General Linear Cameras

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Abstract. We present a General Linear Camera (GLC) model that unifies many previous camera models into a single representation. The GLC model is capable of describing all perspective (pinhole), orthographic, and many multiperspective (including pushbroom and two-slit) cameras, as well as epipolar plane images. It also includes three new and previously unexplored multiperspective linear cameras. Our GLC model is both general and linear in the sense that, given any vector space where rays are represented as points, it describes all 2D affine subspaces (planes) that can be formed by affine combinations of 3 rays. The incident radiance seen along the rays found on subregions of these 2D affine subspaces are a precise definition of a projected image of a 3D scene. The GLC model also provides an intuitive physical interpretation, which can be used to characterize real imaging systems. Finally, since the GLC model provides a complete description of all 2D affine subspaces, it can be used as a tool for first-order differential analysis of arbitrary (higher-order) multiperspective imaging systems.

1 Introduction

Camera models are fundamental to the fields of computer vision and photogrammetry. The classic pinhole and orthographic camera models have long served as the workhorse of 3D imaging applications. However, recent developments have suggested alternative multiperspective camera models [4, 20] that provide alternate and potentially advantageous imaging systems for understanding the structure of observed scenes. Researchers have also recently shown that these multiperspective cameras are amenable to stereo analysis and interpretation [13, 11, 20].

In contrast to pinhole and orthographic cameras, which can be completely characterized using a simple linear model (the classic 3 by 4 matrix [5]), multiperspective cameras models are defined less precisely. In practice, multiperspective cameras models are described by constructions. By this we mean that a system or process is described for generating each specific class. While such physical models are useful for both acquisition and imparting intuition, they are not particularly amenable to analysis.

In this paper we present a unified General Linear Camera (GLC) model that is able to describe nearly all useful imaging systems. In fact, under an appropriate interpretation, it describes all possible linear images. In doing so it provides a single model that unifies existing perspective and multiperspecive cameras.

2 Previous Work

The most common linear camera model is the classic 3 x 4 pinhole camera matrix [5], which combines six extrinsic and five intrinsic camera parameters into single operator that maps homogenous 3D points to a 2D image plane. These mappings are unique down to a scale factor, and the same infrastructure can also be used to describe orthographic cameras. Recently, several researchers have proposed alternative camera representations known as multiperspective cameras which capture rays from different points in space. These multiperspective cameras include pushbroom cameras [4], which collect rays along parallel planes from points swept along a linear trajectory, and two-slit cameras [10], which collect all rays passing through two lines. Zomet et al [20] did an extensive analysis and modelling of two slit(XSlit) multiperspective cameras. However, they discuss the relationship of these cameras to pinhole cameras only for the purpose of image construction, whereas we provide a unifying model.

Multiperspective imaging has also been explored in the field of computer graphics. Examples include multiple-center-of-projection images [12], manifold mosaics [11], and multiperspective panoramas [18]. Most multiperspective images are generated by stitching together parts of pinhole images [18, 12], or slicing through image sequences [11, 20].

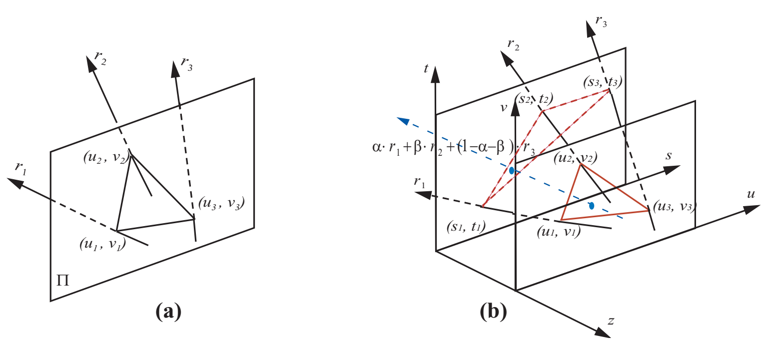

Fig. 1. General Linear Camera Model. a)A GLC is characterized by three rays originated from the image plane. b)It collects all possible affine combination of three rays.

Seitz [13] has analyzed the space of multiperspective cameras to determine those with a consistent epipolar geometry. Their work suggests that some multiperspective images can be used to analyze three-dimensional structure, just as pinhole cameras are commonly used. We focus our attention on a specific class of linear multiperspective cameras, most of which satisfy Seitz's criterion.

Our analysis is closely related to the work of Gu et al [3], which explored the linear structures of 3D rays under a particular 4D mapping known as a two-plane parametrization. This model is commonly used for light field rendering. Their primary focus was on the duality of points and planes under this mapping. They deduced that XSlits are another planar structure within this space, but they do not characterize all of the possible planar structures, nor discuss their analogous camera models.

