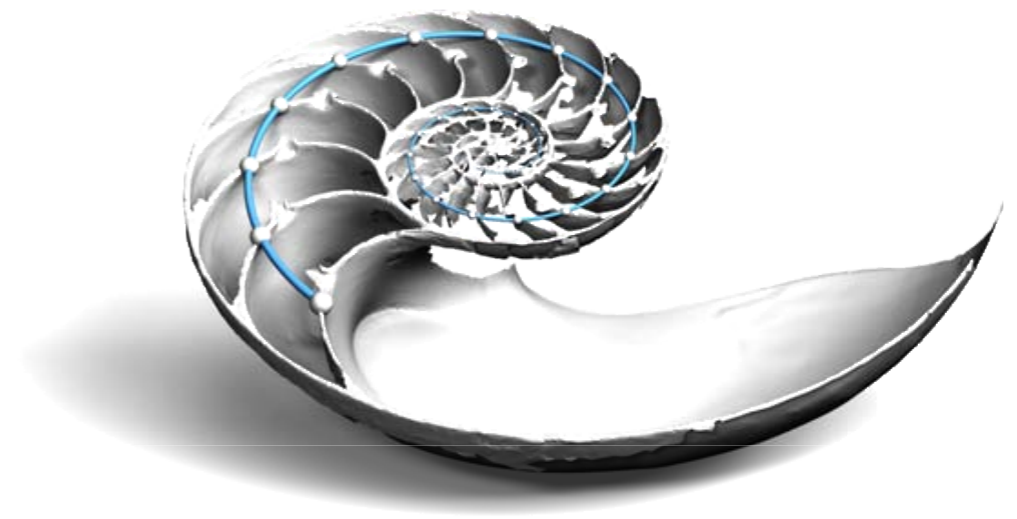
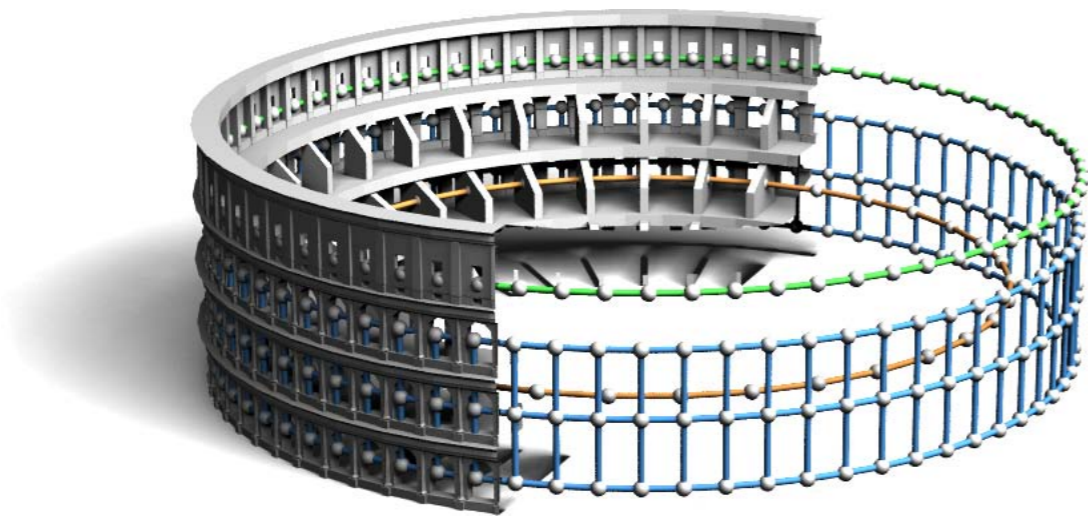


# Discovering Structural Regularity in 3D Geometry



Mark Pauly  
ETH Zurich

***Niloy J. Mitra***  
IIT Delhi

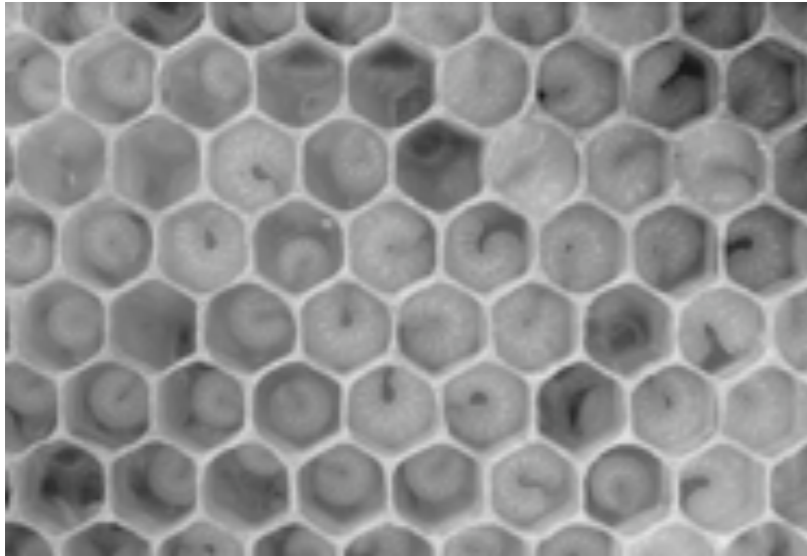
Johannes Wallner  
TU Graz

Helmut Pottmann  
TU Vienna

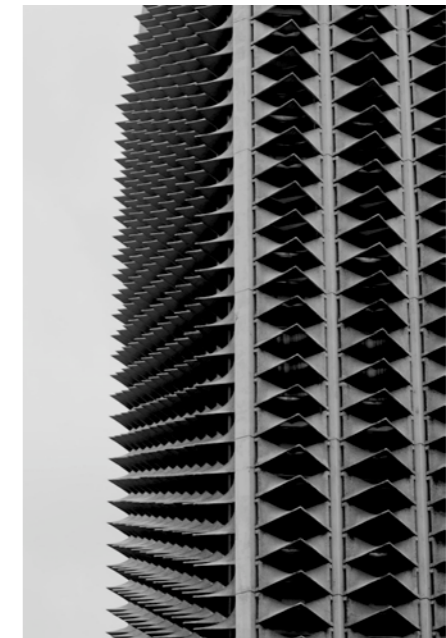
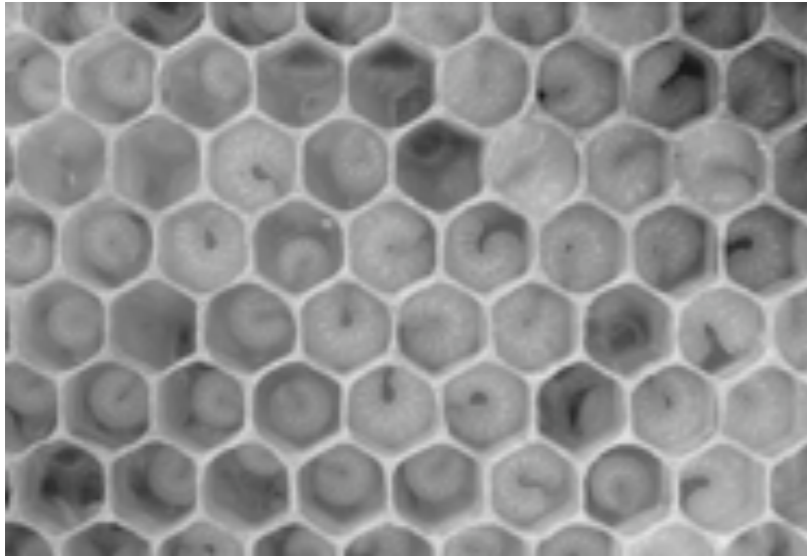
Leonidas Guibas  
Stanford University



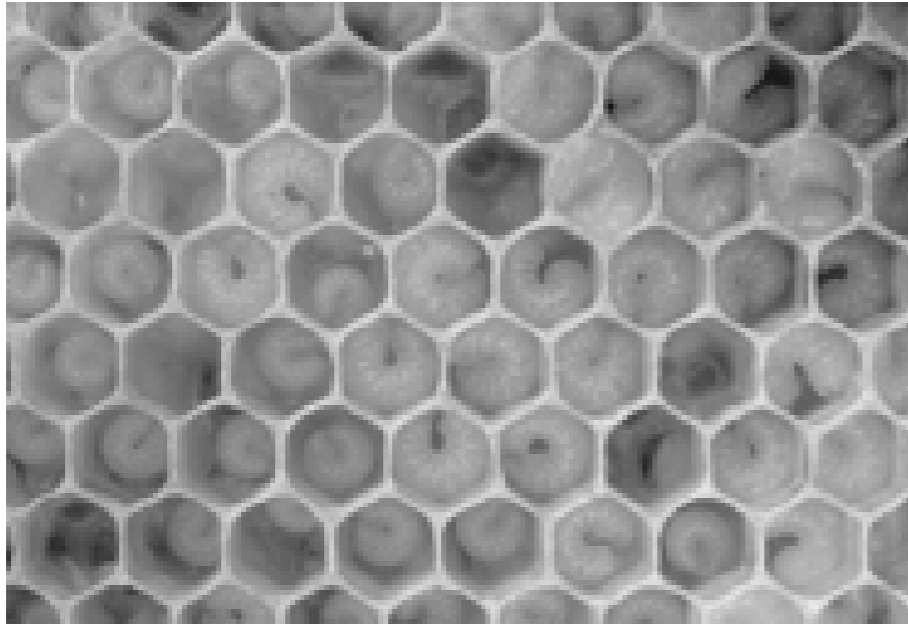
# Regular Structures



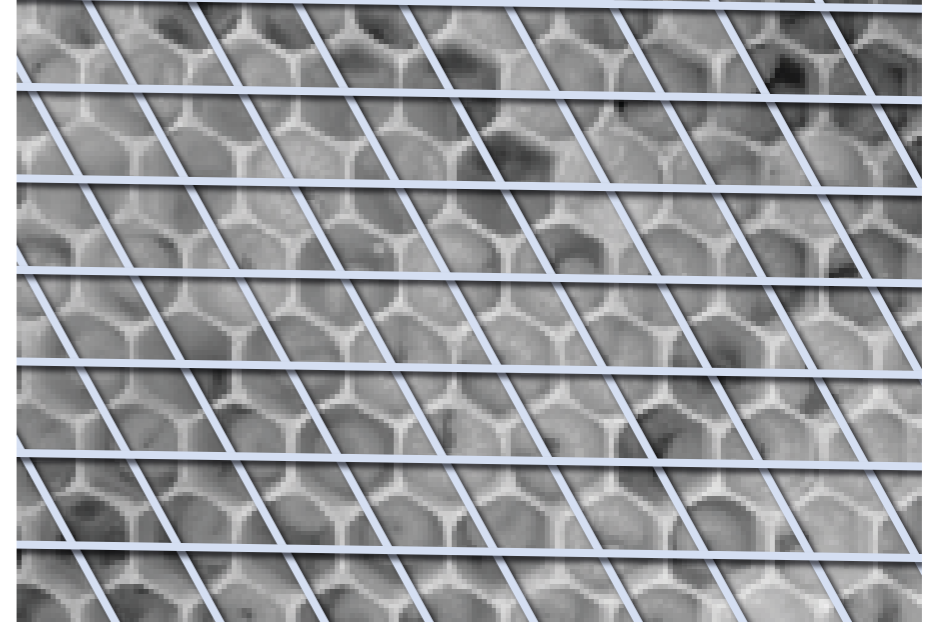
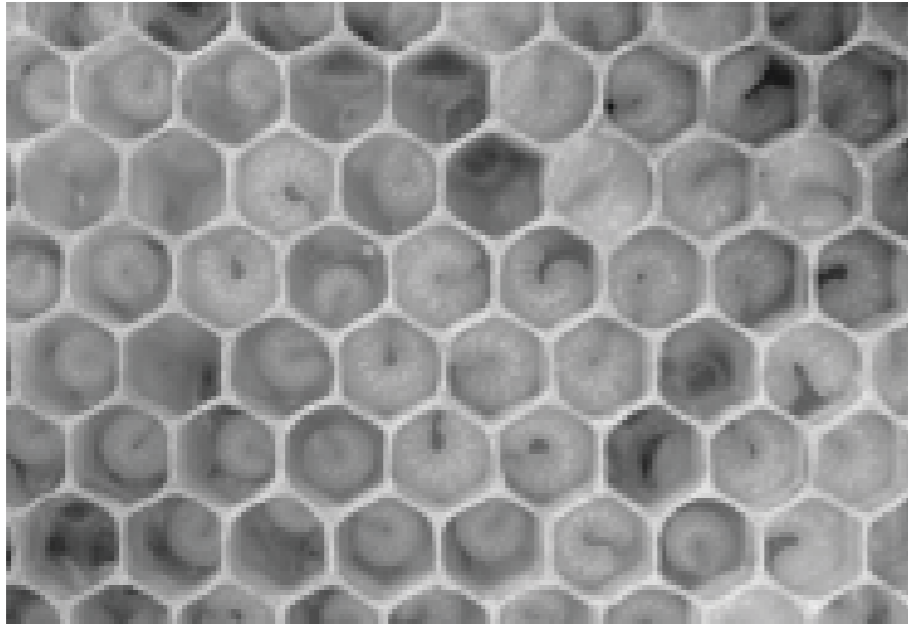
# Regular Structures



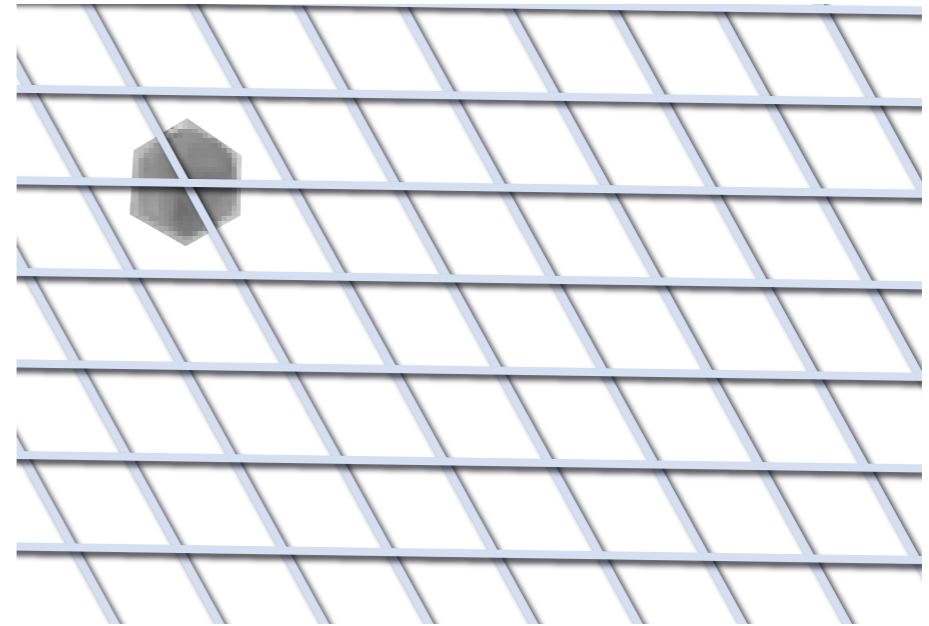
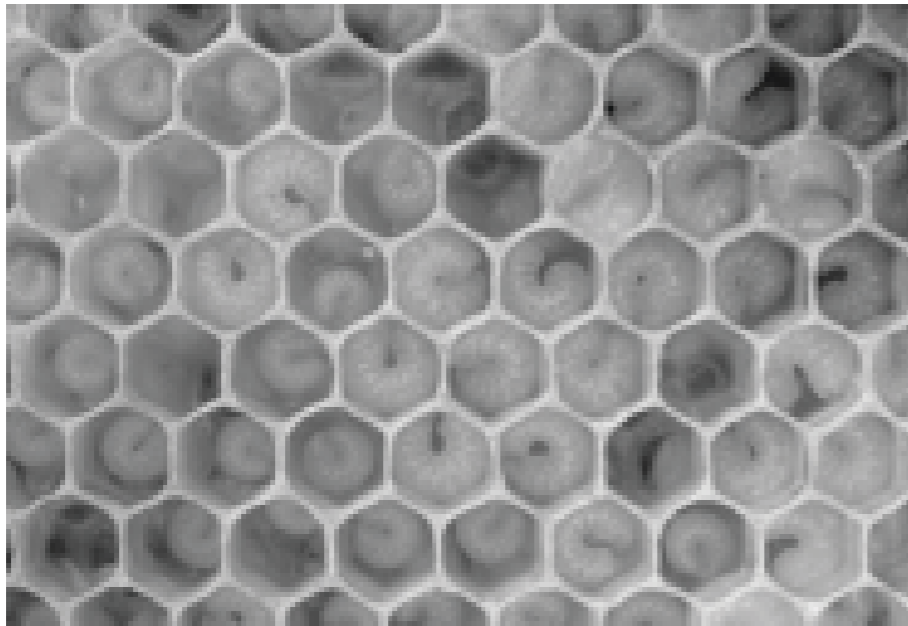
# Regular Structure



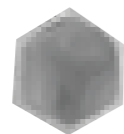
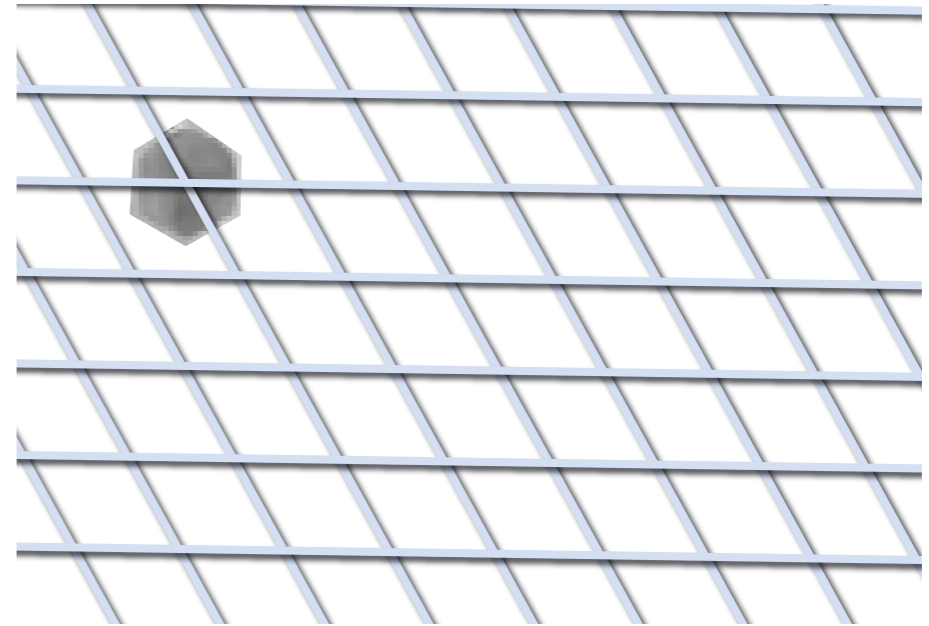
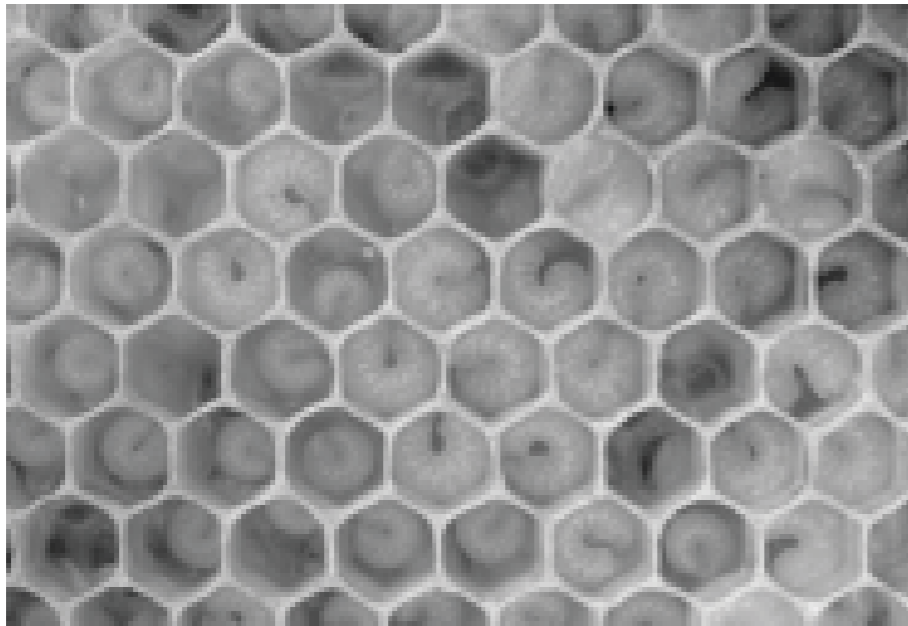
# Regular Structure



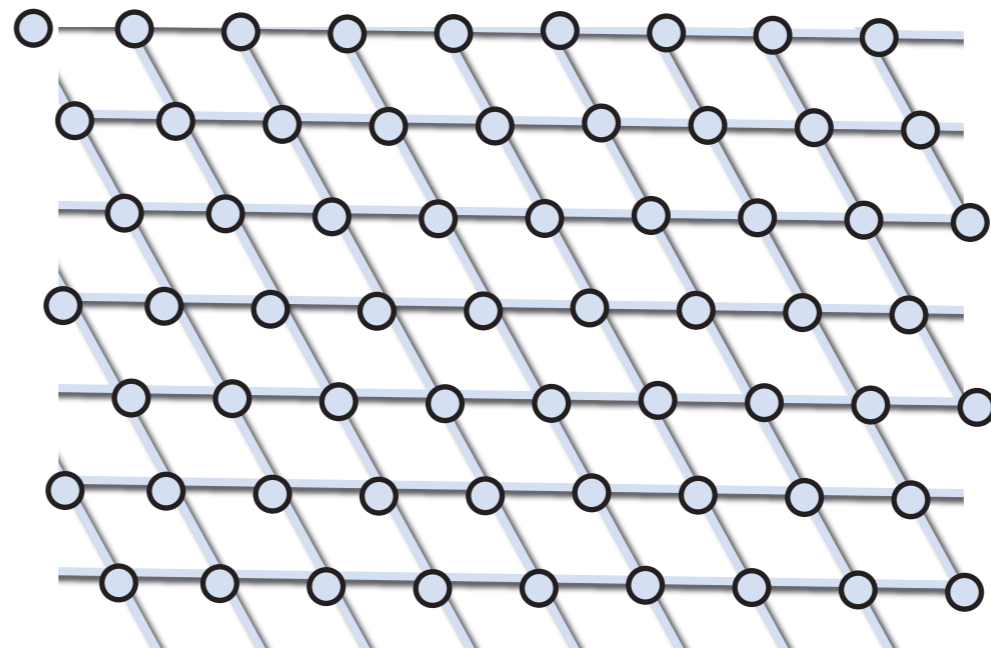
# Regular Structure



# Regular Structure



+



# Motivation





# Motivation

- Regularity  $\longrightarrow$  form, semantics



# Motivation

- Regularity  $\longrightarrow$  form, semantics



# Motivation

- Regularity  $\longrightarrow$  form, semantics



- Scan cleaning, completion

# Motivation

- Regularity  $\longrightarrow$  form, semantics



- Scan cleaning, completion
- Compression

# Motivation

- Regularity  $\longrightarrow$  form, semantics



- Scan cleaning, completion
- Compression
- Geometric edits, synthesis

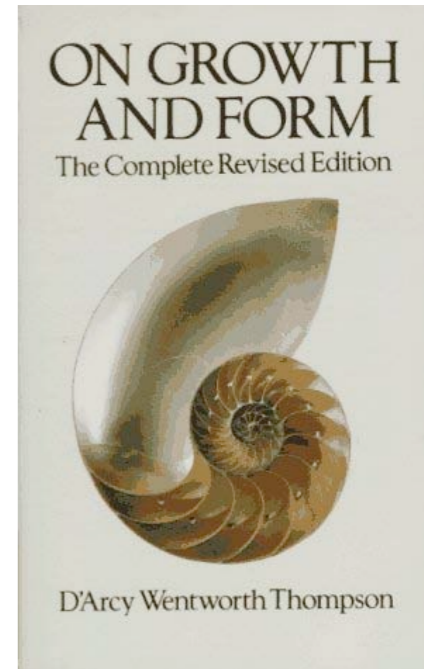
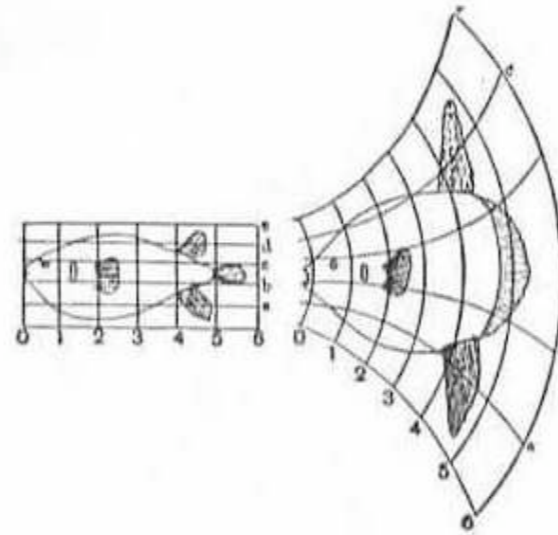
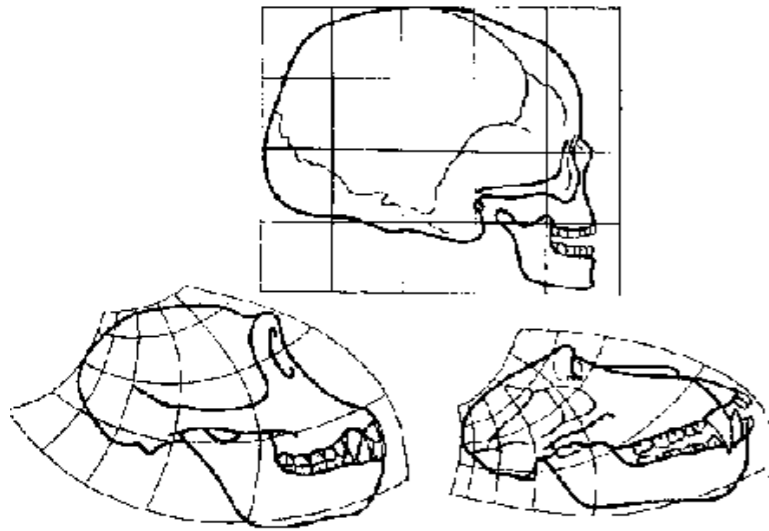
# Motivation

- Regularity  $\longrightarrow$  form, semantics



- Scan cleaning, completion
- Compression
- Geometric edits, synthesis
- Growth laws or design principles

# Inspiration



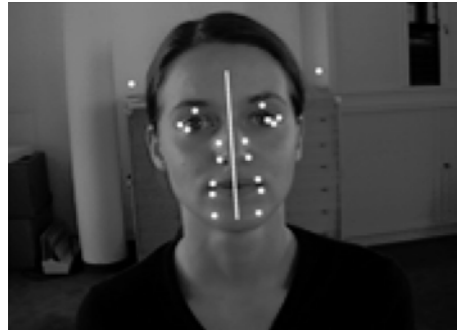
On Growth and Form  
[Thompson 1917]

# Related Work

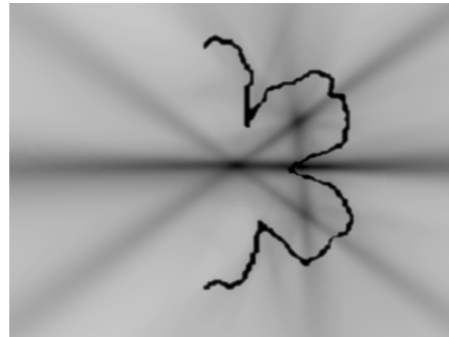
---



# Related Work



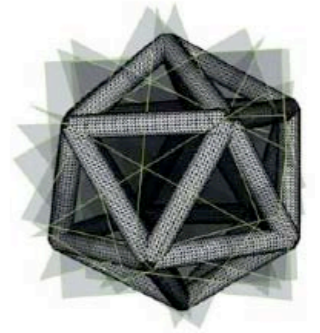
[Podolak et al. '06]



[Loy, Eklundh '06]

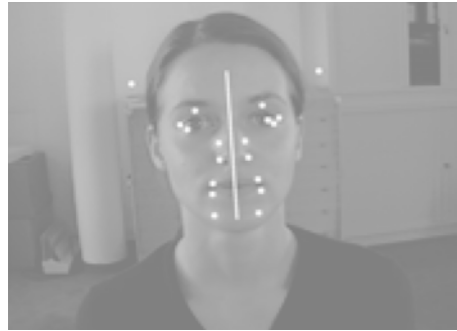


[Mitra et al. '06]



[Martinet et al. '07]

# Related Work



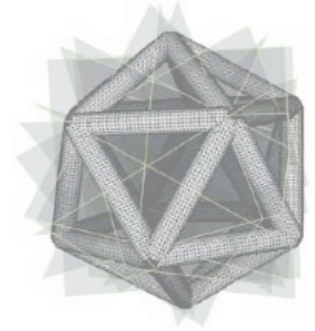
[Podolak et al. '06]



[Loy, Eklundh '06]



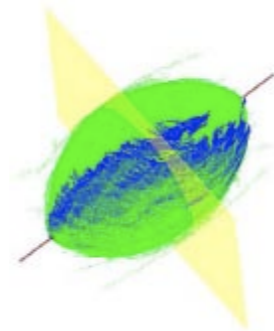
[Mitra et al. '06]



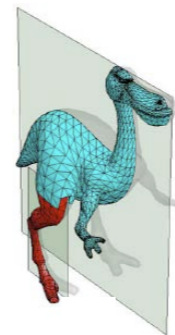
[Martinet et al. '07]



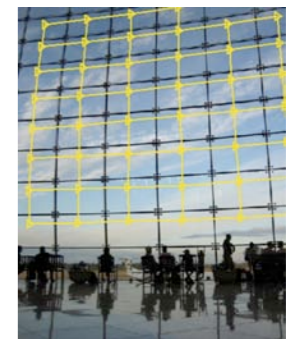
[Funkhouser et al. '05]



[Thrun, Wegbreit '05]

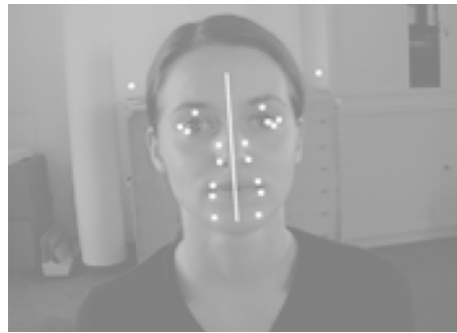


[Shimari et al. '06]



[Liu et al. '08]

# Related Work



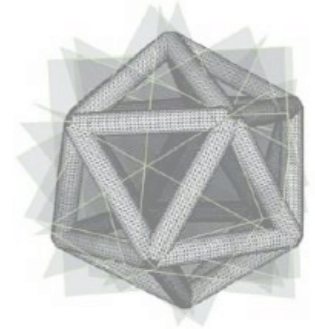
[Podolak et al. '06]



[Loy, Eklundh '06]



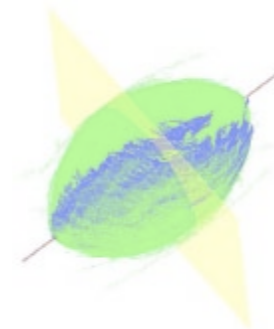
[Mitra et al. '06]



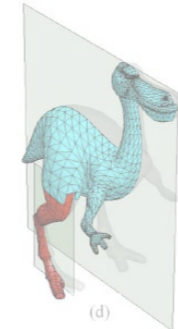
[Martinet et al. '07]



[Funkhouser et al. '05]



[Thrun, Wegbreit '05]



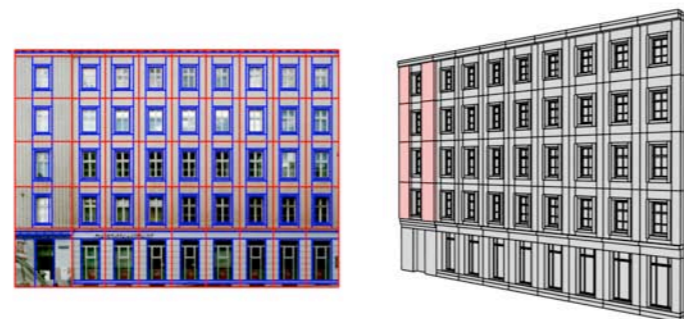
[Shimari et al. '06]



[Liu et al. '08]



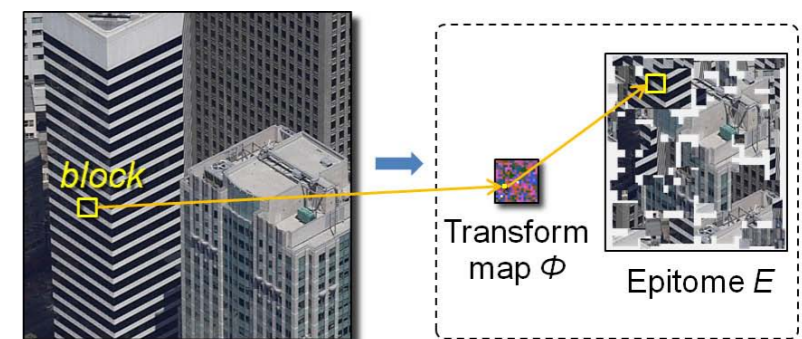
[Hays et al. '06]



[Mueller et al. '07]

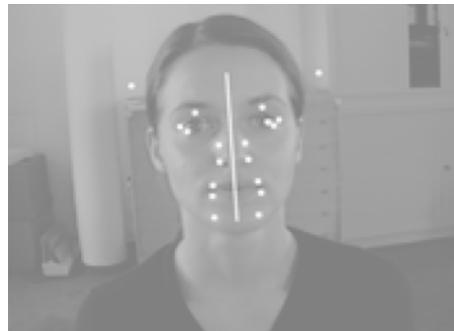


[Baudes et al. '08]



[Wang et al. '08]

# Related Work



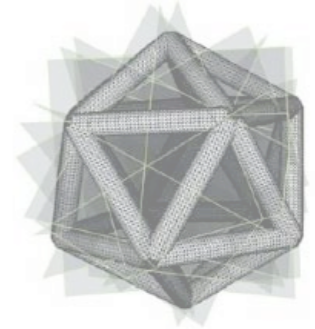
[Podolak et al. '06]



[Loy, Eklundh '06]



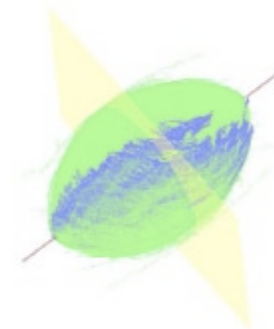
[Mitra et al. '06]



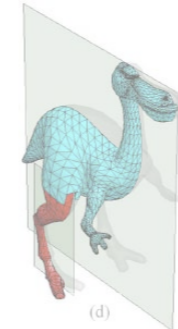
[Martinet et al. '07]



[Funkhouser et al. '05]



[Thrun, Wegbreit '05]



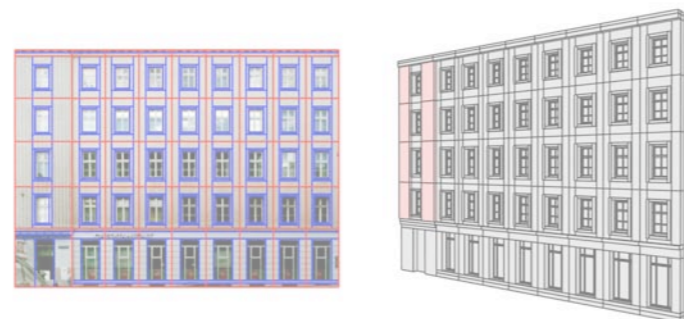
[Shimari et al. '06]



[Liu et al. '08]



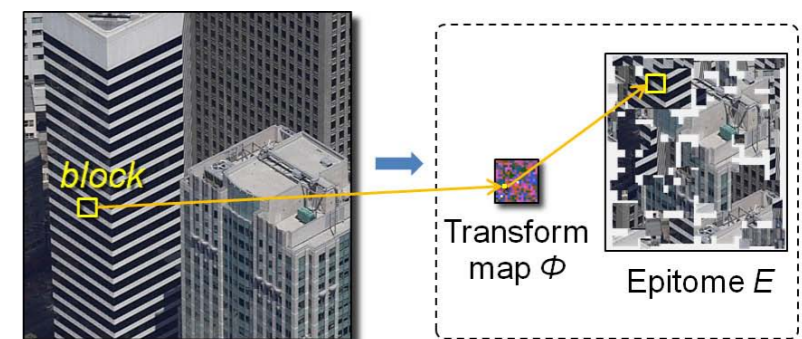
[Hays et al. '06]



[Mueller et al. '07]

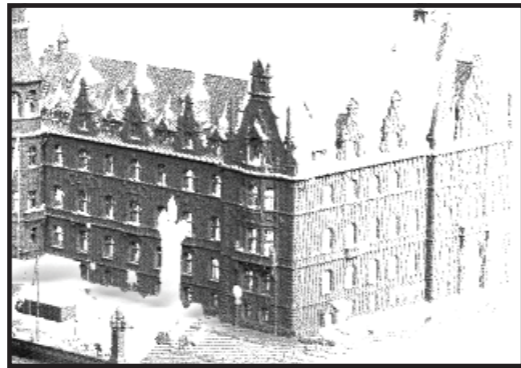


[Baudes et al. '08]



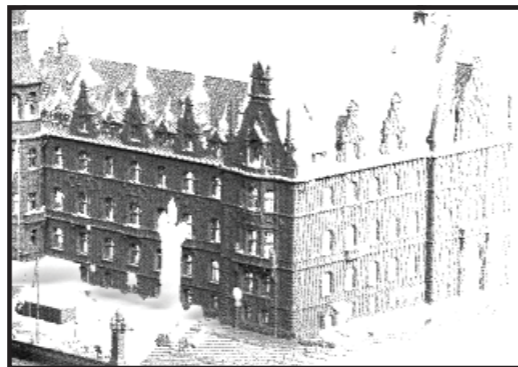
[Wang et al. '08]

# Structure Discovery

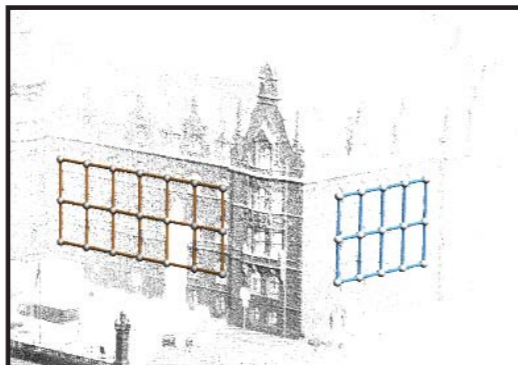
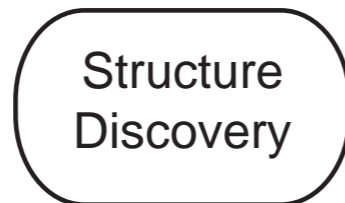


