Overview
How does water make sound?

• Mainly from bubbles
• When bubbles form, they are forced into volume oscillations
• Behave like a damped oscillator
  - Frequency depends on size, shape, and position

  5.0 mm (1.3 kHz) → ○
  2.0 mm (3.3 kHz) → ○
  1.0 mm (6.6 kHz) → ○
  0.5 mm (13. kHz) → ●

[Zheng and James 2009]
Fluid Sound Simulation

Harmonic Fluids
[Zheng & James 2009]

Sounding Liquids
[Moss et al. 2010]

Toward Animating Water with Complex Acoustic Bubbles
[Langlois et al. 2016]
Overview

Fluid Simulation → Frequency Solver → Radiation Solver → Sound Synthesis

- geometry
- frequency
- transfer

Incompressible
Overview

Fluid Simulation ➔ Frequency Solver ➔ Radiation Solver ➔ Sound Synthesis

- geometry
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- transfer
Fluid Simulation Choices

Particles

- FLIP/PIC [Harmonic Fluids]
  - Bubbles are particles - easy to track
  - Stochastic entrainment model, requires hand tuning
  - No splitting/merging of bubbles

[Zheng and James 2009]
Fluid Simulation Choices
(Two phase) Eulerian Simulation

• Define a Volume-of-Fluid field
  - 0 in air, 1 in water

• Other variables are interpolated based on this field

\[ \rho = c \rho_{\text{water}} + (1 - c) \rho_{\text{air}} \]

• Entrainment handled accurately

• Tracking is more difficult

• Sounding Liquids - two options
  - Level sets with particles for tiny bubbles
  - Shallow water equations
Bubble Tracking

- Entrainment
- Collapse
- Advection
- Splitting
- Merging
Implementation: Simultaneous events
Implementation: Resolution limits

- Resolving all bubble sizes can be challenging
- HF - not an issue
- SL - add bubble particles that are coupled with the flow for tiny bubbles
- CAB - add tiny bubbles in the audio domain
Overview

Fluid Simulation → Frequency Solver  
Geometry

Frequency Solver → Radiation Solver  
Frequency

Radiation Solver → Sound Synthesis  
Transfer
Bubble Vibration Model

- Analogy: Spring
  - Bubble is spring force
  - Liquid is mass

\[ v = V(t) - V_0 \]

\[ \ddot{v} + 2\beta \dot{v} + \omega^2 v = \frac{p(t)}{m} \]

\[ \omega^2 = \frac{\kappa}{m} \]
Frequency Modeling Choices

• Minnaert Frequency - HF, SL
  - Developed almost a century ago
  - Isolated, spherical bubbles

• Pitch rise
  - HF - ad-hoc formula based on distance to fluid surface
  - SL - ad-hoc formula based on age of bubble

• CAB - solve for frequency at each time step (computationally expensive)

\[
\omega_m = \sqrt{\frac{3\gamma p_0}{r_0^2 \rho}} - \frac{2\sigma}{r_0^3 \rho}
\]
Effective Mass  [Strasberg 1953]

Kinetic energy: \( E = \frac{1}{2} m \dot{v}^2 \quad \Rightarrow \quad m = \frac{2E}{\dot{v}^2} \)

Bubbles are acoustically compact sources
3.3 kHz
1482 m/s in water
449 mm wavelength

2 mm
Effective Mass  [Strasberg 1953]

Kinetic energy: \[ E = \frac{1}{2} m \dot{v}^2 \implies m = \frac{2E}{\dot{v}^2} \]

Bubbles are acoustically compact sources

Acoustic particle velocity \[ = \nabla \phi \quad \nabla^2 \phi = 0 \]

\[ E = \frac{\rho}{2} \int_\Omega (\nabla \phi)^2 \, d\Omega \]

\[ \int_\Omega (\nabla \phi)^2 \, d\Omega = -\int_{\partial\Omega} \phi \, \partial_n \phi \, dS - \int_\Omega \phi \nabla^2 \phi \, d\Omega \]
Bubble BVP

\[ \nabla^2 \phi = 0 \]
\[ \Omega \]
\[ \partial_n \phi = 0 \]
\[ \Gamma_r \]
\[ \phi = 0 \]
\[ \Gamma_a \]
\[ \phi = 1 \]
\[ \Gamma_b \]
Effective Mass

\[ m = -\frac{\rho}{\dot{v}^2} \int_{\Gamma_b} \phi \, \partial_n \phi \, dS \]
Bubble Capacitance Analogy