Our new camera model only describes the set of rays seen by a particular camera, not their distribution on the image plane. Under this definition pinhole cameras are defined by only 3 parameters (the position of the pinhole in 3D). Homographies and other non-linear mappings of pinhole images (i.e., radial distortion) only change the distribution of rays in the image plane, but do not change the set of rays seen. Therefore, all such mappings are equivalent under our model.

3 General Linear Camera Model

The General Linear Camera (GLC) is defined by three rays that originate from three points $p_1(u_1, v_1)$, $p_2(u_2, v_2)$ and $p_3(u_3, v_3)$ on an image plane Π_{image}, as is shown in Figure 1. A GLC collects radiance measurements along all possible "affine combinations" of these three rays. In order to define this affine combination of rays, we assume a specific ray parametrization.

W.o.l.g, we define Π_{image} to lie on $z = 0$ plane and its origin to coincide with the origin of the coordinate system. From now on, we call Π_{image} as Π_{uv}. In order to parameterize rays, we place a second plane Π_{st} at $z = 1$. All rays not parallel to Π_{st}, Π_{uv} will intersect the two planes at $(s, t, 1)$ and $(u, v, 0)$ respectively. That gives a 4D parametrization of each ray in form (s, t, u, v). This parametrization for rays, called the two-plane parametrization (2PP), is widely used by the computer graphics community for representing light fields and lumigraphs [7, 2]. Under this parametrization, an affine combination of three rays $r_i(s_i, t_i, u_i, v_i)$, $i = 1, 2, 3$, is defined as:

$$r = \alpha \cdot (s_1, t_1, u_1, v_1) + \beta \cdot (s_2, t_2, u_2, v_2) + (1 - \alpha - \beta) \cdot (s_3, t_3, u_3, v_3) \qquad (1)$$

The choice of Π_{st} at $z = 1$, is, of course, arbitrary. One can choose any plane parallel to Π_{uv} to derive an equivalent parametrization. Moreover, these alternate parameterizations will preserve affine combinations of three rays.

Lemma 1. *The affine combinations of any three rays under two different 2PP parameterizations that differ by choice of Π_{st} (i.e., (s, t, u, v) and (s', t', u, v)) are the same.*

Proof. Suppose $\Pi_{s't'}$ is at some arbitrary depth z_0, $z_0 \neq 0$. Consider the transformation of a ray between the default parametrization ($z_0 = 1$) and this new one.

If $r(s,t,u,v)$ and $r(s',t',u,v)$ represent the same ray r in 3D, then $r(s,t,u,v)$ must pass through (s',t',z_0), and there must exist some λ such that

$$\lambda \cdot (s,t,1) + (1-\lambda) \cdot (u,v,0) = (s',t',z_0) \tag{2}$$

Solving for λ, we have

$$s' = s \cdot z_0 + u \cdot (1-z_0), \quad t' = t \cdot z_0 + v \cdot (1-z_0) \tag{3}$$

Since this transformation is linear, and affine combinations are preserved under linear transformation, the affine combinations of rays under our default two-plane parametrization ($z_0 = 1$) will be consistent for parameterizations over alternative parallel planes. Moreover, the affine weights for a particular choice of parallel Π_{st} are general. □

We call the GLC model "linear" because it defines all 2-dimensional affine subspaces in the 4-dimensional "ray space" imposed by a two-plane parametrization. Moreover, these 2D affine subspaces of rays can be considered as images. We refer to the three rays used in a particular GLC as the GLC's *generator rays*. Equivalently, a GLC can be described by the coordinates of two triangles with corresponding vertices, one located on Π_{st}, and the second on Π_{uv}. Unless otherwise specified, we will assume the three generator rays (in their 4D parametrization) are linearly independent. This affine combination of generator rays also preserves linearity, while other parameterizations, such as the 6D *Plücker* coordinates [16], do not [3].

Lemma 2. *If three rays are parallel to a plane Π in 3D, then all affine combinations of them are parallel to Π as well.*

Lemma 3. *If three rays intersect a line l parallel to the image plane, all affine combinations of them will intersect l as well.*

Proof. By lemma 1, we can reparametrize three rays by placing Π_{st} so that it contains l resulting in the same set of affine combinations of the three rays. Because the st plane intersections of the three rays will lie on l, all affine combinations of three rays will have their st coordinates on l, i.e., they will all pass through l. The same argument can be applied to all rays which pass through a given point. □

4 Equivalence of Classic Camera Models

Traditional camera models have equivalent GLC representations.