Input Model

# Structure Discovery

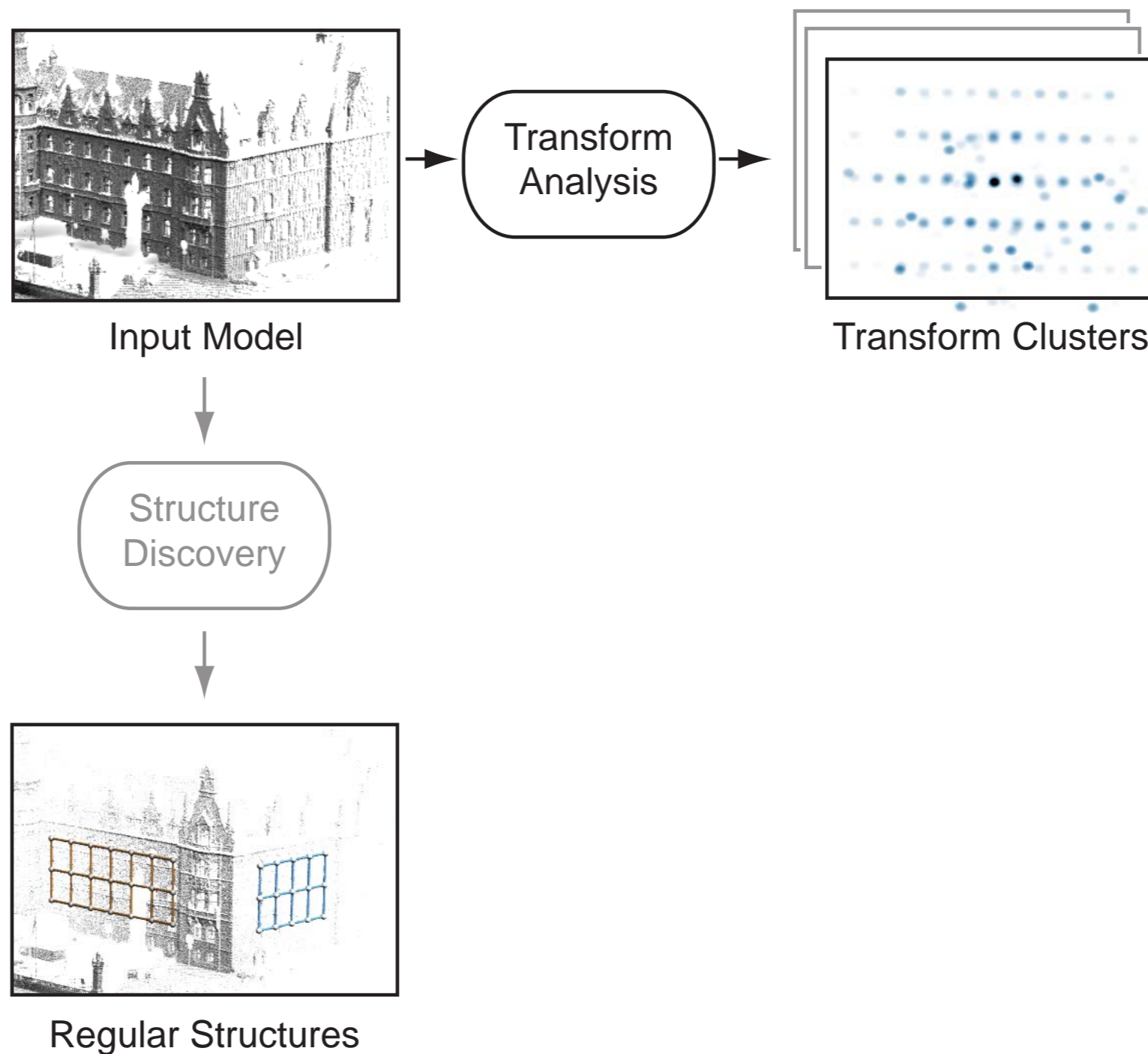


Input Model

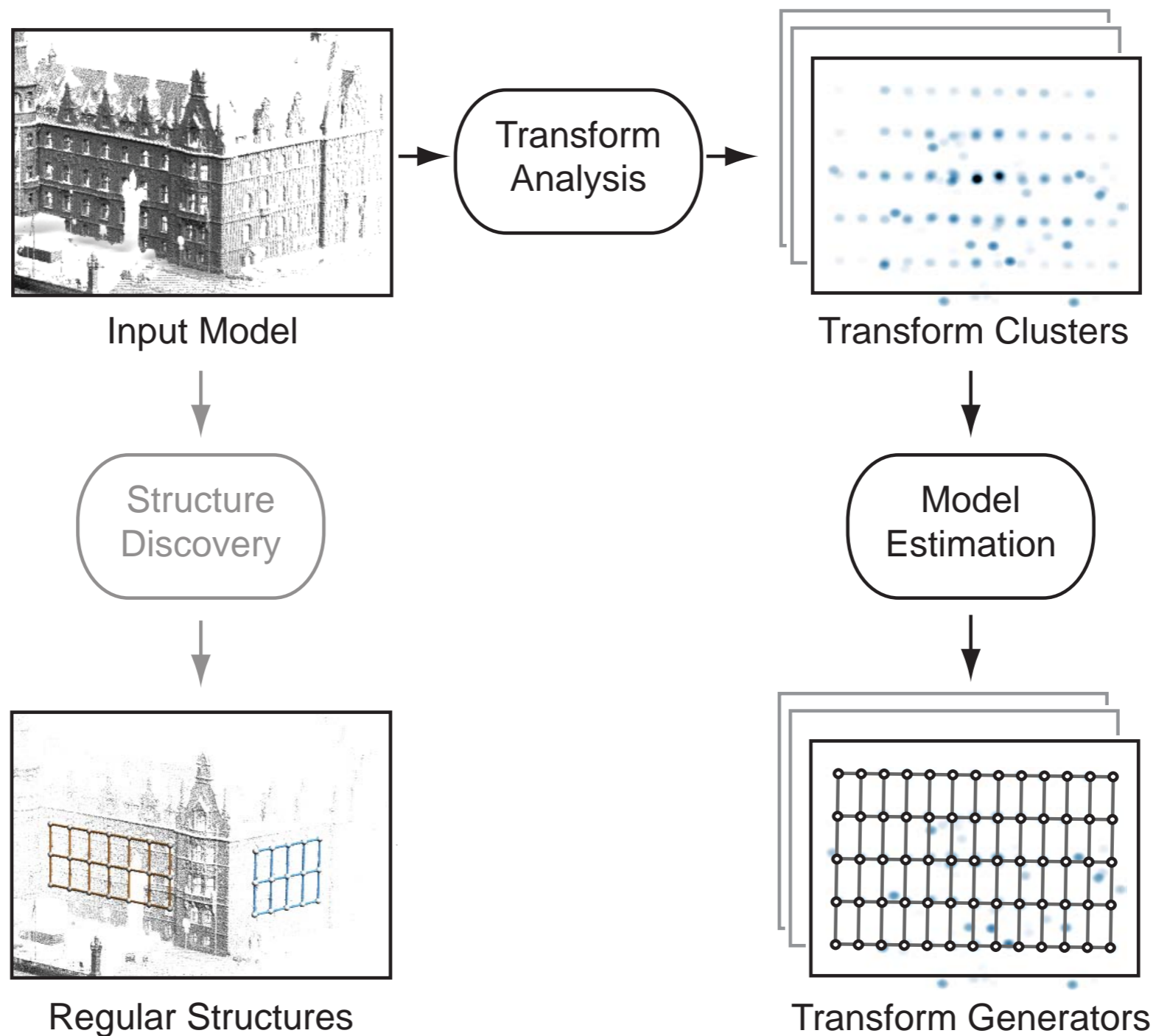


Regular Structures

# Structure Discovery

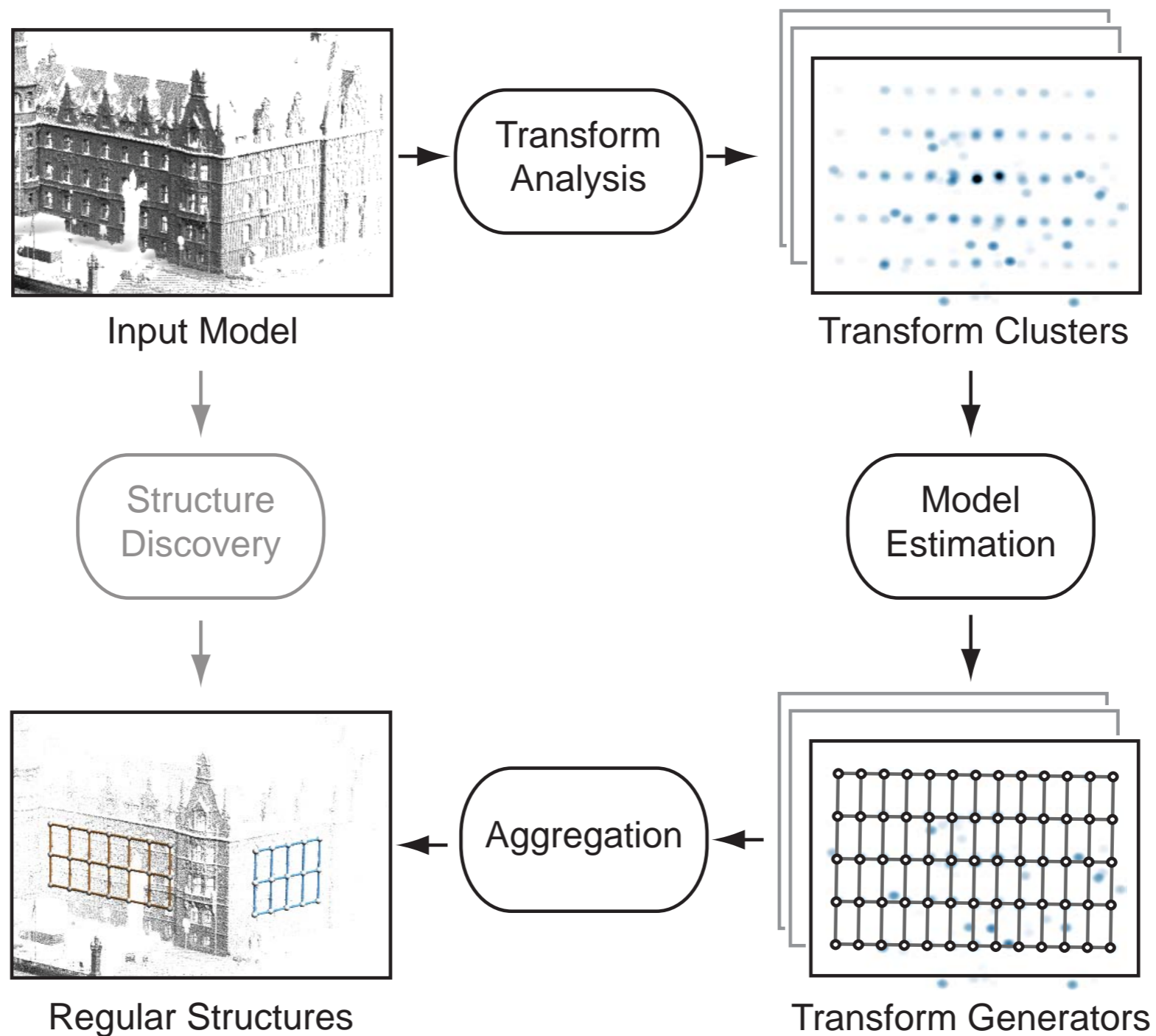


# Structure Discovery

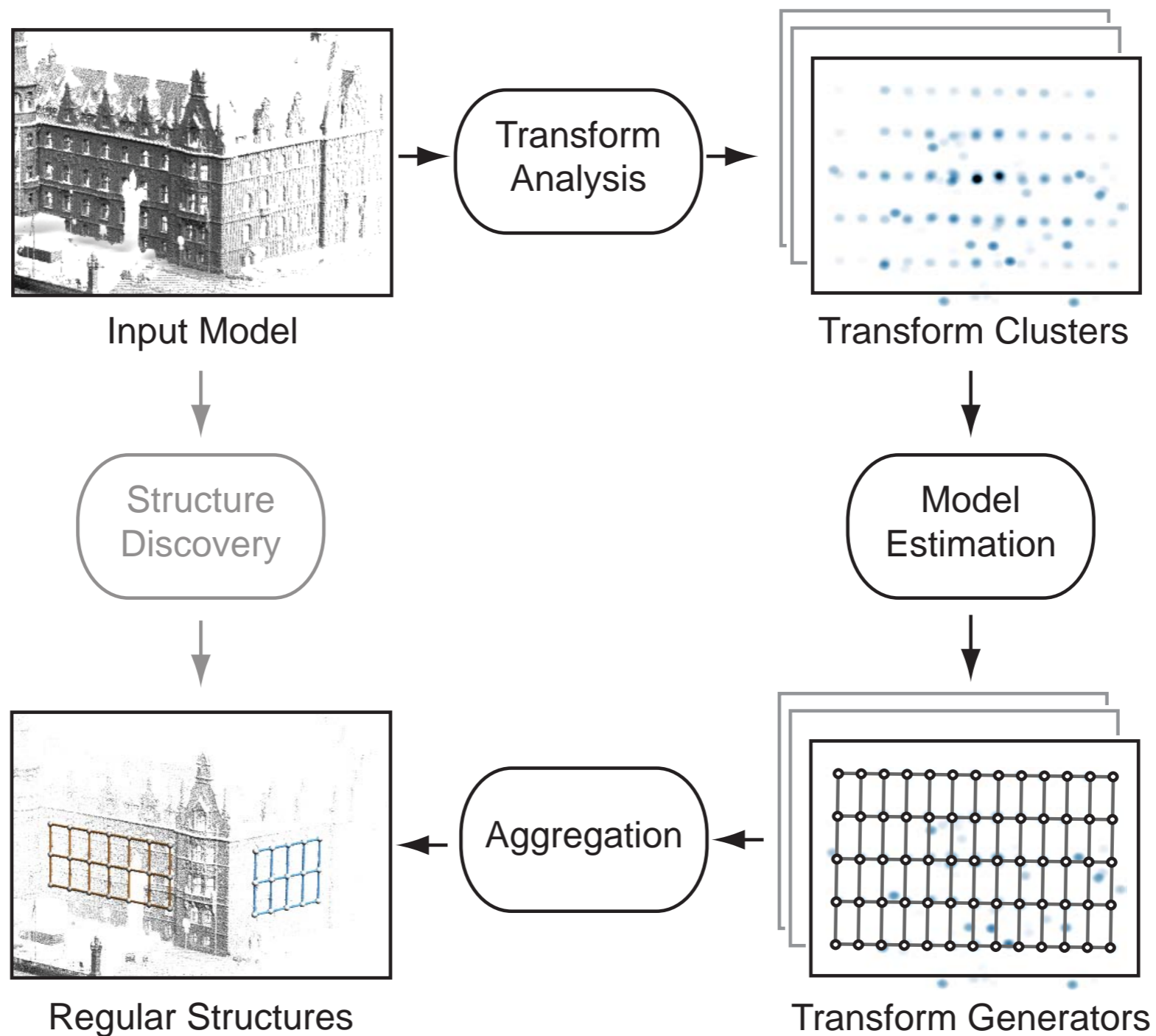




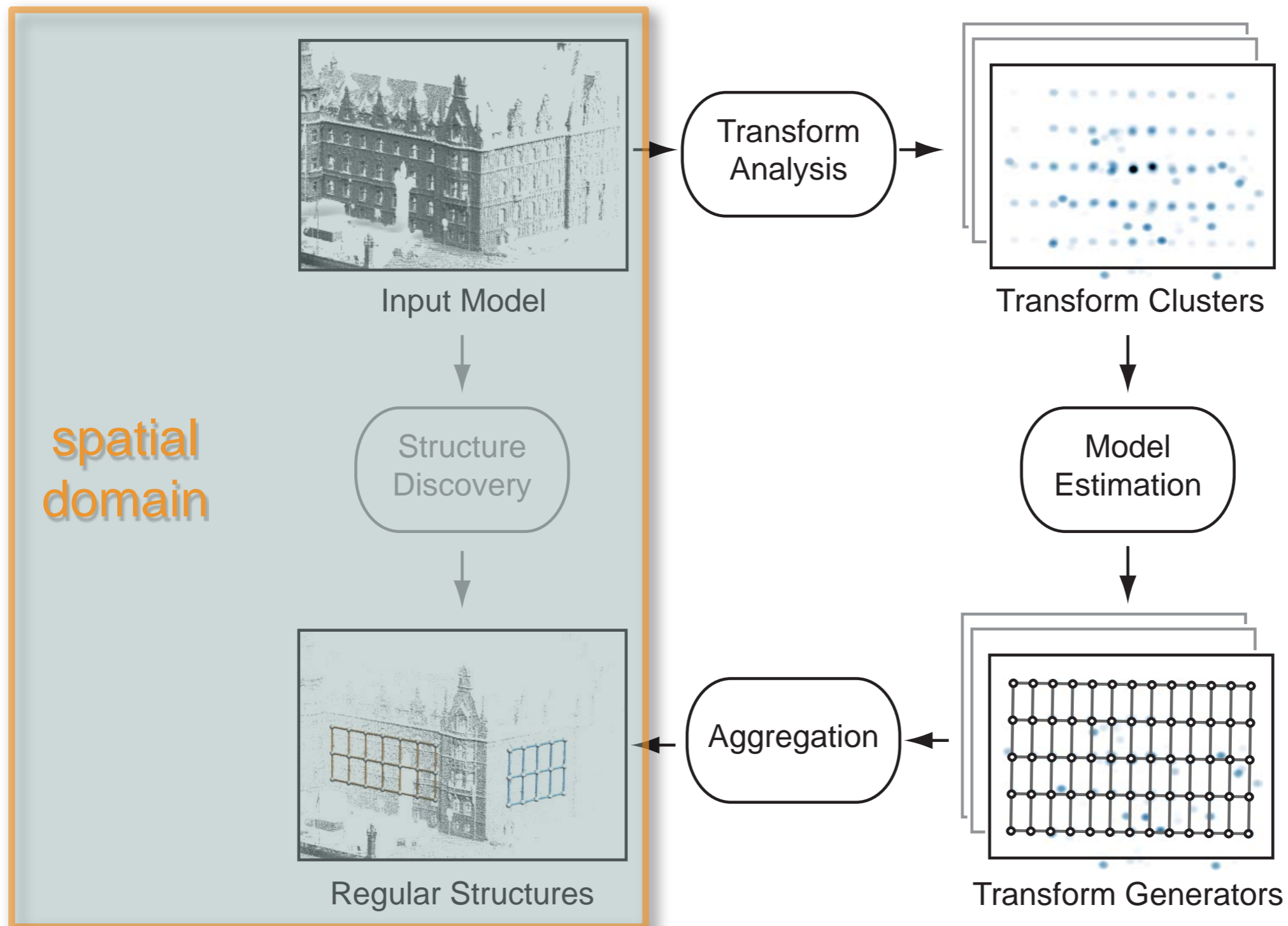
# Structure Discovery



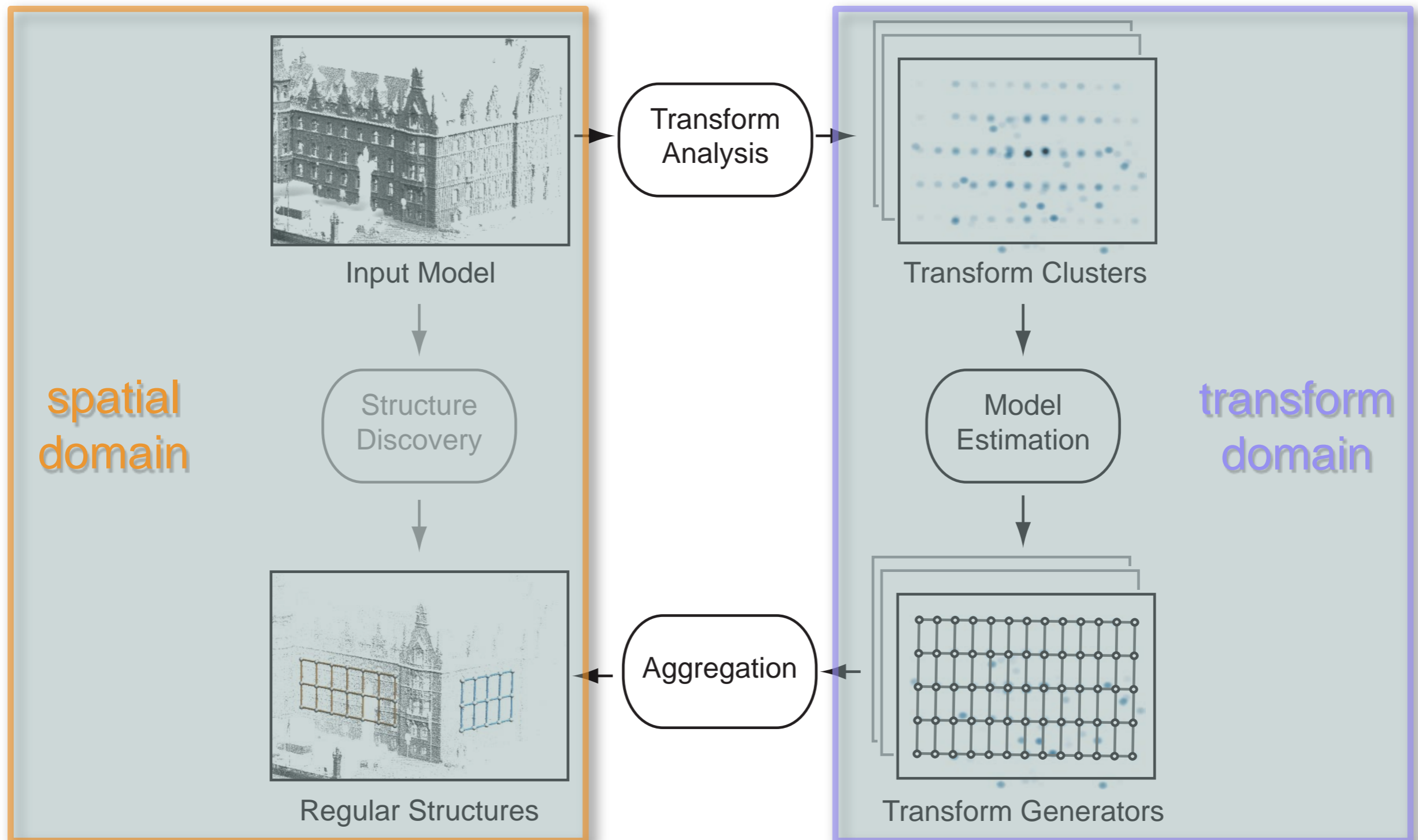
# Structure Discovery



# Structure Discovery



# Structure Discovery

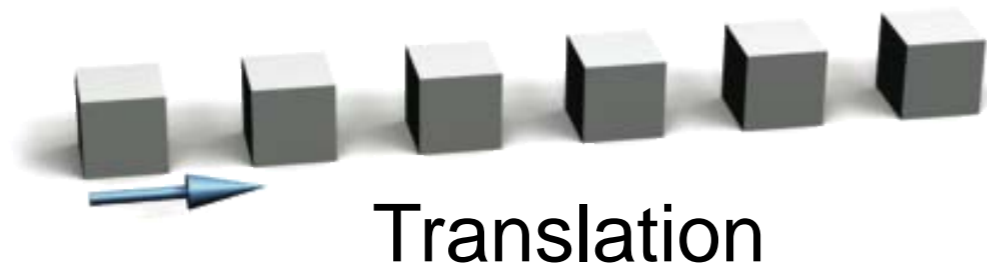


# Repetitive Structures

- Invariance under transformations

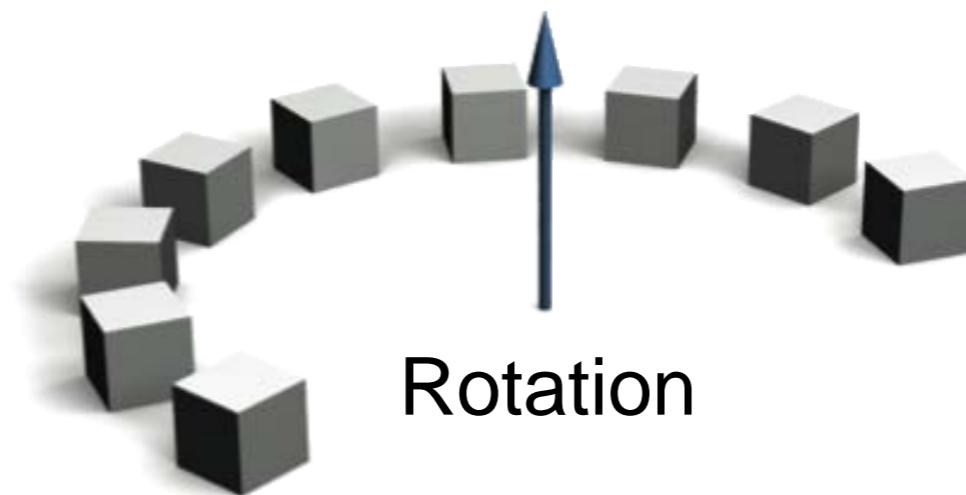
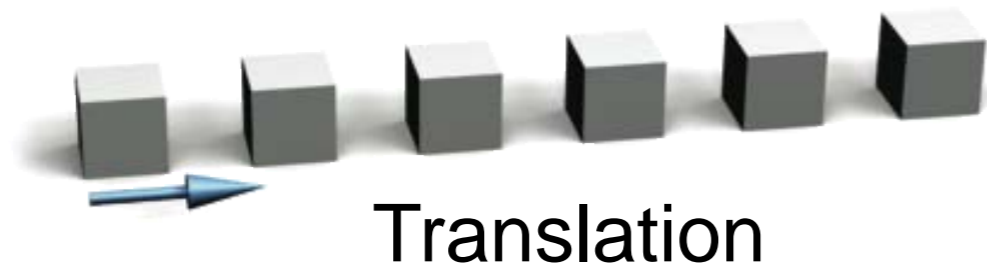
# Repetitive Structures

- Invariance under transformations



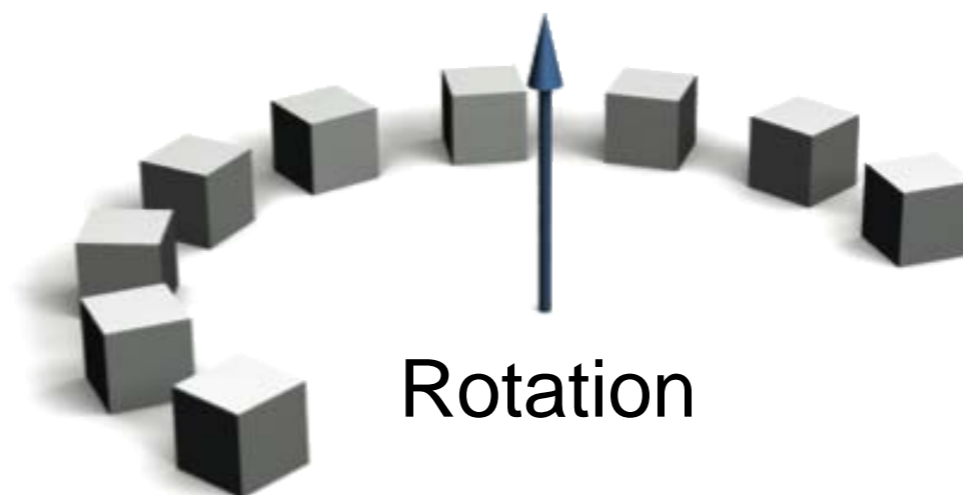
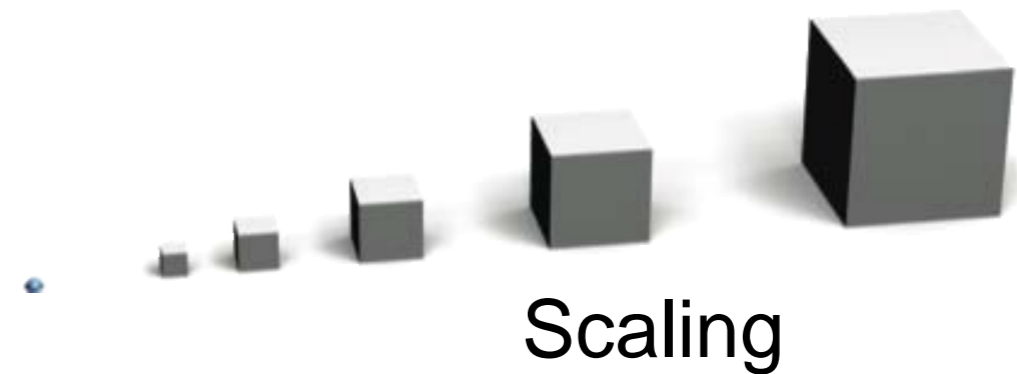
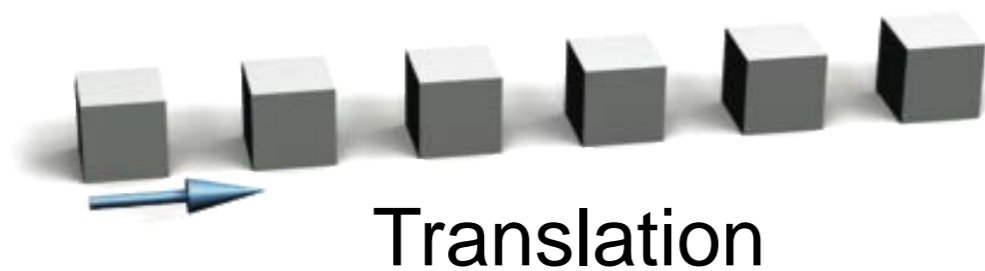
# Repetitive Structures

- Invariance under transformations



# Repetitive Structures

- Invariance under transformations





# Repetitive Structures

- Invariance under transformations



Translation + Rotation

# Repetitive Structures

- Invariance under transformations



Translation + Rotation



Rotation + Scaling

# Repetitive Structures

- Invariance under transformations



Translation + Rotation



Rotation + Scaling

1-parameter patterns

# Repetitive Structures

- Invariance under transformations



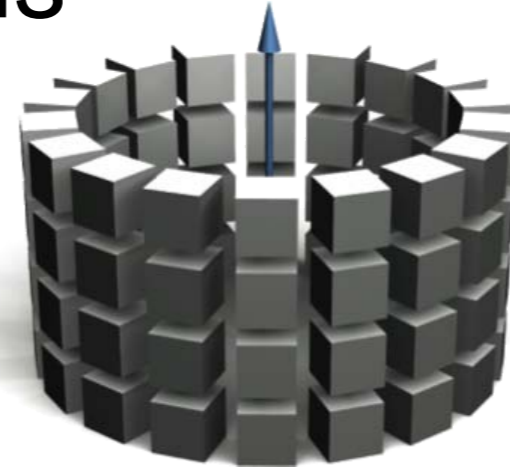
Translation  $\times$  Translation

# Repetitive Structures

- Invariance under transformations



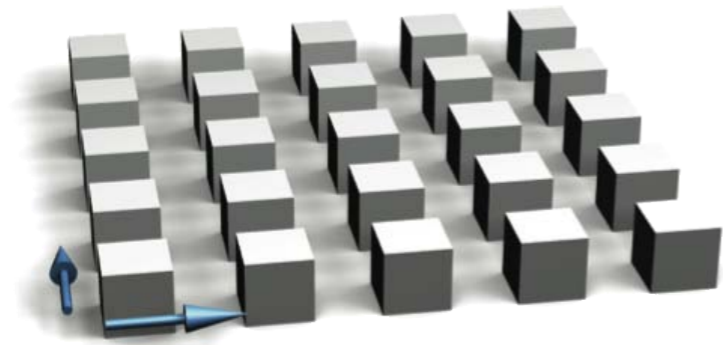
Translation  $\times$  Translation



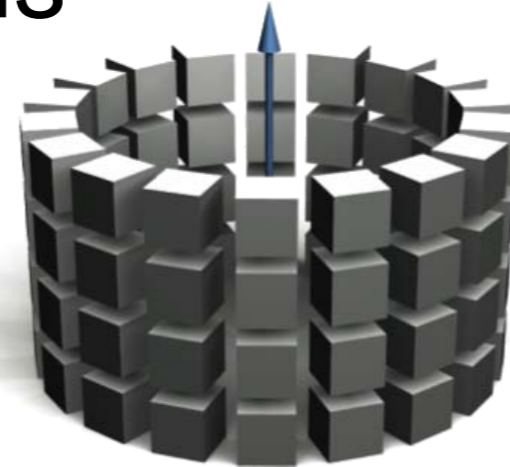
Translation  $\times$  Rotation

# Repetitive Structures

- Invariance under transformations



Translation  $\times$  Translation



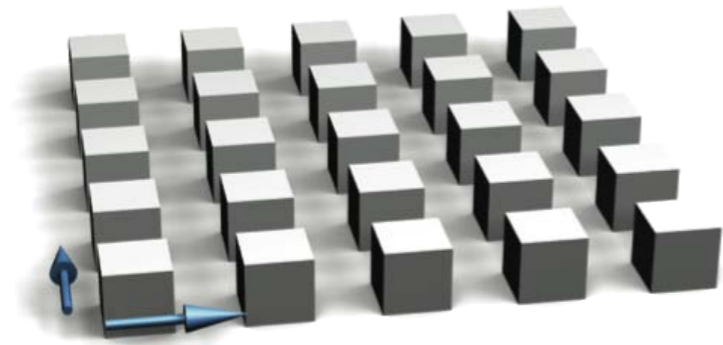
Translation  $\times$  Rotation



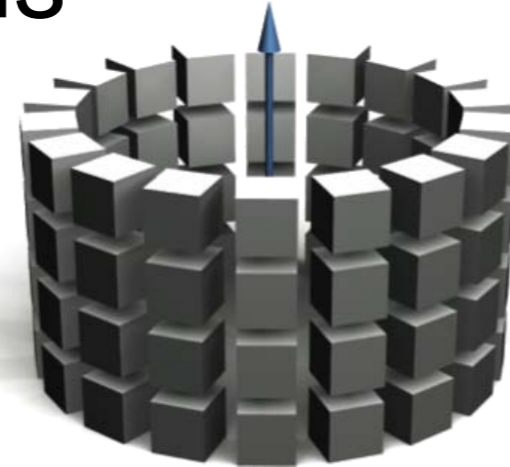
Rotation  $\times$  Scaling

# Repetitive Structures

- Invariance under transformations



Translation  $\times$  Translation



Translation  $\times$  Rotation

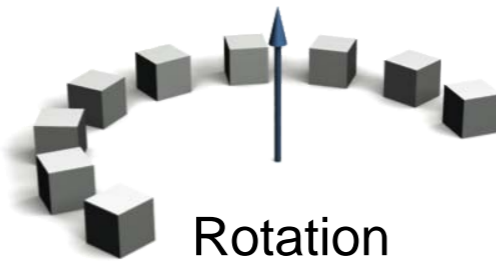
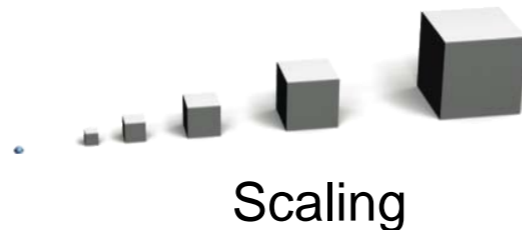
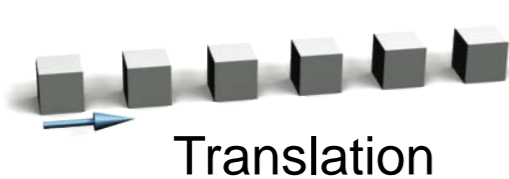


Rotation  $\times$  Scaling

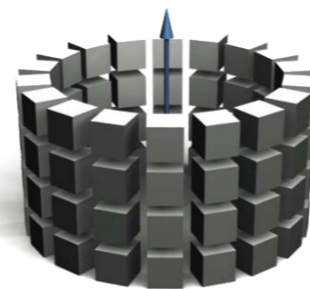
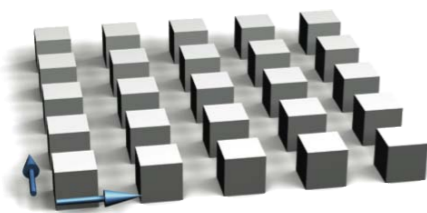
2-parameter ***commutative*** patterns

# Repetitive Structures

- 1-parameter groups

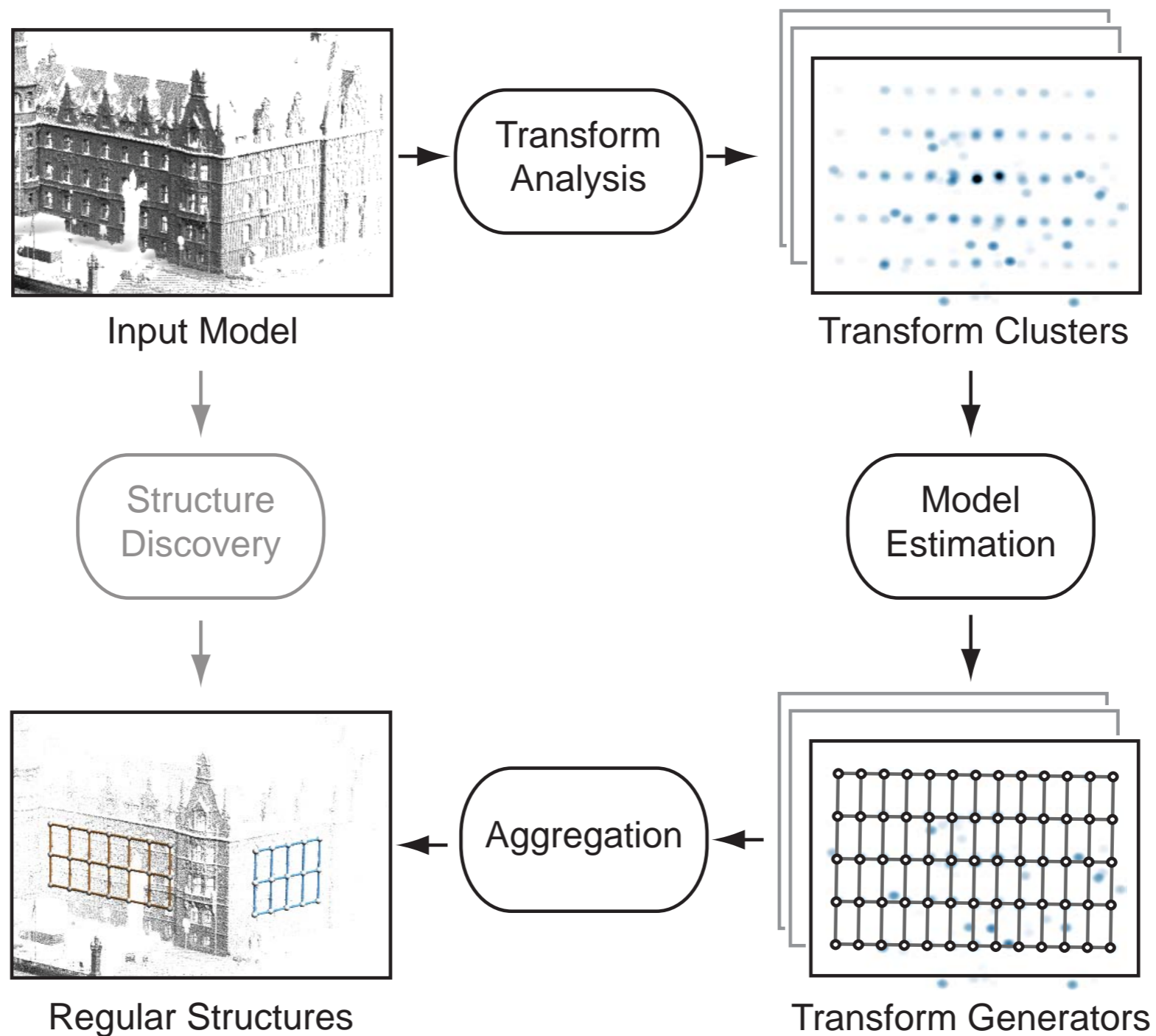


- *Commutative* 2-parameter groups

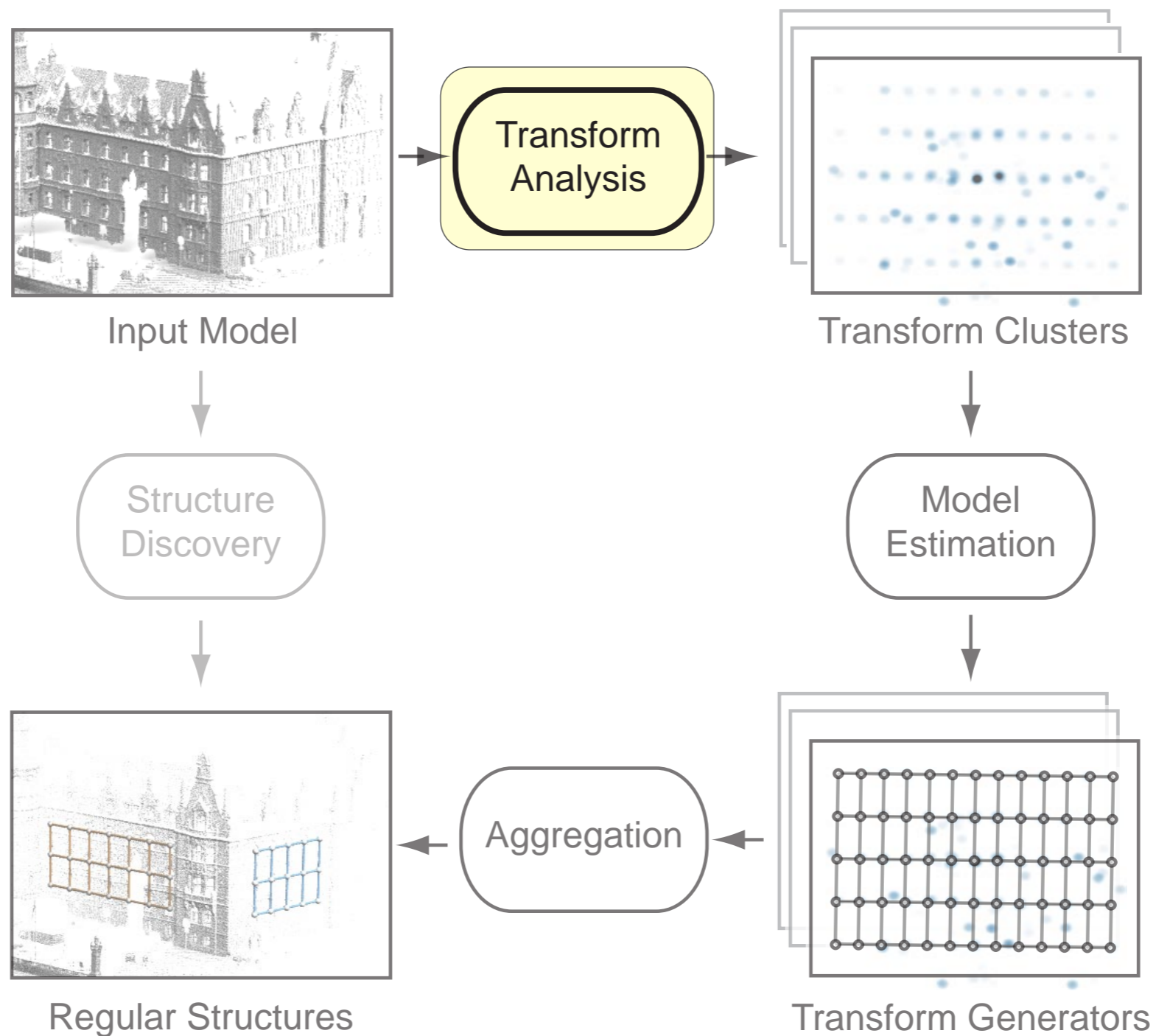




# Structure Discovery



# Structure Discovery



# Transformations



# Transformations [Mitra et al. '06]



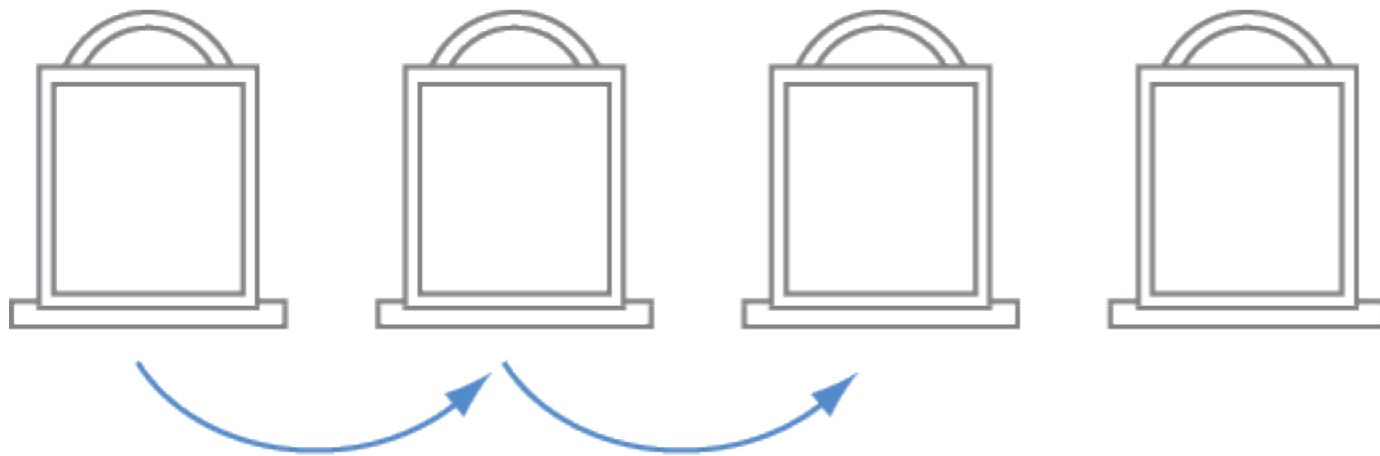
# Transformations



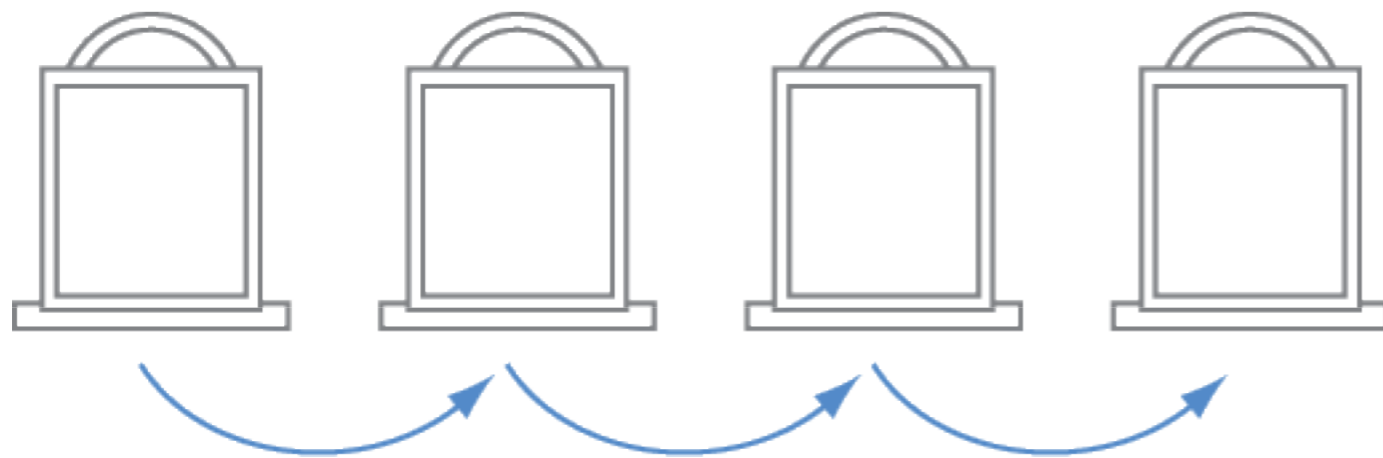
# Transformations



# Transformations

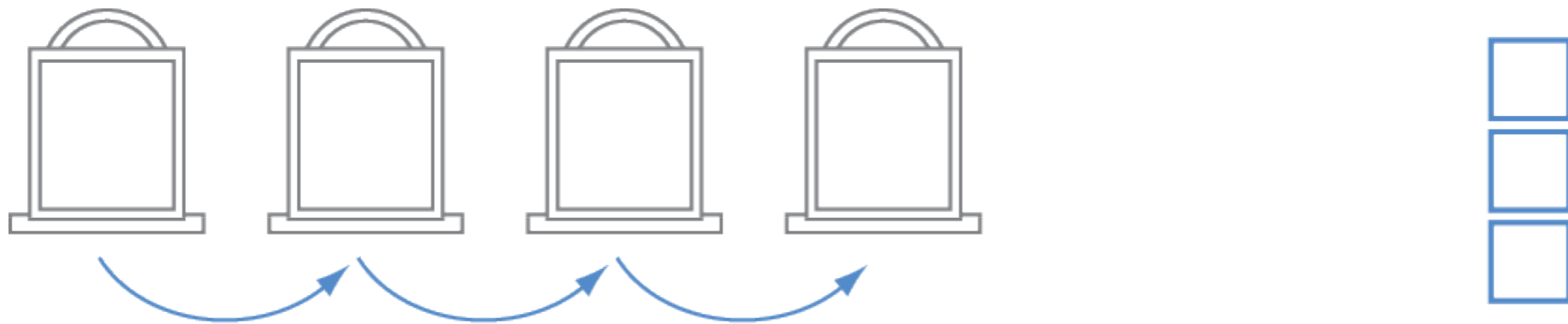


# Transformations

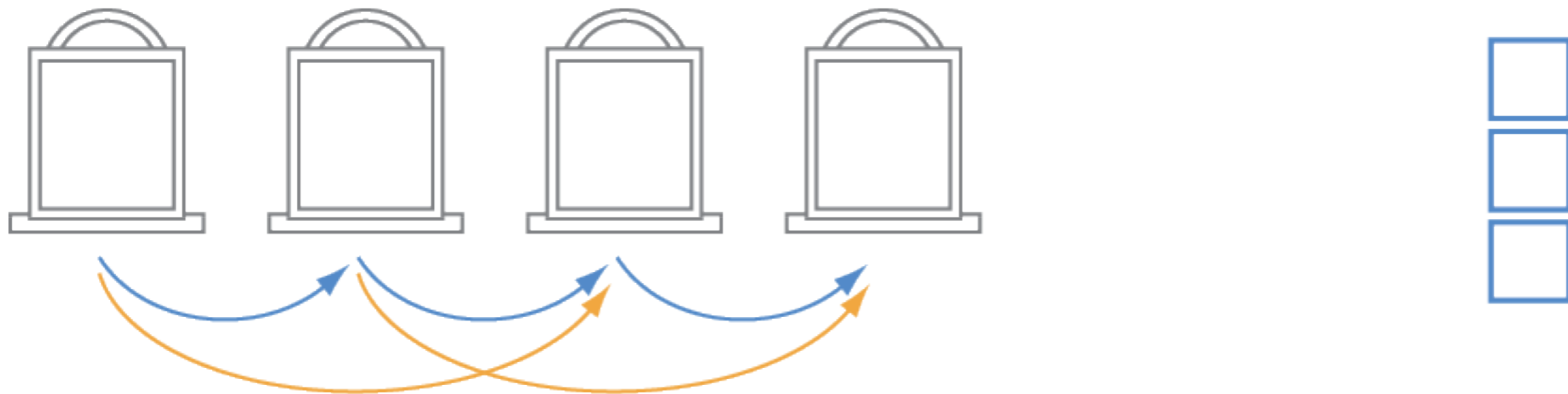




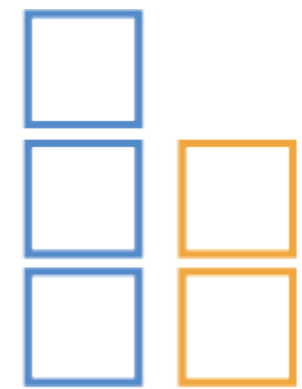
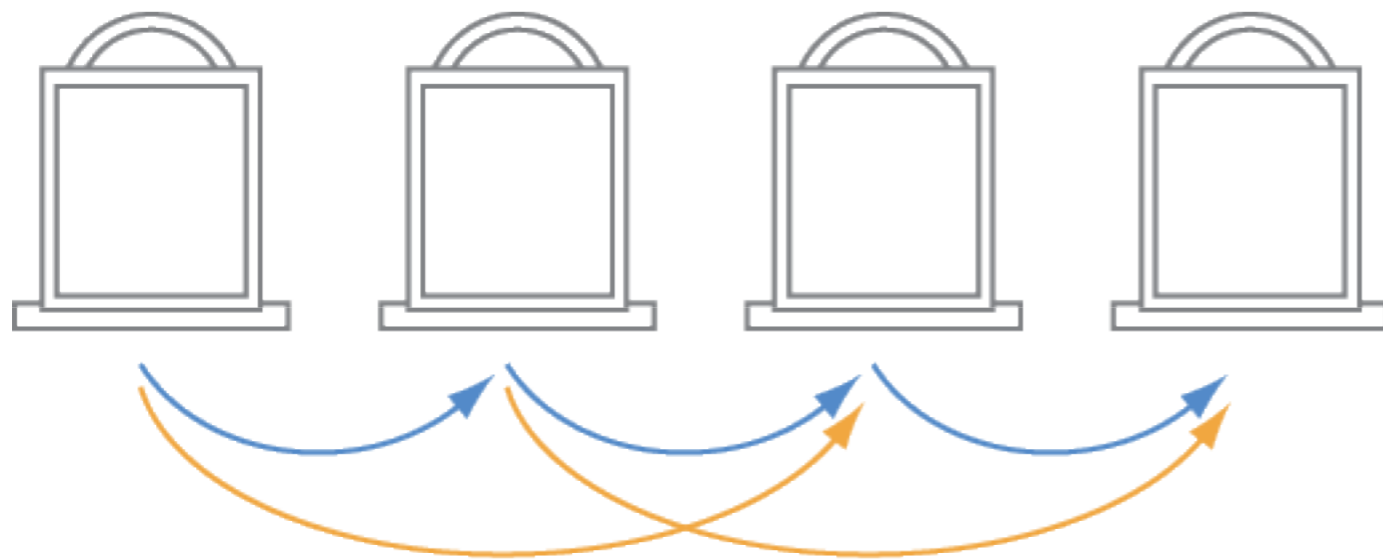
# Transformations



