Discovering Structural Regularity in 3D Geometry

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Regular Structures
Regular Structures
Regular Structure
Regular Structure
Regular Structure
Regular Structure

Discovering Structural Regularity in 3D Geometry

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Motivation
Motivation

• Regularity $\rightarrow$ form, semantics
Motivation

• Regularity → form, semantics
Motivation

- Regularity → form, semantics
- Scan cleaning, completion
Motivation

• Regularity → form, semantics

• Scan cleaning, completion

• Compression
Motivation

• Regularity → form, semantics

• Scan cleaning, completion

• Compression

• Geometric edits, synthesis
Motivation

• Regularity → form, semantics

• Scan cleaning, completion
• Compression
• Geometric edits, synthesis
• Growth laws or design principles
Inspiration

On Growth and Form
[Thompson 1917]
Related Work
Related Work

[Podolak et al. `06]

[Loy, Eklundh `06]

[Mitra et al. `06]

[Martinet et al. `07]
Related Work

- [Podolak et al. `06]
- [Loy, Eklundh `06]
- [Mitra et al. `06]
- [Martinet et al. `07]
- [Funkhouser et al. `05]
- [Thrun, Wegbreit `05]
- [Shimari et al. `06]
- [Liu et al. `08]
Related Work

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[Hays et al. `06]  [Mueller et al. `07]  [Baudes et al. `08]  [Wang et al. `08]
Related Work

[Mitra et al. '06]

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[Loy, Eklundh '06]

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[Hays et al. '06]

[Shimari et al. '06]

[Baudes et al. '08]

[Wang et al. '08]
Structure Discovery

Input Model
Structure Discovery

Input Model

Structure Discovery

Regular Structures
Structure Discovery

Input Model → Transform Discovery → Transform Clusters

Regular Structures
Structure Discovery

Input Model → Transform Analysis → Transform Clusters → Model Estimation → Transform Generators

- Structure Discovery
- Regular Structures
Structure Discovery

Transform Analysis

Input Model

Structure Discovery

Transform Clusters

Model Estimation

Aggregation

Regular Structures

Transform Generators

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SIGGRAPH 2008
Structure Discovery

Input Model → Structure Discovery → Regular Structures

Transform Analysis → Transform Clusters → Model Estimation → Transform Generators

Aggregation
Structure Discovery

Input Model

Transform Analysis

Transform Clusters

Model Estimation

Aggregation

Transform Generators

Regular Structures

spatial domain

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Structure Discovery

Input Model

Structure Discovery

Regular Structures

Transform Analysis

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Transform Generators

spatial domain

Transform domain

Aggregation

Discovering Structural Regularity in 3D Geometry

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Repetitive Structures

• Invariance under transformations
Repetitive Structures

- Invariance under transformations

Translation
Repetitive Structures

- Invariance under transformations

Translation

Rotation
Repetitive Structures

- Invariance under transformations

Translation

Rotation

Scaling
Repetitive Structures

- Invariance under transformations

Translation + Rotation
Repetitive Structures

• Invariance under transformations

Translation + Rotation

Rotation + Scaling
Repetitive Structures

- Invariance under transformations

Translation + Rotation  
Rotation + Scaling

1-parameter patterns
Repetitive Structures

- Invariance under transformations

Translation $\times$ Translation
Repetitive Structures

- Invariance under transformations

Translation × Translation

Translation × Rotation
Repetitive Structures

- Invariance under transformations

  Translation $\times$ Translation

  Translation $\times$ Rotation

  Rotation $\times$ Scaling
Repetitive Structures

• Invariance under transformations

Translation $\times$ Translation

Translation $\times$ Rotation

Rotation $\times$ Scaling

2-parameter commutative patterns
Repetitive Structures

- 1-parameter groups
  - Translation
  - Rotation
  - Scaling

- Commutative 2-parameter groups
  - Translation × Translation
  - Translation × Rotation
  - Rotation × Scaling
Structure Discovery

Input Model

Structure Discovery

Regular Structures

Transform Analysis

Model Estimation

Transform Clusters

Transform Generators

Aggregation

Discovering Structural Regularity in 3D Geometry
Structure Discovery

Input Model → Transform Analysis → Transform Clusters

Structure Discovery → Model Estimation → Aggregation

Regular Structures → Transform Generators

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Transformations
Transformations [Mitra et al. `06]
Transformations
Transformations
Transformations
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Transformations

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Transformations
Transformations

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Transformations
Transformations

$n^2$ transformations
Transformations

- Spatial domain
- Transformation space

$n^2$ transformations
Transformations

spatial domain

transformation space

spatial domain
Transformations

spatial domain

transformation space

Discovering Structural Regularity in 3D Geometry
Transformations

spatial domain

transformation space
Model Estimation
Model Estimation

density plot of pair-wise transformations
Model Estimation

density plot of pair-wise transformations

origin
Model Estimation

density plot of pair-wise transformations
Model Estimation

cluster centers
Transform Mapping
Transform Mapping

\[ G_1^i \cdot G_1^j \rightarrow \{ ig_1 + jg_2 \} \]
Transform Mapping

\[ G_i^1 \cdot G_j^1 \rightarrow \{ ig_1 + jg_2 \} \]

\[ I \rightarrow \{ 0 \} \]
Transform Mapping

\[ G_i \cdot G_j \rightarrow \{ ig_1 + jg_2 \} \]

\[ I \rightarrow \{ 0 \} \]

Translation x Translation

\[ T \rightarrow (t_1, t_2) \]

Rotation x Scaling

\[ T \rightarrow (\theta, \log s) \]