Pinhole camera: By definition, all rays of a pinhole camera pass through a single point, C in 3D space (the center of projection). Any three linearly independent rays from C will the intersect Π_{uv} and Π_{st} planes to form two triangles. These triangles will be similar and have parallel corresponding edges, as shown in Figure 2(a). Furthermore, any other ray, r, through C will intersect

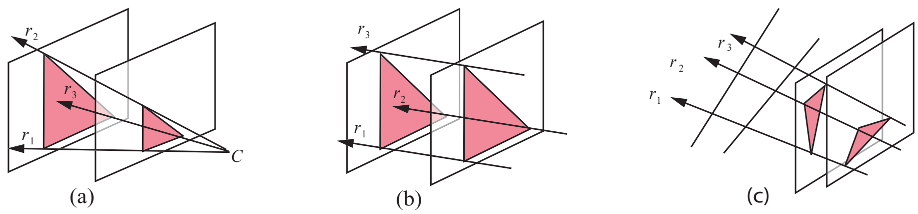

Fig. 2. Classic camera models represented as GLC. (a)Two similar triangles on two planes define a pinhole camera; (b)Two parallel congruent triangles define an orthographic camera; (c)Three rays from an XSlit camera.

Π_{uv} and Π_{st} planes at points $\dot{p}_{uv}$, and $\dot{q}_{st}$. These points will have the same affine coordinates relative to the triangle vertices on their corresponding planes, and r has the same affine coordinates as these two points.

Orthographic camera: By definition, all rays on an orthographic camera have the same direction. Any three linearly independent rays from an orthographic camera intersect parallel planes at the vertices of congruent triangles with parallel corresponding edges, as shown in Figure 2(b). Rays connecting the same affine combination of these triangle vertices, have the same direction as the 3 generator rays, and will, therefore, originate from the same orthographic camera.

Pushbroom camera: A pushbroom camera sweeps parallel planes along a line l collecting those rays that pass through l. We refer to this family of parallel planes as Π^*. We choose Π_{uv} parallel to l but not containing l, and select a non-degenerate set of generator rays (they intersect Π_{uv} in a triangle). By Lemma 2 and 3, all affine combinations of the three rays must all lie on Π^* parallel planes and must also pass through l and, hence, must belong to the pushbroom camera. In the other direction, for any point $\dot{p}$ on Π_{uv}, there exist one ray that passes through $\dot{p}$, intersects l and is parallel to Π^*. Since $\dot{p}$ must be some affine combination of the three vertexes of the uv triangle, r must lie on the corresponding GLC. Furthermore, because all rays of the pushbroom camera will intersect Π_{uv}, the GLC must generate equivalent rays.

XSlit camera: By definition, an XSlit camera collects all rays that pass through two non-coplanar lines. We choose Π_{uv} to be parallel to both lines but to not contain either of them. One can then pick a non-degenerate set of generator rays and find their corresponding triangles on Π_{st} and Π_{uv}. By Lemma 3, all affine combinations of these three rays must pass through both lines and hence must belong to the XSlit camera. In the other direction, authors of XSlit [10, 20] have shown that each point $\dot{p}$ on the image plane Π_{uv}, maps to a unique ray r in an XSlit camera. Since $\dot{p}$ must be some affine combination of the three vertexes of the uv triangle, r must belong to the GLC. The GLC hence must generate equivalent rays as the XSlit camera.

Epipolar Plane Image: EPI [1] cameras collect all rays that lie on a plane in 3D space. We therefore can pick any three linearly independent rays on the plane as generator rays. Affine combinations of these rays generate all possible rays on the plane,so long as they are linearly independent. Therefore a GLC can also represent Epipolar Plane Images.

5 Characteristic Equation of GLC

Although we have shown that a GLC can represent most commonly used camera models, the representation is not unique (i.e., three different generator rays can define the same camera). In this section we develop a criterion to classify general linear cameras. One discriminating characteristic of affine ray combinations is whether or not all rays pass through a line in 3D space. This characteristic is fundamental to the definition of many multi-perspective cameras. We will use this criteria to define the characteristic equation of general linear cameras.

Recall that any 2D affine subspace in 4D can be defined as affine combinations of three points. Thus, GLC models can be associated with all possible planes in the 4D since GLCs are specified as affine combinations of three rays, whose duals in 4D are the three points.

Lemma 4. *Given a non-EPI, non-pinhole GLC, if all camera rays pass through some line l, not at infinity, in 3D space, then l must be parallel to Π_{uv}.*

Proof. We demonstrate the contrapositive. If l is not parallel to Π_{uv}, and all rays on a GLC pass through l, then we show the GLC must be either an EPI or a pinhole camera.