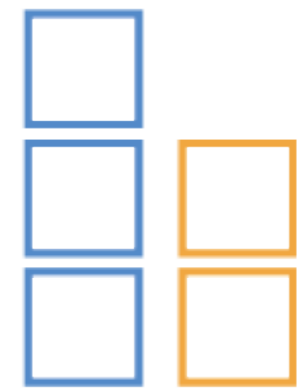
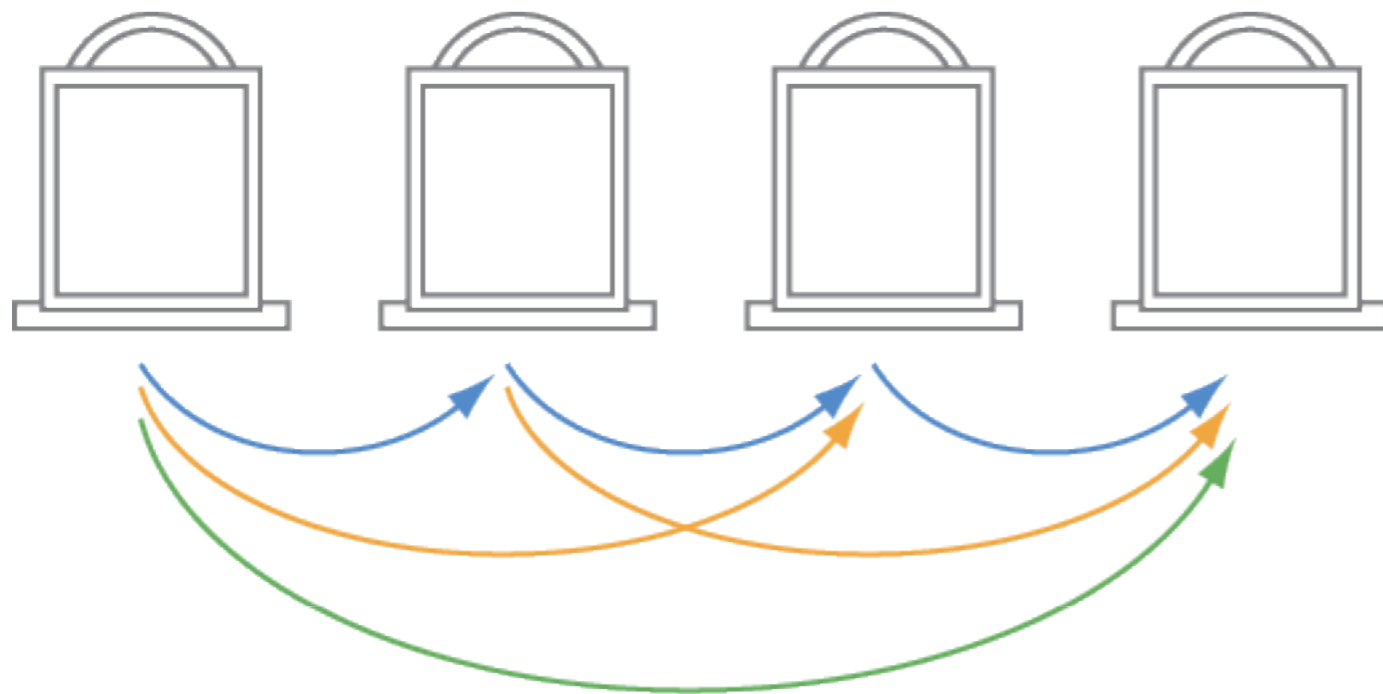
# Transformations



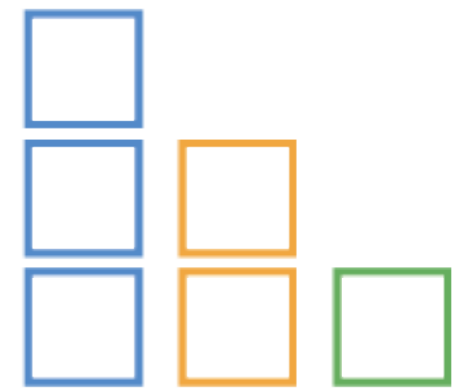
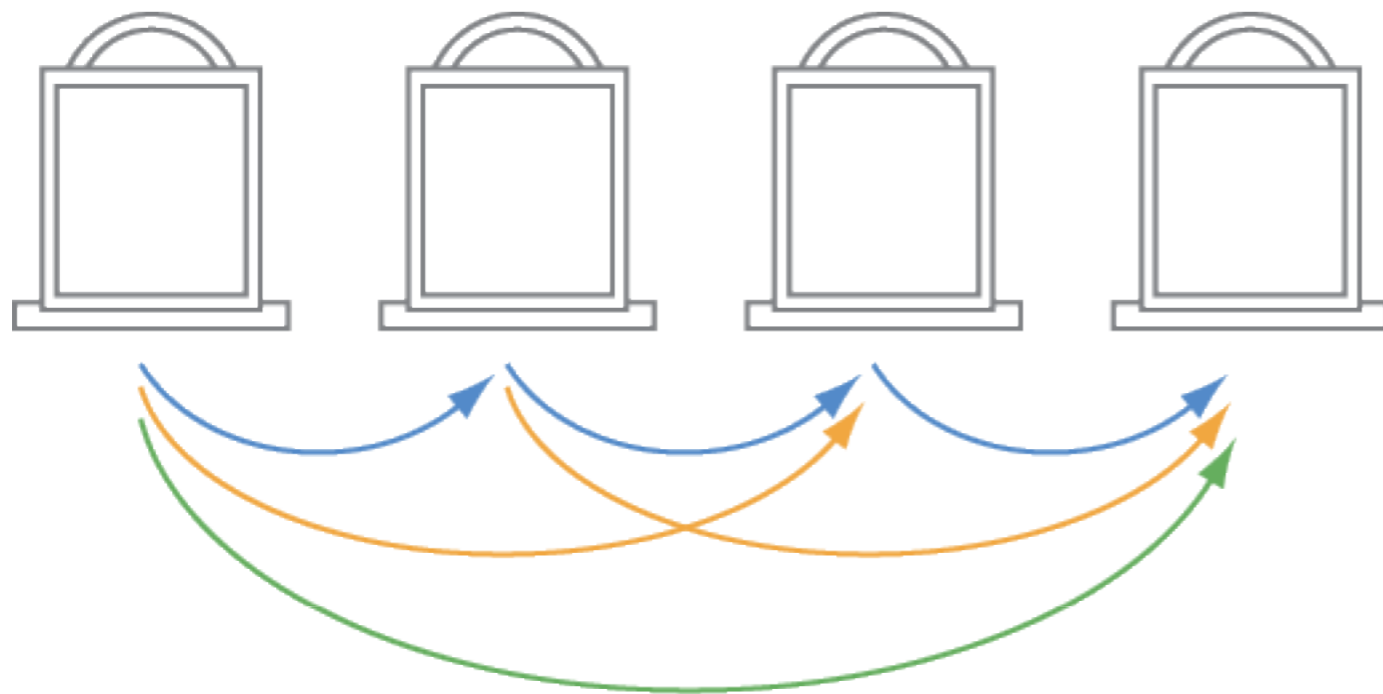
# Transformations



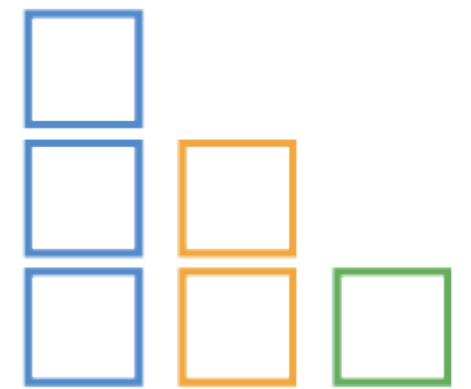
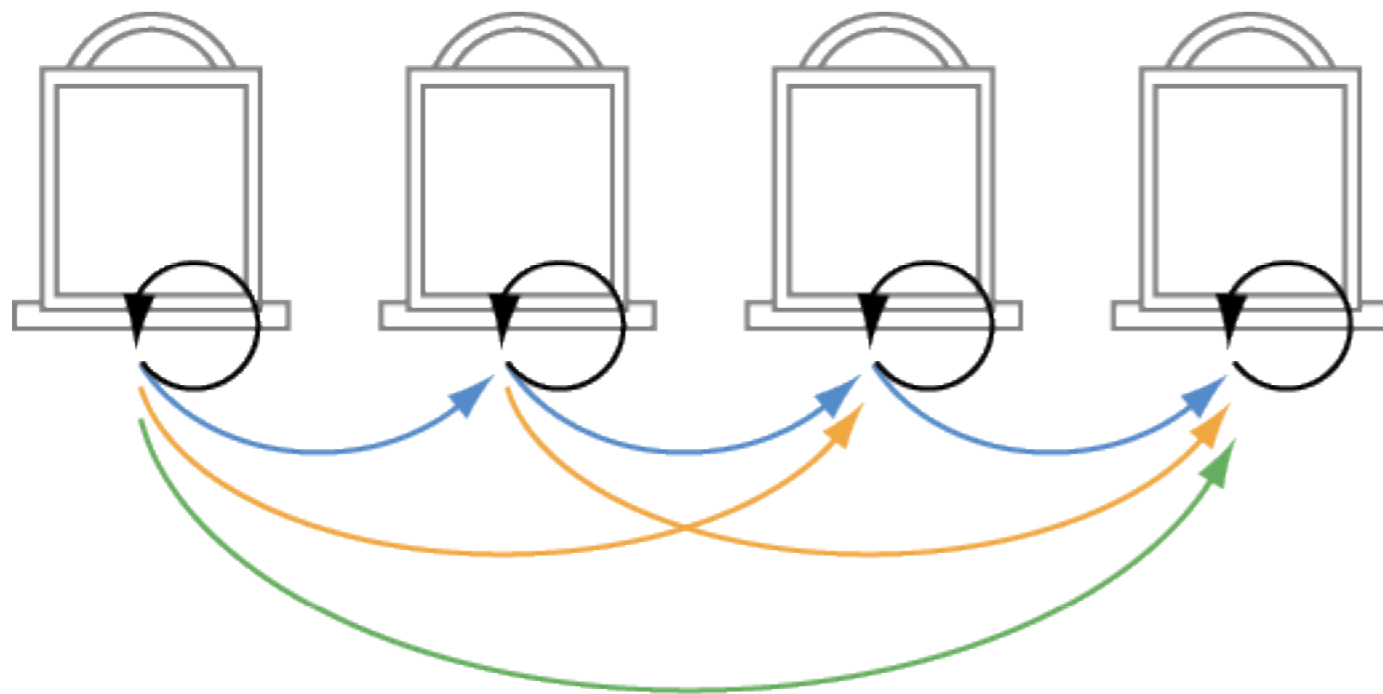
# Transformations



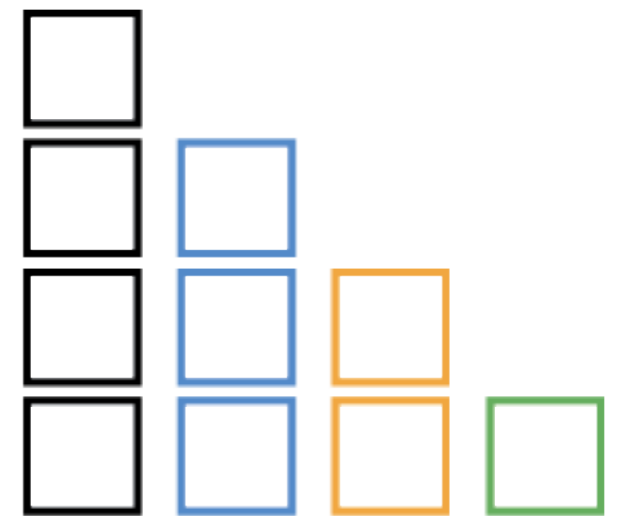
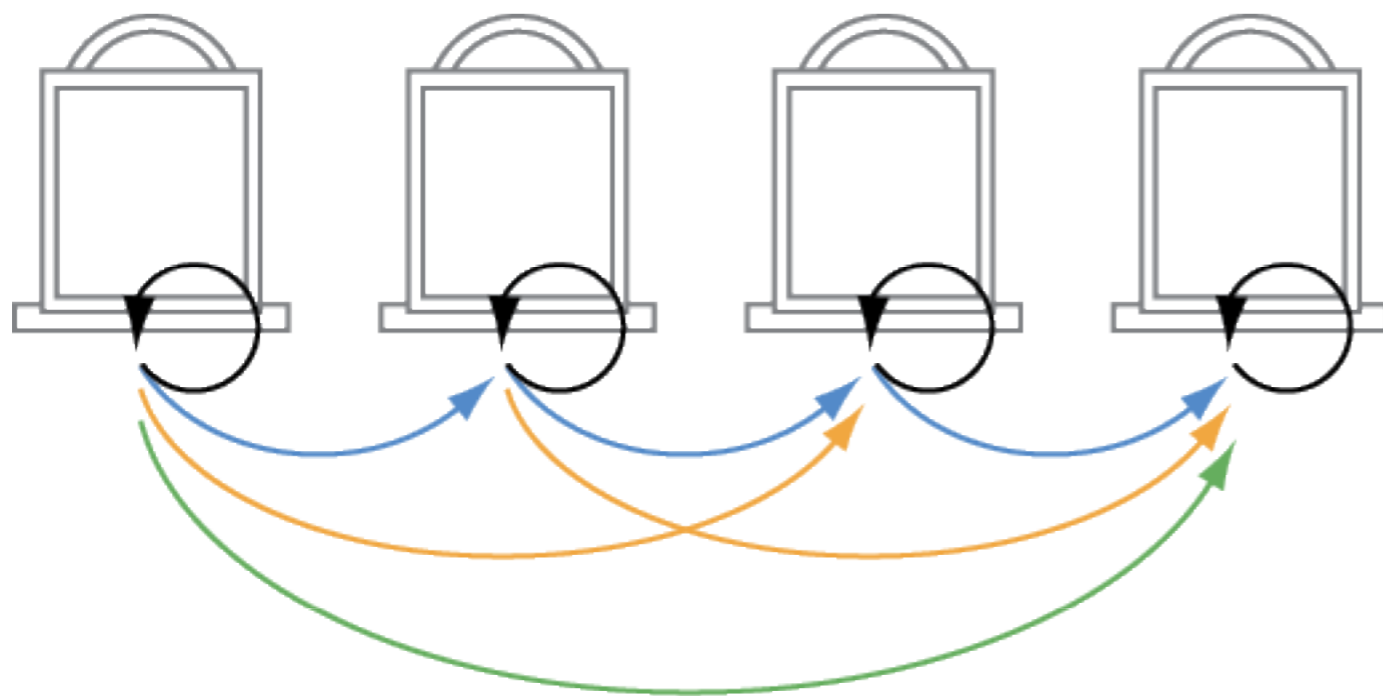
# Transformations



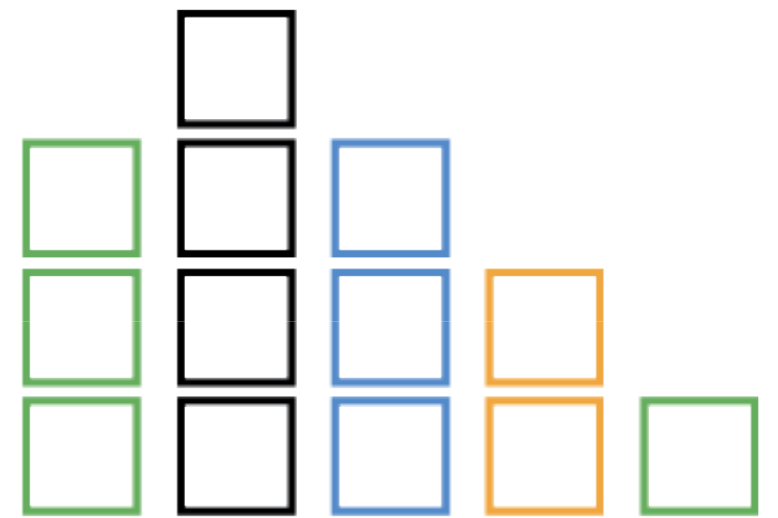
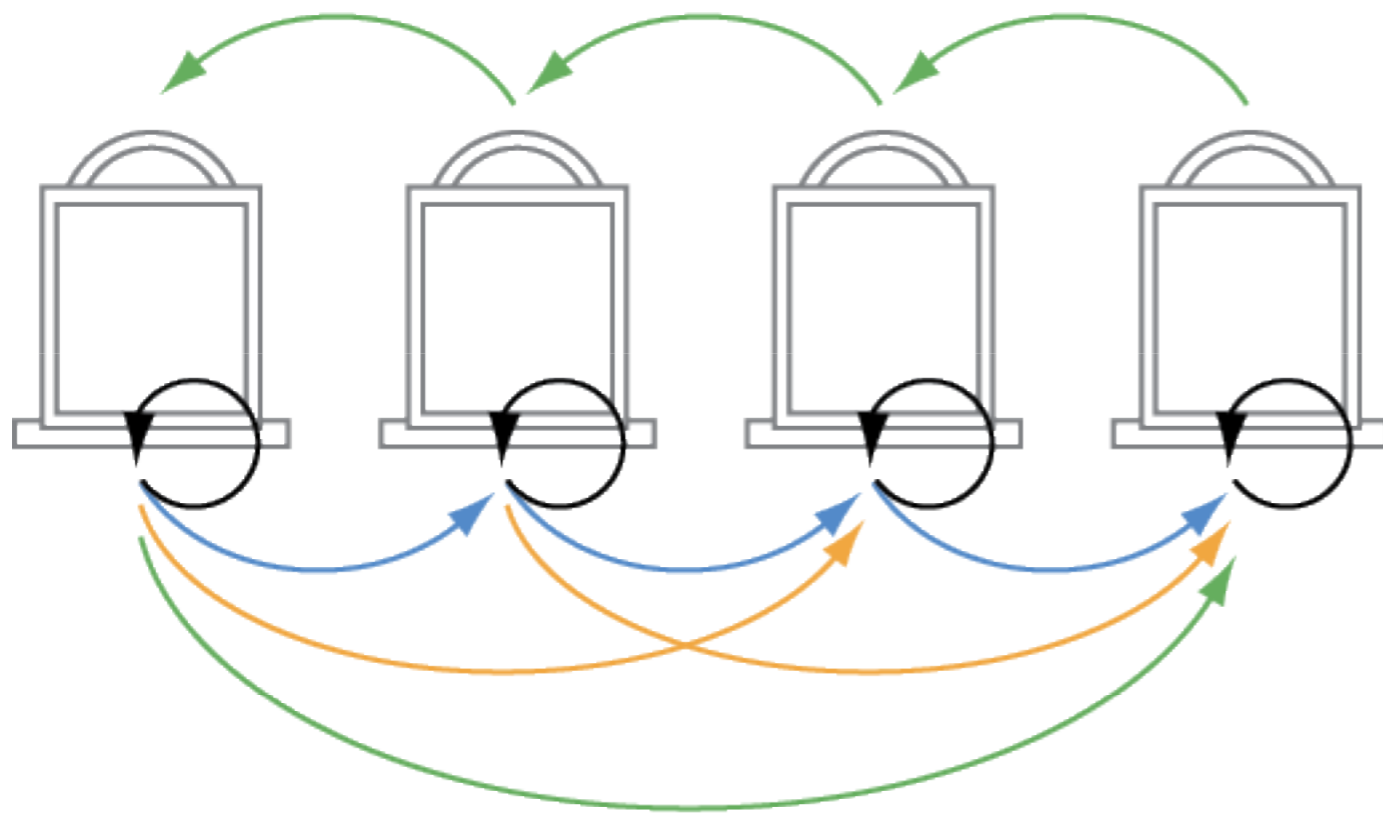
# Transformations



# Transformations

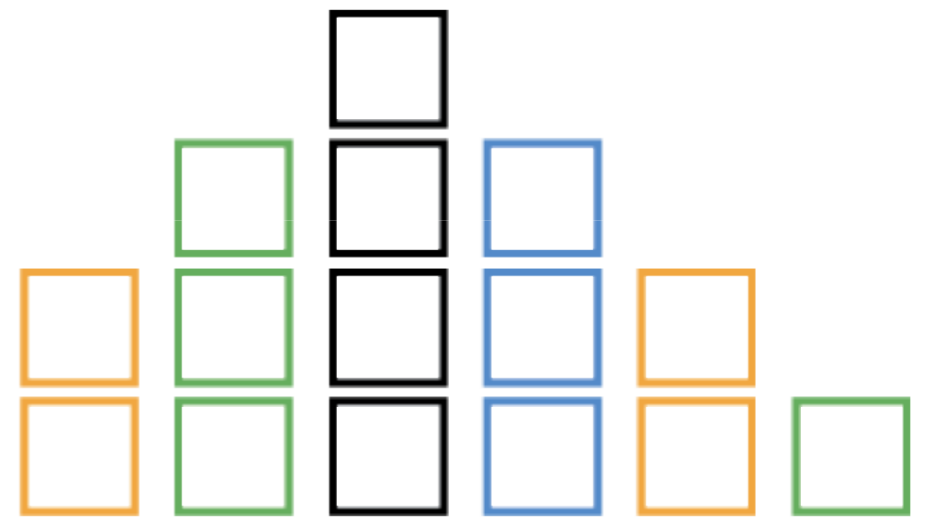
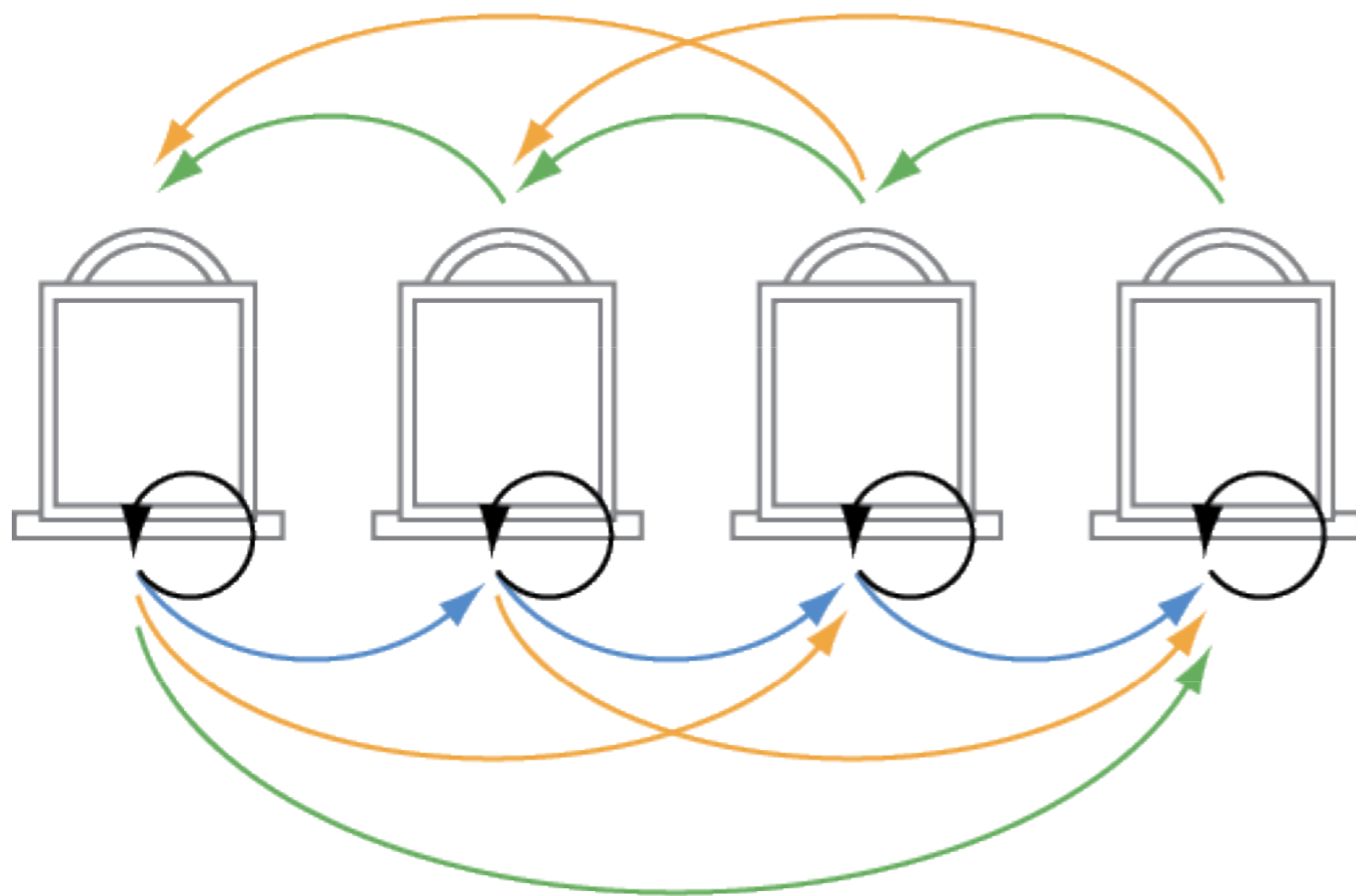


# Transformations

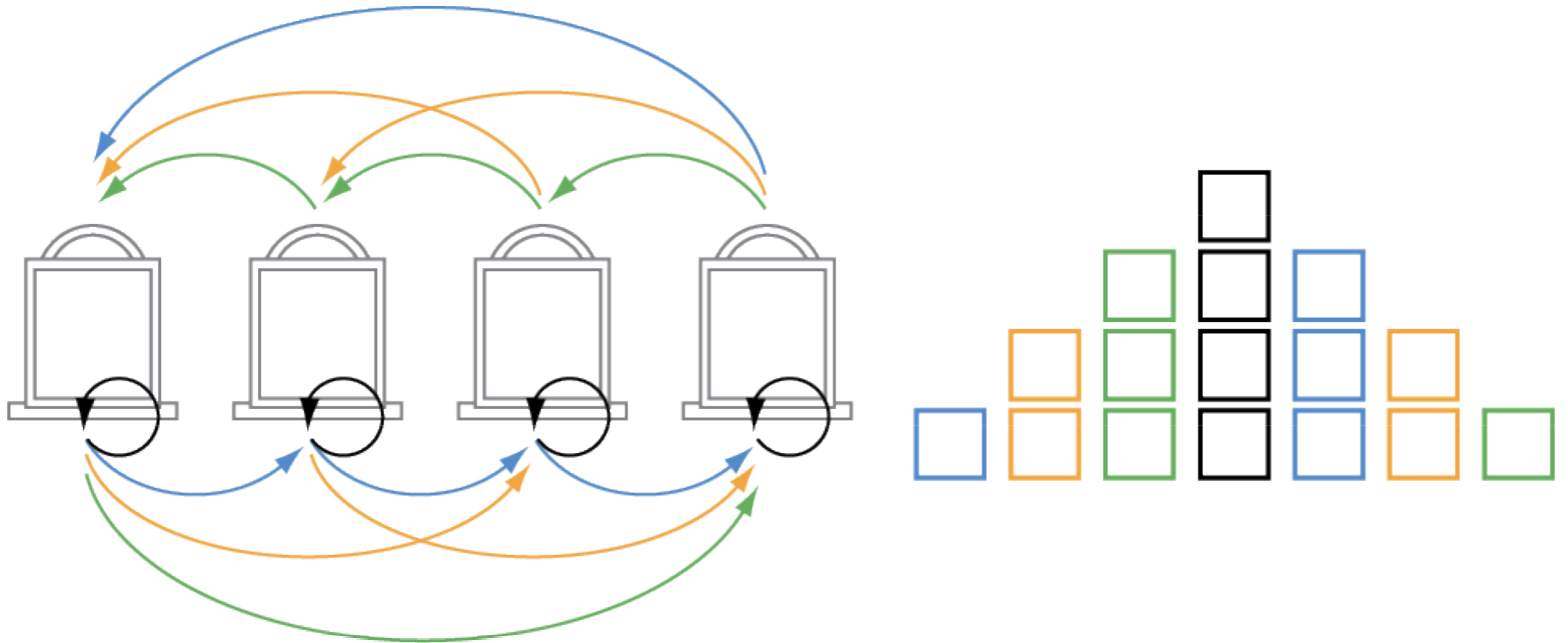




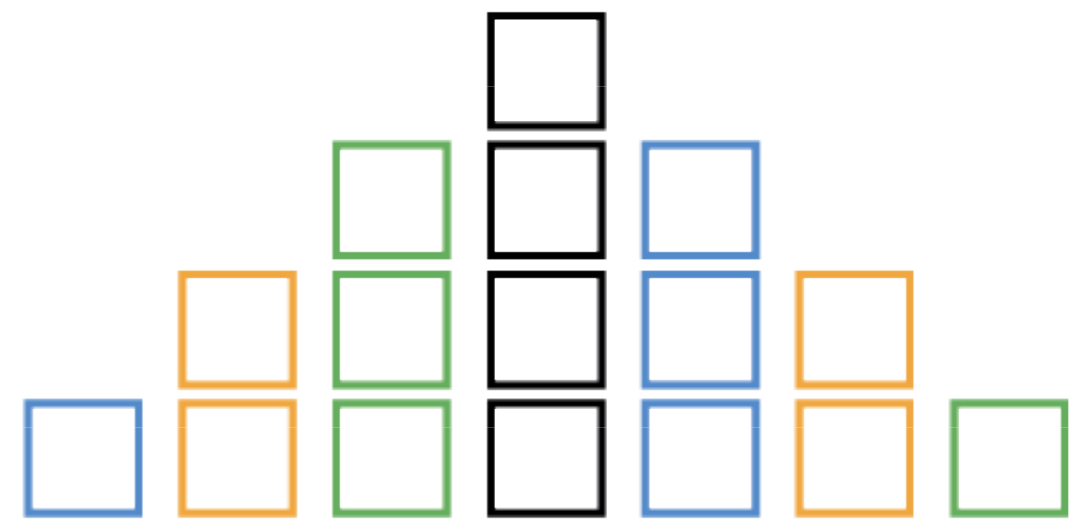
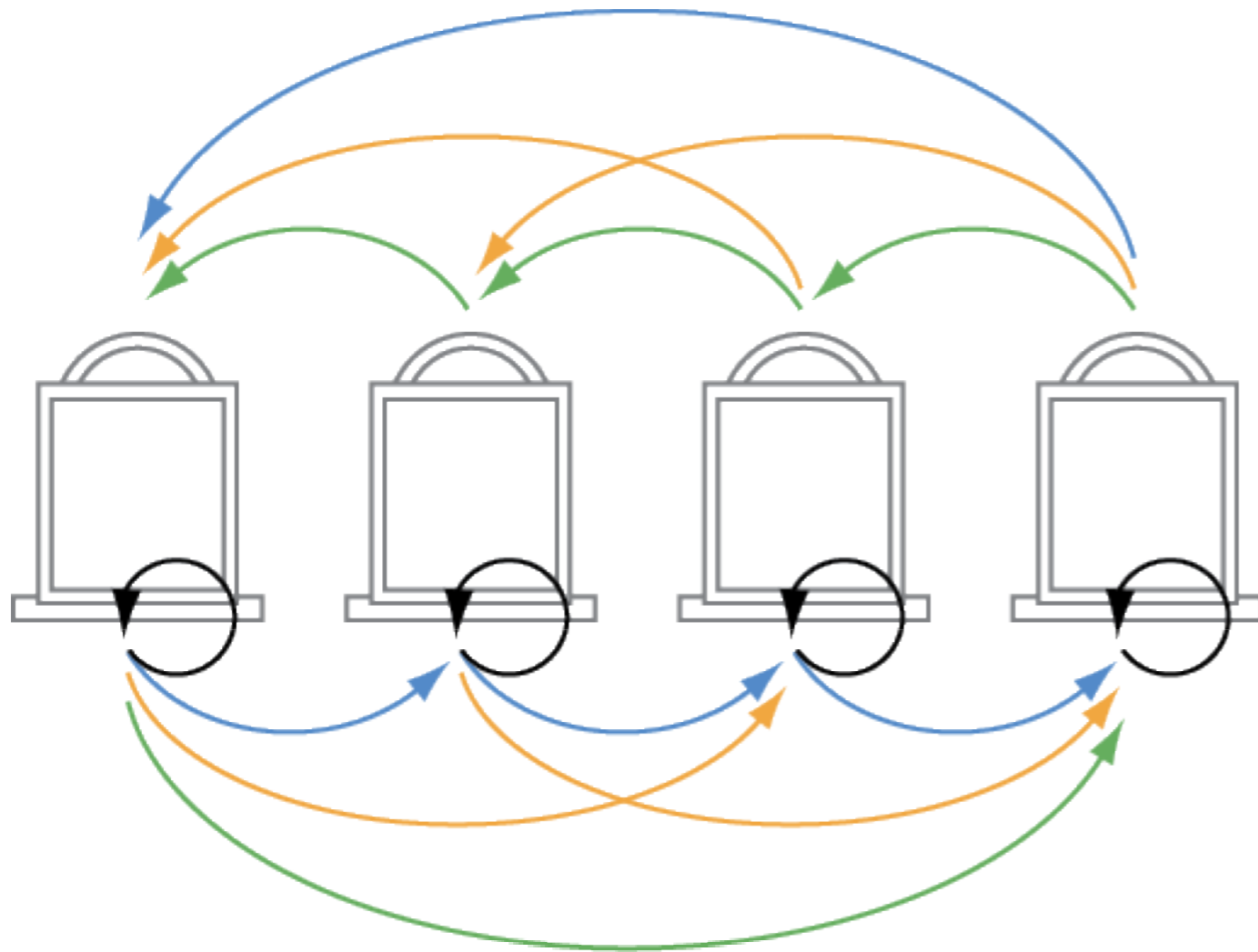
# Transformations



