On the Impact of Ground Sound
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Introduction
Many sound synthesis examples in animation model impacts between moving objects and the ground. During object-ground collisions, three types of sound are emitted:

- Object emits ringing sound from resonant modes
- Object emits a transient acceleration noise upon impact
- Ground emits a transient sound upon impact

Previous works [3][4][5] model the first two sounds and omit the third. Through physical simulation, we study the relative importance of the ground sound.

Our work:
- Studies how material properties affect ground sound relevance
- Proposes an interactive method to synthesize ground sound
- Proposes an “acoustic shaper” for fine-difference time-domain (FDTD) simulations to incorporate ground sound

Background: Ground Vibration Model
We model the ground surface vibration by solving Lamb’s problem, and then we use it to drive sound propagation into the air.

Lamb’s problem statement: Given an elastic half-space (the ground), find the surface displacement ($u_0$) in response to an instantaneous point load ($f(t)$).

Pekeris[12] derived an analytical expression for the displacement. Unfortunately, it has a singularity at each waveform. In this figure, the solution at 1 m away is in blue, while our temporal regularization is labeled in the other colors.

The real vertical displacement response ($u(r,t)$) is the following:

$$u(r,t) = 0.5 - j\frac{\rho c^2}{4\pi \eta} e^{\frac{-j\omega t}{c\rho}} e^{-j\frac{(\omega^2 + k^2)rt}{c^2}}$$

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where $\omega$, $c$, and $\kappa$ are the wave number in the following:

$$\omega = \sqrt{k^2 + \frac{\omega^2}{c^2}}$$

$\kappa = \kappa_0 - j\kappa_1$

Ground Surface Displacement over Time

Method
Temporal Regularization: To smooth the singularities, we convolve the solution in time with a fourth-order smoothed delta ($f_j$):

$$g_j(t) = c_r e^{\frac{-j\omega t}{c\rho}} e^{-j\frac{(\omega^2 + k^2)rt}{c^2}}$$

$$f_j(t) = 2g_j(t) - g_j(t) - g_j(t-\tau).$$

We chose $f_j$ so that:
- It approximates a delta as $\epsilon \to 0$
- It approximates the Hertzian half-sine contact force profile,
- It is smooth enough to eliminate the singularities, and
- The final result is a closed-form expression, for quick evaluation.

Impulse Profile: Our smoothed delta approximates the Hertz half-sine contact force, setting epsilon ($\epsilon$) based on the contact timescale:

$$u_r = c_r e^{\frac{-j\omega t}{c\rho}} e^{-j\frac{(\omega^2 + k^2)rt}{c^2}}$$

where $a_0$, $m$, $E^*$, $\nu$, are the object’s local radius of curvature, mass, effective stiffness, impulse, and normal impact velocity.

Ground Sound Synthesis: We use the Rayleigh integral (Eq 12) for direct sound synthesis in our material properties studies, and we add a smooth acoustic shaper to the FDTD wavevector in [9] for animation scenarios.

$$p(r,t) = \rho f_0 \int_{\mathbb{R}^3} \frac{\rho_f f_\epsilon(t)}{2\pi R^3} e^{-i\epsilon} dr.$$ (12)

Results: Validation and Sound Synthesis

Results: Material and Listening Angle Dependence
Consider a ball dropped from a fixed height onto the ground.
- Listening Angle (θ): Observe in this plot that overhead listening angles receive more ball sound, while lower elevation angles receive more ground sound.
- Ball density ($\rho_b$), ground stiffness ($E$): For fixed initial drop height, the ground sound amplitude is proportional to $\rho_b/E$, while the ball sound is unaffected. See
- Ground speed of shear waves ($c_s$): The ball sound does not depend on $c_s$, while the ground amplitude increases linearly with $c_s$ until a knee threshold $c_k$

Theoretical Relative Intensities (dB) of Ground to Ball Sound, Measured Overhead

Theoretical Relative Intensities (dB) of Ground to Ball Sound, 5° above Ground

Discussion and Conclusion
We found the following three properties affect ground sound importance:
- Object density (denser objects $\to$ louder ground)
- Ground stiffness (softer grounds $\to$ louder ground)
- Listening angle (lower elevation angles $\to$ louder ground)

This is only important when the object’s modal ringing noise, which is louder in large objects, is not audible.

Future work directions:
- Model resonant modes in floors with finite depth and buildings
- Regularize the response to tangential forces incurred by contact friction
- Derive an analytical approximation for the final sound based on listening angle

References
[3] Iwanski and James, “Modeling and rendering for virtual drop sound based on physical model of rigid body,” Proc. of the Intl. Conf. on Digital Audio Effects (DAF’17), 2018