Assume the three rays pass through at least two distinct points on l, otherwise, they will be on a pinhole camera, by Lemma 3. If l is not parallel, then it must intersect Π_{st}, Π_{uv} at some point $(s_0, t_0, 1)$ and $(u_0, v_0, 0)$. Gu et al [3] has shown all rays passing through l must satisfy the following bilinear constraints

$$(u - u_0)(t - t_0) - (v - v_0)(s - s_0) = 0 \tag{4}$$

We show that the only GLCs that satisfy this constraint are EPIs or pinholes.

All 2D affine subspaces in (s, t, u, v) can be written as the intersection of two linear constraints $A_i \cdot s + B_i \cdot t + C_i \cdot u + D_i \cdot v + E_i = 0$, $i = 1, 2$. In general we can solve these two equations for two variables, for instance, we can solve for u-v as

$$u = A'_1 \cdot s + B'_1 \cdot t + E'_1 \ , \ v = A'_2 \cdot s + B'_2 \cdot t + E'_2 \tag{5}$$

Substituting u and v into the bilinear constraint (4), we have

$$(A'_1 \cdot s + B'_1 \cdot t + E'_1 - u_0)(t - t_0) = (A'_2 \cdot s + B'_2 \cdot t + E'_2 - v_0)(s - s_0) \tag{6}$$

This equation can only be satisfied for all s and t if $A'_1 = B'_2$ and $B'_1 = A'_2 = 0$, therefore, equation (5) can be rewritten as $u = A' \cdot s + E'_1$ and $v = A' \cdot t + E'_2$. Gu et al [3] have shown all rays in this form must pass through a 3D point P (P cannot be at infinity, otherwise all rays have uniform directions and cannot all pass through any line l, not at infinity). Therefore all rays must lie on a 3D plane that passes through l and finite P. The only GLC camera in which all rays lie on a 3D plane is an EPI. If the two linear constraints are singular in u and v, we can solve for s-t, and similar results hold.

If the two linear constraints cannot be solved for u-v or s-t but can be solved for u-s or v-t, then a similar analysis results in equations of two parallel lines, one on Π_{st}, the other on Π_{uv}. The set of rays through two parallel lines must lie on an EPI. □

Lemma 3 and 4 imply that given a GLC, we need only consider if the three generator rays pass through some line parallel to Π_{st}. We use this relationship to define the characteristic equation of a GLC.

The three generator rays in a GLC correspond to the following 3D lines:

$$r_i = \lambda_i \cdot (s_i, t_i, 1) + (1 - \lambda_i) \cdot (u_i, v_i, 0) \quad i = 1, 2, 3$$

The three rays intersect some plane $\Pi_{z=\lambda}$ parallel to Π_{uv} when $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$. By Lemma 3, all rays on the GLC pass through some line l on $\Pi_{z=\lambda}$ if the three generator rays intersect l. Therefore, we only need to test if there exist any λ so that the three intersection points of the generator rays with $\Pi_{z=\lambda}$ lie on a line. A necessary and sufficient condition for 3 points on a constant z-plane to be co-linear is that they have zero area on that plane. This area is computed as follows (Note: the value of z is unnecessary):

$$\begin{vmatrix} (\lambda \cdot s_1 + (1-\lambda) \cdot u_1) & (\lambda \cdot t_1 + (1-\lambda) \cdot v_1) & 1 \\ (\lambda \cdot s_2 + (1-\lambda) \cdot u_2) & (\lambda \cdot t_2 + (1-\lambda) \cdot v_2) & 1 \\ (\lambda \cdot s_3 + (1-\lambda) \cdot u_3) & (\lambda \cdot t_3 + (1-\lambda) \cdot v_3) & 1 \end{vmatrix} = 0 \tag{7}$$

Notice equation (7) is a quadratic equation in λ of the form

$$A \cdot \lambda^2 + B \cdot \lambda + C = 0 \tag{8}$$

where

$$A = \begin{vmatrix} s_1 - u_1 & t_1 - v_1 & 1 \\ s_2 - u_2 & t_2 - v_2 & 1 \\ s_3 - u_3 & t_3 - v_3 & 1 \end{vmatrix}, \ B = \begin{vmatrix} s_1 & v_1 & 1 \\ s_2 & v_2 & 1 \\ s_3 & v_3 & 1 \end{vmatrix} - \begin{vmatrix} t_1 & u_1 & 1 \\ t_2 & u_2 & 1 \\ t_3 & u_3 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ u_3 & v_3 & 1 \end{vmatrix}, \ C = \begin{vmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ u_3 & v_3 & 1 \end{vmatrix}$$

We call equation (8) the characteristic equation of a GLC. Since the characteristic equation can be calculated from any three rays, one can also evaluate the characteristic equation for EPI and pinhole cameras. The number of solutions of the characteristic equation implies the number of lines that all rays on a GLC pass through. It may have 0, 1, 2 or infinite solutions. The number of solutions depends on the denominator A and the quadratic discriminant $\Delta = B^2 - 4AC$.