# Transformations

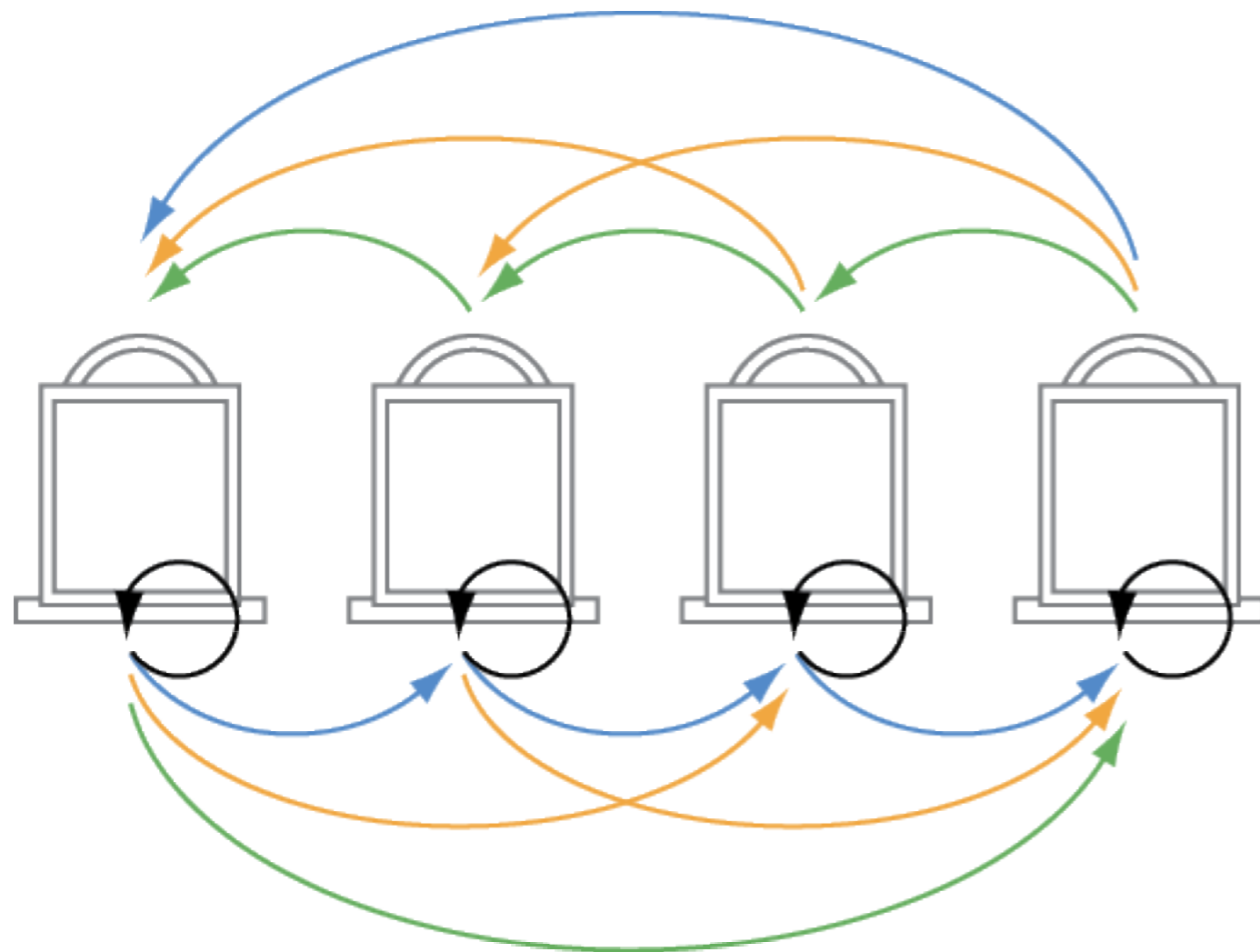


# Transformations

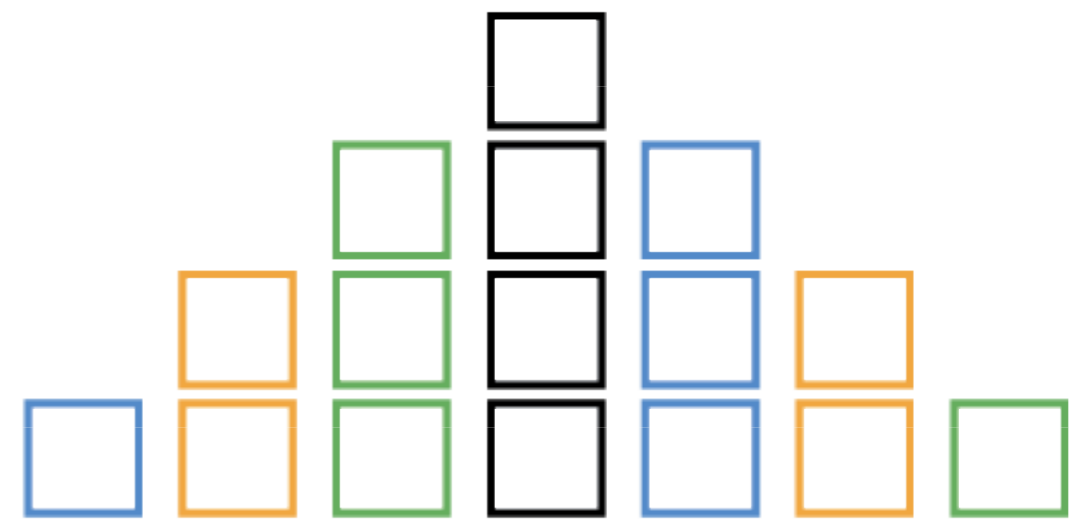


$n^2$  transformations

# Transformations



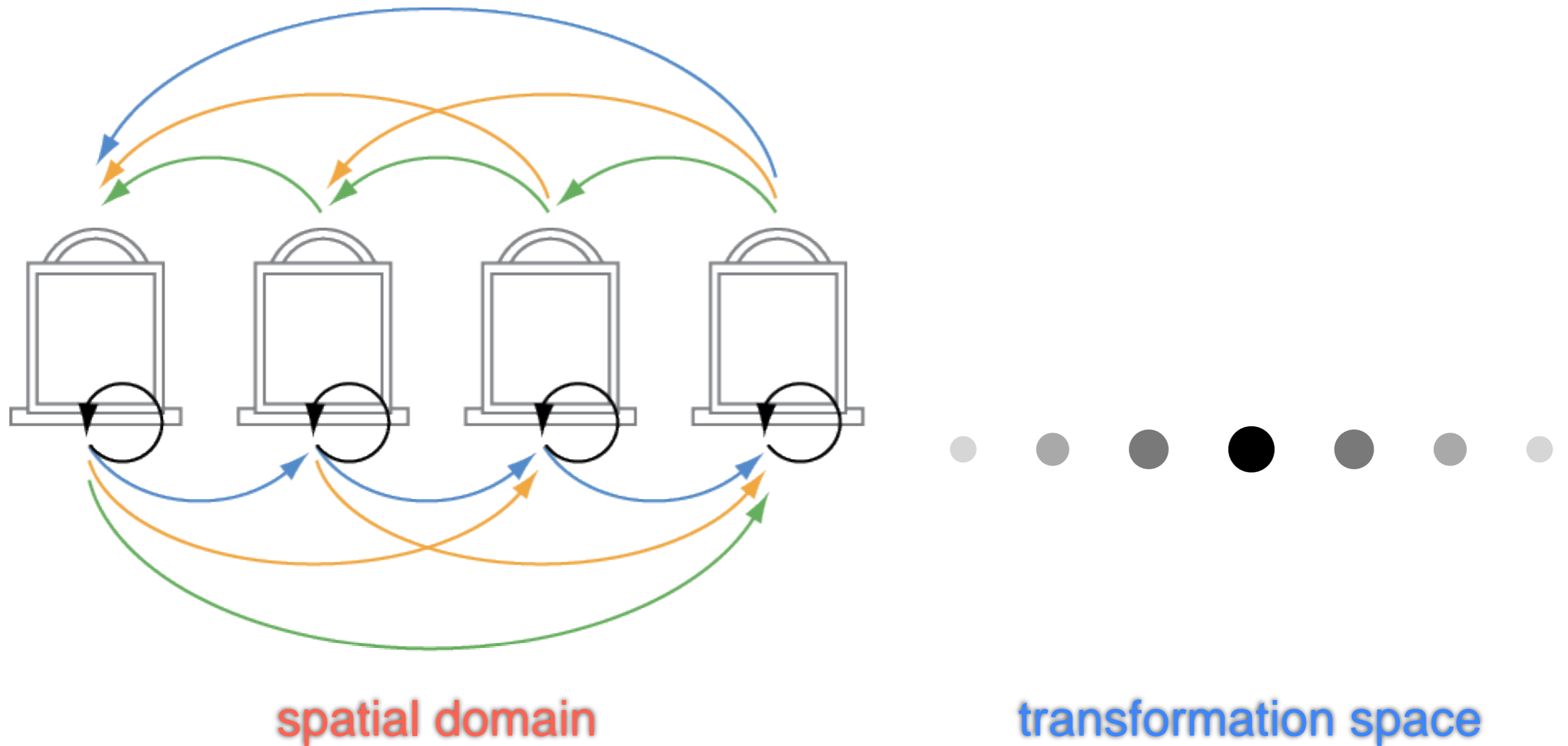
spatial domain



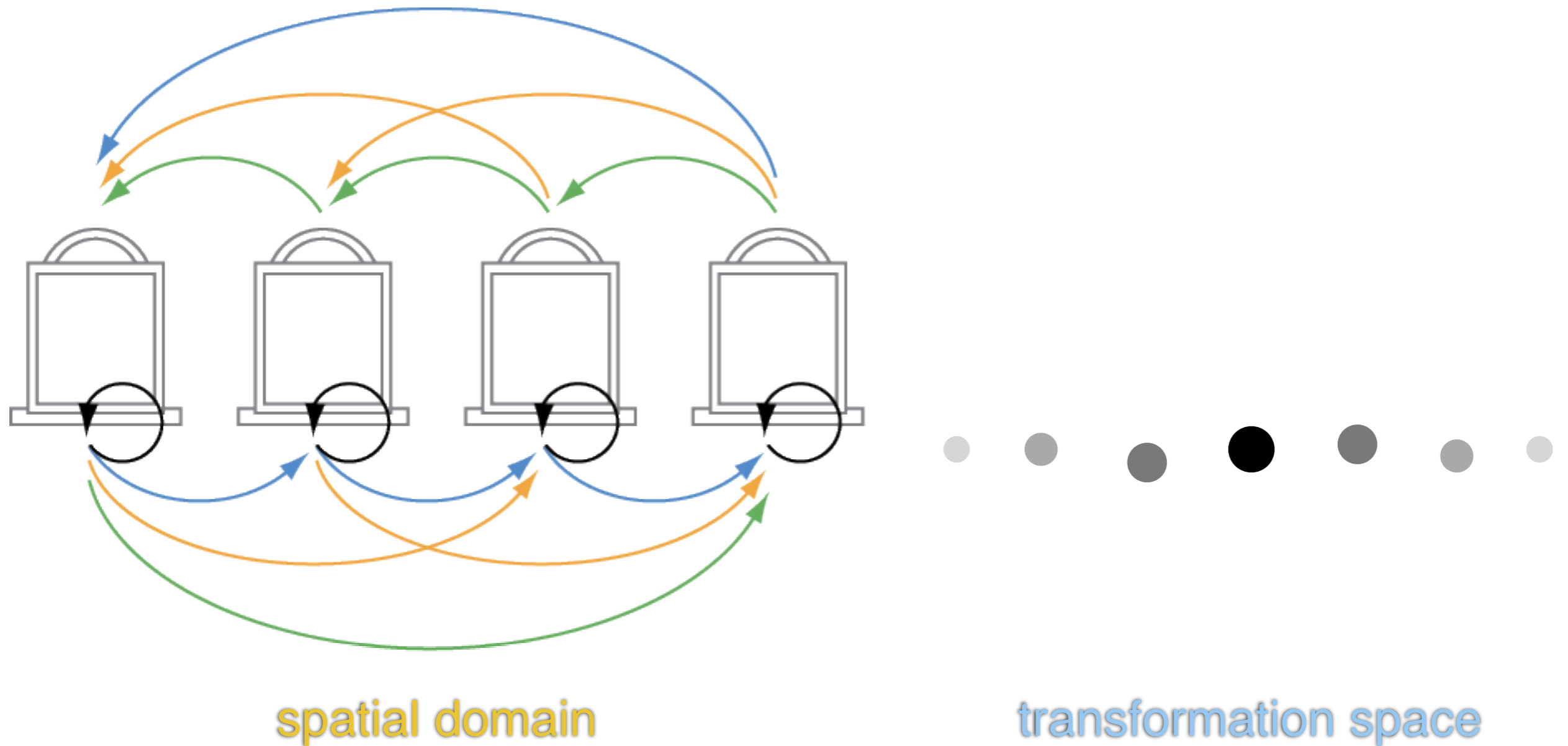
$n^2$  transformations

transformation space

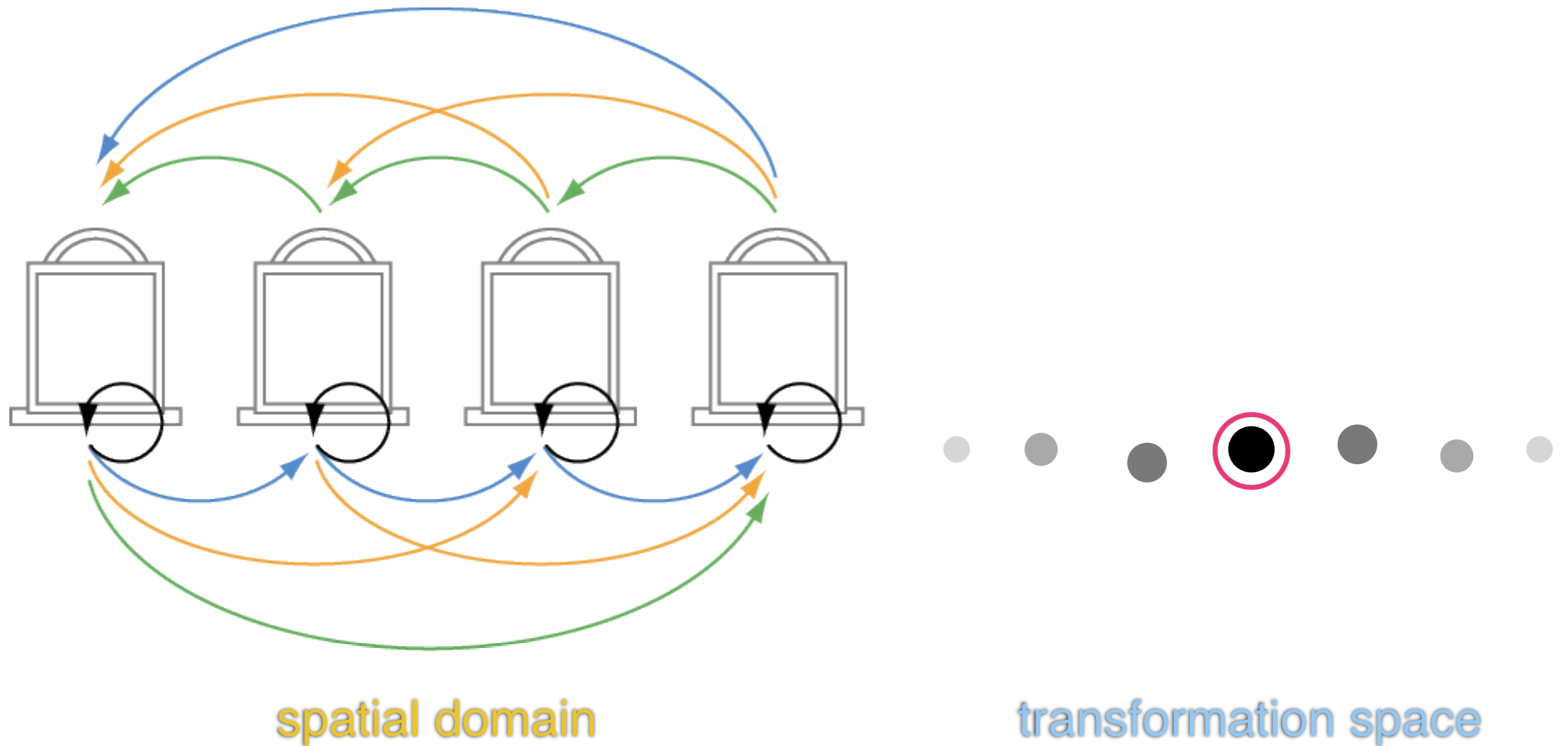
# Transformations



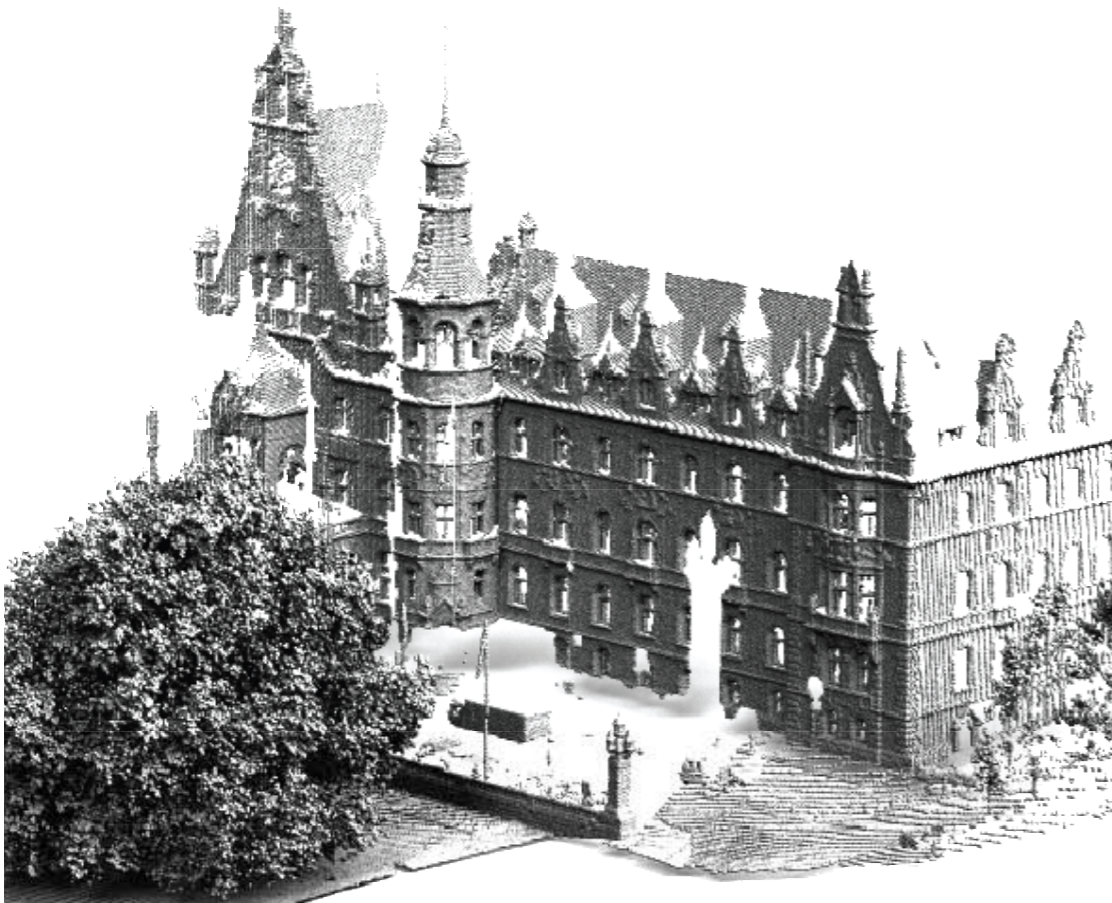
# Transformations



# Transformations

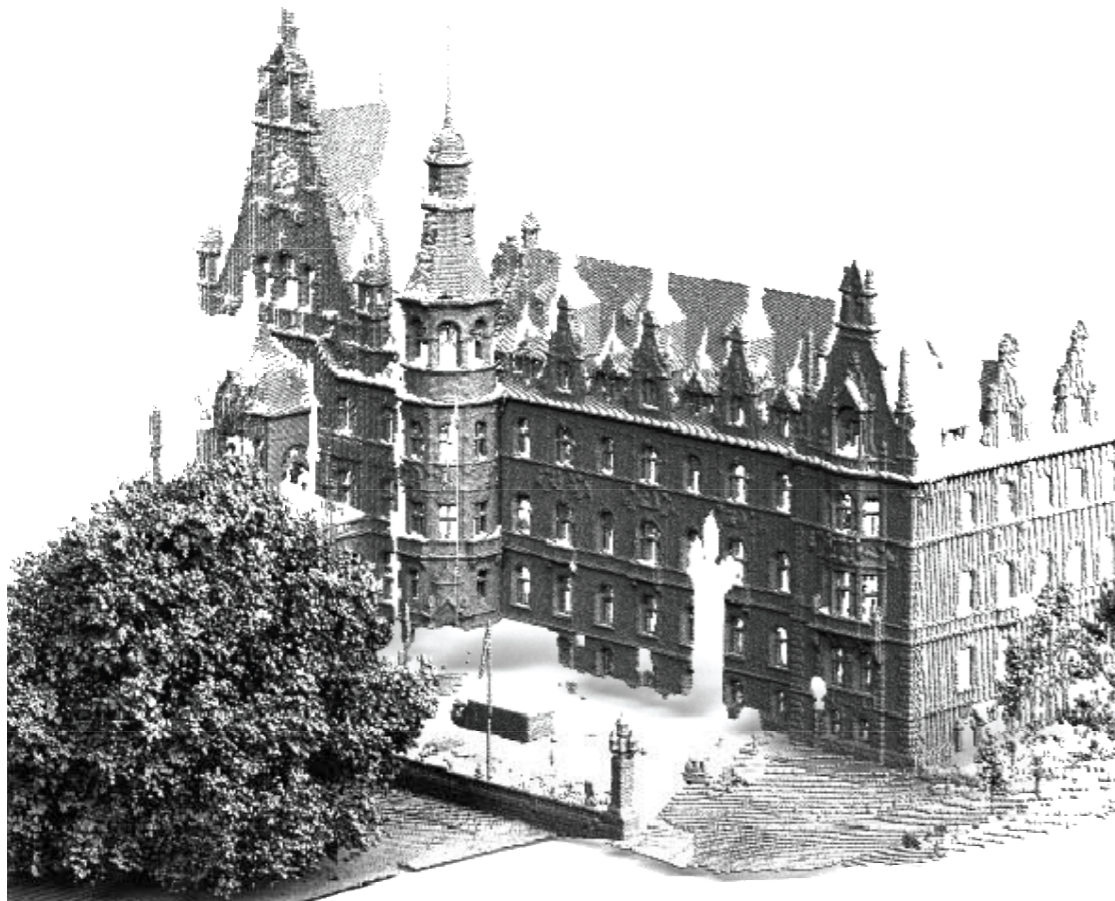


# Model Estimation



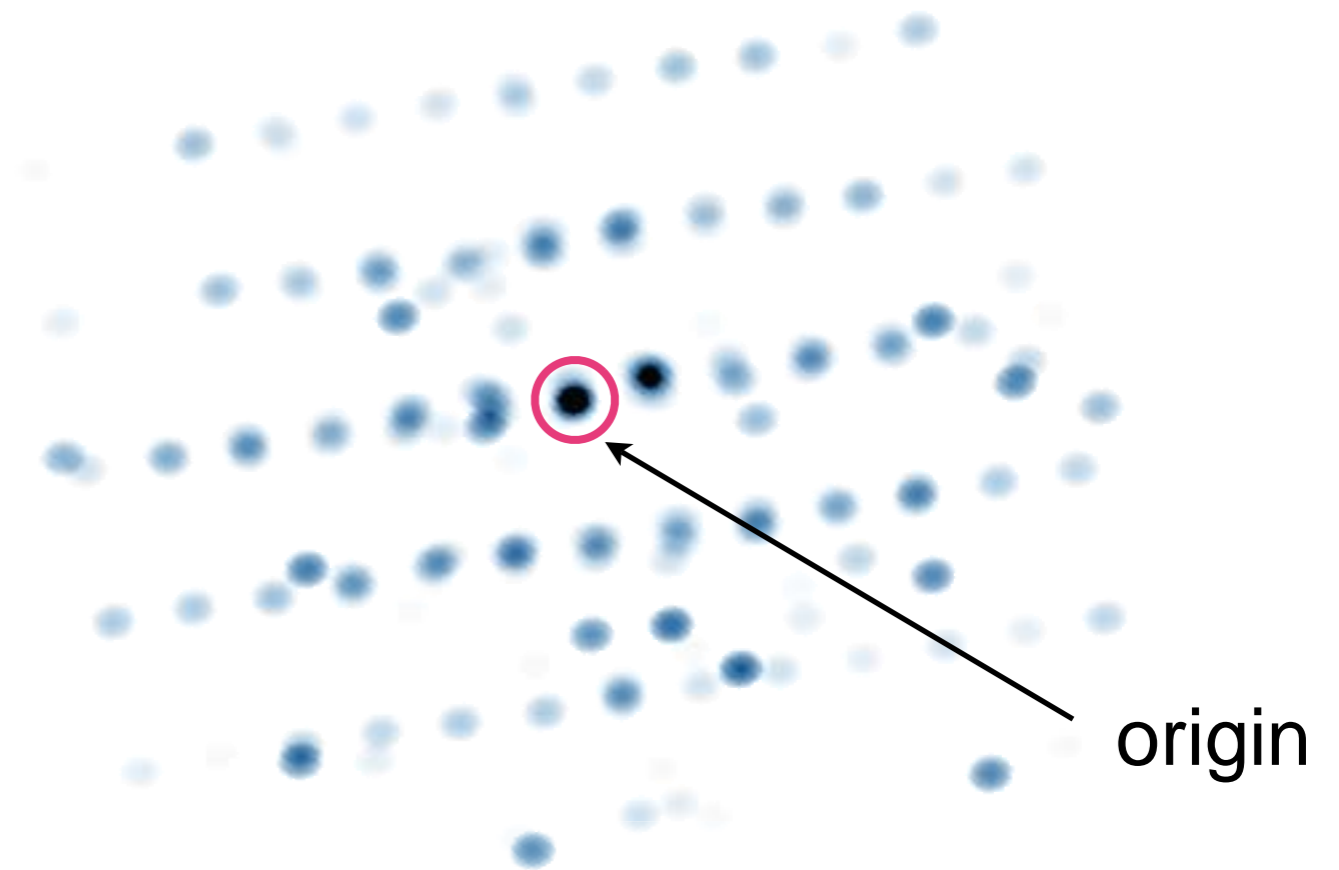


# Model Estimation



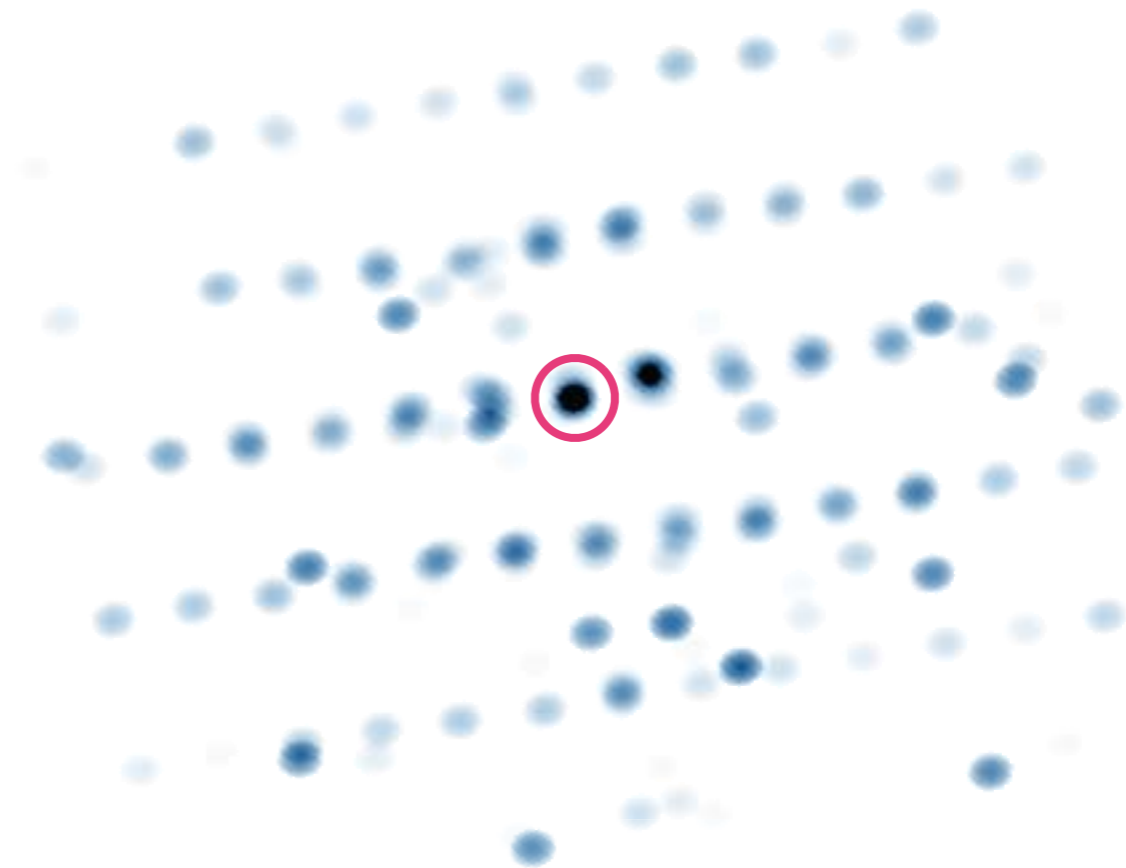
density plot of  
pair-wise transformations

# Model Estimation



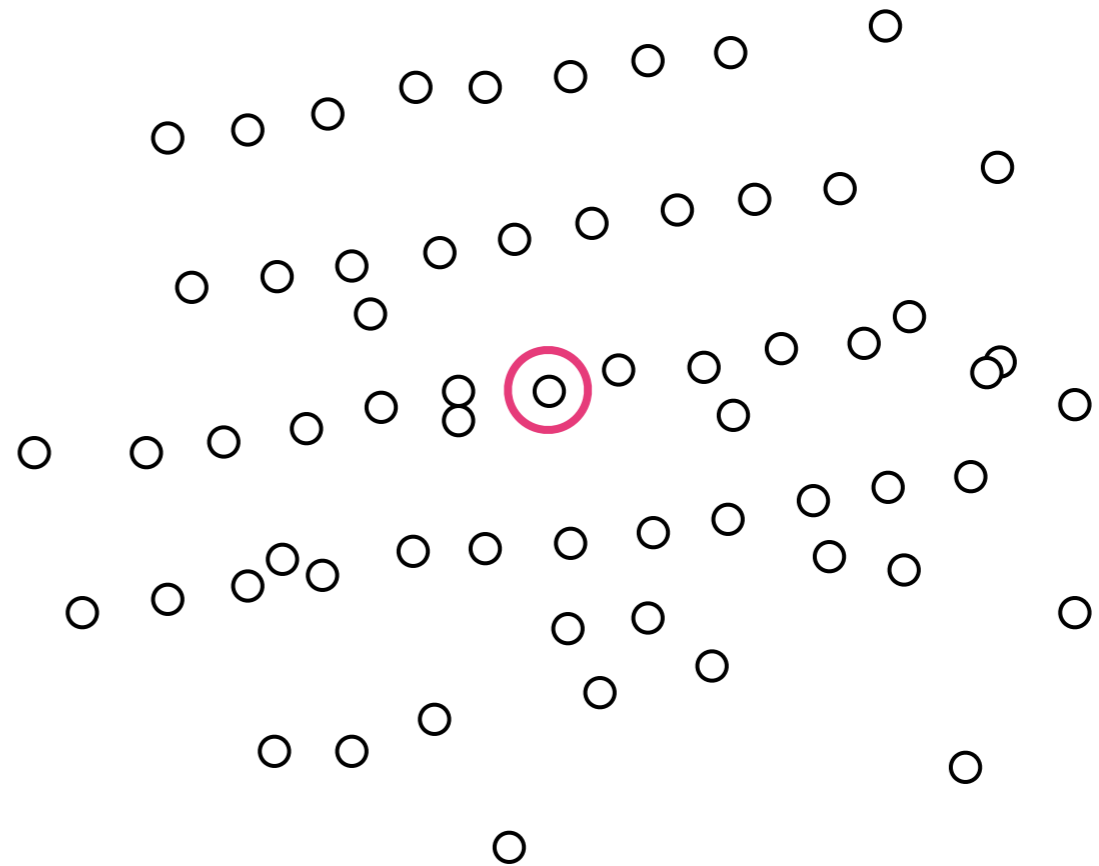
density plot of  
pair-wise transformations

# Model Estimation



density plot of  
pair-wise transformations

# Model Estimation



cluster centers

# Transform Mapping

# Transform Mapping

$$\mathbf{G}_1^i \cdot \mathbf{G}_1^j \rightarrow \{i\mathbf{g}_1 + j\mathbf{g}_2\}$$

# Transform Mapping

$$\mathbf{G}_1^i \cdot \mathbf{G}_1^j \rightarrow \{i\mathbf{g}_1 + j\mathbf{g}_2\}$$

$$\mathbf{I} \rightarrow \{\mathbf{0}\}$$

# Transform Mapping

$$\mathbf{G}_1^i \cdot \mathbf{G}_1^j \rightarrow \{i\mathbf{g}_1 + j\mathbf{g}_2\}$$

$$\mathbf{I} \rightarrow \{\mathbf{0}\}$$

Translation x Translation

$$T \rightarrow (t_1, t_2)$$

Rotation x Scaling

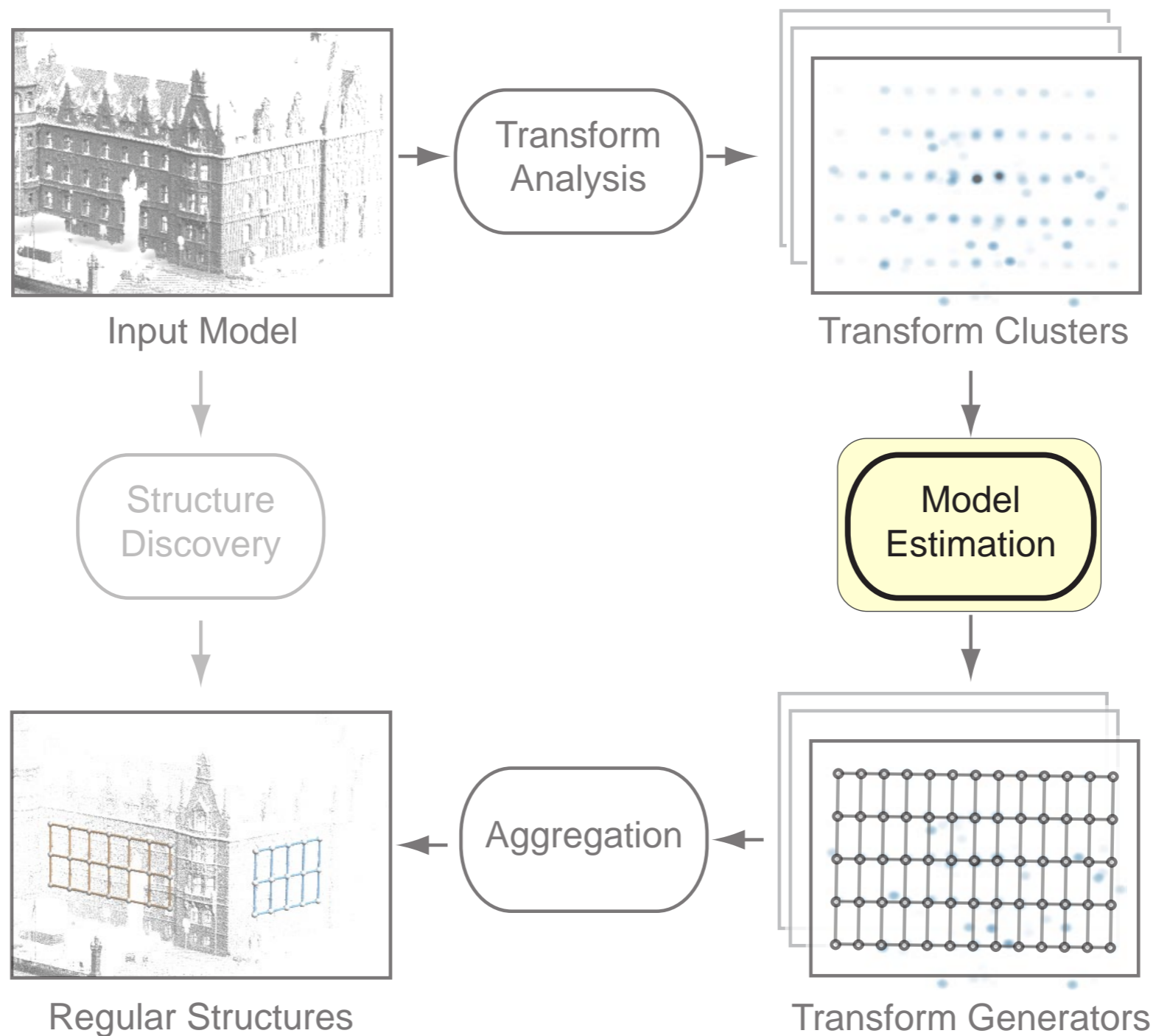
$$T \rightarrow (\theta, \log s)$$

Translation x Rotation

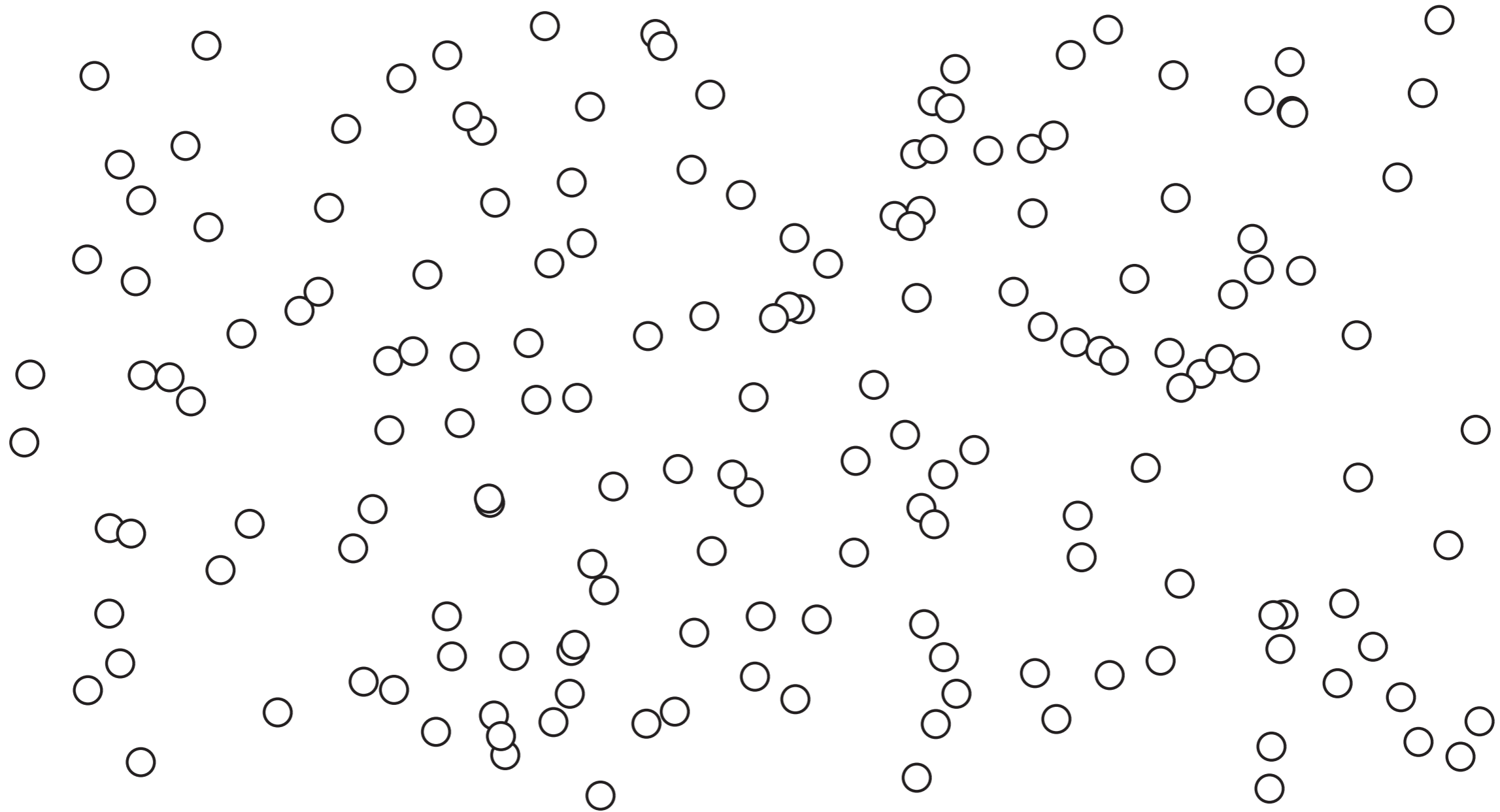
$$T \rightarrow (t, \theta)$$



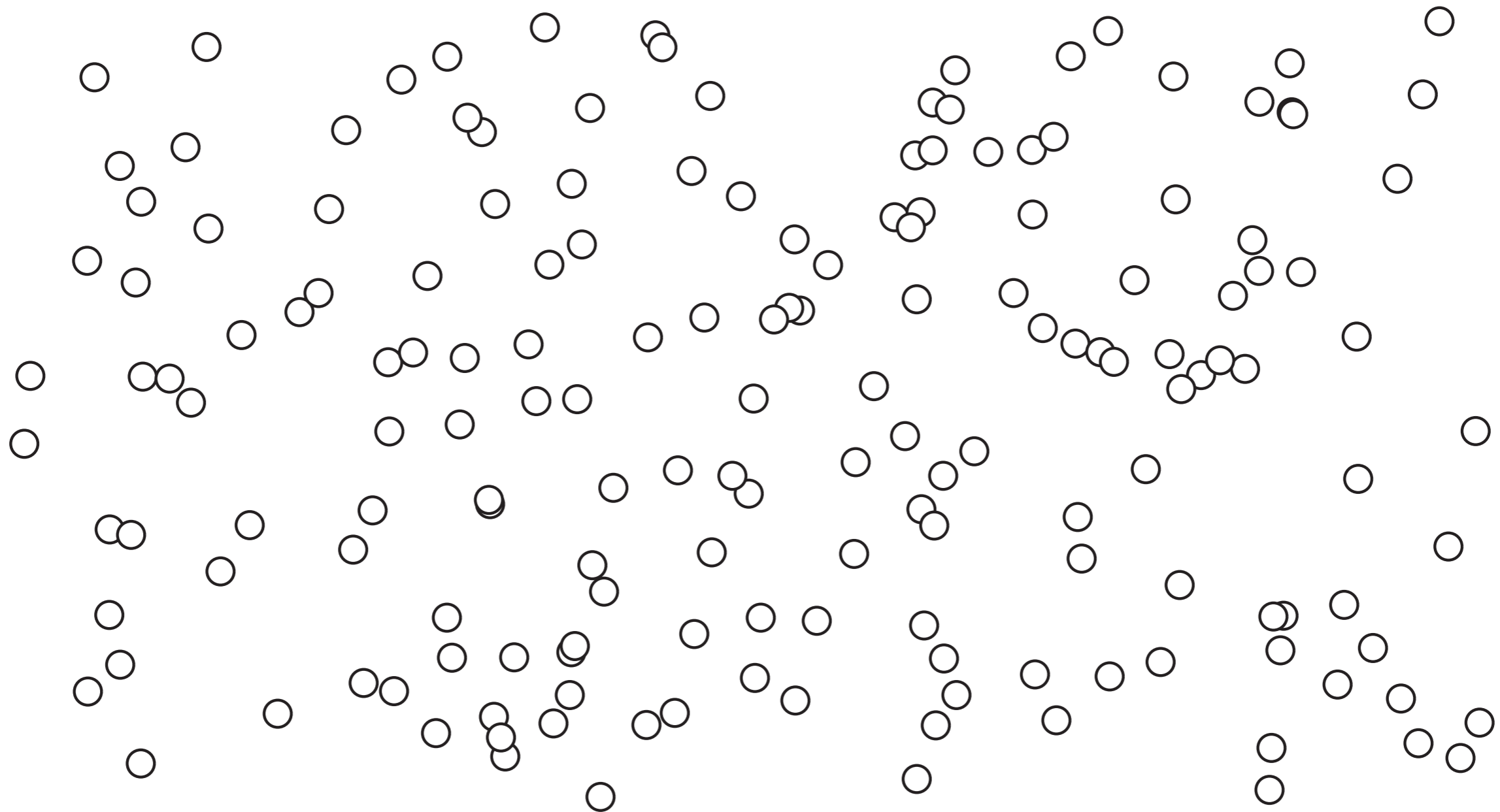
# Structure Discovery



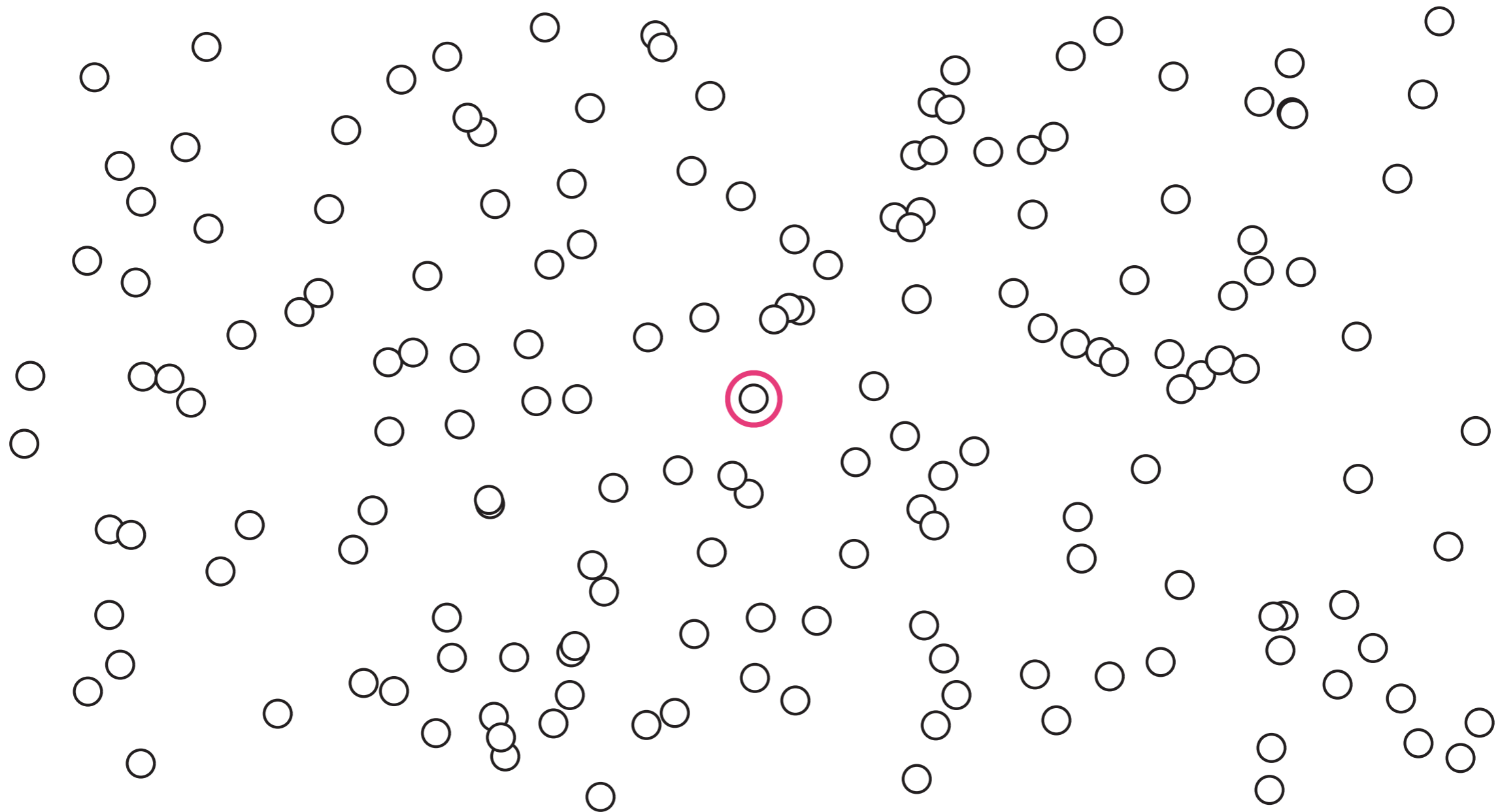
# Model Estimation



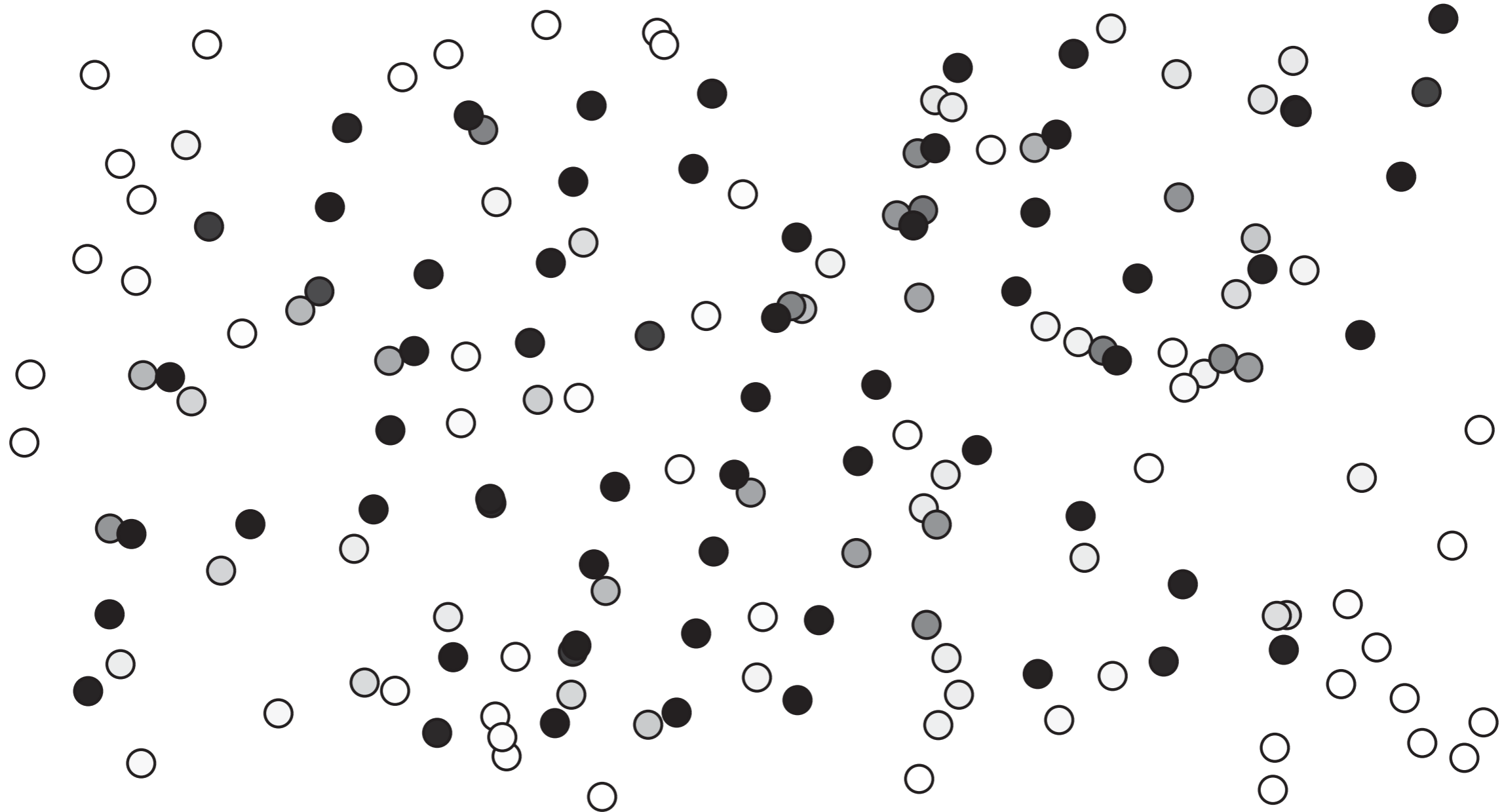
# Is there a Pattern?