• Effective bubble capacitance $C$:

\[ m = \frac{\rho \phi_b}{\dot{\phi}} = \frac{\rho}{4\pi C} \]

\[ 4\pi C = \frac{\dot{\phi}}{\phi_b} \]

\[ \omega^2 = \frac{\kappa}{m} = \frac{4\pi \gamma P_0}{\rho V_0} C \]

• Equivalent to electrostatic capacitance
  - $\phi$ is electrostatic potential
  - $\dot{\phi}$ is flux

• Generalized frequency model:

• Reproduces classical Minnaert frequency for spherical bubbles
Frequency vs Bubble Position

![Color map of frequency vs bubble position with values ranging from 720 Hz to 1440 Hz. The map shows a gradient from blue to red, indicating frequency changes across the bubble position.]
Frequency Rise

Radius: 5.8mm
Minneart Frequency: 567 Hz
Cross-section view

\[
f = 651 \ (1)
\]
\[
f = 745 \ (1.14)
\]
\[
f = 825 \ (1.27)
\]
\[
f = 1117 \ (1.72)
\]
Shape Dependence

\[ f = 1209 \ (1) \]
\[ t = 0 \ (\text{ms}) \]

\[ f = 1158 \ (0.96) \]
\[ t = 3 \]

\[ f = 1103 \ (0.91) \]
\[ t = 6 \]

\[ f = 1034 \ (0.85) \]
\[ t = 13 \]

Radius: 2.4mm  Minneart Frequency: 1347 Hz
Transfer Function

\[ p_1 q_1(t) + p_2 q_2(t) + p_3 q_3(t) + \cdots \]

Final Sound:

\[ \sum_{\text{for each bubble}} p_i q_i(t) \]
Helmholtz Problems

\[(\nabla^2 + k^2(x))P(x; x_b) = S_b \delta(x - x_b)\]
Acoustic Radiation Importance

- Frequency dependent
- Spatially and temporally varying

[Zheng & James 2009]
Harmonic Fluids: Dual-domain approximation

Estimate surface vibration

\[(\nabla^2 + k_f^2)P_f = S_b \delta(x - x_b), \quad x \in \Omega_f\]

Estimate sound radiation

\[(\nabla^2 + k_a^2)P_a = 0, \quad x \in \Omega_a\]
Complex Acoustic Bubbles

Boundary conditions

\[ \nabla^2 \phi + k_a^2 \phi = 0 \]

\[ \Omega_a \]

\[ \nabla^2 \phi = 0 \]

\[ \Omega \]

\[ \phi = 0 \]

\[ \Gamma_a \]

\[ \phi = 1 \]

\[ \Gamma_b \]

\[ \partial_n \phi = 0 \]

\[ \Gamma_r \]
Time-Domain Effects

Air microphone

Hydrophone
Implementation - Meshing

T-junction

T-junction
Implementation - Meshing

- Too many triangles == slow
- Adaptive simplification
  - Resolve bubbles near interfaces
- Watch out for bad triangles
- Resolution constraints
  - Need to resolve waves in Helmholtz solves
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Frequency Solver → Radiation Solver

Radiation Solver → Sound Synthesis

Connections:
- geometry
- frequency
- transfer
Oscillator integration

\[ \ddot{v} + 2\beta \dot{v} + \omega^2 v = \frac{p(t)}{m} \]

- Numerical integration necessary
- Midpoint method or RK4 work well
- Sum oscillations from all bubbles, possibly weighted with transfer functions
Implementation: Bubble tracking through events
Loosely related but fun fact!

Bubble net hunting

Swim bladder
[en.wikipedia.org/wiki/Swim_bladder]

Humpback whale
[en.wikipedia.org/wiki/Humpback_whale]