Physically Based Sound for Computer Animation and Virtual Environments

Thin Shells

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Crash!
Nonlinear mode coupling
Nonlinear mode coupling

**Linear Frequency Response**

**Nonlinear Frequency Response**

Linear Response

Nonlinear Response
Thin-Shell Dynamics

- Thin-shell membrane + bending energy
  \[
  W_m = \frac{Y h}{2(1 - \nu^2)} \left[ (1 - \nu) \text{tr}(\epsilon_m^2) + \nu \text{tr}(\epsilon_m)^2 \right]
  \]
  \[
  W_b = \frac{Y h^3}{24(1 - \nu^2)} \left[ (1 - \nu) \text{tr}(\epsilon_b^2) + \nu \text{tr}(\epsilon_b)^2 \right]
  \]

- Internal energy + force
  \[
  E(u) = \int_S W(u; X) \, dS_X = \sum_{i=1}^{N_\Delta} A_i W_i(u)
  \]
  \[
  f_{int} = \nabla_u E(u) = \sum_{i=1}^{N_\Delta} A_i \nabla_u W_i(u)
  \]
Dimension Model Reduction
(a.k.a. “subspace integration”)
Dimensional Model Reduction

\[ M \ddot{u} + \int f(u) = f_{ext} \]

Substitute \( u = Uq \)

Project \( U^T \)

Identity

\[ U^T M U \ddot{q} + U^T \int f(Uq) = U^T f_{ext} \]

\[ \ddot{q} + f(q) = f_{ext} \]

Reduced Equations of Motion

[Bathe 1996; Krysl, Lall, Marsden 2001]
Reduced Internal Forces: Linear Modal Analysis

\[ f(q) = U^T f^{int} (Uq) \]

\[
\begin{pmatrix}
\omega_1^2 & q_1 \\
\omega_2^2 & q_2 \\
\vdots & \vdots \\
\omega_r^2 & q_r
\end{pmatrix}
\]

- **Linear force terms.**
- \( r \) decoupled oscillators
- Fast \( O(r) \) time-stepping using IIR filters
  - [Hamming 1983; van den Doel and Pai]
Reduced Internal Forces

\[ f(q) = U^T f^{int} (Uq) \]

Slow \( O(rN) \) computation

(but less stiff, so larger explicit time-steps)

[Bathe 1996; Krysl, Lall, Marsden 2001]
Reduced Internal Forces

\[ O(rN) \]

\[ \begin{align*}
U & \quad \quad \quad U^T \\
 q & \quad \quad \quad f
\end{align*} \]

[Bathe 1996; Krysl, Lall, Marsden 2001]
Optimizing Cubature for Efficient Integration of Subspace Deformations (with Steven An and Ted Kim)

\[ f(q) = -\nabla_q E(q) \]
\[ = \int_\Omega g(X; q) \; d\Omega_X \]
\[ \approx \sum_{i=1}^{n} w_i \; g(X_i; q) \]
Optimizing Cubature for Efficient Integration of Subspace Deformations
(with Steven An and Ted Kim)

- Fast, non-linear subspace forces
- Complex geometry
- Non-linear hyperelastic materials
- Scalable training pre-process:
  - INPUT: Subspace model, training poses
  - OUTPUT: Fast $O(r^2)$ subspace force model
Thin-Shell Cubature

- Subspace dynamics
  \[ \ddot{q} + \mathbf{D} \dot{q} + \tilde{f}_{int}(q) = \tilde{f}_{ext} \]

- Subspace forces
  \[ \tilde{f}_{int}(q) = U^T f_{int}(Uq) = \sum_{i=1}^{N \Delta} A_i \ g_i(q) \approx \sum_{i \in C} w_i \ g_i(q) \]
Thin-Shell Cubature

Trash can (200 modes) with 800-feature cubature scheme
Results
## Model Statistics