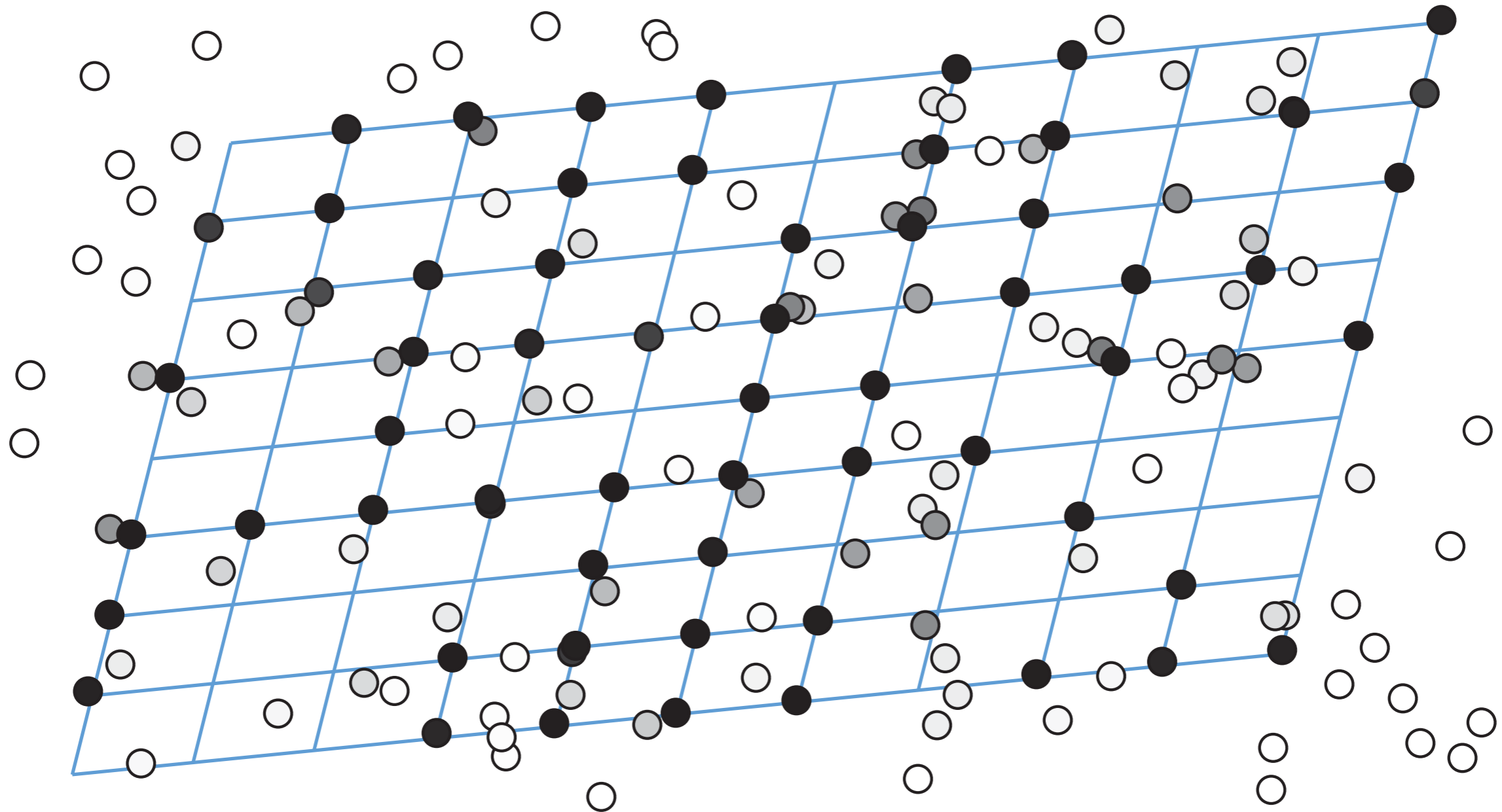
# Is there a Pattern?



# Yes, there is!



# Yes, there is!



# Model Estimation

# Model Estimation

- Global, non-linear optimization



# Model Estimation

- Global, non-linear optimization
  - *simultaneously* detects *outliers* and *grid structure*

# Model Estimation

- Grid fitting
  - *input*: cluster centers

$$C = \{\mathbf{c}_1, \dots, \mathbf{c}_n\}$$

# Model Estimation

- Grid fitting
  - *input*: cluster centers

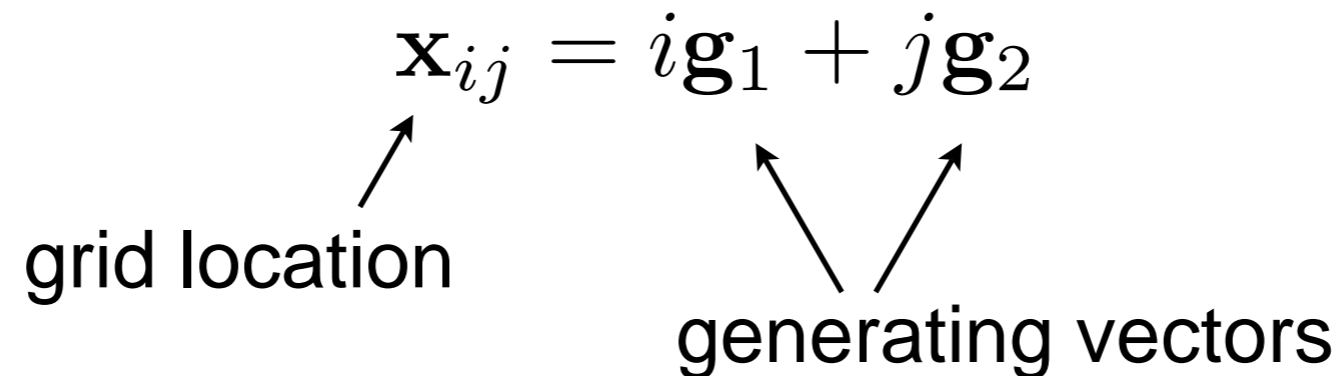
$$C = \{\mathbf{c}_1, \dots, \mathbf{c}_n\}$$

- *unknowns*: grid generators

$$\mathbf{x}_{ij} = i\mathbf{g}_1 + j\mathbf{g}_2$$

grid location

generating vectors



# Model Estimation

- Grid fitting
  - *input*: cluster centers

$$C = \{\mathbf{c}_1, \dots, \mathbf{c}_n\}$$

- *unknowns*: grid generators

$$\mathbf{x}_{ij} = i\mathbf{g}_1 + j\mathbf{g}_2$$

grid location  $\nearrow$

$\nwarrow$   $\nearrow$  generating vectors

$i \in [-n, n]$   
 $j \in [-m, m]$

# Model Estimation

- Fitting terms

$$E_{C \rightarrow X} = \sum_{k=1}^{|C|} \|\mathbf{c}_k - \mathbf{x}(k)\|^2$$

cluster center      closest grid point

$$E_{X \rightarrow C} = \sum_i \sum_j \|\mathbf{x}_{ij} - \mathbf{c}(i, j)\|^2$$

grid point      closest cluster center

# Model Estimation

- Fitting terms

$$E_{C \rightarrow X} = \sum_{k=1}^{|C|} \beta_k^2 \|\mathbf{c}_k - \mathbf{x}(k)\|^2$$

data confidence

cluster center

closest grid point

$$E_{X \rightarrow C} = \sum_i \sum_j \alpha_{ij}^2 \|\mathbf{x}_{ij} - \mathbf{c}(i, j)\|^2$$

grid confidence

grid point

closest cluster center

# Model Estimation

- Fitting terms

$$E_{C \rightarrow X} = \sum_{k=1}^{|C|} \beta_k^2 \|\mathbf{c}_k - \mathbf{x}(k)\|^2$$

$$E_{X \rightarrow C} = \sum_i \sum_j \alpha_{ij}^2 \|\mathbf{x}_{ij} - \mathbf{c}(i, j)\|^2$$

- Data and grid confidence terms

$$E_\alpha = \sum_i \sum_j (1 - \alpha_{ij}^2)^2 \quad E_\beta = \sum_k (1 - \beta_k^2)^2$$

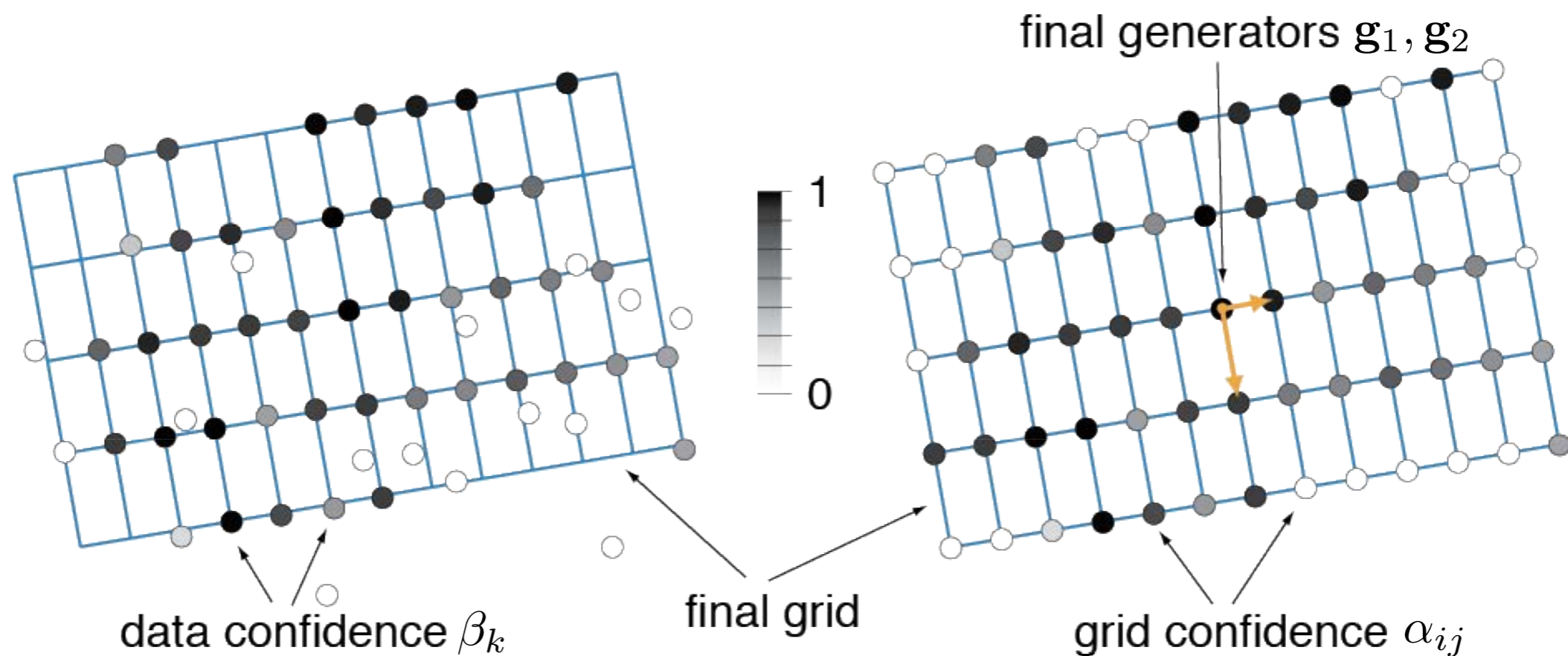
# Model Estimation

- Global, non-linear optimization
  - *simultaneously* detects *outliers* and *grid structure*

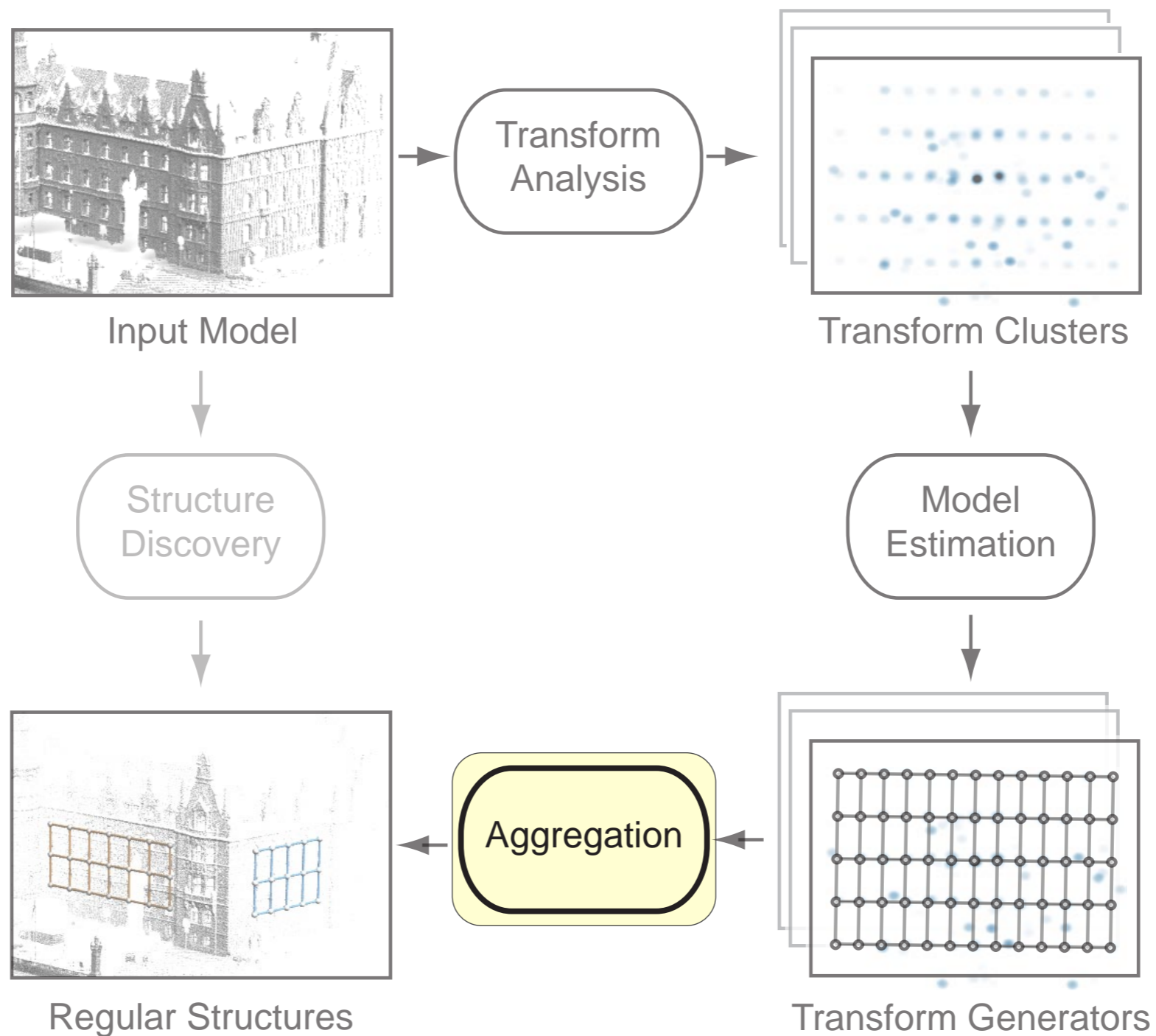


# Model Estimation

- Global, non-linear optimization
  - *simultaneously* detects *outliers* and *grid structure*

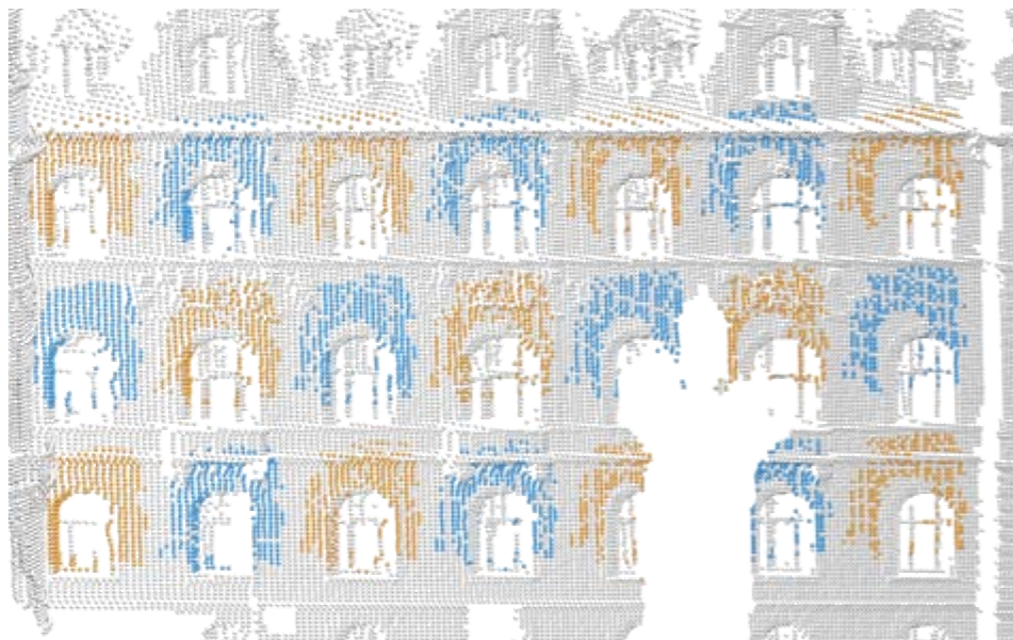


# Structure Discovery



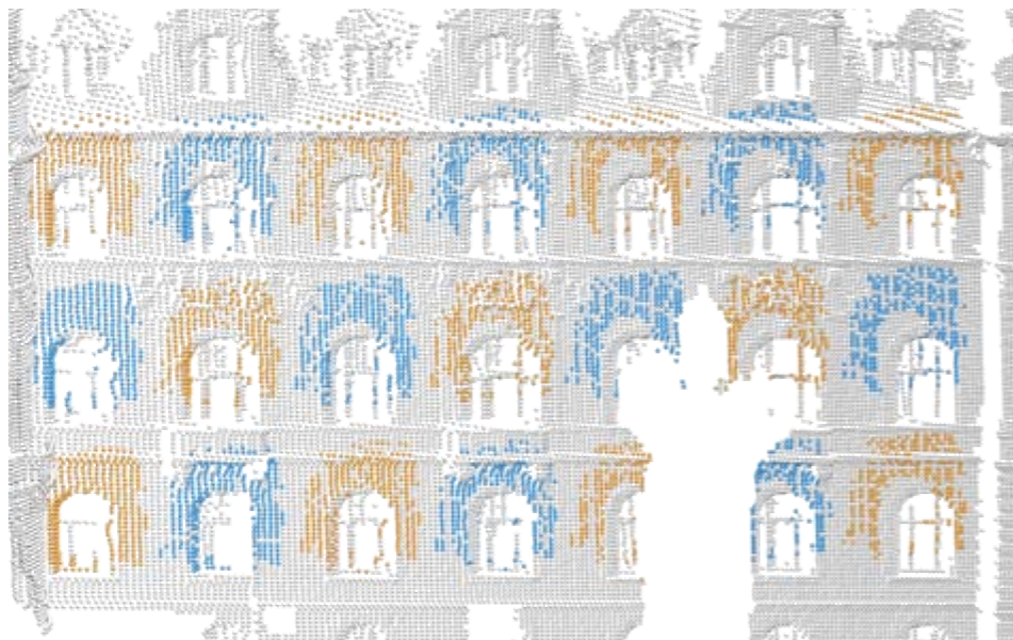
# Aggregation

- Region-growing to extract repetitive elements



# Aggregation

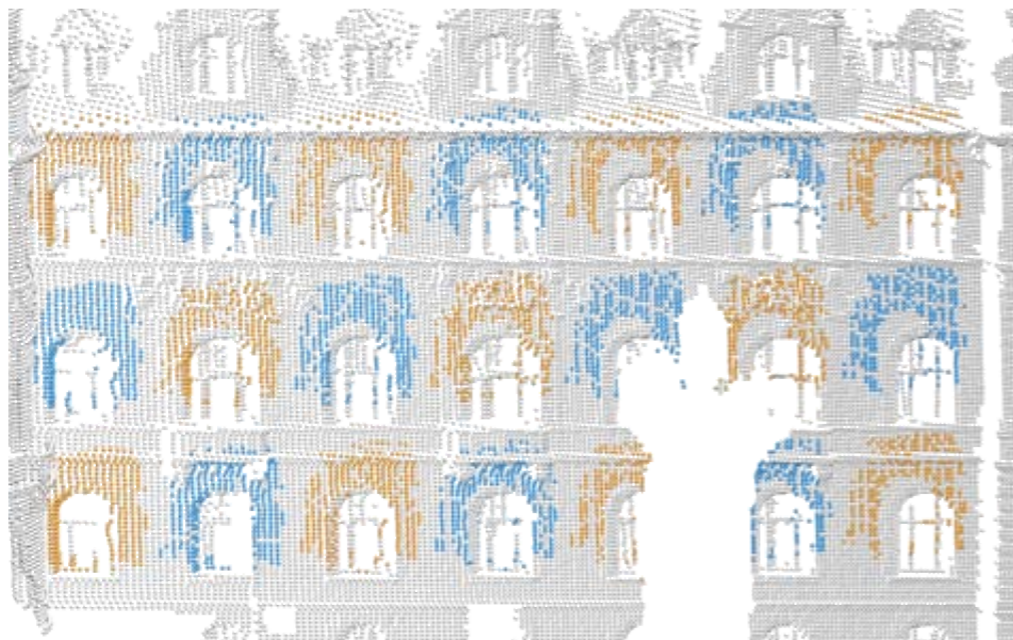
- Region-growing to extract repetitive elements
- Simultaneous registration



# Aggregation

- Region-growing to extract repetitive elements
- Simultaneous registration

$$\mathbf{H}_+ \approx \mathbf{H} + \epsilon \mathbf{D} \cdot \mathbf{H}$$

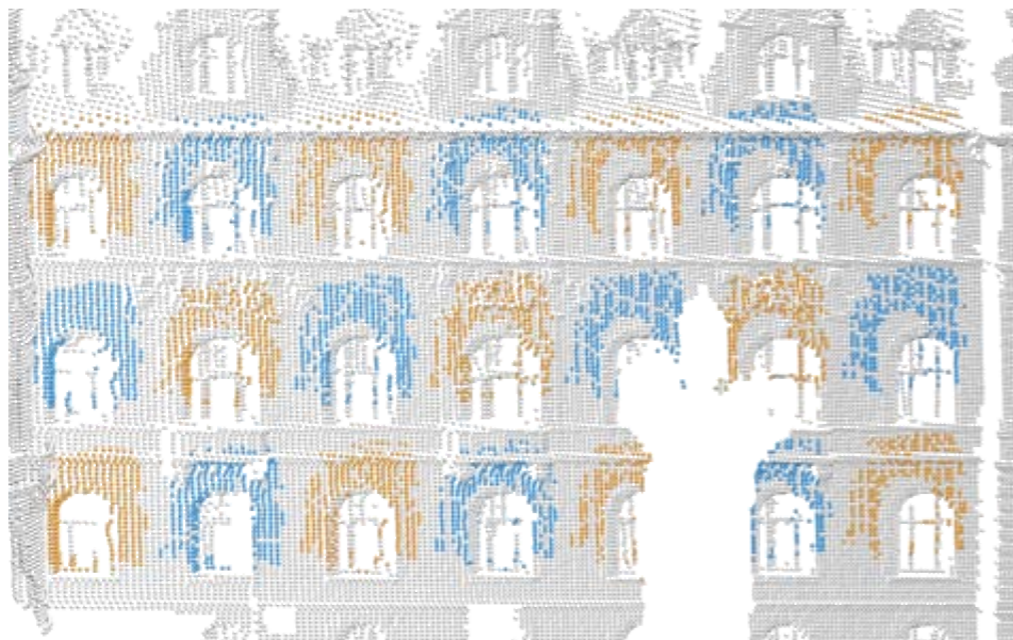


# Aggregation

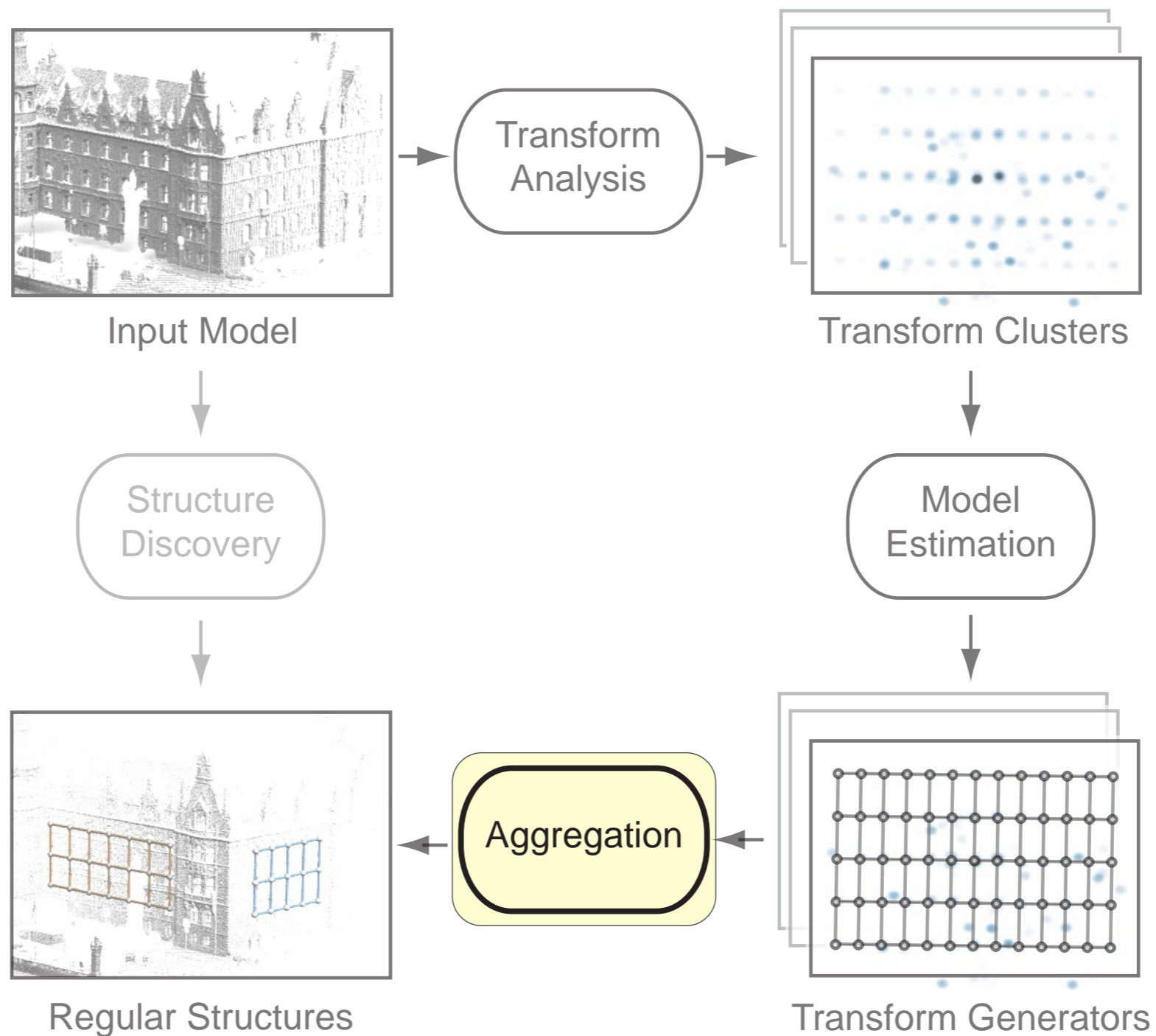
- Region-growing to extract repetitive elements
- Simultaneous registration

$$\mathbf{H}_+ \approx \mathbf{H} + \epsilon \mathbf{D} \cdot \mathbf{H}$$

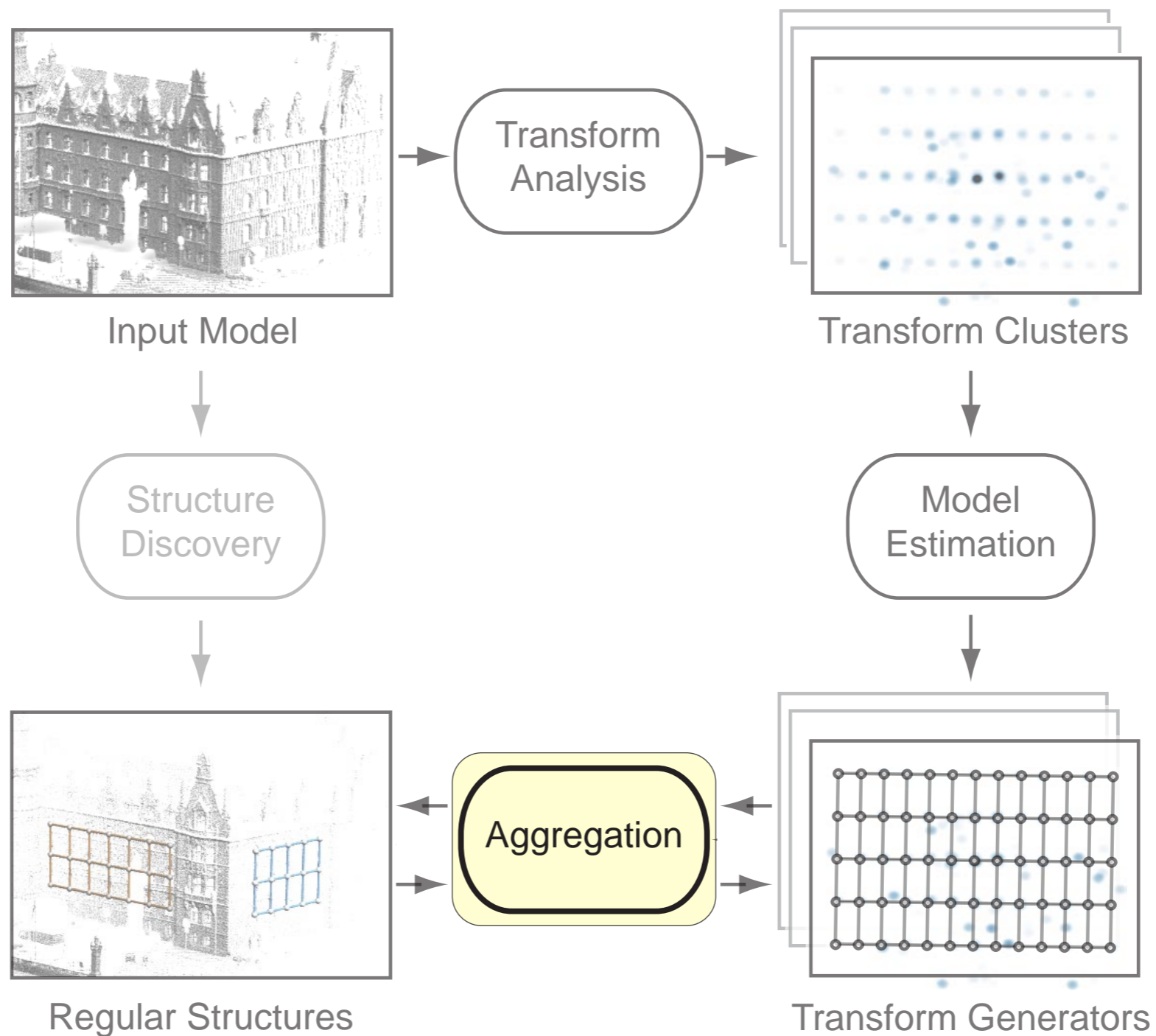
$$T_+^k \approx (\mathbf{H} + \epsilon \mathbf{D} \cdot \mathbf{H})^k$$



# Structure Discovery



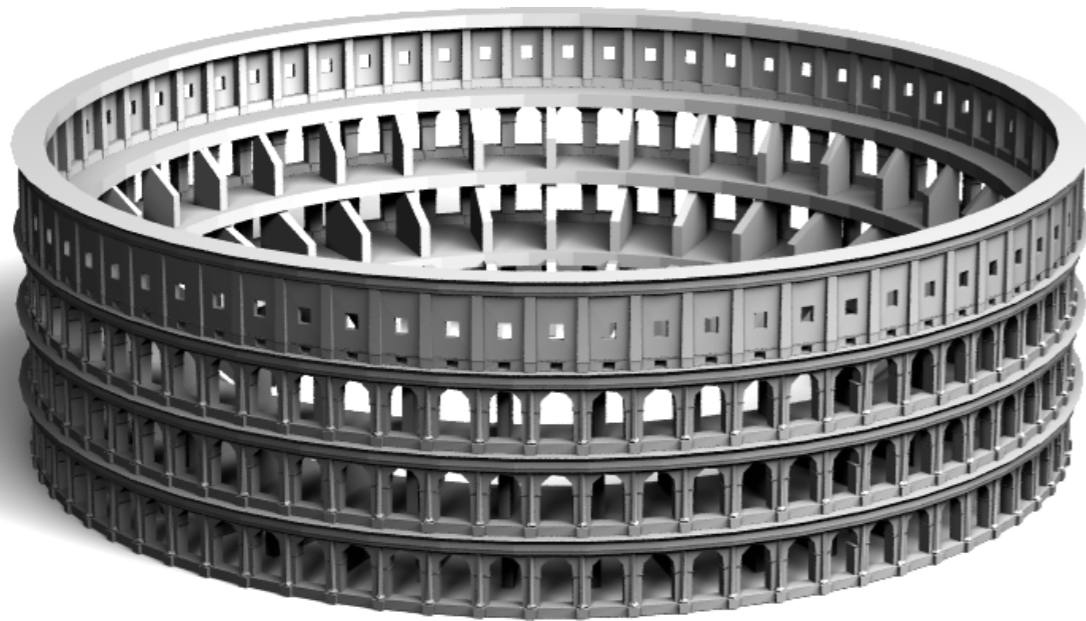
# Structure Discovery



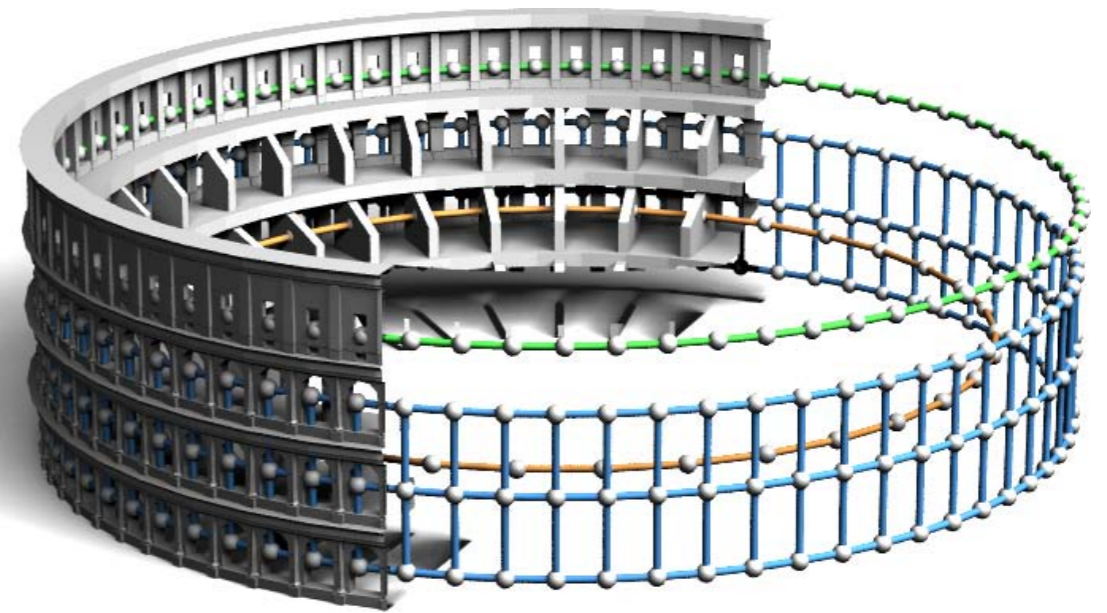
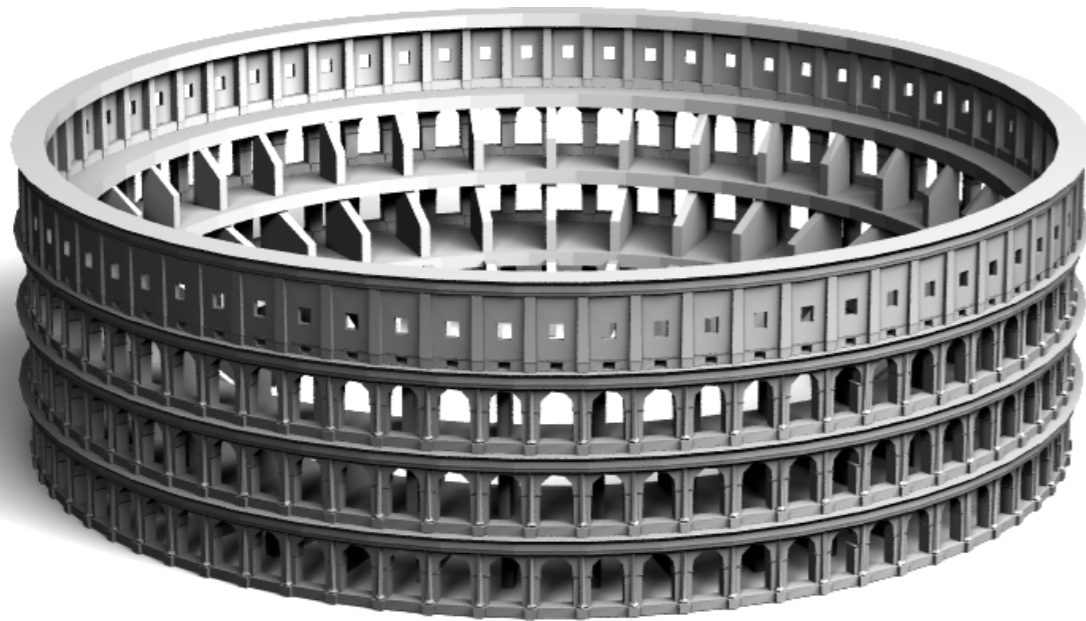


# Results and Applications

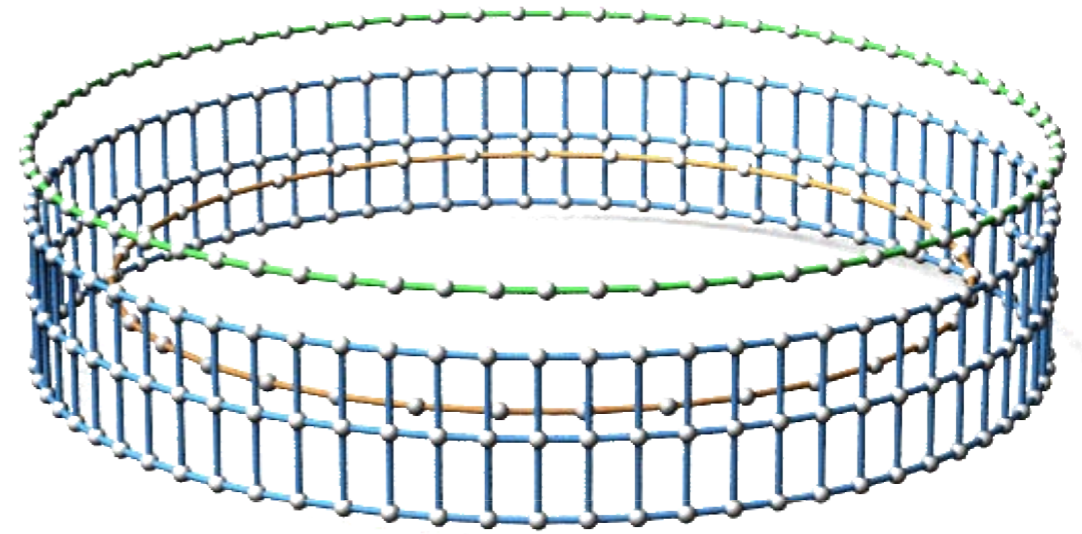
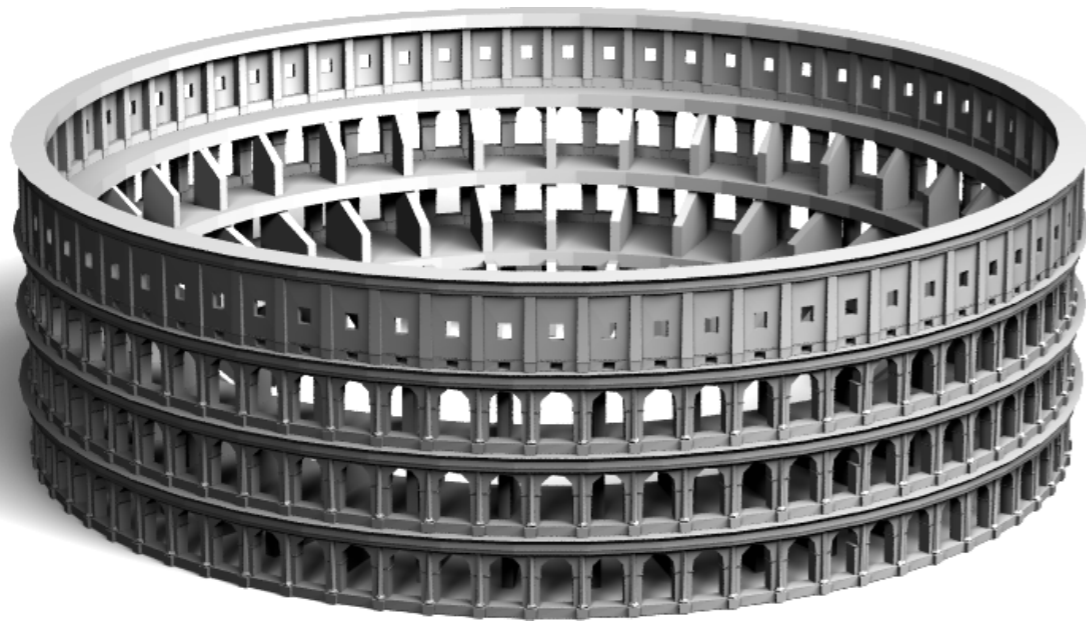
# Amphitheater



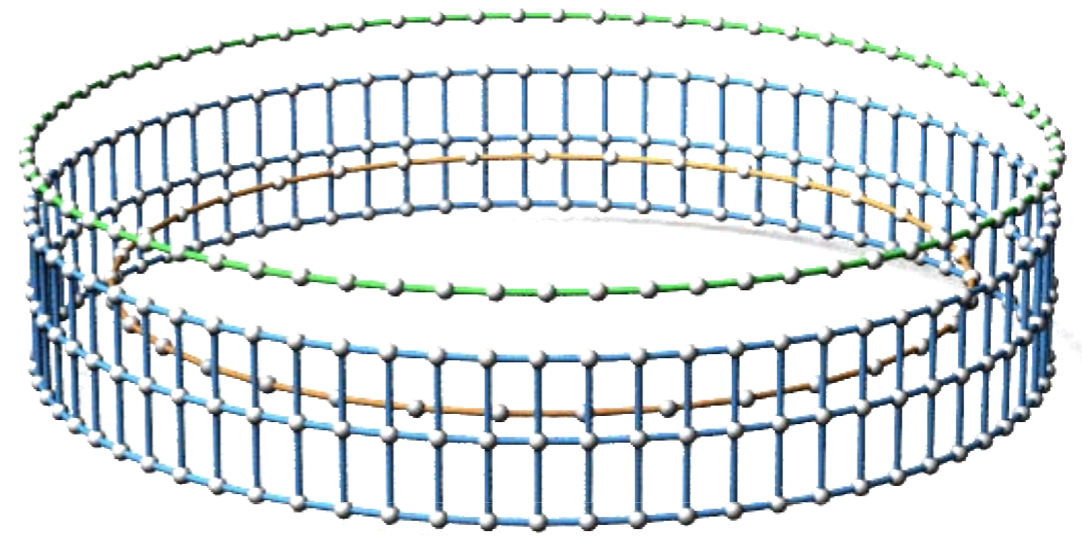
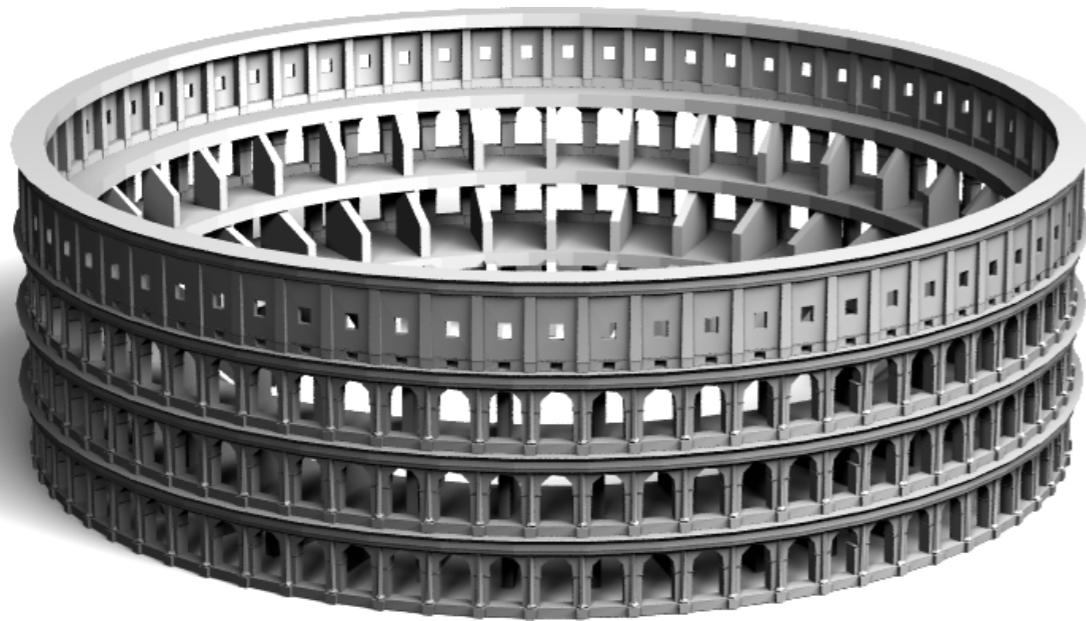
# Amphitheater