We note that the characteristic equation is invariant to translations in 4D. Equivalently, translations of the two triangles formed by generator rays $(s'_i, t'_i) = (s_i + T_s, t_i + T_t)$, $(u'_i, v'_i) = (u_i + T_u, v_i + T_v)$, $i = 1, 2, 3$, do not change the coefficients A, B and C of equation (8).

6 Characterizing Classic Camera Models

In this section, we show how to identify standard camera models using the characteristic equation of 3 given generator rays.

Lemma 5. *Given a GLC, three generator rays, and its characteristic equation* $A \cdot \lambda^2 + B \cdot \lambda + C = 0$*, then all rays are parallel to some plane if and only if* $A = 0$.

Proof. Notice in the matrix used to calculate A, row i is the direction $\boldsymbol{d}_i$ of ray r_i. Therefore A can be rewritten as $A = (\boldsymbol{d}_1 \times \boldsymbol{d}_2) \cdot \boldsymbol{d}_3$. Hence $A = 0$ if and only if $\boldsymbol{d}_1$, $\boldsymbol{d}_2$ and $\boldsymbol{d}_3$ are parallel to some 3D plane. And by Lemma 2, all affine combinations of these rays must also be parallel to that plane if $A = 0$. □

6.1 $A = 0$ case

When $A = 0$, the characteristic equation degenerates to a linear equation, which can have 1, 0, or an infinite number of solutions. By Lemma 5, all rays are parallel to some plane. Only three standard camera models satisfy this condition: pushbroom, orthographic, and EPI.

All rays of a pushbroom lie on parallel planes and pass through one line, as is shown in Figure 4(a). A GLC is a pushbroom camera if and only if $A = 0$ and the characteristic equation has 1 solution.

All rays of an orthographic camera have the same direction and do not all simultaneously pass through any line l. Hence its characteristic equation has no solution. The zero solution criteria alone, however, is insufficient to determine if a GLC is orthographic. We show in the following section that one can *twist* an orthographic camera into bilinear sheets by rotating rays on parallel planes, as is shown in Figure 4(b), and still maintain that all rays do not pass through a common line. In Section 3, we showed that corresponding edges of the two congruent triangles of an orthographic GLC must be parallel. This parallelism is captured by the following expression:

$$\frac{(s_i - s_j)}{(t_i - t_j)} = \frac{(u_i - u_j)}{(v_i - v_j)} \quad i, j = 1, 2, 3 \;\; \textit{and} \;\; i \neq j \tag{9}$$

We call this condition the *edge-parallel* condition. It is easy to verify that a GLC is orthographic if and only if $A = 0$, its characteristic equation has no solution, and it satisfies the *edge-parallel* condition.

Rays of an EPI camera all lie on a plane and pass through an infinite number of lines on the plane. In order for a characteristic equation to have infinite number of solutions when $A = 0$, we must also have $B = 0$ and $C = 0$. This is not surprising, because the intersection of the epipolar plane with Π_{st} and Π_{uv} must be two parallel lines and it is easy to verify $A = 0$, $B = 0$ and $C = 0$ if and only if the corresponding GLC is an EPI.

6.2 $A \neq 0$ case

When $A \neq 0$, the characteristic equation becomes quadratic and can have 0, 1, or 2 solutions, which depends on the characteristic equation's discriminant Δ. We show how to identify the remaining two classical cameras, pinhole and XSlit cameras in term of A and Δ.

All rays in a pinhole camera pass through the center of projection (COP). Therefore, any three rays from a pinhole camera, if linearly independent, cannot all be parallel to any plane, and by Lemma 4, $A \neq 0$. Notice that the roots of the characteristic equation correspond to *the depth* of the line that all rays pass through, hence the characteristic equation of a pinhole camera can only have one solution that corresponds to the depth of the COP, even though there exists an infinite number of lines passing through the COP. Therefore, the characteristic equation of a pinhole camera must satisfy $A \neq 0$ and $\Delta = 0$. However, this condition alone is insufficient to determine if a GLC is pinhole. In the following section, we show that there exists a camera where all rays lie on pencil of planes sharing a line, as shown in Figure 4(c), which also satisfies these conditions. One can, however, reuse the *edge-parallel* condition to verify if a GLC is pinhole. Thus a GLC is pinhole, if and only if $A \neq 0$, has one solution, and it satisfies *edge-parallel* condition.