<table>
<thead>
<tr>
<th>Model</th>
<th>L (m)</th>
<th>tri</th>
<th>vtx</th>
<th>N</th>
<th>modes</th>
<th>freq (kHz)</th>
<th>material</th>
<th>ν</th>
<th>Y (GPa)</th>
<th>h (mm)</th>
<th>α</th>
<th>β (10^9)</th>
<th>kL</th>
<th>Error_cuba</th>
<th>kL Error_cuba</th>
<th>Δt (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trash Can</td>
<td>0.75</td>
<td>77536</td>
<td>38833</td>
<td>116499</td>
<td>200</td>
<td>0.071 – 4.43</td>
<td>Steel</td>
<td>0.30</td>
<td>190</td>
<td>2</td>
<td>0.5</td>
<td>75</td>
<td>800</td>
<td>10.3%</td>
<td>0.98 – 61</td>
<td>1/44100</td>
</tr>
<tr>
<td>Trash Lid</td>
<td>0.55</td>
<td>34312</td>
<td>17286</td>
<td>51858</td>
<td>200</td>
<td>0.112 – 6.79</td>
<td>Steel</td>
<td>0.30</td>
<td>190</td>
<td>2</td>
<td>0.5</td>
<td>75</td>
<td>800</td>
<td>11.5%</td>
<td>1.1 – 68</td>
<td>1/44100</td>
</tr>
<tr>
<td>Water Bottle</td>
<td>0.46</td>
<td>28658</td>
<td>14418</td>
<td>43254</td>
<td>300</td>
<td>0.116 – 3.59</td>
<td>Polycarb</td>
<td>0.37</td>
<td>2.4</td>
<td>2.25</td>
<td>0.5</td>
<td>400</td>
<td>900</td>
<td>10.7%</td>
<td>0.98 – 48</td>
<td>1/44100</td>
</tr>
<tr>
<td>Recycling Bin</td>
<td>0.61</td>
<td>109568</td>
<td>54945</td>
<td>164835</td>
<td>300</td>
<td>0.062 – 2.21</td>
<td>Polycarb</td>
<td>0.37</td>
<td>2.4</td>
<td>5</td>
<td>4.0</td>
<td>300</td>
<td>1200</td>
<td>15.7%</td>
<td>0.70 – 30</td>
<td>1/44100</td>
</tr>
<tr>
<td>Cymbal</td>
<td>0.50</td>
<td>61952</td>
<td>31104</td>
<td>93312</td>
<td>500</td>
<td>0.061 – 9.94</td>
<td>Bronze</td>
<td>0.33</td>
<td>124</td>
<td>0.7</td>
<td>1.0</td>
<td>6.25</td>
<td>1500</td>
<td>10.7%</td>
<td>0.57 – 92</td>
<td>1/88200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Modes r</th>
<th>Modal Analysis</th>
<th>Cubature Precomp.</th>
<th>Timestep Cost</th>
<th>Simulation Cost (per second of audio)</th>
<th>FFAT Precomp. (average time/mode)</th>
<th>FFAT Eval (all modes, M = 4)</th>
<th>FFAT Storage (floats, M = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trash Can</td>
<td>200</td>
<td>569 s</td>
<td>2.49 hr</td>
<td>16.1 ms</td>
<td>714 s</td>
<td>109.2 min</td>
<td>0.151 ms</td>
<td>56 MB</td>
</tr>
<tr>
<td>Trash Lid</td>
<td>200</td>
<td>170 s</td>
<td>1.87 hr</td>
<td>14.6 ms</td>
<td>642 s</td>
<td>85.5 min</td>
<td>0.151 ms</td>
<td>113 MB</td>
</tr>
<tr>
<td>Water Bottle</td>
<td>300</td>
<td>314 s</td>
<td>4.31 hr</td>
<td>23.6 ms</td>
<td>1026 s</td>
<td>25.6 min</td>
<td>0.227 ms</td>
<td>54 MB</td>
</tr>
<tr>
<td>Recycling Bin</td>
<td>300</td>
<td>2332 s</td>
<td>9.65 hr</td>
<td>27.8 ms</td>
<td>1224 s</td>
<td>48.0 min</td>
<td>0.227 ms</td>
<td>25 MB</td>
</tr>
<tr>
<td>Cymbal</td>
<td>500</td>
<td>1155 s</td>
<td>3.88 hr</td>
<td>44.3 ms</td>
<td>3900 s</td>
<td>318 min</td>
<td>0.378 ms</td>
<td>512 MB</td>
</tr>
</tbody>
</table>

**Table 2: Representative Timings:** All timings are for a single 2.66GHz Xeon X5355 processor core, except “Cubature Precomp” which used 8 cores.
Harmonic Shells
A Practical Nonlinear Sound Model for Near-Rigid Thin Shells

Jeffrey Chadwick
Steven An
Doug James
Cornell University

SIGGRAPH ASIA 2009
Thin-shell Cubature

TRASH CAN $r=200$, $T=2000$

TRASH LID $r=200$, $T=2000$
# Cubature Error Comparison

<table>
<thead>
<tr>
<th>Error</th>
<th>Time per Timestep</th>
<th>Speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>167ms</td>
<td></td>
</tr>
<tr>
<td>6%</td>
<td>28ms</td>
<td>6.1x</td>
</tr>
<tr>
<td>10%</td>
<td>16ms</td>
<td>10x</td>
</tr>
<tr>
<td>15%</td>
<td>10ms</td>
<td>17x</td>
</tr>
</tbody>
</table>

Traditional subspace integration [Krysl et al. 2001] (shown previously)
## Comparison to unreduced dynamics

<table>
<thead>
<tr>
<th>Unreduced Dynamics (Explicit Newmark)</th>
<th>Reduced Dynamics (11% cubature error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>~90 hrs / second of simulated sound</td>
<td>17 min / second of simulated sound</td>
</tr>
<tr>
<td>(This 5 sec clip took ~19 days to compute.)</td>
<td>10 sec clip</td>
</tr>
<tr>
<td></td>
<td>441,000 time steps in 2.8 hours</td>
</tr>
</tbody>
</table>
Comparison to Other Methods

1. Nonlinear dynamics + Transfer
   i.e., “Harmonic Shells”

2. Linear dynamics + Transfer
   e.g., [James et al. 2006]

3. Linear dynamics + Monopole
   e.g., [O'Brien et al. 2002; Boneel et al. 2008]

4. Nonlinear dynamics + Monopole