Translation x Rotation

\[ T \rightarrow (t, \theta) \]
Structure Discovery

Input Model

Transform Analysis

Transform Clusters

Model Estimation

Aggregation

Regular Structures

Transform Generators
Model Estimation
Is there a Pattern?
Is there a Pattern?
Yes, there is!
Yes, there is!
Model Estimation
Model Estimation

• Global, non-linear optimization
Model Estimation

• Global, non-linear optimization
  – *simultaneously* detects *outliers* and *grid structure*
Model Estimation

• Grid fitting
  – *input:* cluster centers

\[ C = \{c_1, \ldots, c_n\} \]
Model Estimation

- Grid fitting
  - \textit{input}: cluster centers
    \[ C = \{ c_1, \ldots, c_n \} \]
  - \textit{unknowns}: grid generators

\[ x_{ij} = ig_1 + jg_2 \]

grid location \hspace{1cm} generating vectors
Model Estimation

- **Grid fitting**
  - *input*: cluster centers
    \[ C = \{ c_1, \ldots, c_n \} \]
  - *unknowns*: grid generators
    \[ x_{ij} = ig_1 + jg_2 \]
    \[ i \in [-n, n] \]
    \[ j \in [-m, m] \]
Model Estimation

- Fitting terms

\[ E_{C \rightarrow X} = \sum_{k=1}^{\left| C \right|} \left\| c_k - x(k) \right\|^2 \]

\text{closest grid point} \quad \text{cluster center}

\[ E_{X \rightarrow C} = \sum_i \sum_j \left\| x_{ij} - c(i, j) \right\|^2 \]

\text{grid point} \quad \text{closest cluster center}
Model Estimation

- Fitting terms

\[ E_{C \rightarrow X} = \sum_{k=1}^{\left| C \right|} \beta_k^2 \| c_k - x(k) \|^2 \]

\[ E_{X \rightarrow C} = \sum_i \sum_j \alpha_{i,j}^2 \| x_{i,j} - c(i,j) \|^2 \]
Model Estimation

• Fitting terms

\[ E_{C \rightarrow X} = \sum_{k=1}^{\mid C \mid} \beta_k^2 \| c_k - x(k) \|^2 \]

\[ E_{X \rightarrow C} = \sum_i \sum_j \alpha_{ij}^2 \| x_{ij} - c(i, j) \|^2 \]

• Data and grid confidence terms

\[ E_\alpha = \sum_i \sum_j (1 - \alpha_{ij}^2)^2 \quad E_\beta = \sum_k (1 - \beta_k^2)^2 \]
Model Estimation

• Global, non-linear optimization
  – *simultaneously* detects *outliers* and *grid structure*
Model Estimation

• Global, non-linear optimization
  – \textit{simultaneously} detects \textit{outliers} and \textit{grid structure}

\[ \alpha_{ij}, \beta_k \]

\[ \text{final grid}, \quad \text{grid confidence } \alpha_{ij}, \quad \text{data confidence } \beta_k \]
Structure Discovery

Input Model -> Transform Analysis -> Transform Clusters

Structure Discovery -> Model Estimation

Regular Structures -> Aggregation

Transform Generators
Aggregation

• Region-growing to extract repetitive elements
Aggregation

- Region-growing to extract repetitive elements
- Simultaneous registration
Aggregation

- Region-growing to extract repetitive elements
- Simultaneous registration

\[ H_+ \approx H + \epsilon D \cdot H \]
Aggregation

- Region-growing to extract repetitive elements
- Simultaneous registration

\[
H_+ \approx H + \epsilon D \cdot H
\]

\[
T^k_+ \approx (H + \epsilon D \cdot H)^k
\]
Structure Discovery

Input Model → Transform Analysis → Transform Clusters

Structure Discovery → Model Estimation

Regular Structures ← Aggregation ← Transform Generators
Structure Discovery

Input Model

Structure Discovery

Regular Structures

Transform Analysis

Transform Clusters

Model Estimation

Transform Generators

Aggregation
Results and Applications
Amphitheater
Amphitheater

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Amphitheater

Rot: 72

Rot x Trans: 72 x 3

Rot: 35

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Robustness
Robustness
Robustness

100%  61°

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Robustness

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Robustness
Geometry Synthesis

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Geometry Synthesis

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Geometry Synthesis

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Discovering Structural Regularity in 3D Geometry
Chambord Castle
Chambord Castle
Chambord Castle
Chambord Castle [Mitra et al. `06]
Chambord Castle
Chambord Castle
Chambord Castle
Outdoor Scan
Outdoor Scan
Outdoor Scan
Scan Completion

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Scan Completion

Discovering Structural Regularity in 3D Geometry
Chambered Nautilus
Chambered Nautilus
Chambered Nautilus
Chambered Nautilus
Scan Completion
Geometry Synthesis
Geometry Synthesis

Discovering Structural Regularity in 3D Geometry
Geometry Synthesis

Discovering Structural Regularity in 3D Geometry
Geometry Synthesis

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Discovering Structural Regularity in 3D Geometry

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Observations
Observations

- Warped structures
Observations

• Warped structures
• Size of grid vs accuracy
Observations

- Warped structures
- Size of grid vs accuracy
- Choice of parameters
Structure Discovery

Input Model → Transform Analysis → Transform Clusters → Model Estimation → Transform Generators → Aggregation → Regular Structures
Structure Discovery

- Algorithm is fully automatic
Structure Discovery

- Algorithm is fully automatic
- Requires no prior information on size, shape, or location of repetitive elements
Structure Discovery

- Algorithm is fully automatic
- Requires no prior information on size, shape, or location of repetitive elements
- Robust, efficient, independent of dimension

→ general tool for scientific data analysis
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Thank You
Thank You