Rays of an XSlit camera pass through two slits and, therefore, the characteristic equation of a GLC must have at least two distinct solutions. Furthermore, Pajdla [10] has shown all rays of an XSlit camera cannot pass through lines other than its two slits, therefore, the characteristic equation of an XSlit camera has exactly two distinct solutions. Thus, a GLC is an XSlit if and only if $A \neq 0$ and $\Delta > 0$.

7 New Multiperspective Camera Models

The characteristic equation also suggests three new multiperspective camera types that have not been previously discussed. They include 1)twisted orthographic: $A = 0$, the equation has no solution, and all rays do not have uniform direction; 2)pencil camera: $A \neq 0$ and the equation has one root, but all rays do not pass through a 3D point; 3)bilinear camera: $A \neq 0$ and the characteristic equation has no solution. In this section, we give a geometric interpretation of these three new camera models.

Before describing these camera models, however, we will first discuss a helpful interpretation of the spatial relationships between the three generator rays. An affine combination of two 4D points defines a 1-dimensional affine subspace. Under 2PP, a 1-D affine subspaces corresponds to a bilinear surface S in 3D that contains the two rays associated with each 4D point. If these two rays intersect or have the same direction in 3D space, S degenerates to a plane. Next, we consider the relationship between ray r_3 and S. We define r_3 to be parallel to S if and only if r_3 has the same direction as some ray $r \in S$. This definition of parallelism is quite different from conventional definitions. In particular, if r_3

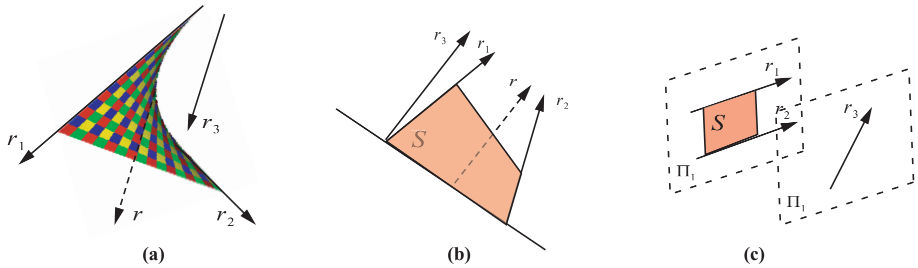

Fig. 3. Bilinear Surfaces. (a) r_3 is parallel to S; (b) r_3 is parallel to S, but still intersects S; (c) r_3 is not parallel to S, and does not intersect S either.

is parallel to S, r_3 can still intersect S. And if r_3 is not parallel to S, r_3 still might not intersect S, Figure 3(b) and (c) show examples of each case.

This definition of parallelism, however, is closely related to A in the characteristic equation. If r_3 is parallel to S, by definition, the direction of r_3 must be some linear combination of the directions of r_1 and r_2, and, therefore, $A = 0$ by Lemma 5. $A = 0$, however, is not sufficient to guarantee r_3 is parallel to S. For instance, one can pick two rays with uniform directions so that $A = 0$, yet still have the freedom to pick a third so that it is not parallel to the plane, as is shown in Figure 3(c).

The number of solutions to the characteristic equation is also closely related to the number of intersections of r_3 with S. If r_3 intersects the bilinear surface $S(r_1, r_2)$ at P, then there exists a line l, where $P \in l$, that all rays pass through. This is because one can place a constant-z plane that passes through P and intersects r_1 and r_2 at Q and R. It is easy to verify that P, Q and R lie on a line and, therefore, all rays must pass through line PQR. Hence r_3 intersecting $S(r_1, r_2)$ is a sufficient condition to ensure that all rays pass through some line. It further implies if the characteristic equation of a GLC has no solution, no two rays in the camera intersect. GLCs whose characteristic equation has no solution are examples of the oblique camera from [9].

