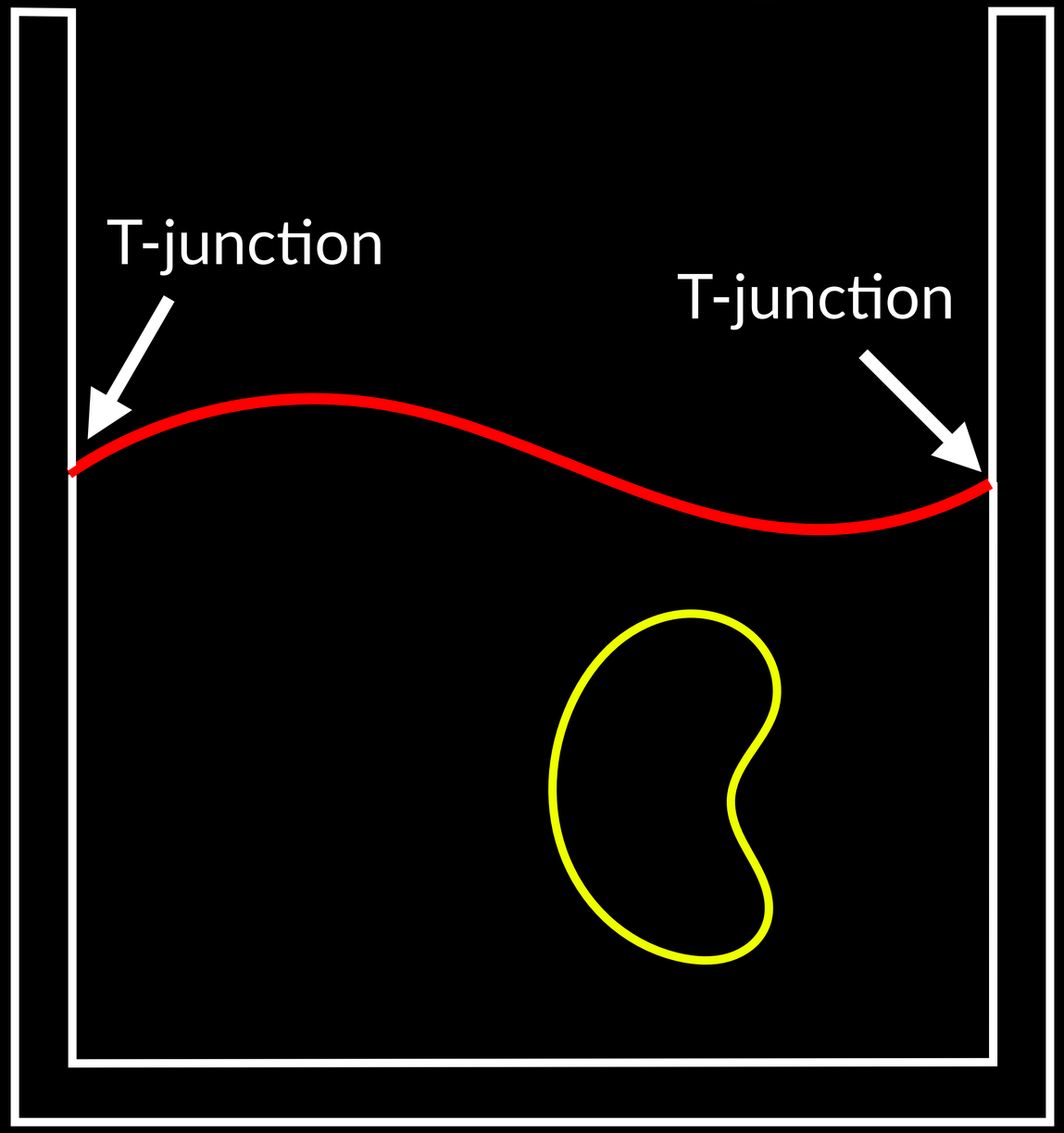
Time-Domain Effects

Air microphone

Hydrophone

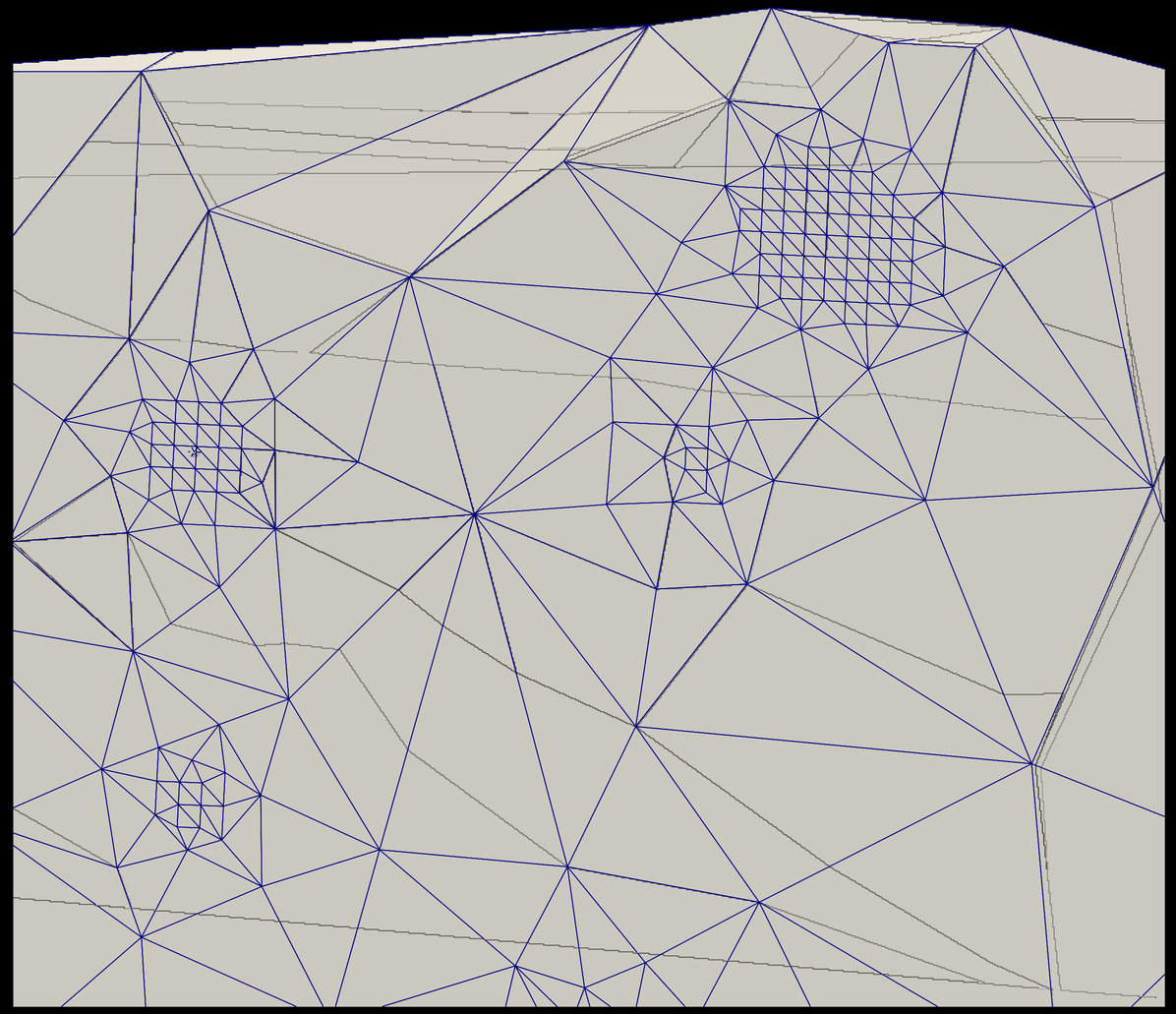


Implementation - Meshing

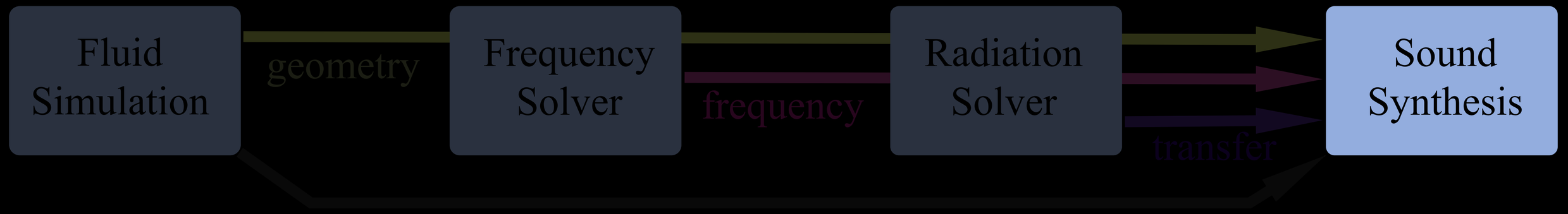


Implementation - Meshing

- Too many triangles == slow
- Adaptive simplification
  - Resolve bubbles near interfaces
- Watch out for bad triangles
