

On the Impact of Ground Sound: Supplemental

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1 No branch cut crossings when $0 \leq \nu < 0.2631$

We show here that our regularized solution, detailed in Appendix A, does not cross a principal branch cut when $0 \leq \nu < 0.2631$.

For these ν , κ_j are real, and $\kappa_2, \kappa_3 < a$ [1]. By inspection of (26), this means $s > \alpha$ in \mathcal{V}_ϵ , $\alpha > s$ in \mathcal{W}_ϵ , and $\epsilon, s, \alpha > 0$. The principal branch cut for both \sqrt{z} and $\log(z)$ are at the negative real line.

$Z_\epsilon(t', \alpha)$: The radicand of Z_ϵ never approaches the negative real line: the only way to achieve zero imaginary part is for $t' = 0$, when the radicand is a positive real number.

The first log in $\mathcal{V}_\epsilon(t', s, \alpha)$ and $\mathcal{W}_\epsilon(t', s, \alpha)$: $\epsilon + s - it'$ has a positive real part, so it never crosses the negative real line.

Second log in $\mathcal{V}_\epsilon(t', s, \alpha)$: because Z_ϵ is a square root, it has positive real part. Therefore the entire expression inside the log has negative imaginary part, never crossing the negative real line.

Second log in $\mathcal{W}_\epsilon(t', s, \alpha)$: When $t' \geq \alpha^2/s$, the imaginary part of Z_ϵ is negative, meaning the entire expression inside the log is in the third and fourth quadrants. When $t' < \alpha^2/s$, the real part of the expression inside the log is positive, meaning it is in the first and fourth quadrants. Overall, the expression only lives in the first, third, and fourth quadrants, implying that it cannot cross the negative real line.

We have shown that no branch crossings occur for any pair of (t', s) in our solution.

References

- [1] Eduardo Kausel, *Fundamental solutions in elastodynamics: a compendium*, Cambridge University Press, 2006.

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