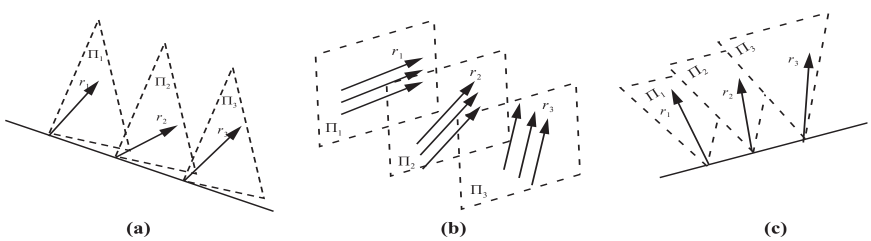

Fig. 4. Pushbroom, Twisted Orthographic, and Pencil Cameras. (a) A pushbroom camera collects rays on a set of parallel planes passing through a line; (b) A twisted orthographic camera collects rays with uniform directions on a set of parallel planes; (c) A pencil camera collects rays on a set of non-parallel planes that share a line.

7.1 New Camera Models

Our GLC model and its characteristic equation suggests 3 new camera types that have not been previously described.

Twisted Orthographic Camera: The characteristic equation of the twisted orthographic camera satisfies $A = 0$, has no solution, and its generators do not satisfy the *edge-parallel* condition. If r_1, r_2 and r_3 are linearly independent, no solution implies r_3 will not intersect the bilinear surface S. In fact, no two rays intersect in 3D space. In addition, $A = 0$ also implies that all rays are parallel to some plane Π in 3D space, therefore the rays on each of these parallel planes must have uniform directions as is shown in Figure 4(b). Therefore, twisted orthographic camera can be viewed as twisting parallel planes of rays in an orthographic camera along common bilinear sheets.

Pencil Camera: The characteristic equation of a pencil camera satisfies $A \neq 0$, has one solution and the generators do not satisfy the *edge-parallel* condition. In Figure 4(c), we illustrate a sample pencil camera: rays lie on a pencil of planes that share line l. In a pushbroom camera, all rays also pass through a single line. However, pushbroom cameras collect rays along planes transverse to l whereas the planes of a pencil camera contains l (i.e., lie in the pencil of planes through l), as is shown in Figure 4(a) and 4(c).

Bilinear Camera: By definition, the characteristic equation of a bilinear camera satisfies $A \neq 0$ and the equation has no solution ($\Delta < 0$). Therefore, similar to twisted orthographic cameras, no two rays intersect in 3D in a bilinear camera. In addition, since $A \neq 0$, no two rays are parallel either. Therefore, any two rays in a bilinear camera form a non-degenerate bilinear surface, as is shown in Figure 3(a). The complete classification of cameras is listed in Table 1.

Table 1. Characterize General Linear Cameras by Characteristic Equation

Characteristic Equation	2 Solution	1 Solution	0 Solution	Inf. Solution
$A \neq 0$	XSlit	Pencil/Pinhole†	Bilinear	Ø
$A = 0$	Ø	Pushbroom	Twisted/Ortho.†	EPI

†: A GLC satisfying *edge-parallel* condition is pinhole($A \neq 0$) or orthographic ($A = 0$).

7.2 All General Linear Cameras

Recall that the characteristic equation of a GLC is invariant to translation, therefore we can translate (s_1, t_1) to $(0, 0)$ to simplify computation. Furthermore, we assume the uv triangle has canonical coordinates $(0, 0)$, $(1, 0)$ and $(0, 1)$. This gives:

$$A = s_2 t_3 - s_3 t_2 - s_2 - t_3 + 1, \quad \Delta = (s_2 - t_3)^2 + 4 s_3 t_2 \tag{10}$$

The probability that $A = 0$ is very small, therefore, pushbroom, orthographic and twisted orthographic cameras are a small subspace of GLCs. Furthermore since s_2, t_2, s_3 and t_3 are independent variables, we can, by integration, determine that approximately two thirds of all possible GLCs are XSlit, one third are bilinear cameras, and remainders are other types.

8 Example GLC Images

In Figure 5, we compare GLC images of a synthetic scene. The distortions of the curved isolines on the objects illustrate various multi-perspective effects of GLC cameras. In Figure 6, we illustrate GLC images from a 4D light field. Each GLC is specified by three generator rays shown in red. By appropriately transforming the rays on the image plane via a 2D homography, most GLCs generate easily interpretable images. In Figure 7, we choose three desired rays from different pinhole images and fuse them into a multiperspective bilinear GLC image.