# Amphitheater



# Amphitheater



Rot: 72

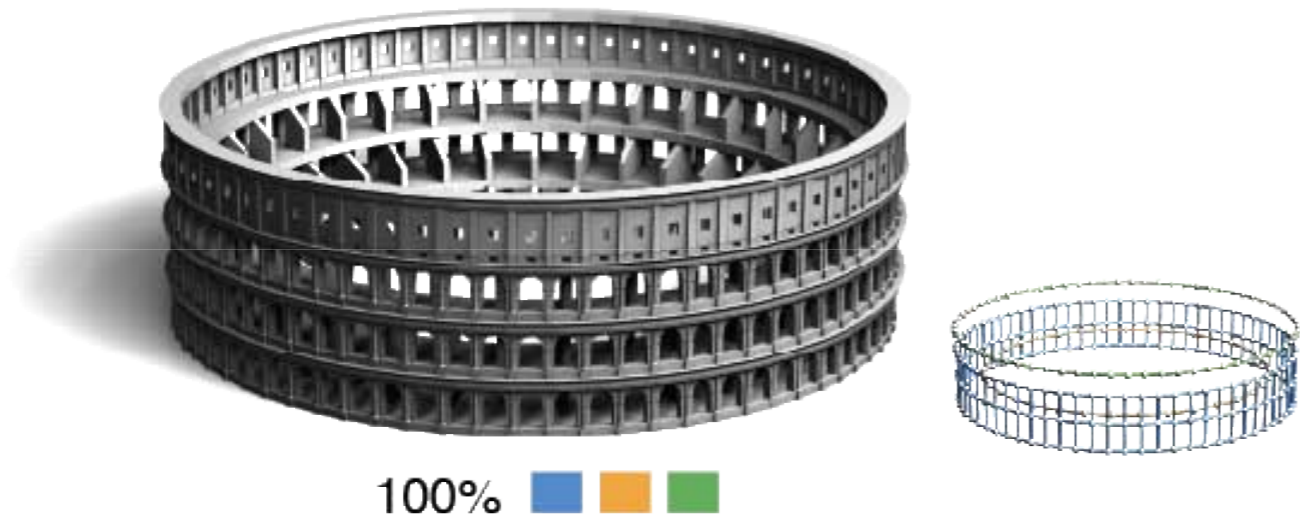


Rot x Trans: 72 x 3

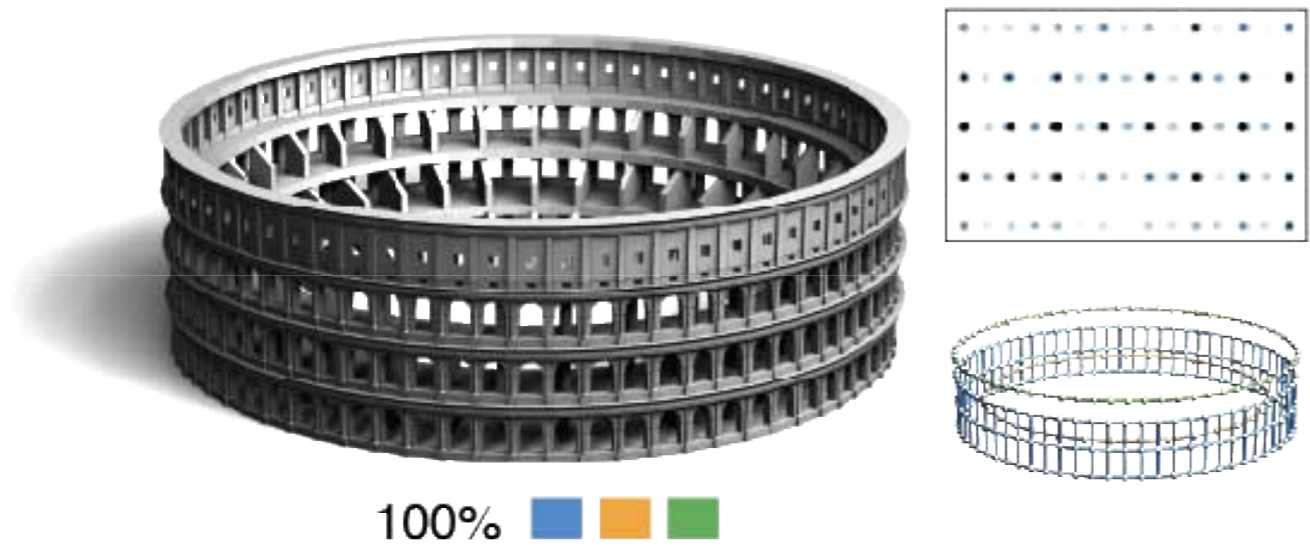


Rot: 35

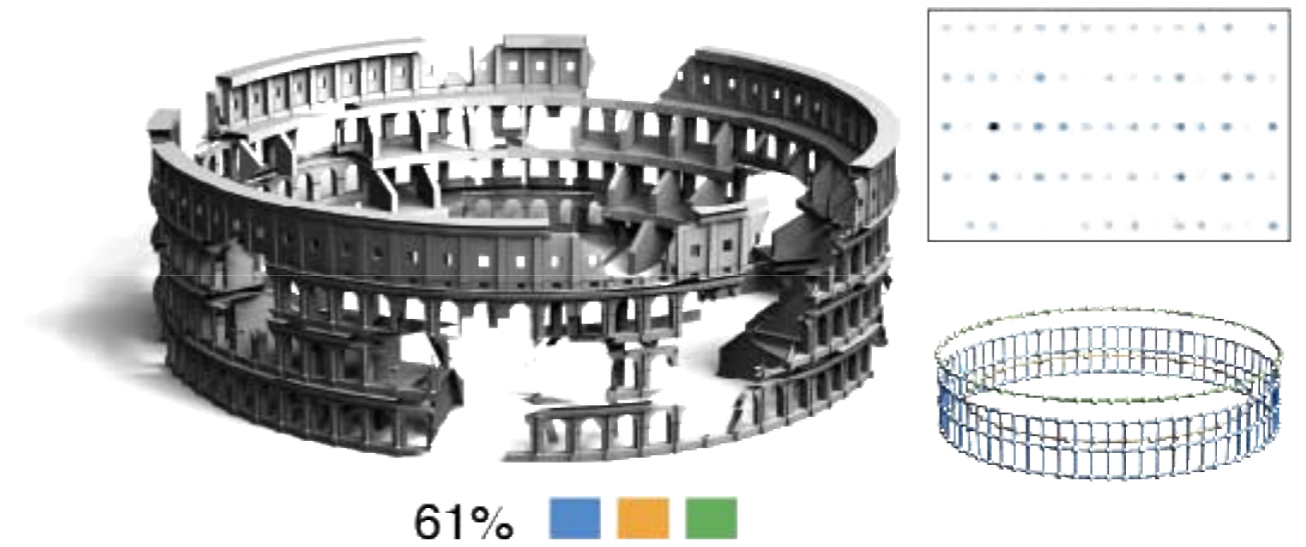
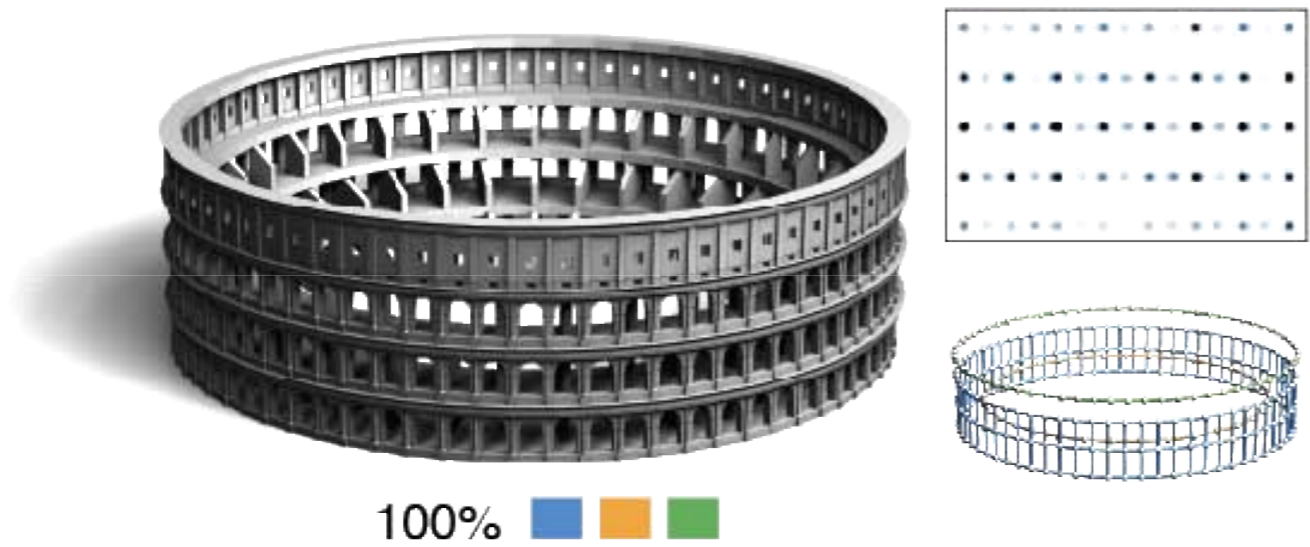
# Robustness



# Robustness

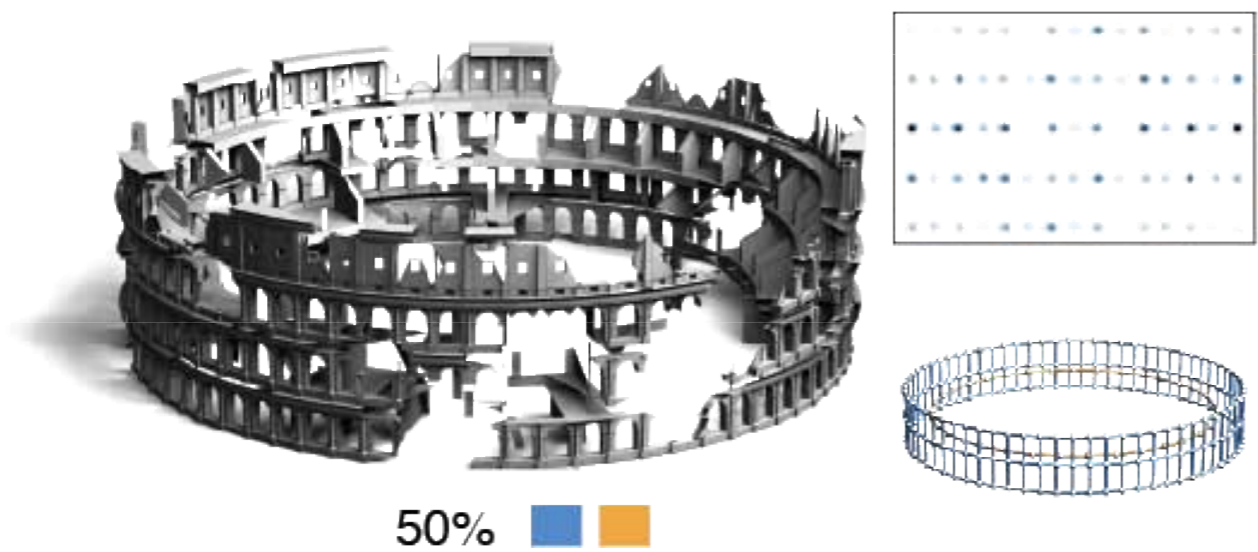
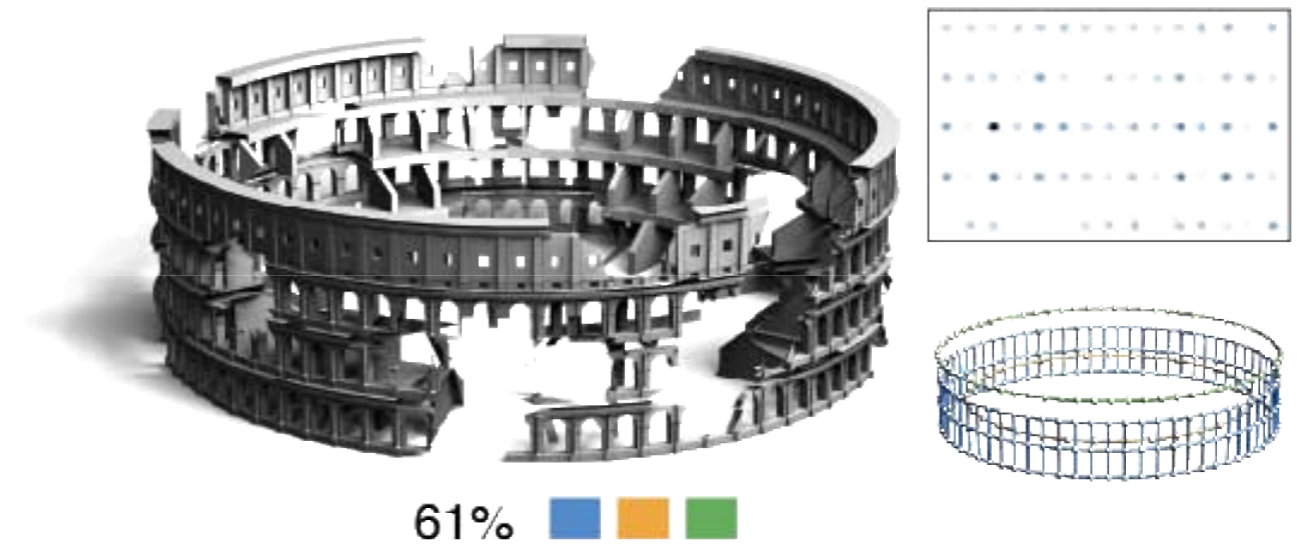
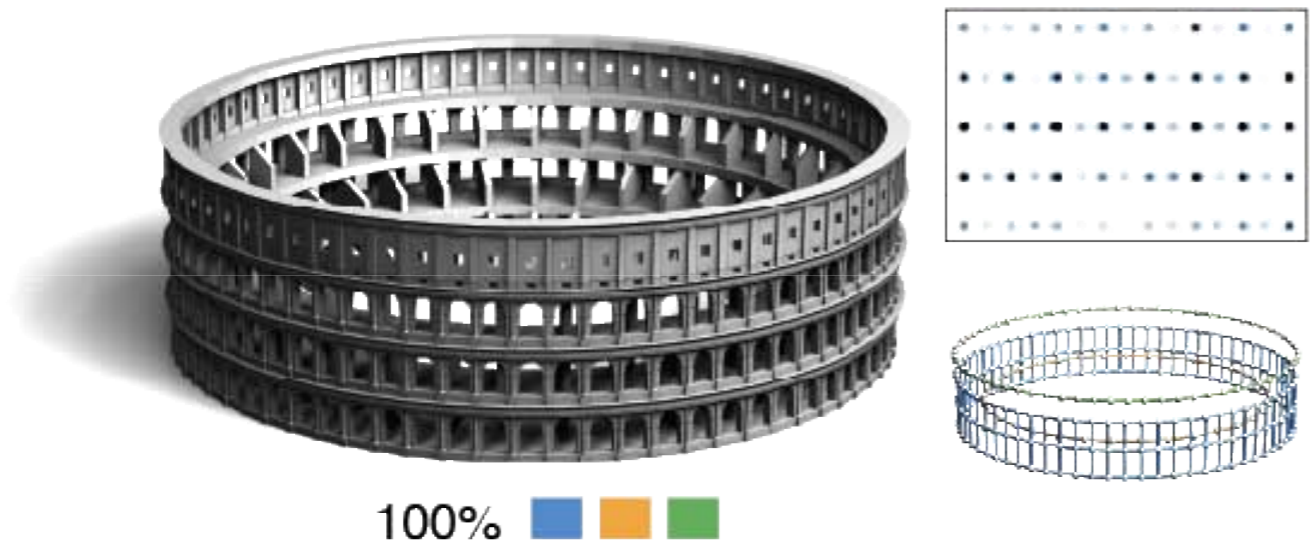


# Robustness

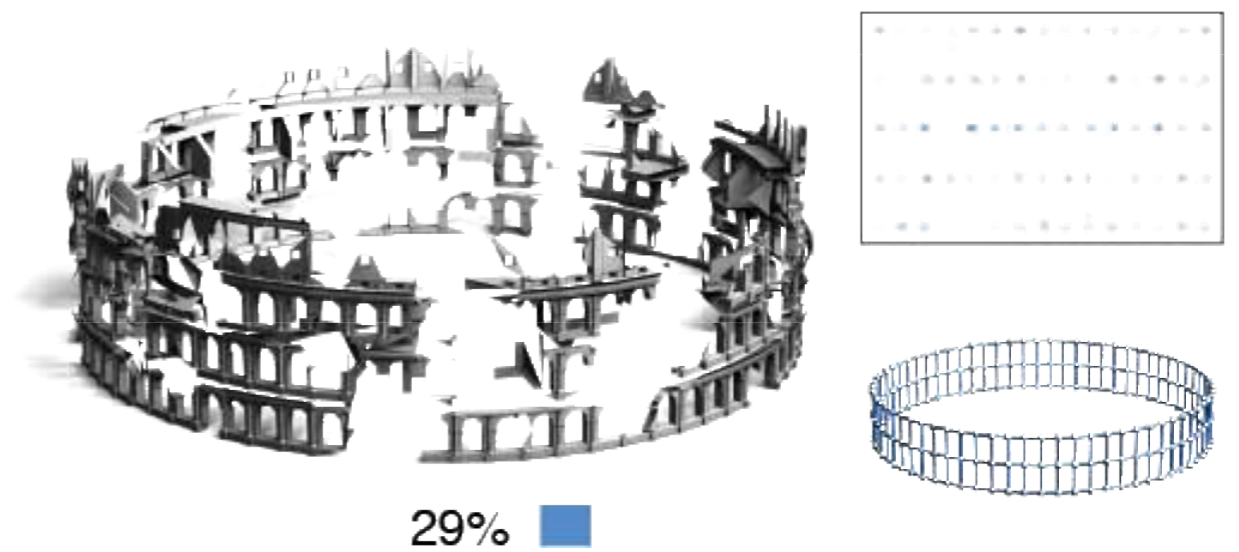
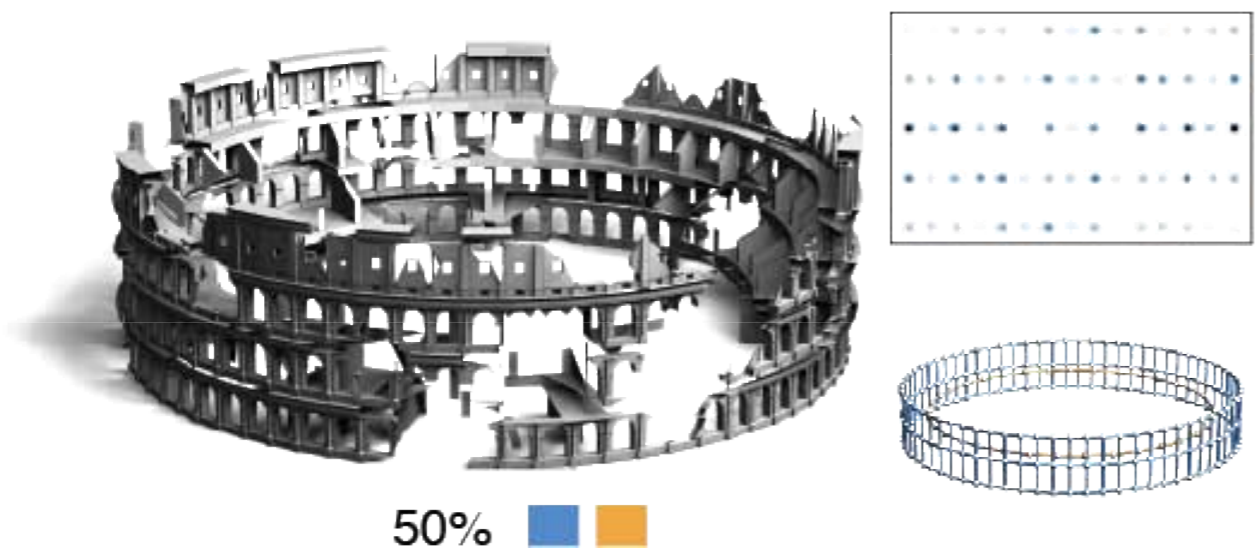
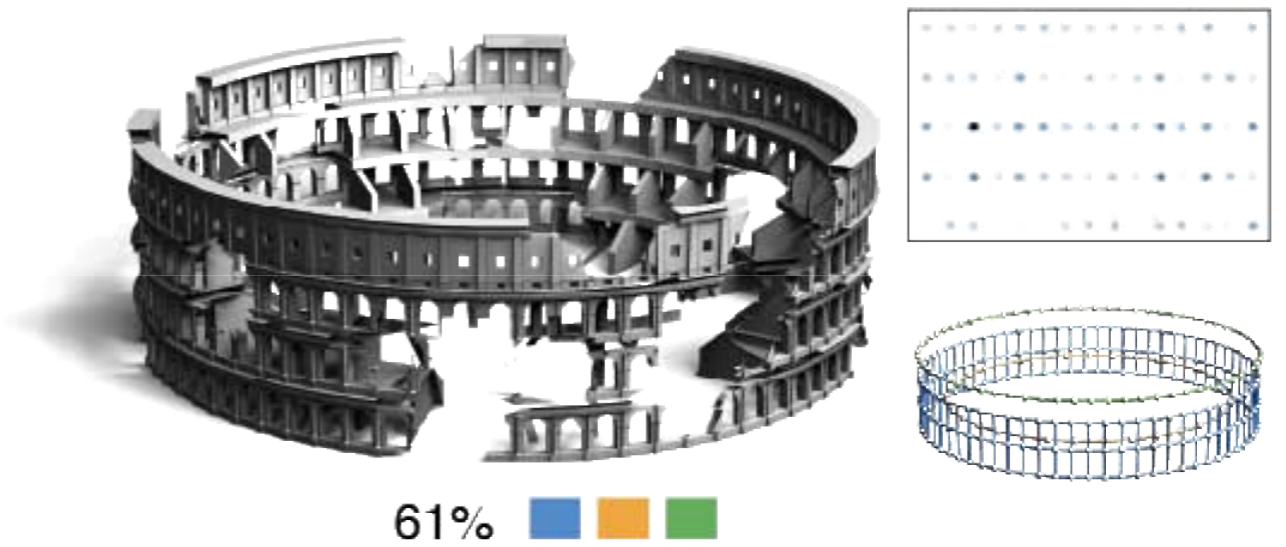
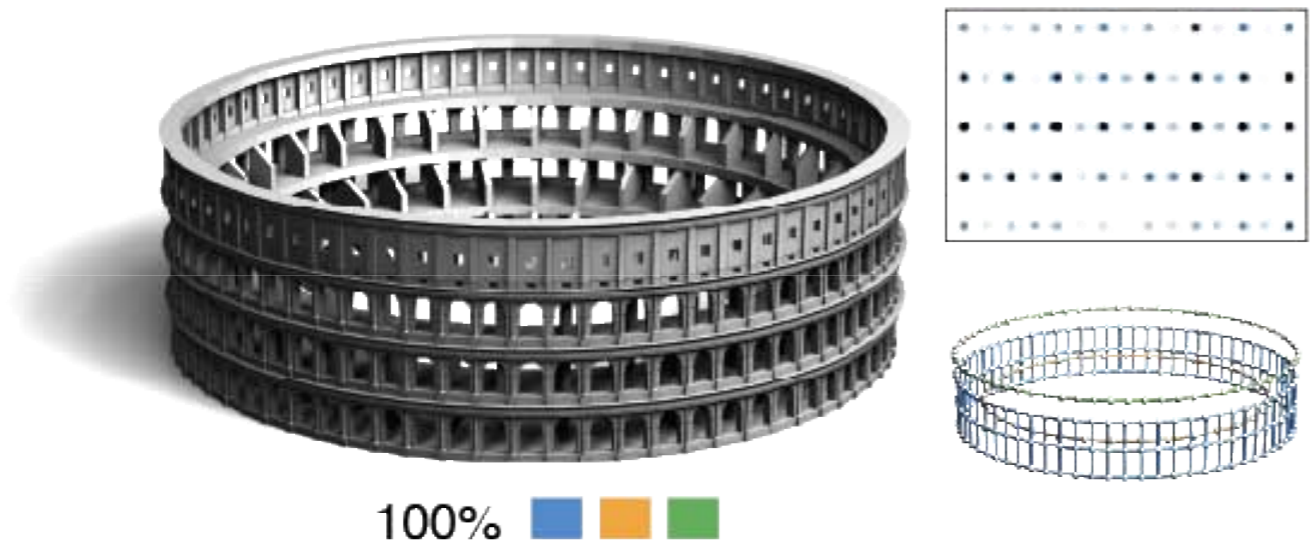




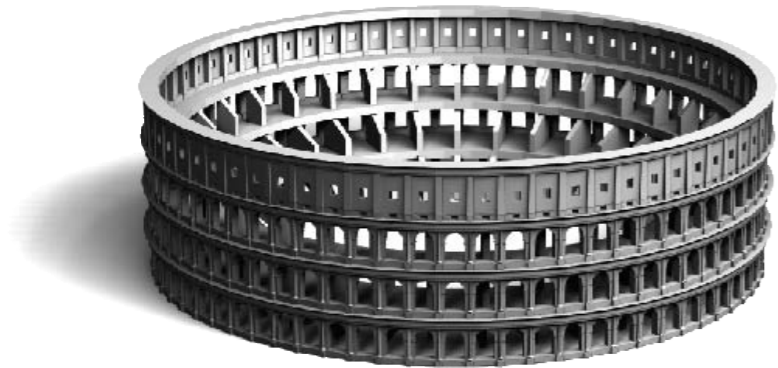
# Robustness



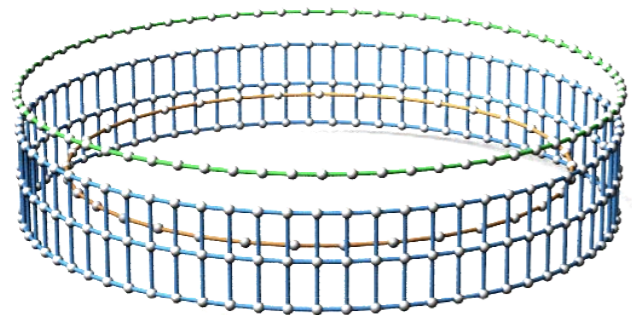
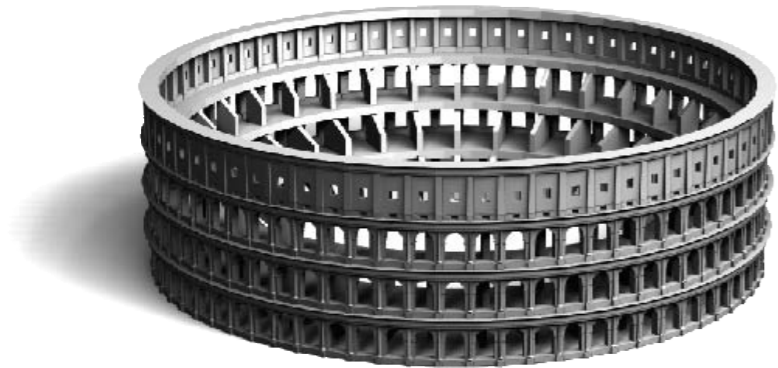
# Robustness



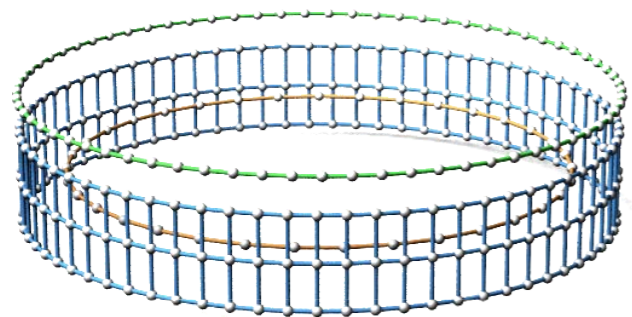
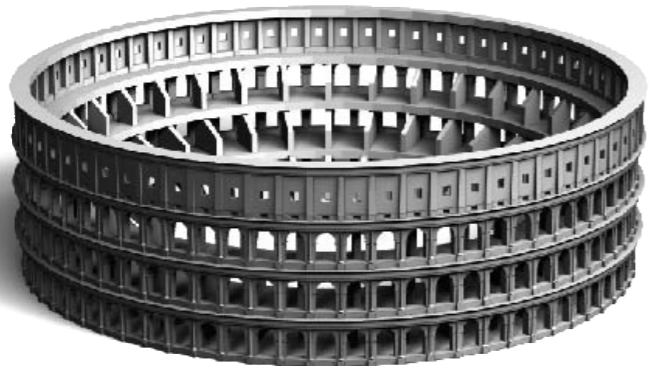
# Geometry Synthesis



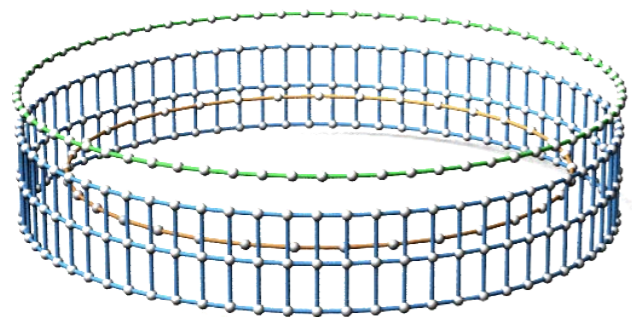
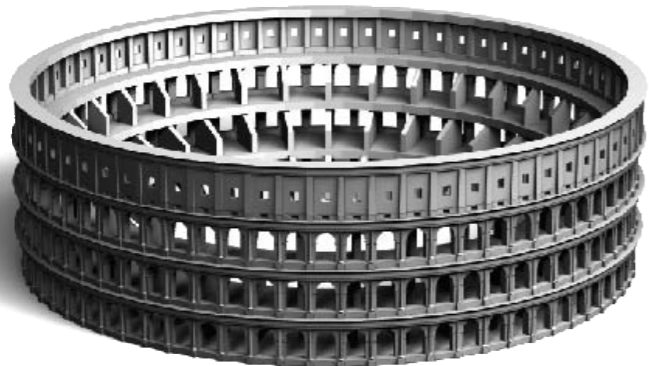
# Geometry Synthesis



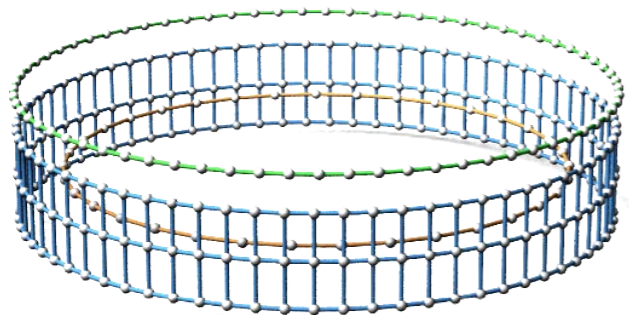
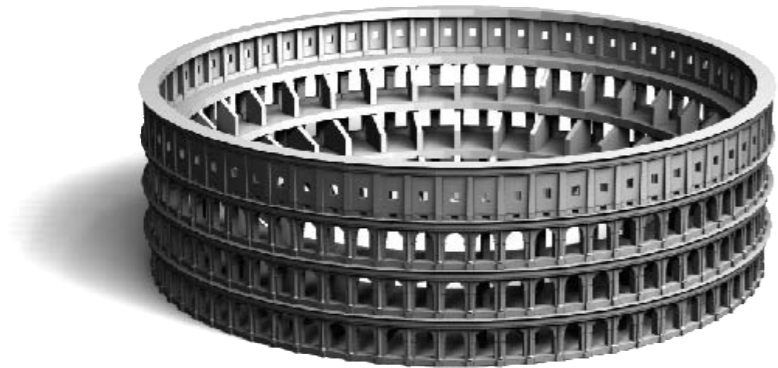
# Geometry Synthesis



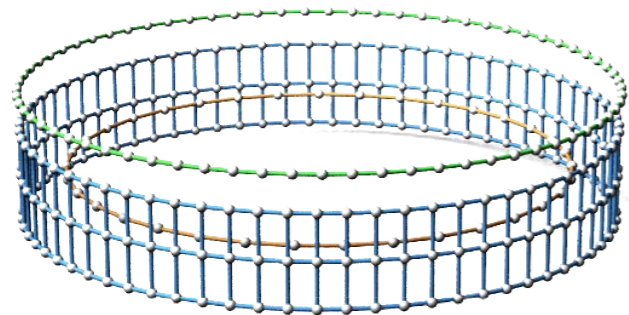
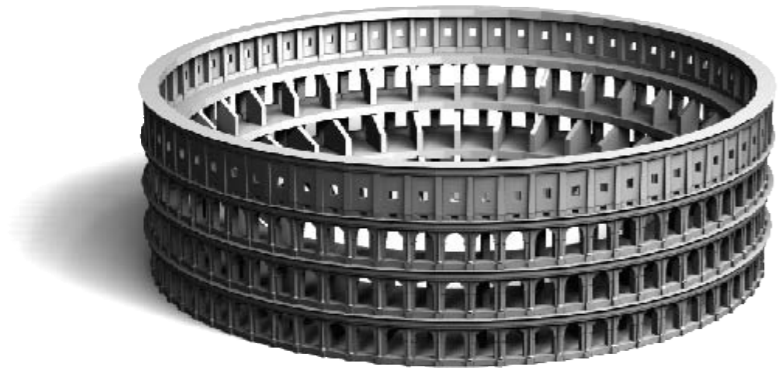
# Geometry Synthesis



# Geometry Synthesis

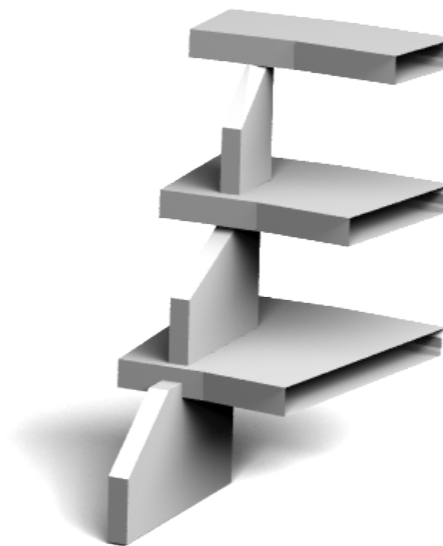
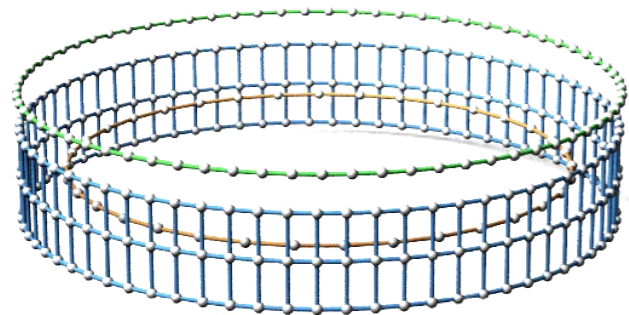
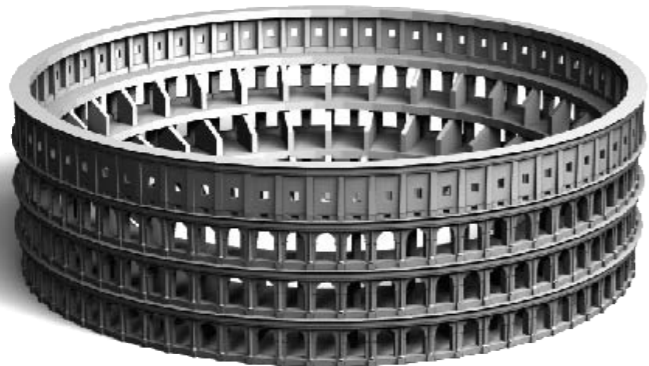


# Geometry Synthesis

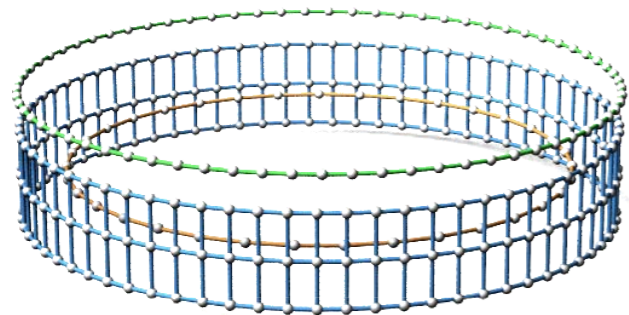
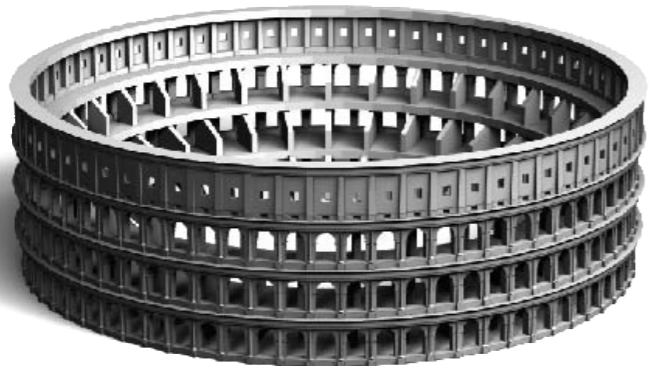




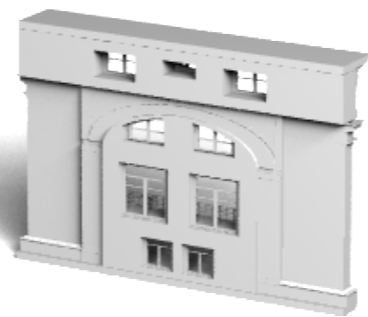
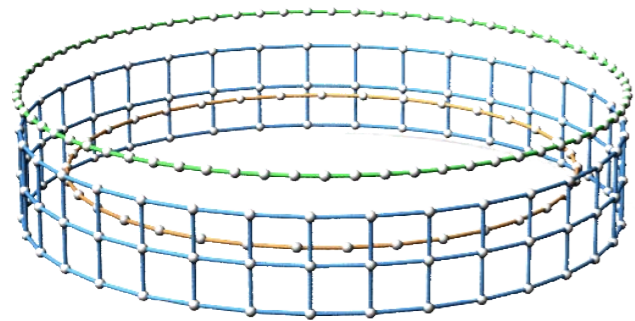
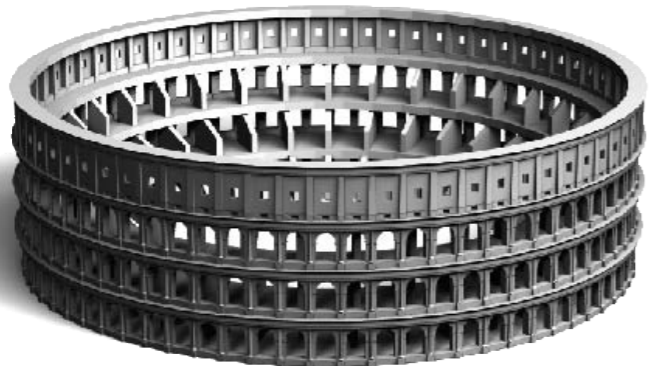
# Geometry Synthesis



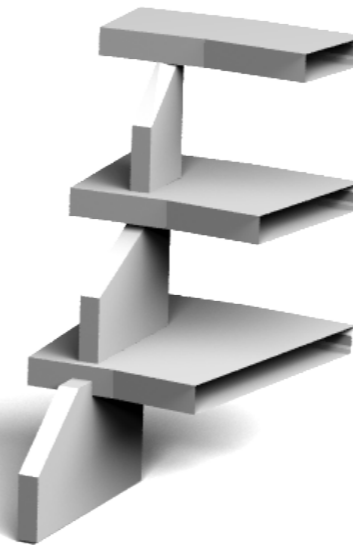
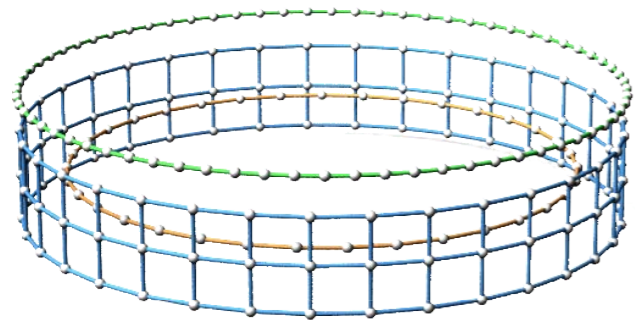
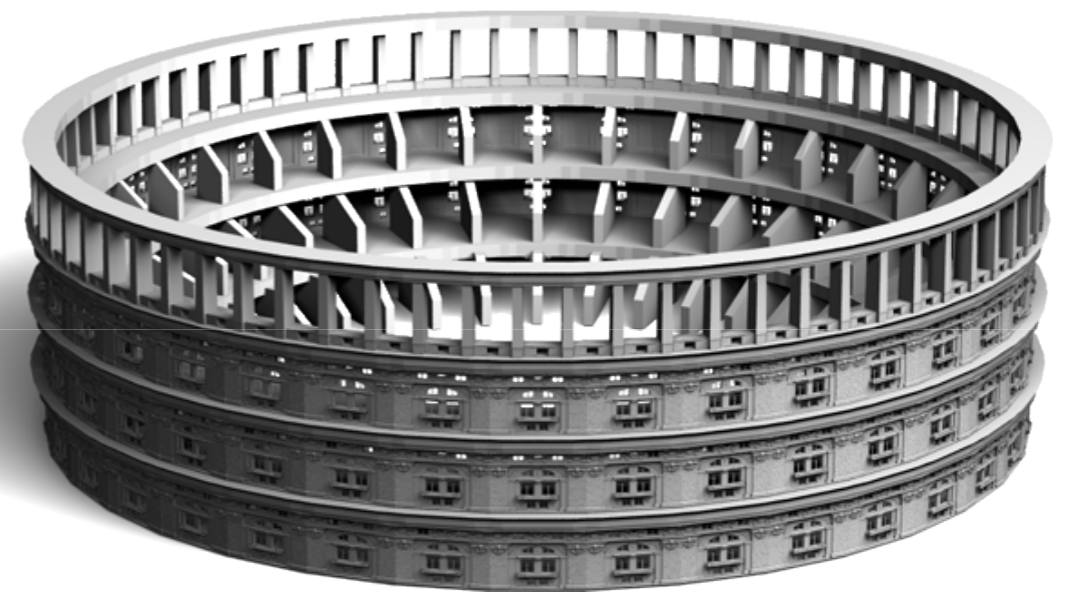
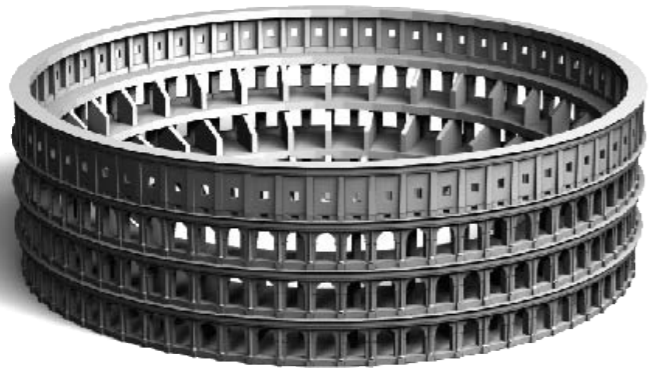
# Geometry Synthesis