- Resolution constraints
  - Need to resolve waves in Helmholtz solves



Overview


Fluid Simulation
geometry
Frequency Solver
frequency
Radiation Solver
transfer
Sound Synthesis

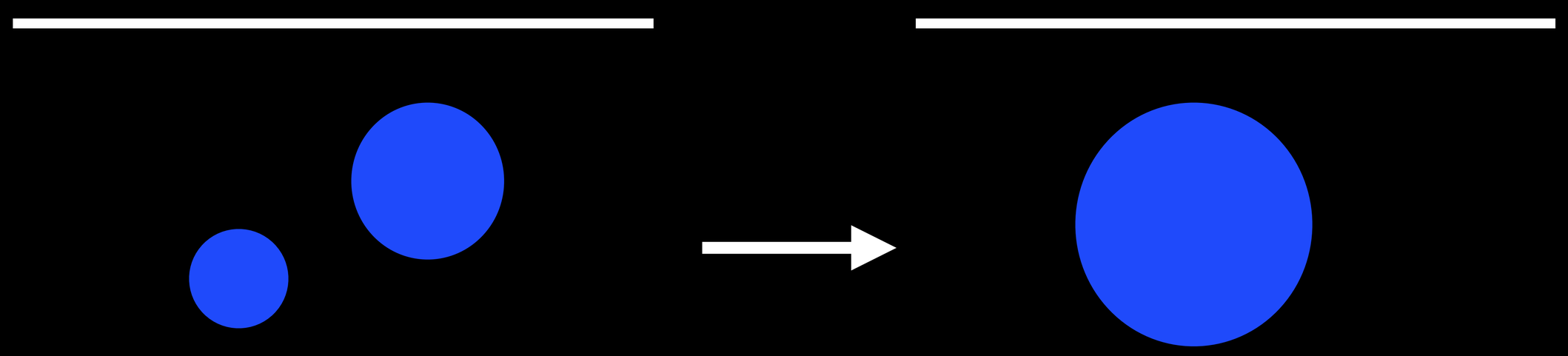
Oscillator integration

$$\ddot{v} + 2\beta\dot{v} + \omega^2 v = \frac{p(t)}{m}$$

Time varying

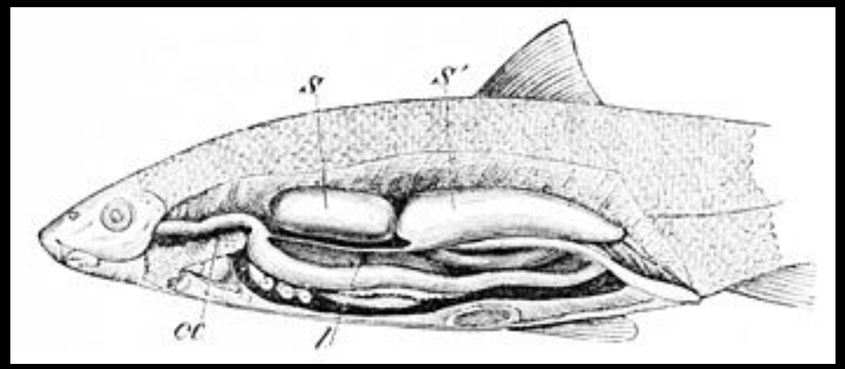
- Numerical integration necessary
- Midpoint method or RK4 work well
- Sum oscillations from all bubbles, possibly weighted with transfer functions

Implementation: Bubble tracking through events



Loosely related but fun fact!

Bubble net hunting



Swim bladder

[en.wikipedia.org/wiki/Swim_bladder]

[en.wikipedia.org/wiki/Humpback_whale]