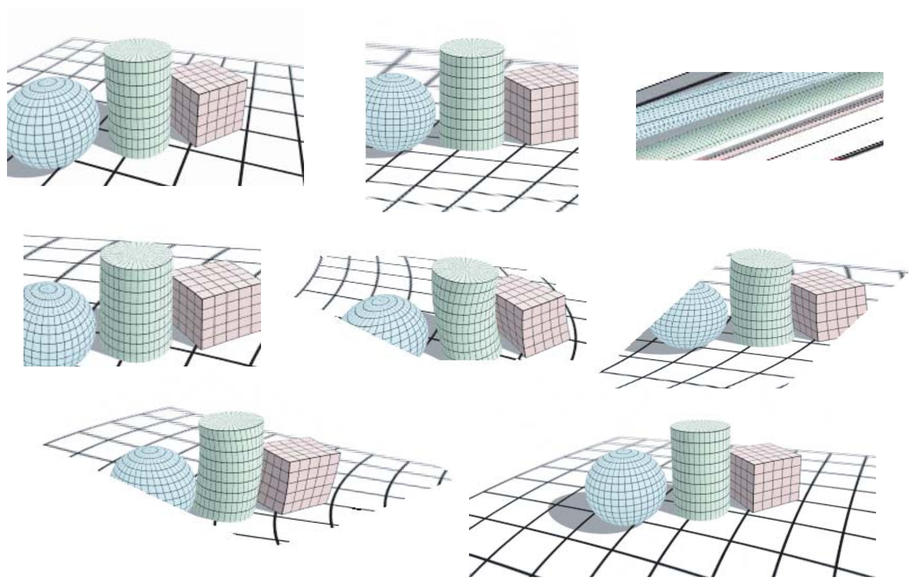

Fig. 5. Comparison between synthetic GLC images. From left to right, top row: a pinhole, an orthographic and an EPI; middle row: a pushbroom, a pencil and an twisted orthographic; bottom row: a bilinear and an XSlit.

9 Conclusions

We have presented a General Linear Camera (GLC) model that unifies perspective (pinhole), orthographic and many multiperspective (including pushbroom

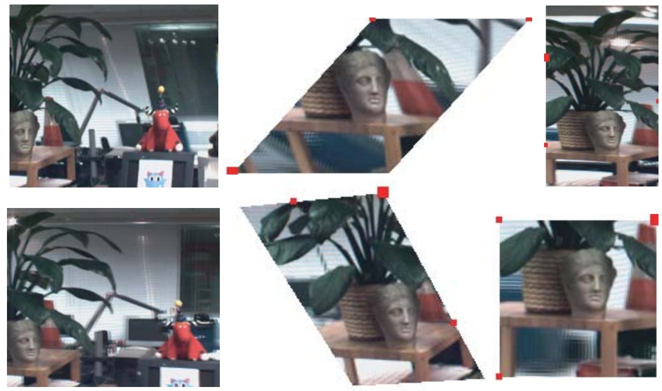

Fig. 6. GLC images created from a light field. Top row: a pencil, bilinear, and pushbroom image. Bottom row: an XSlit, twisted orthographic, and orthographic image.

and two-slit) cameras, as well as Epipolar Plane Images (EPI). We have also introduced three new linear multiperspective cameras that have not been previously explored: they are twisted orthographic, pencil and bilinear cameras. We have further deduced the characteristic equation for every GLC from its three generator rays and have shown how to use it to classify GLCs into eight canonical camera models.

The GLC model also provides an intuitive physical interpretation between lines, planar surfaces and bilinear surfaces in 3D space, and can be used to characterize real imaging systems like mirror reflections on curved surface. Since GLCs describes all possible 2D affine subspaces in 4D ray space, they can used be as a tool for first-order differential analysis of these high-order multiperspective imaging systems. GLC images can be rendered directly by ray tracing a synthetic scene, or by cutting through pre-captured light fields. By appropriately organizing rays, all eight canonical GLCs generate interpretable images similar to pinhole and orthographic cameras. Furthermore, we have shown one can fuse desirable features from different perspectives to form any desired multiperspective image.

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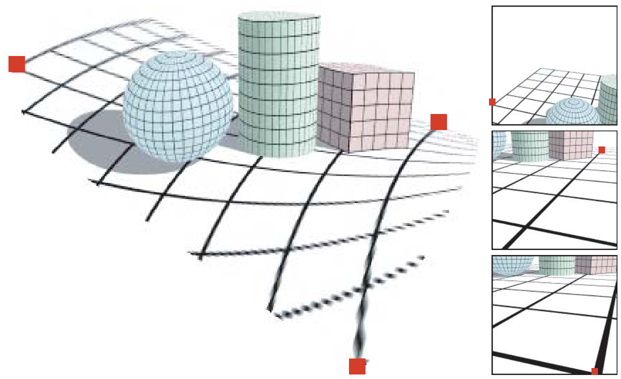

Fig. 7. A multiperspective bilinear GLC image synthesized from three pinhole cameras shown on the right. The generator rays are highlighted in red.

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