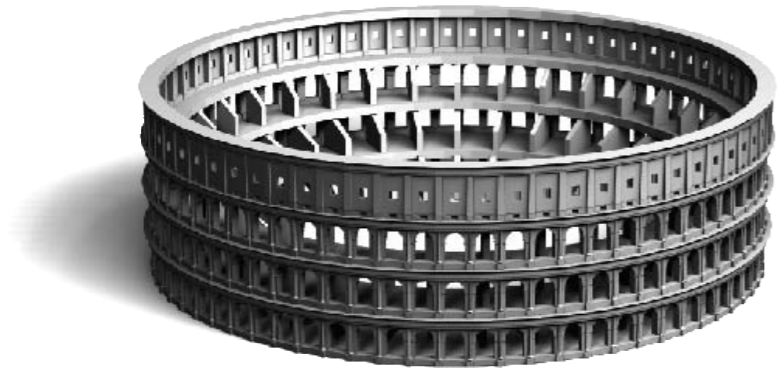
# Geometry Synthesis



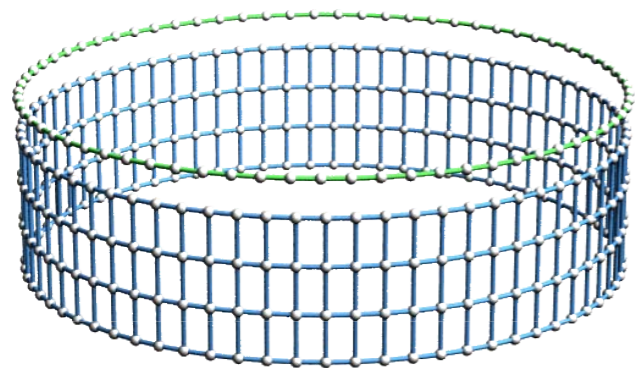
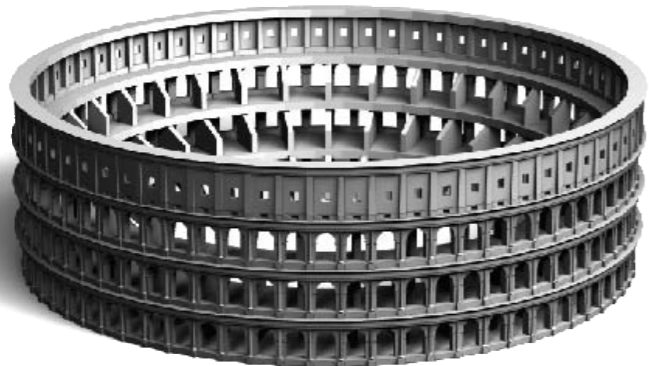
# Geometry Synthesis



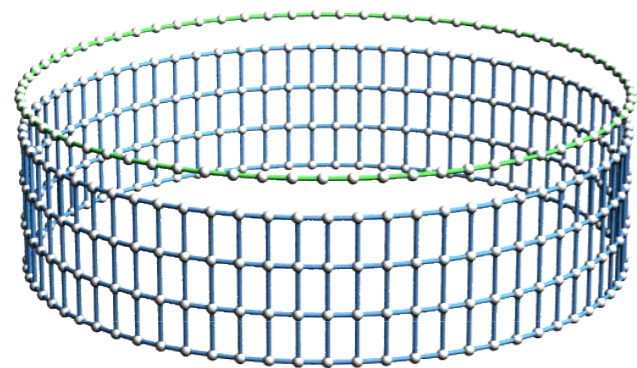
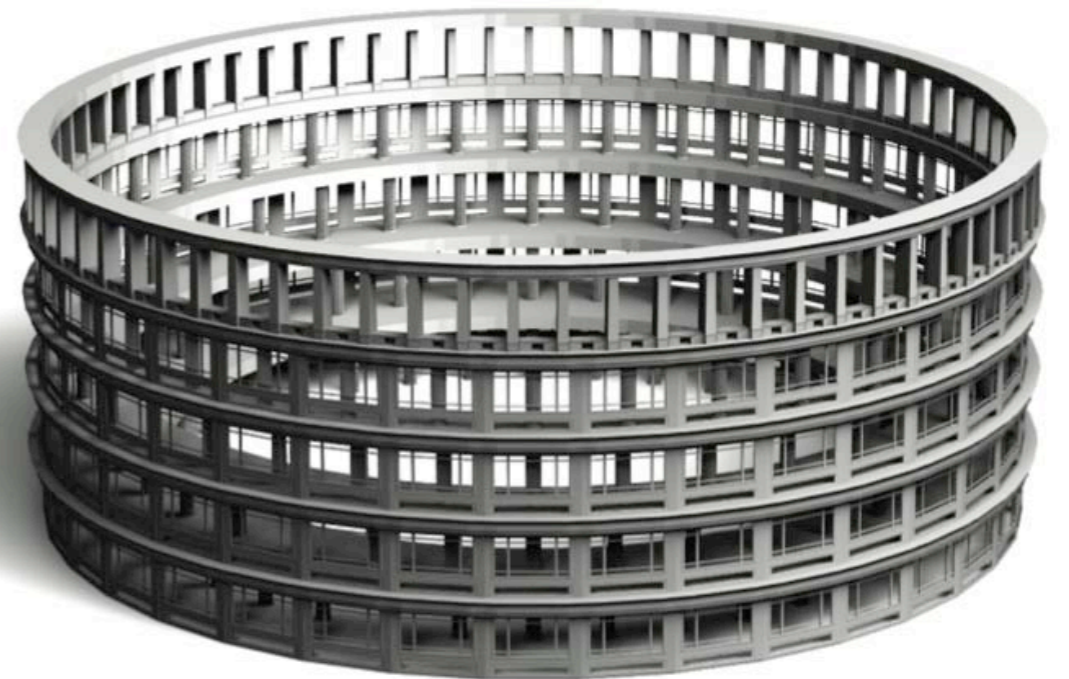
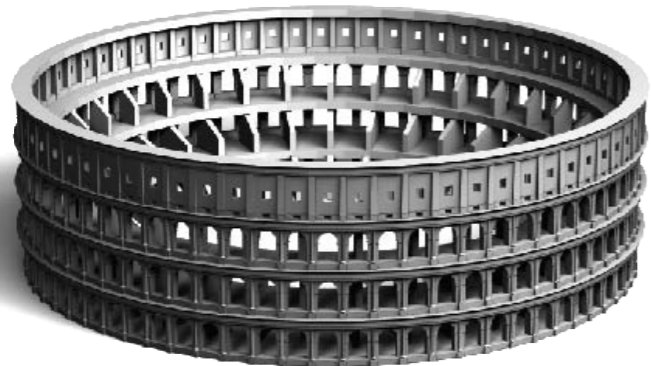
# Geometry Synthesis



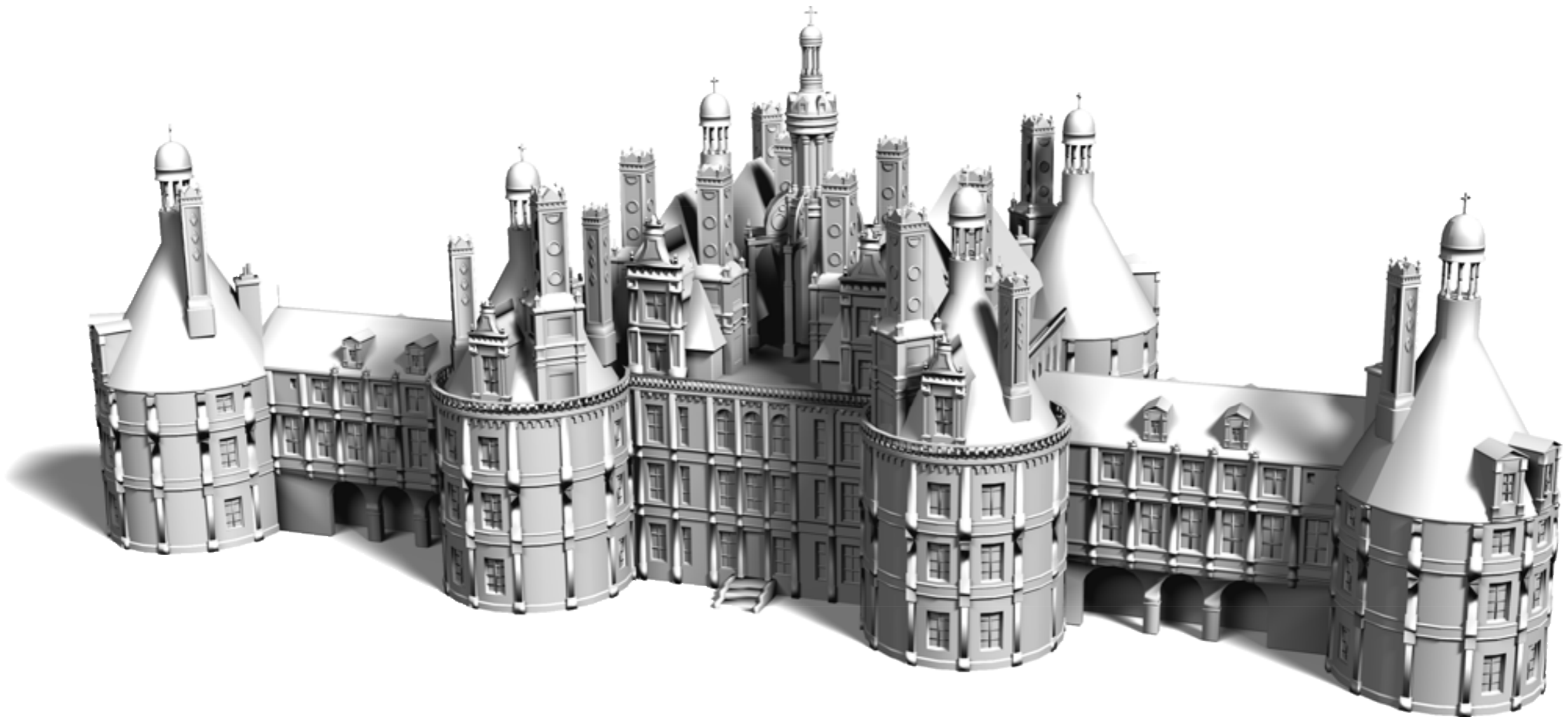
# Geometry Synthesis



# Geometry Synthesis

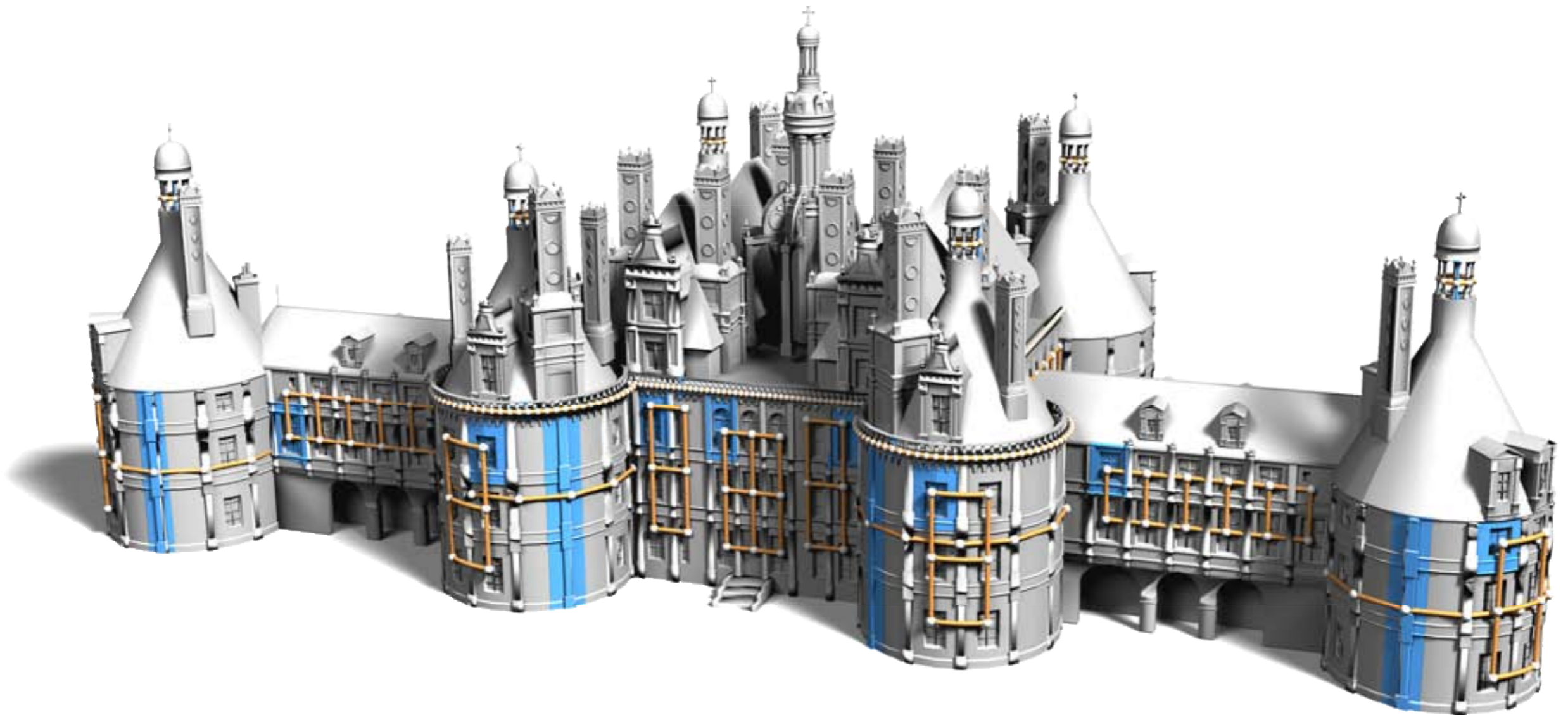


# Chambord Castle

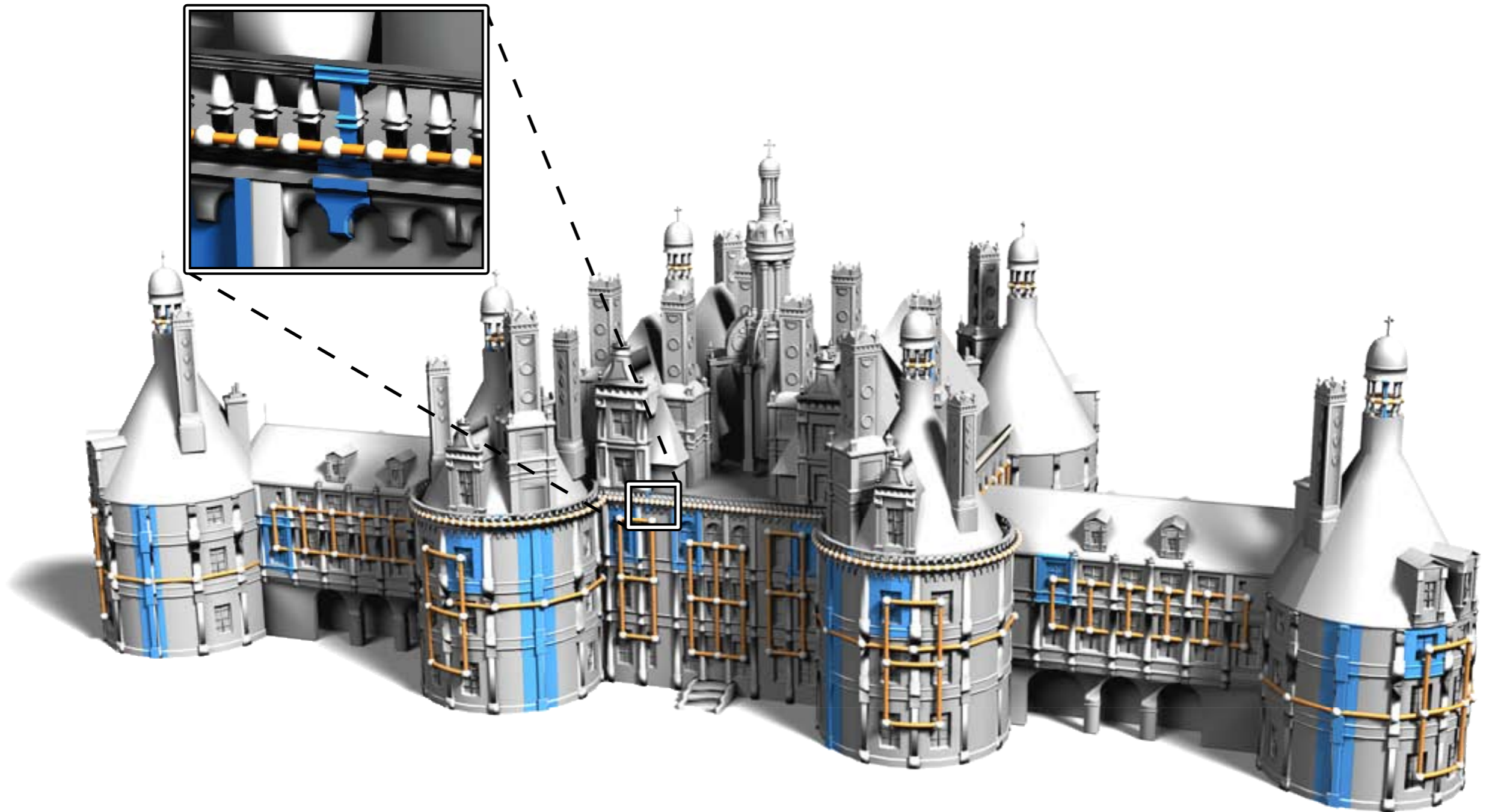




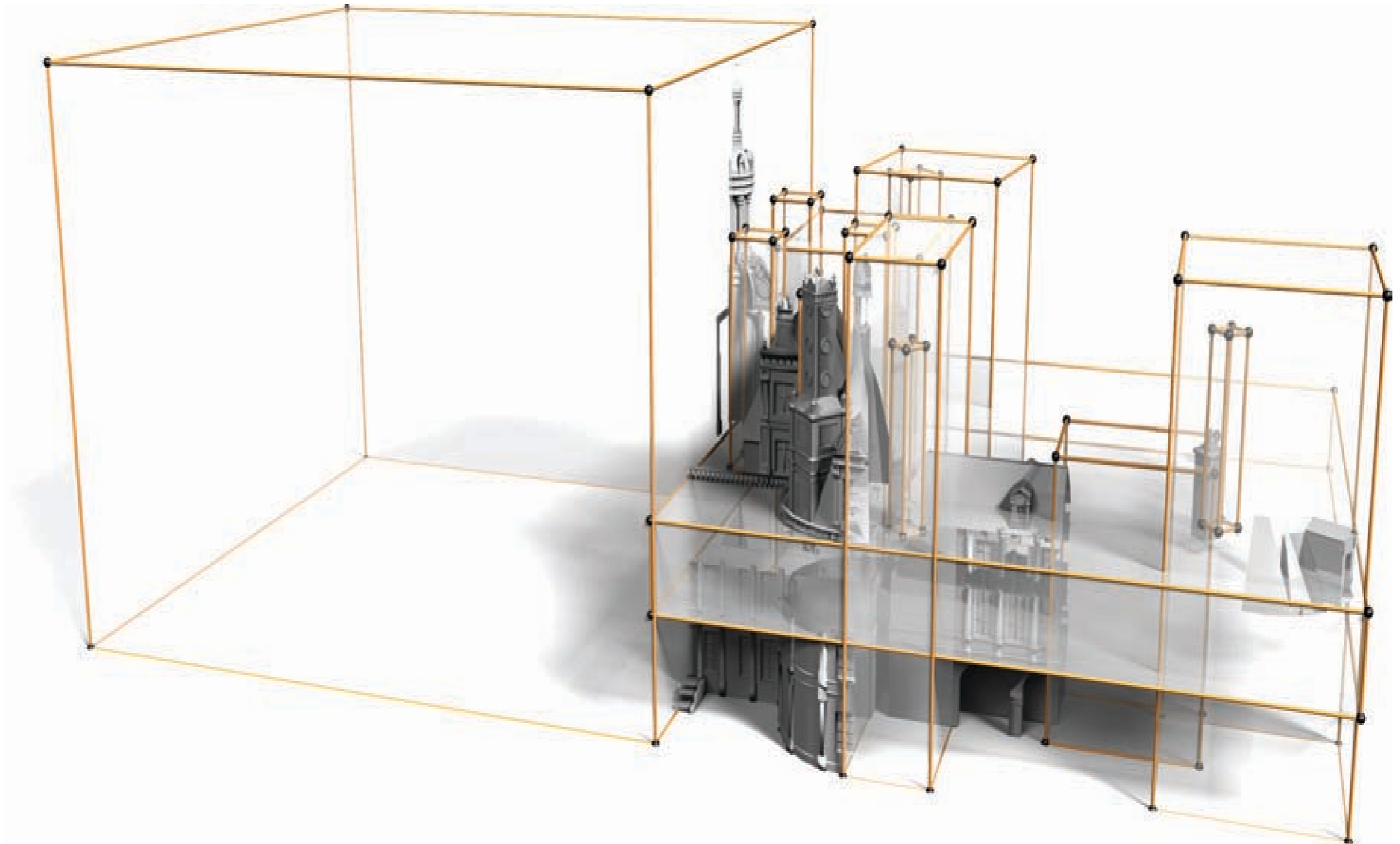
# Chambord Castle



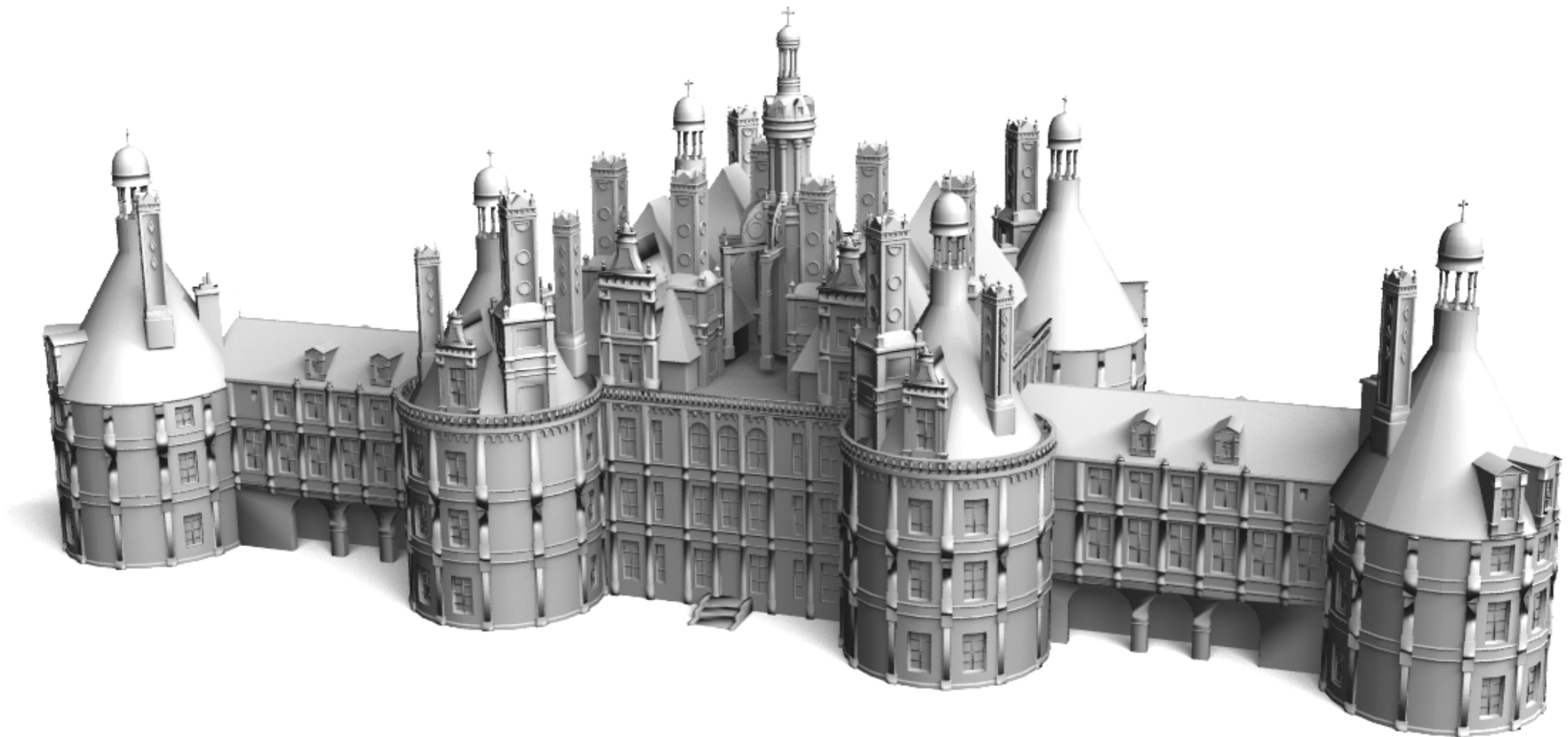
# Chambord Castle



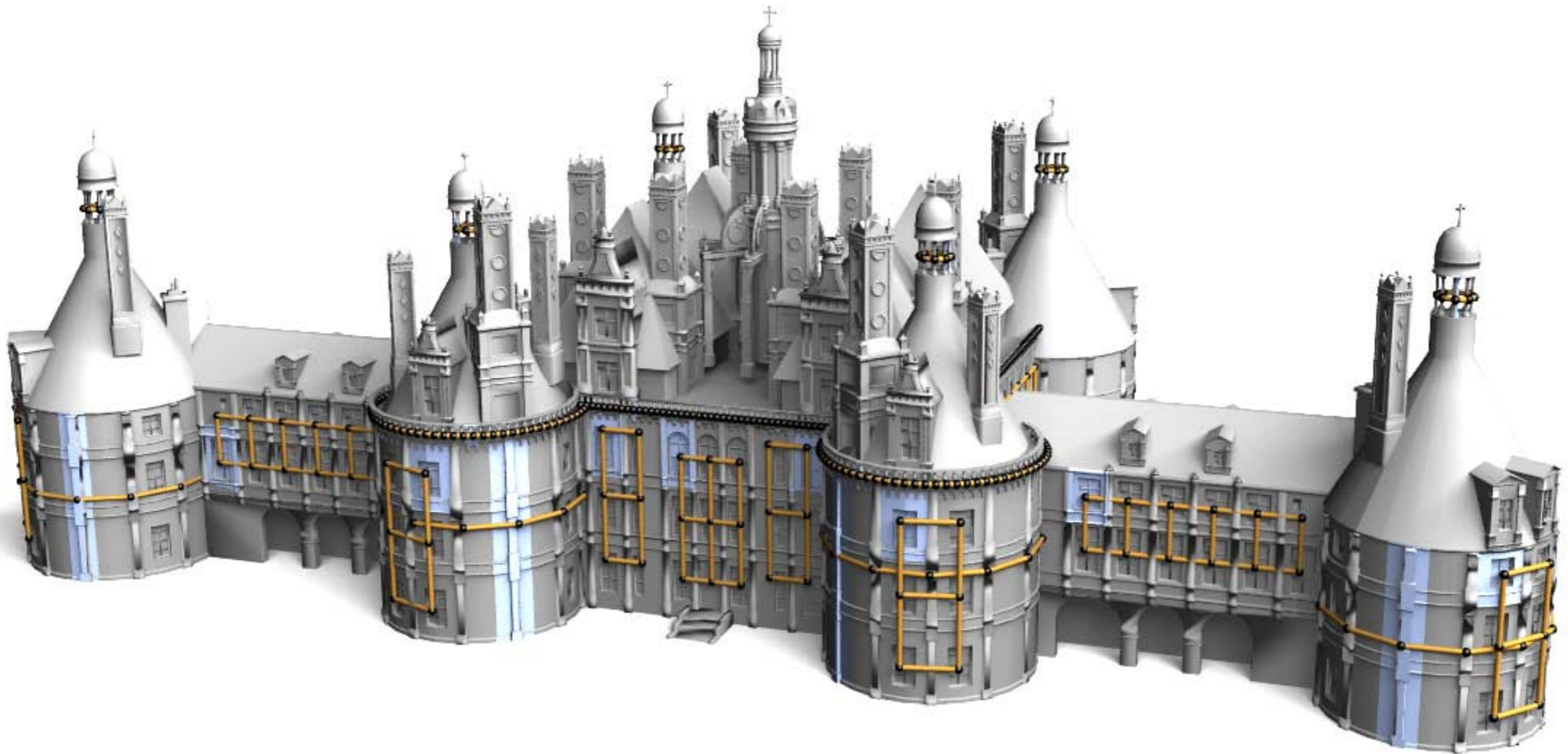
# Chambord Castle [Mitra et al. '06]



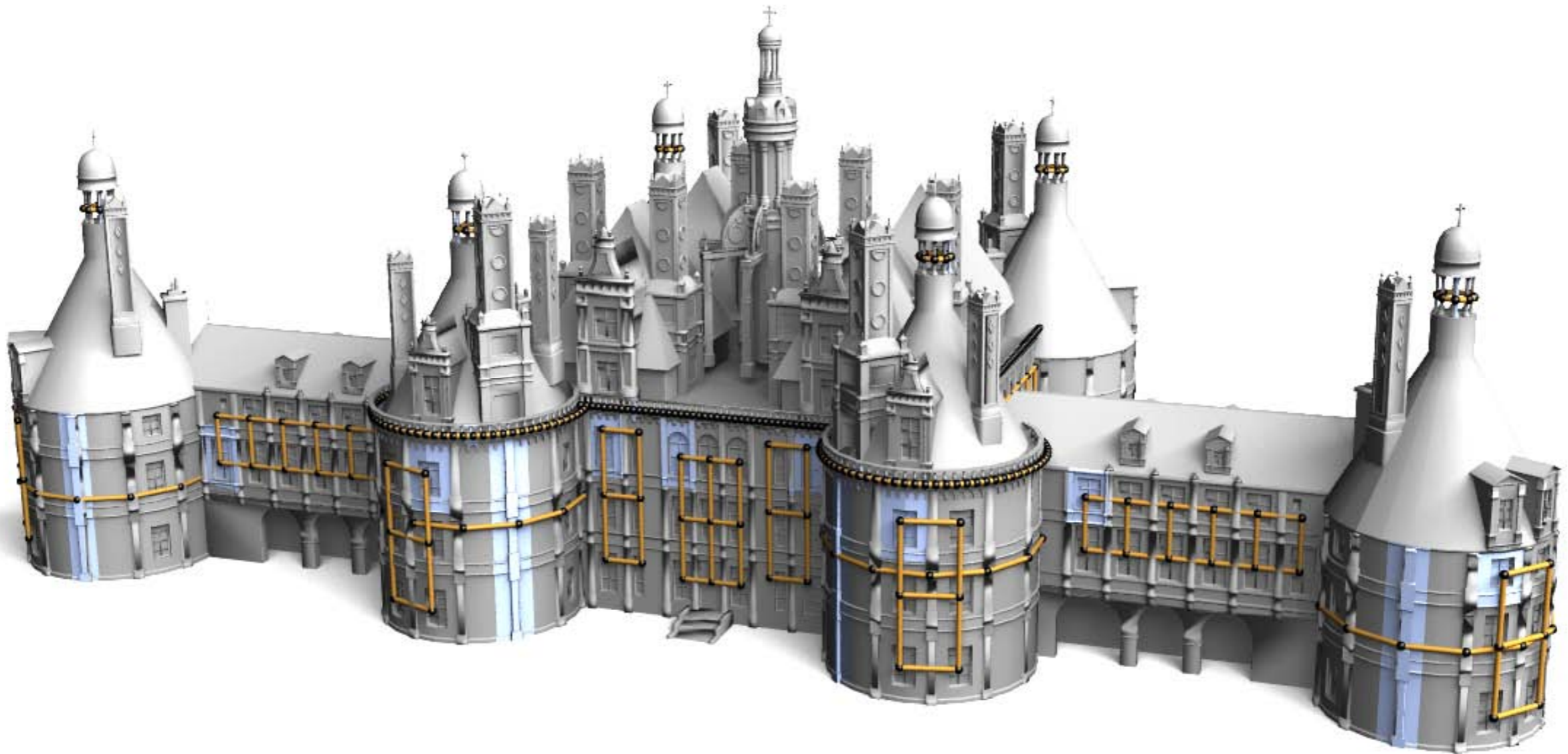
# Chambord Castle



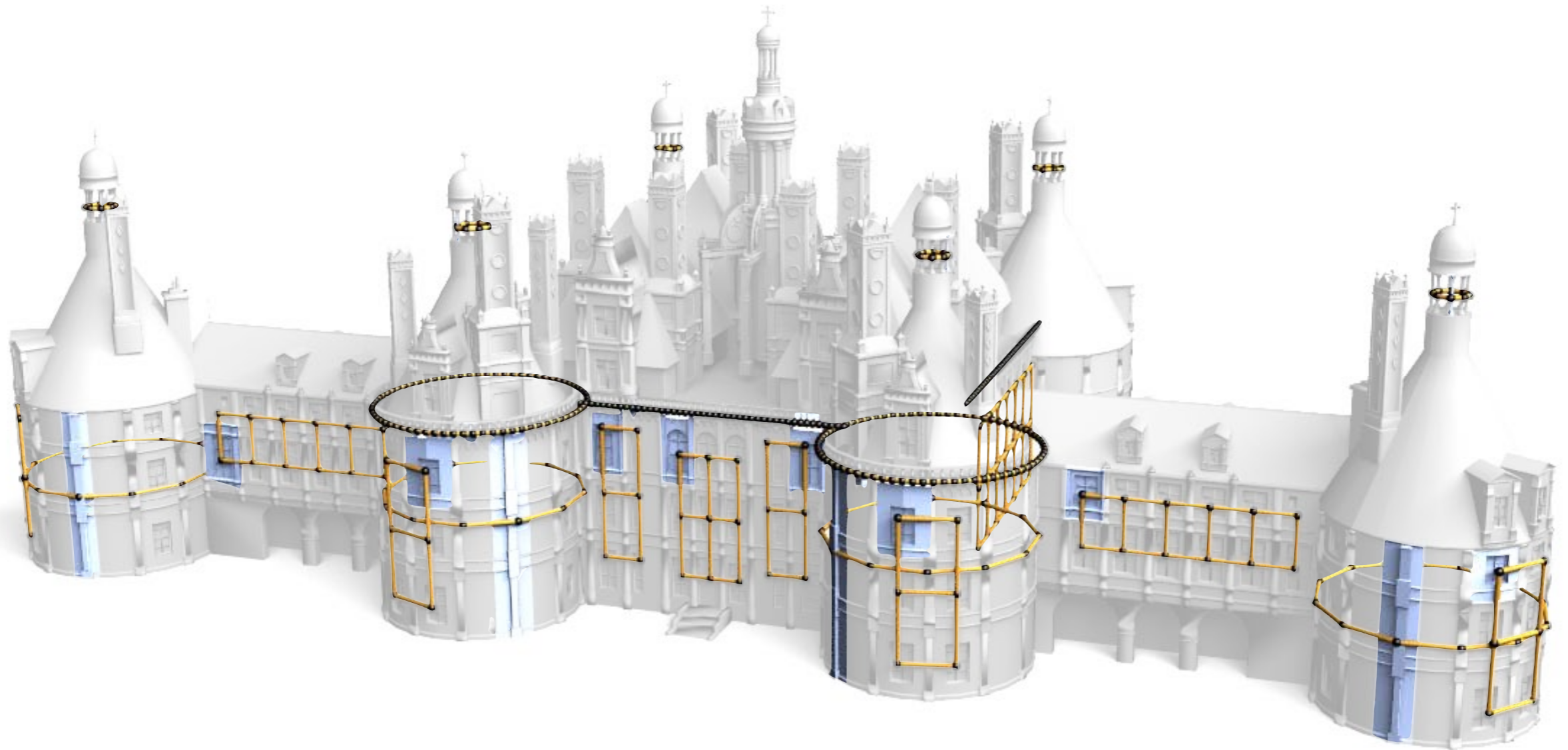
# Chambord Castle



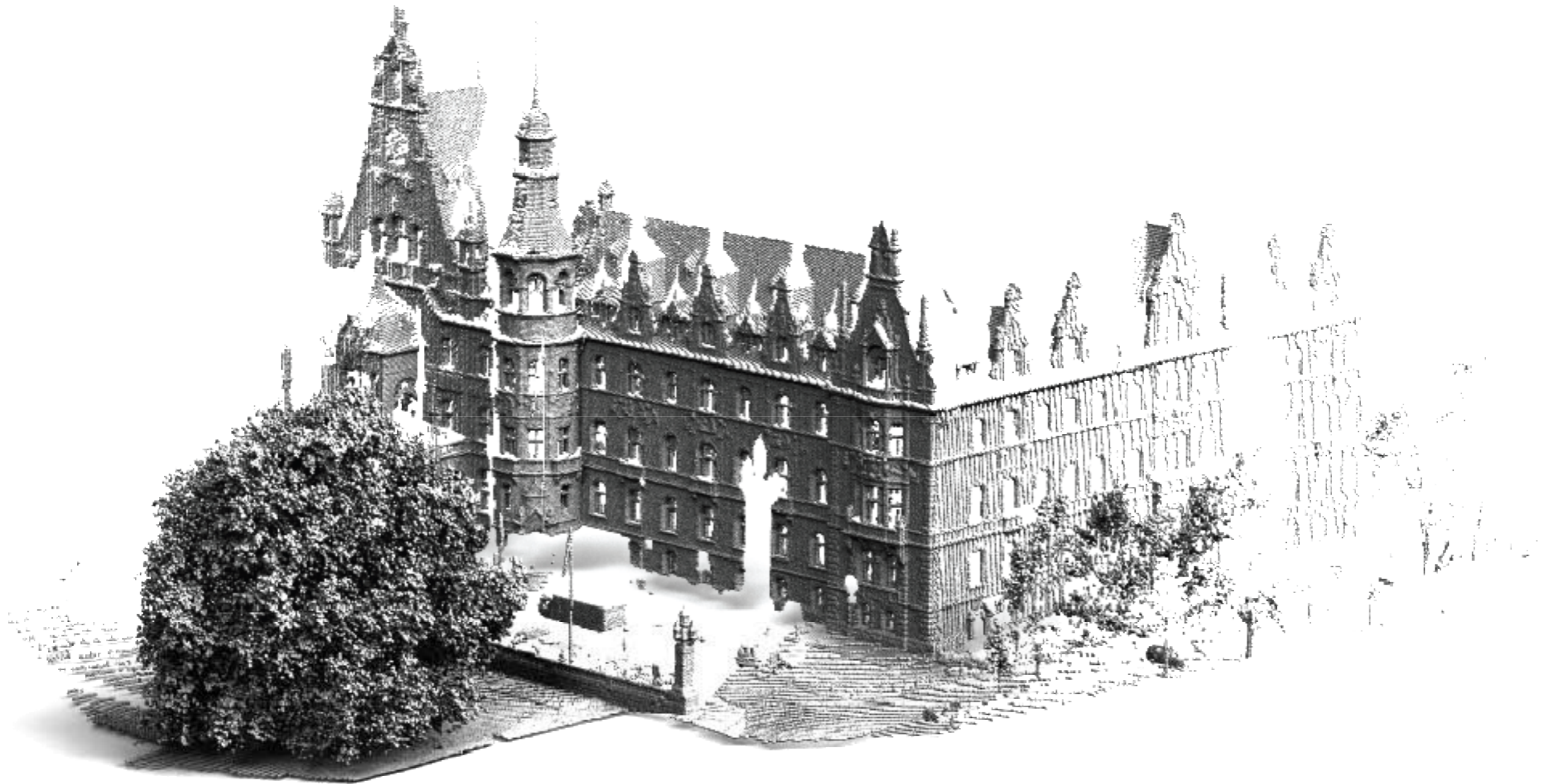
# Chambord Castle



# Chambord Castle

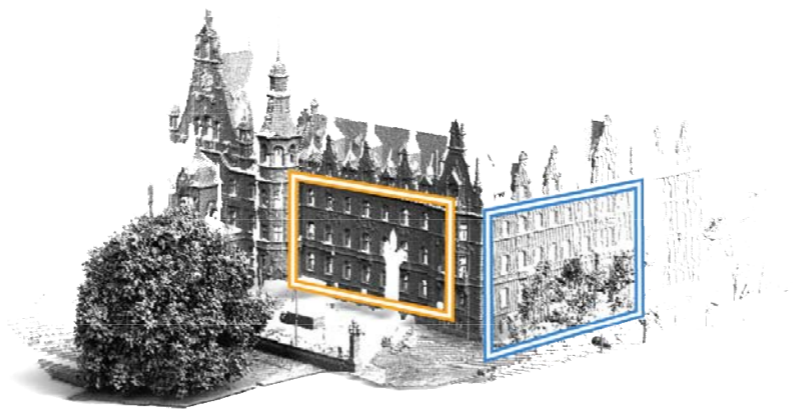


# Outdoor Scan

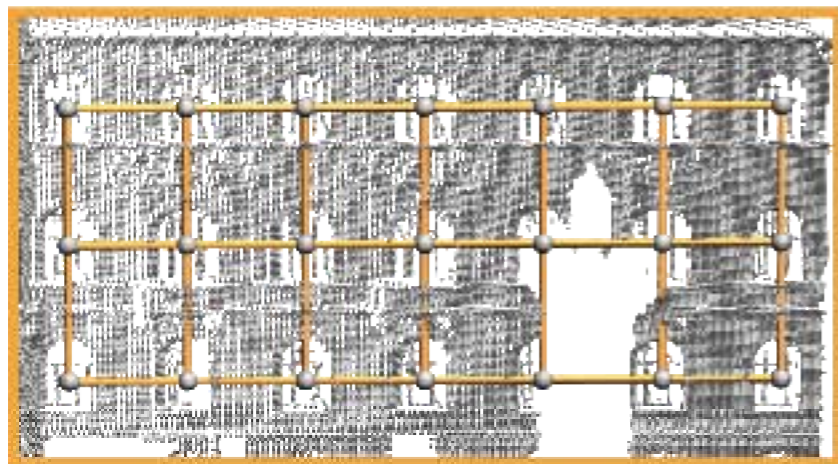
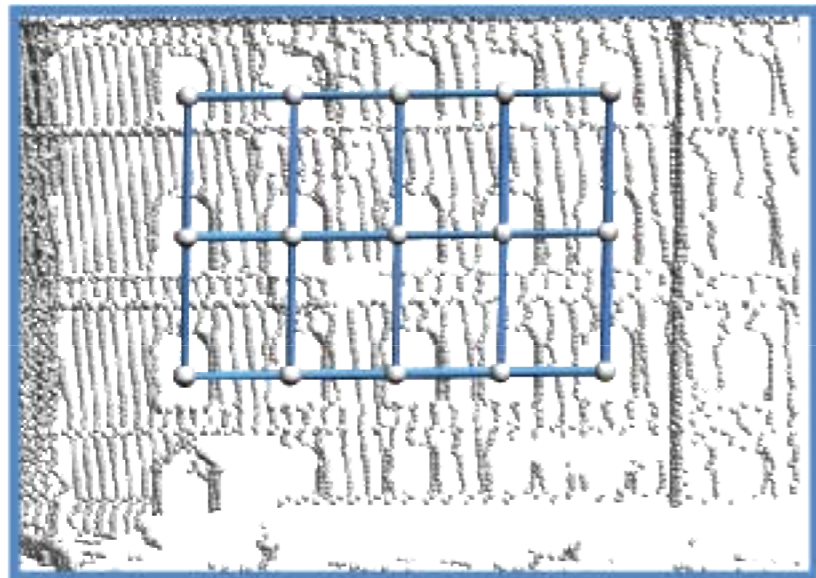
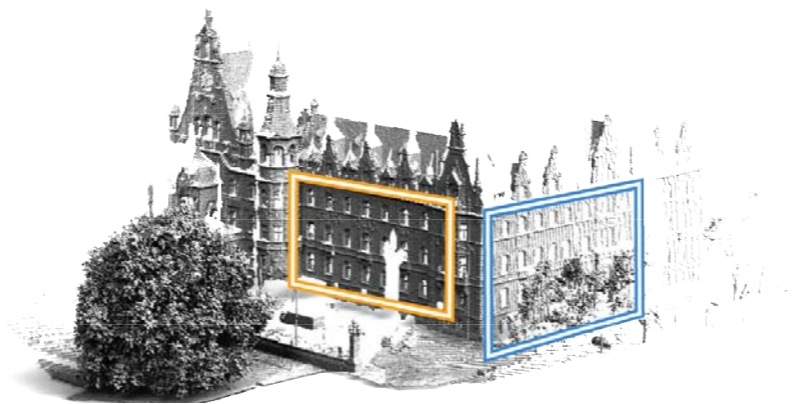




