

$$\text{In[1]:= } g[t\_]= \frac{\epsilon / \pi}{t^2 + \epsilon^2};$$

$$\text{IntegrandV}[s\_]= \frac{g[t - s]}{\sqrt{s^2 - \alpha^2}};$$

$$\text{IntegrandW}[s\_]= \frac{g[t - s]}{\sqrt{\alpha^2 - s^2}};$$

$$\text{In[4]:= } Z = \sqrt{\alpha^2 + (\epsilon - \mathbf{i} t)^2};$$

$$Z\text{conj} = \sqrt{\alpha^2 + (\epsilon + \mathbf{i} t)^2};$$

(\* V: the inside of Re() and its conjugate \*)

$$V\text{in}[s\_]= \frac{1}{\pi Z} \left( -\text{Log}[\epsilon - \mathbf{i} (t - s)] + \text{Log}[\alpha^2 - (t + \mathbf{i} \epsilon) s - \mathbf{i} Z \sqrt{s^2 - \alpha^2}] \right);$$

$$V\text{conj}[s\_]= \frac{1}{\pi Z\text{conj}} \left( -\text{Log}[\epsilon + \mathbf{i} (t - s)] + \text{Log}[\alpha^2 - (t - \mathbf{i} \epsilon) s + \mathbf{i} Z\text{conj} \sqrt{s^2 - \alpha^2}] \right);$$

$$V[s\_]= \frac{1}{2} (V\text{in}[s] + V\text{conj}[s]); (* \text{real part} *)$$

(\* verify that V is the antiderivative of IntegrandV \*)

$$\text{FullSimplify}[D[V[s], s] - \text{IntegrandV}[s]]$$

Out[9]= 0

(\* W: the inside of Im() and its conjugate \*)

$$W\text{in}[s\_]= \frac{-1}{\pi Z} \left( -\text{Log}[\epsilon - \mathbf{i} (t - s)] + \text{Log}[\alpha^2 - (t + \mathbf{i} \epsilon) s + Z \sqrt{\alpha^2 - s^2}] \right);$$

$$W\text{conj}[s\_]= \frac{-1}{\pi Z\text{conj}} \left( -\text{Log}[\epsilon + \mathbf{i} (t - s)] + \text{Log}[\alpha^2 - (t - \mathbf{i} \epsilon) s + Z\text{conj} \sqrt{\alpha^2 - s^2}] \right);$$

$$W[s\_]= \frac{1}{2 \mathbf{i}} (W\text{in}[s] - W\text{conj}[s]); (* \text{imaginary part} *)$$

(\* verify that W is the antiderivative of IntegrandW \*)

$$\text{FullSimplify}[D[W[s], s] - \text{IntegrandW}[s]]$$

Out[13]= 0