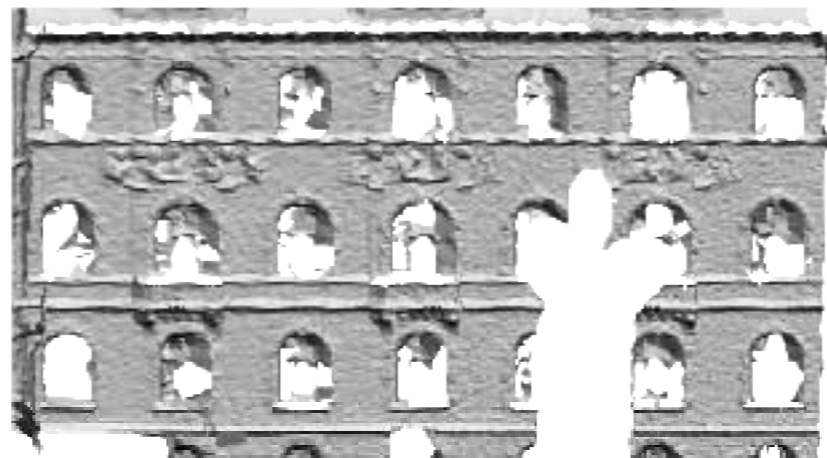
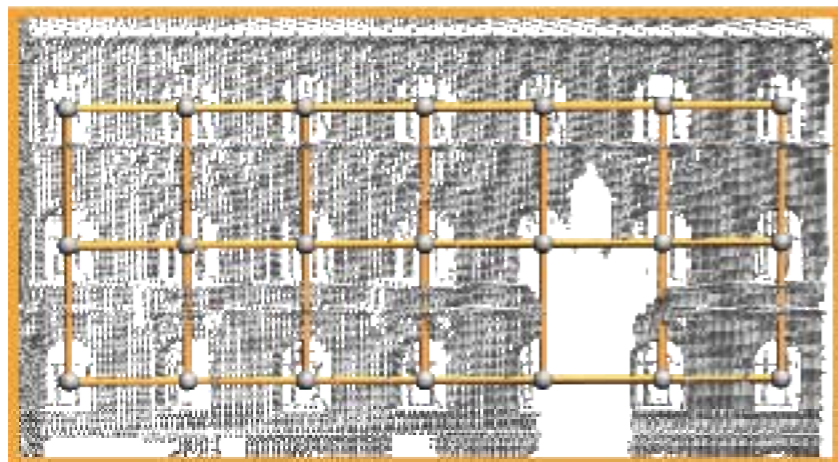
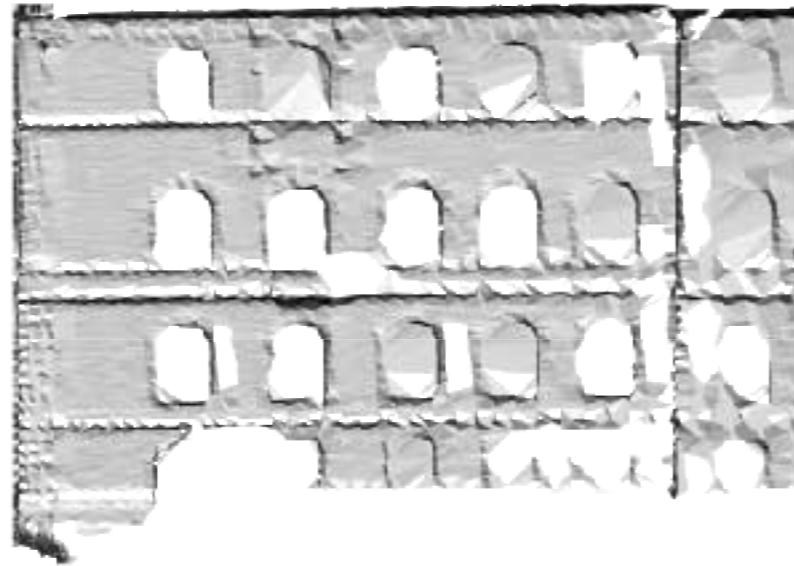
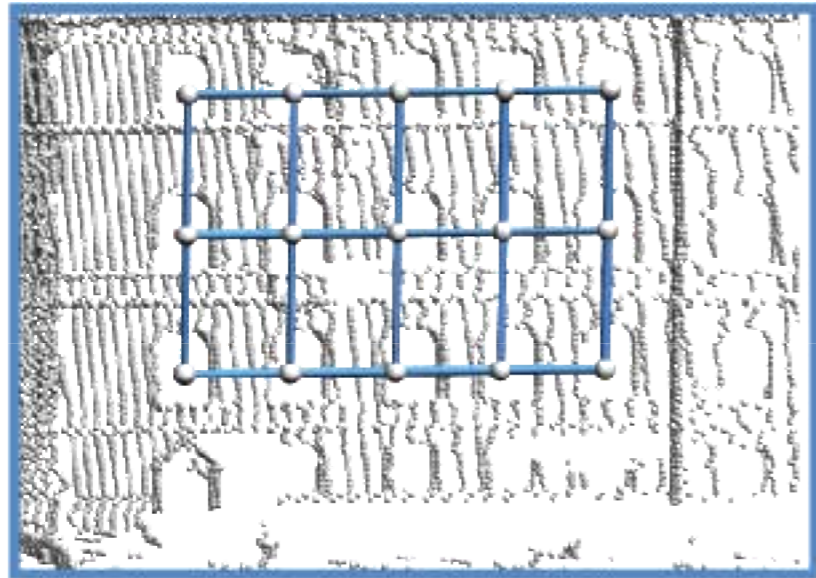
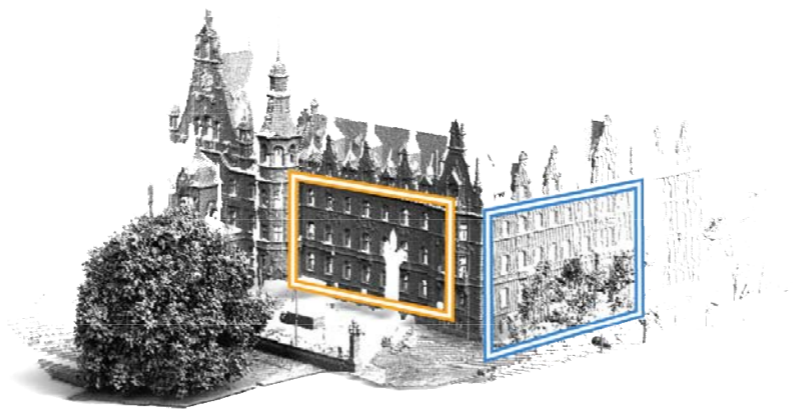
# Outdoor Scan



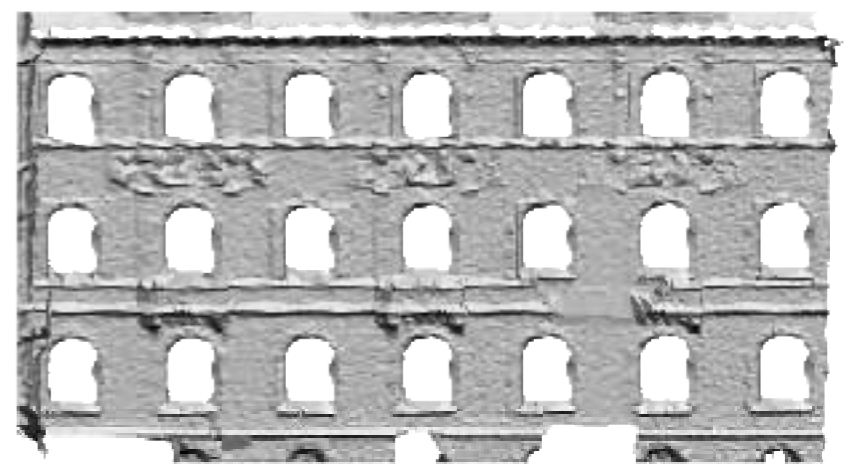
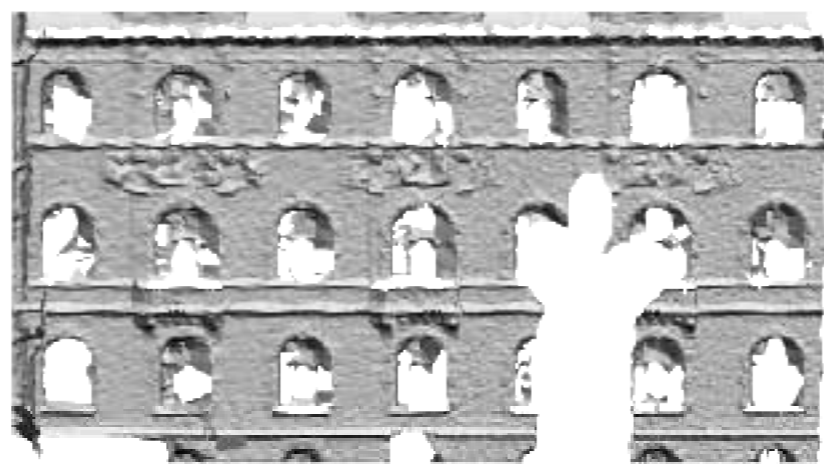
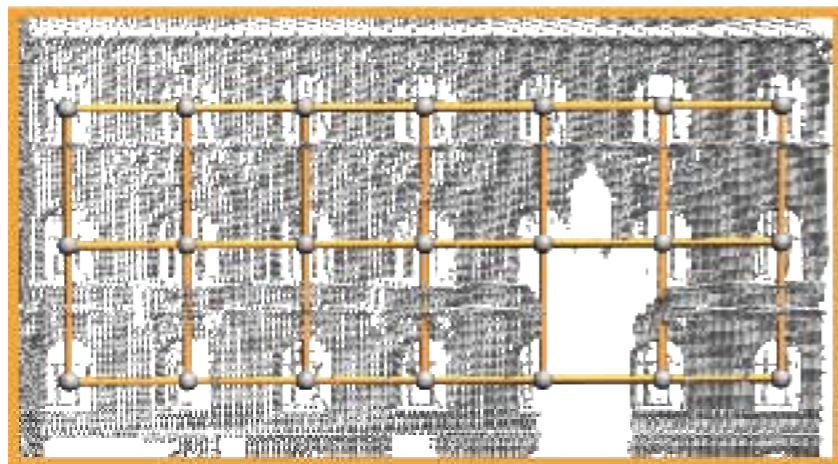
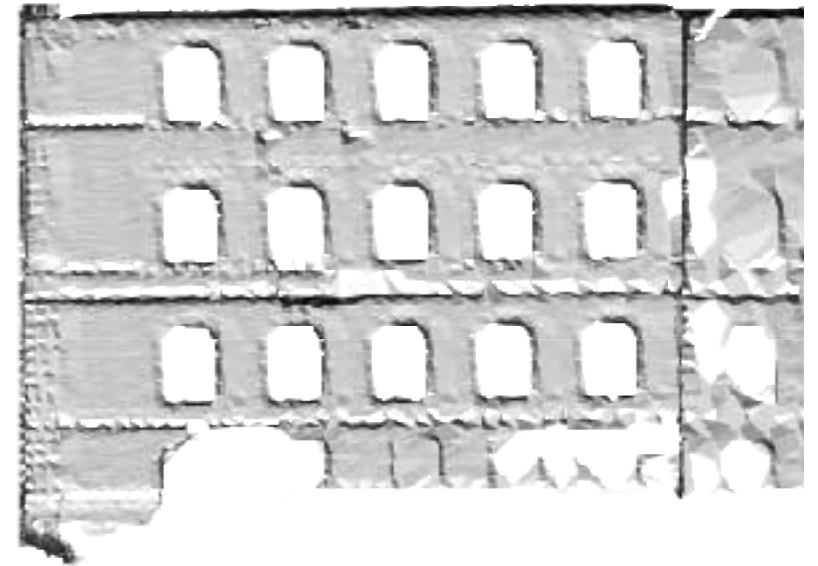
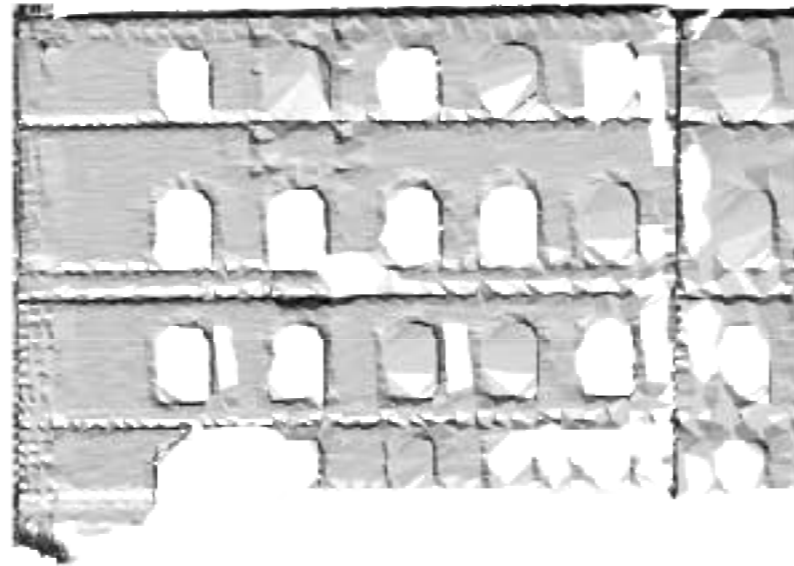
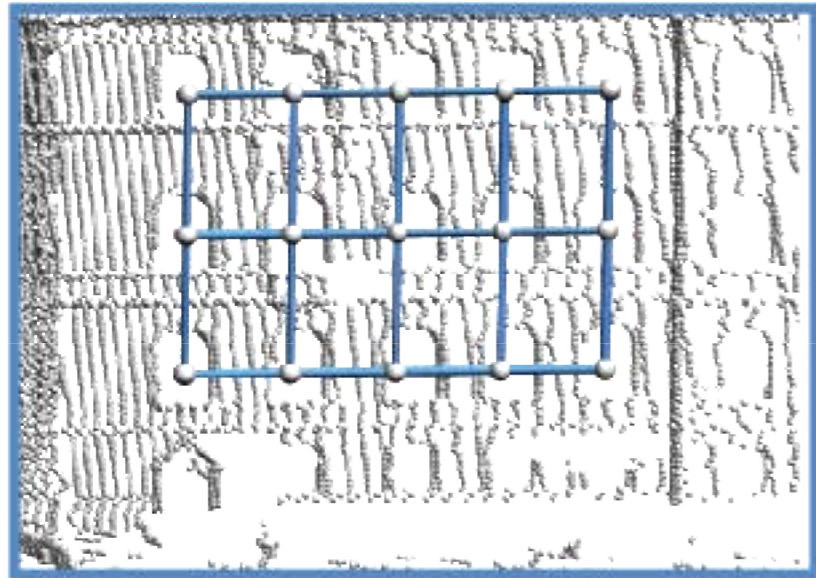
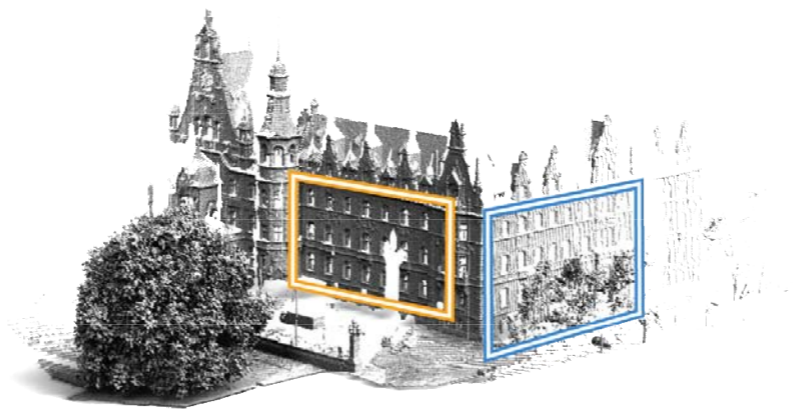
# Outdoor Scan



# Scan Completion



# Scan Completion



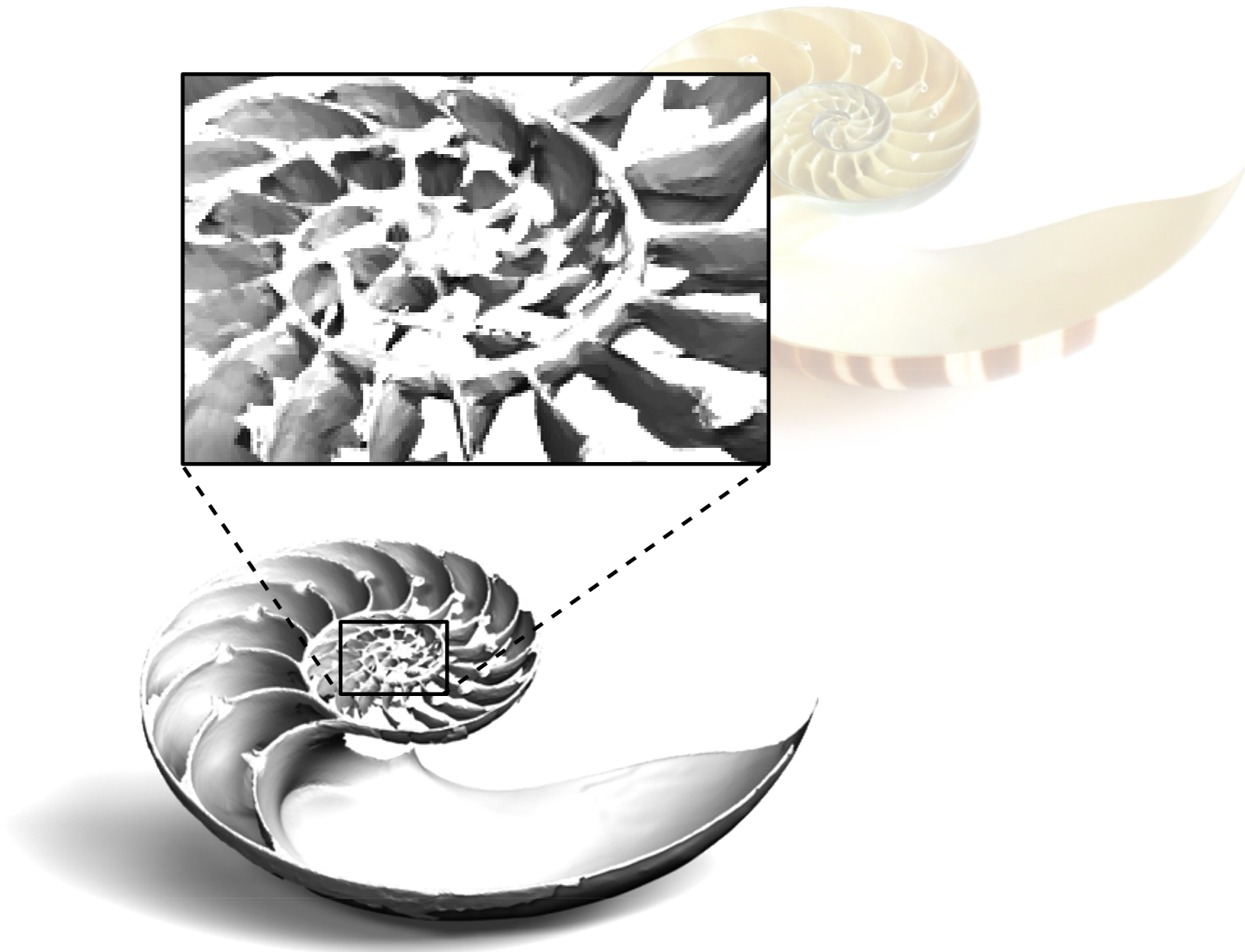
# Chambered Nautilus



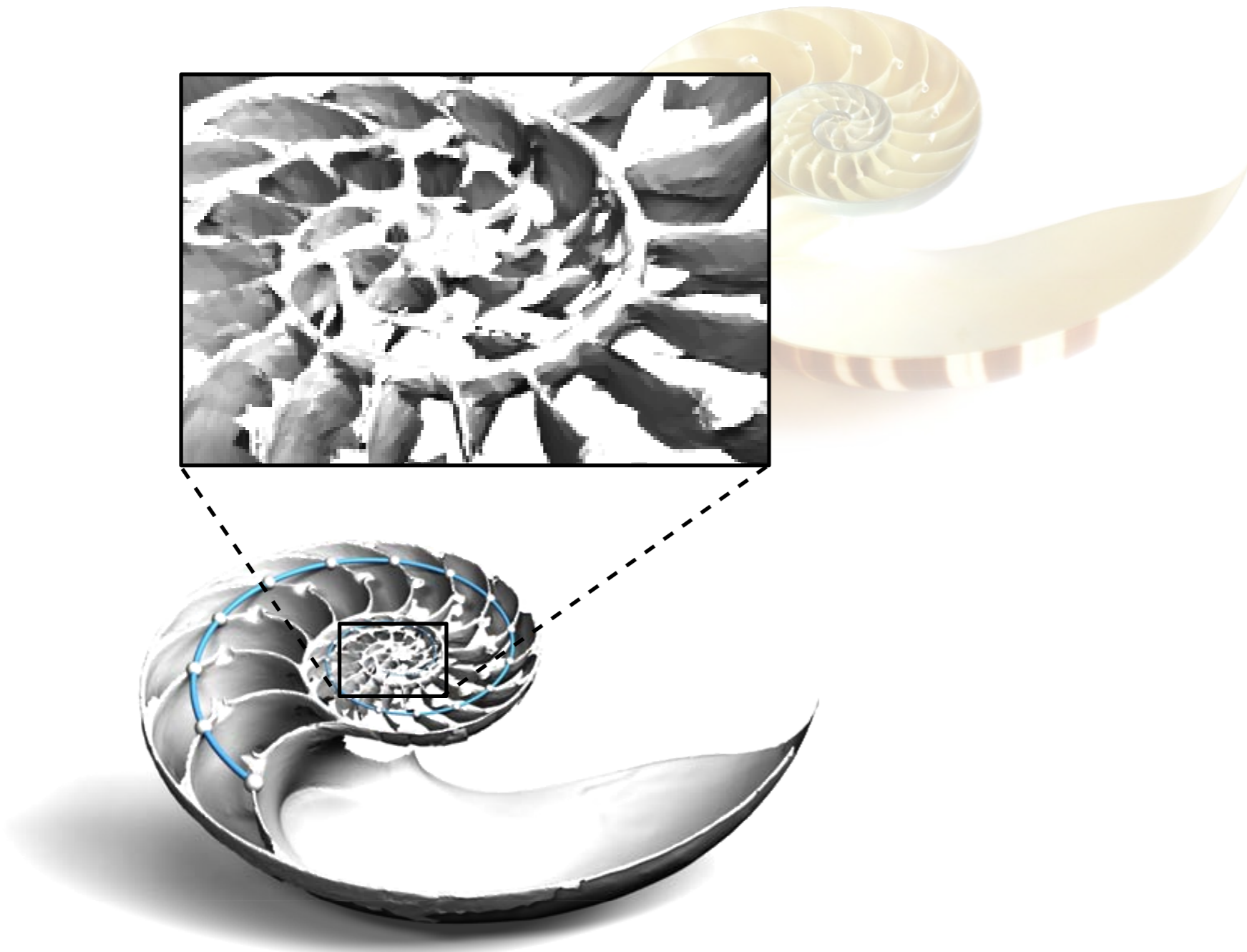
# Chambered Nautilus



# Chambered Nautilus

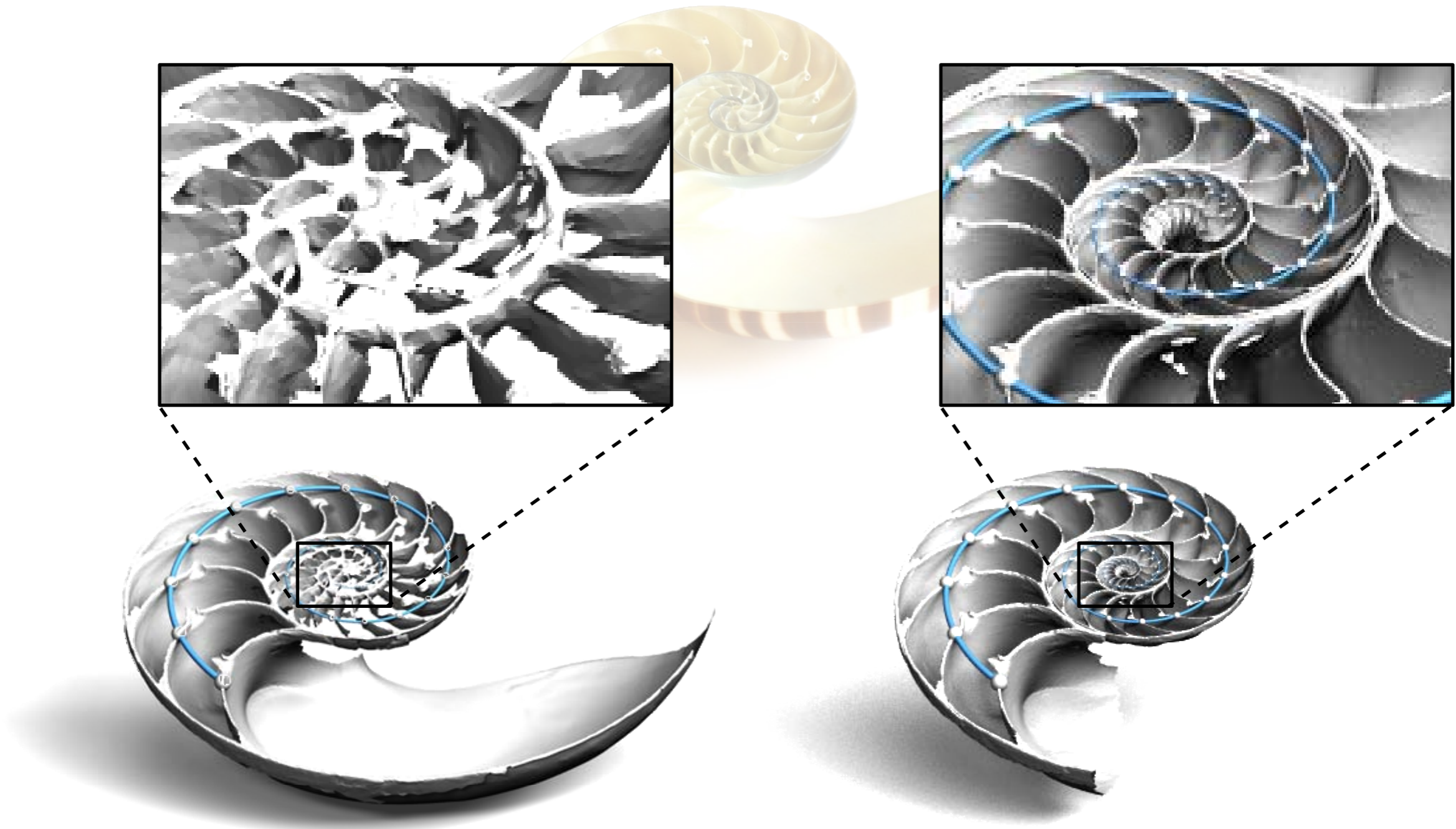


# Chambered Nautilus



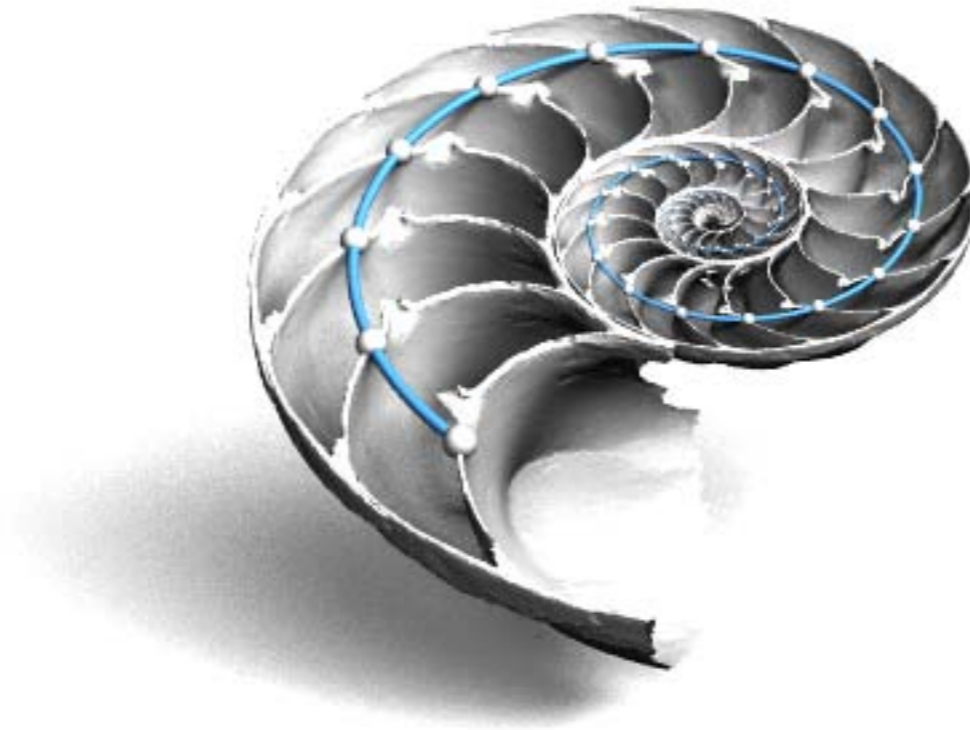


# Scan Completion

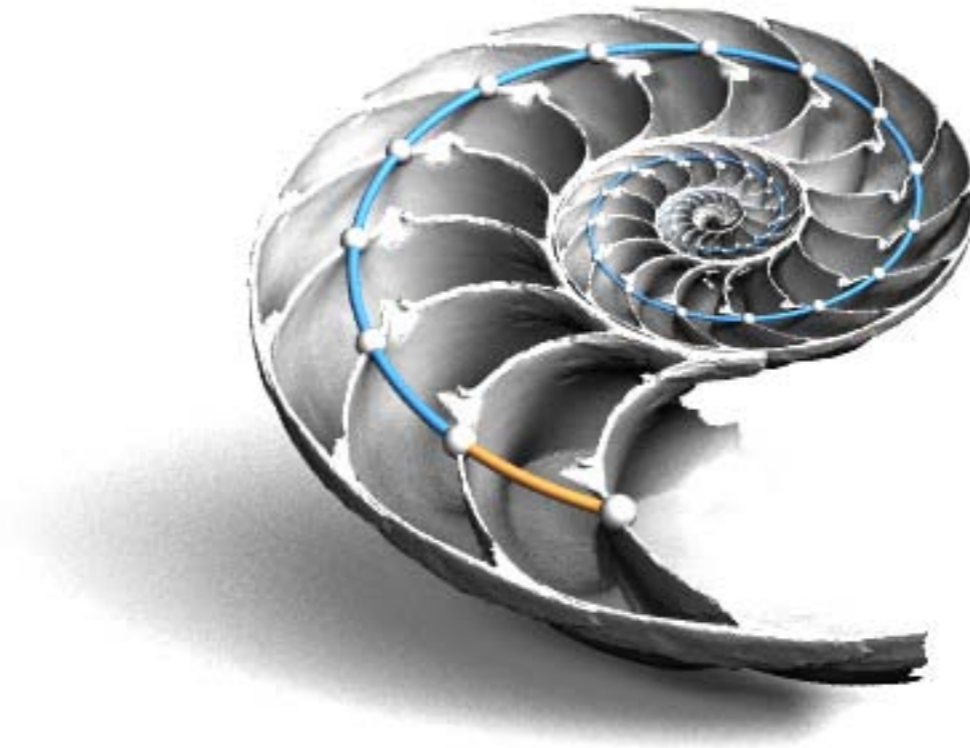


# Geometry Synthesis

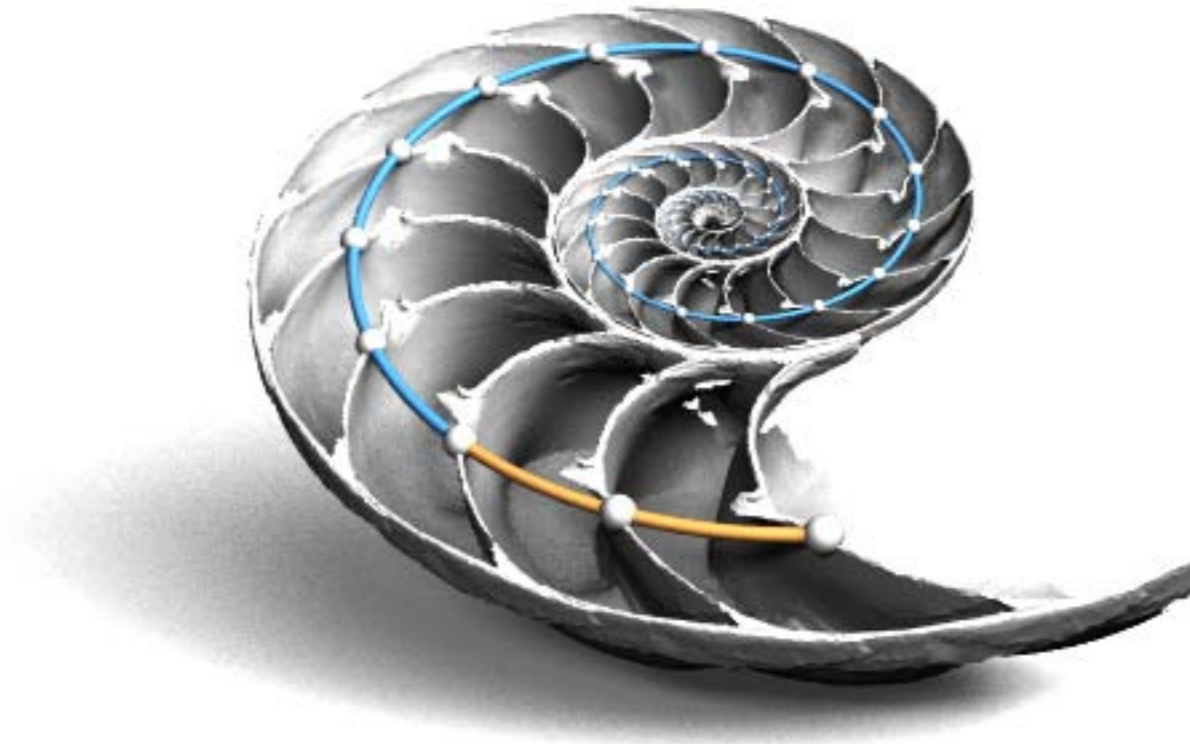
# Geometry Synthesis



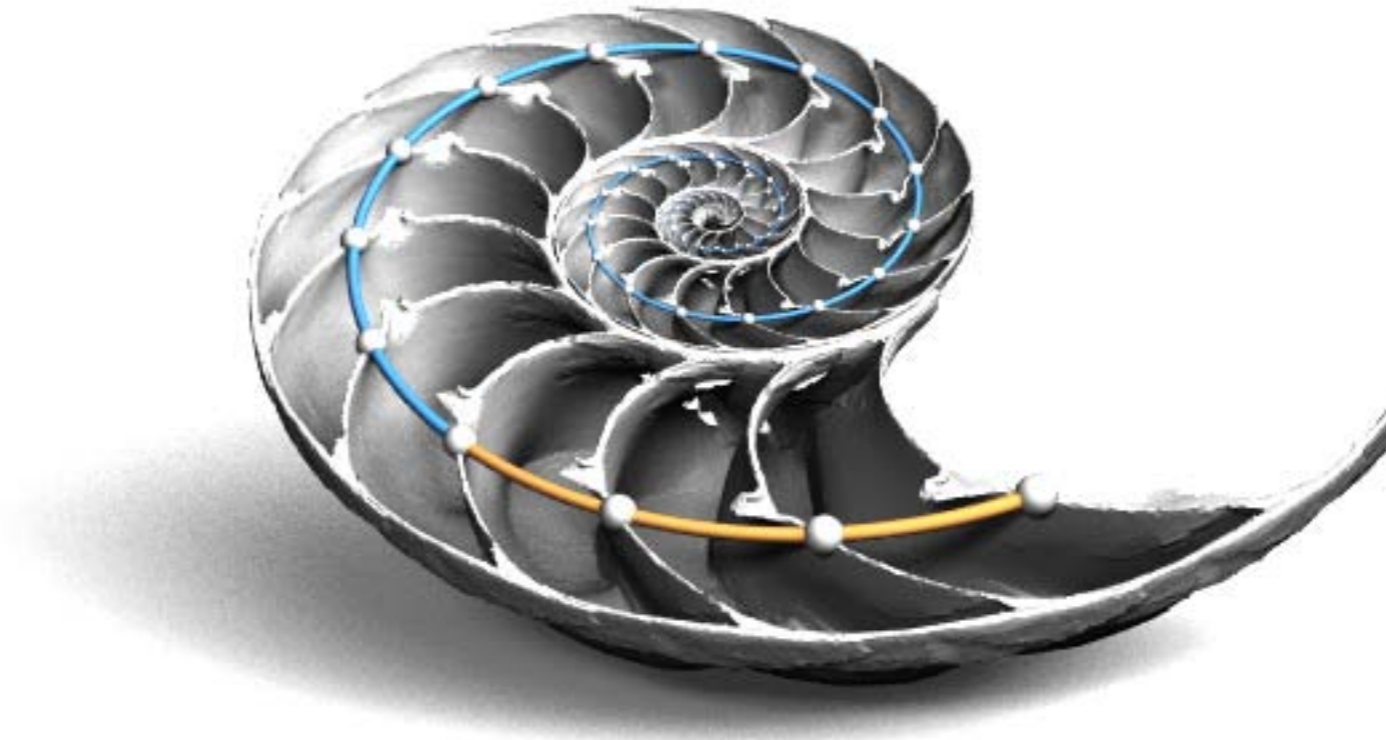
# Geometry Synthesis



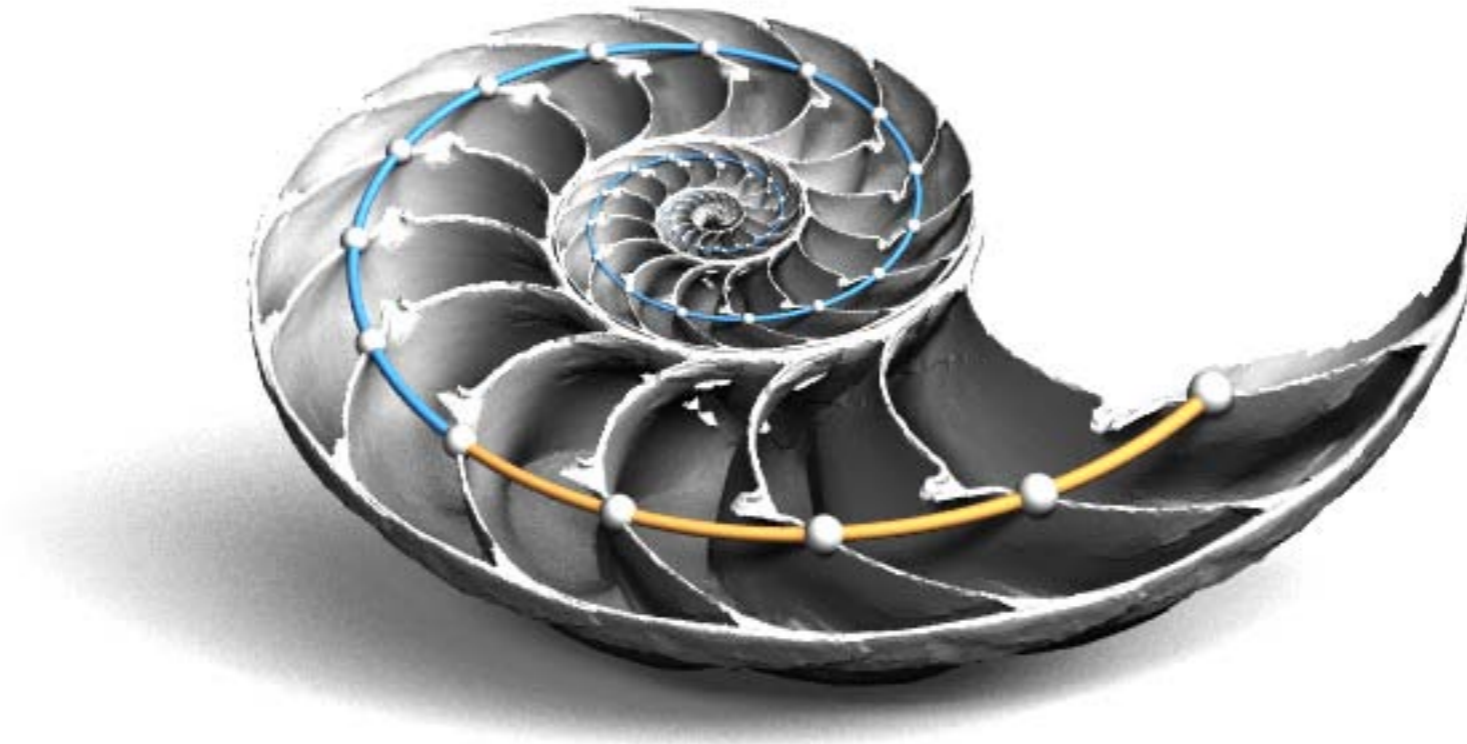
# Geometry Synthesis



# Geometry Synthesis



# Geometry Synthesis



# Observations





# Observations



- Warped structures

# Observations



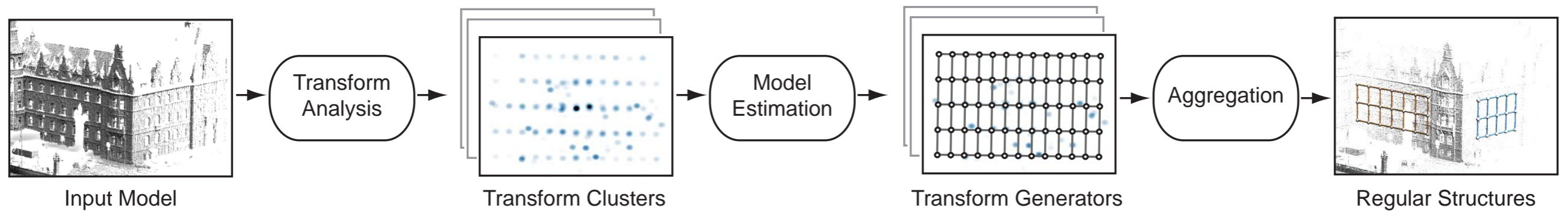
- Warped structures
- Size of grid vs accuracy

# Observations



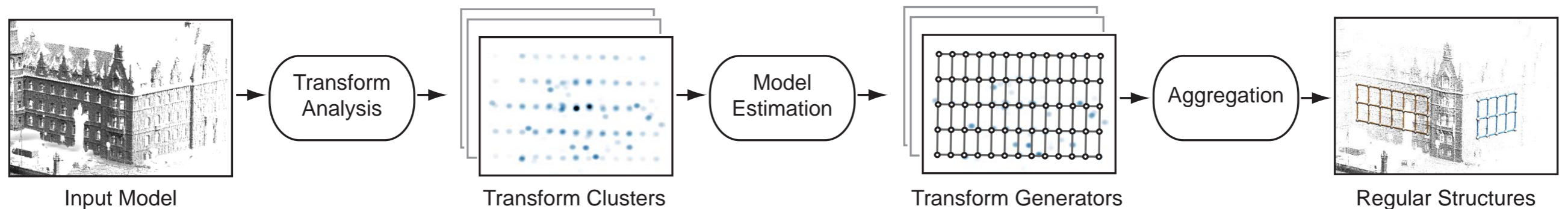
- Warped structures
- Size of grid vs accuracy
- Choice of parameters

# Structure Discovery



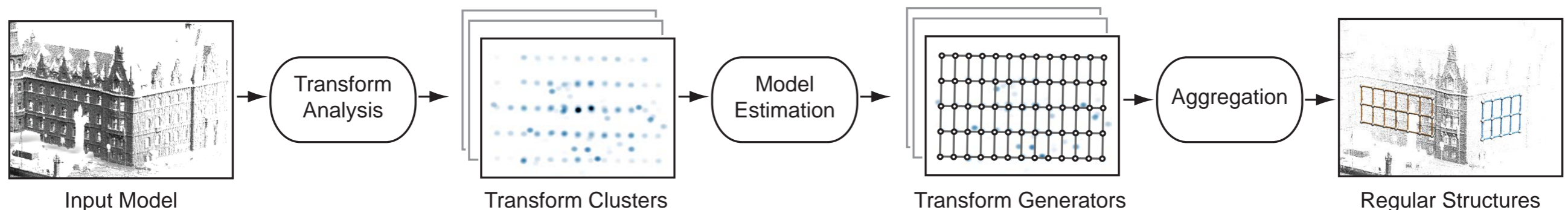
# Structure Discovery

- Algorithm is fully automatic



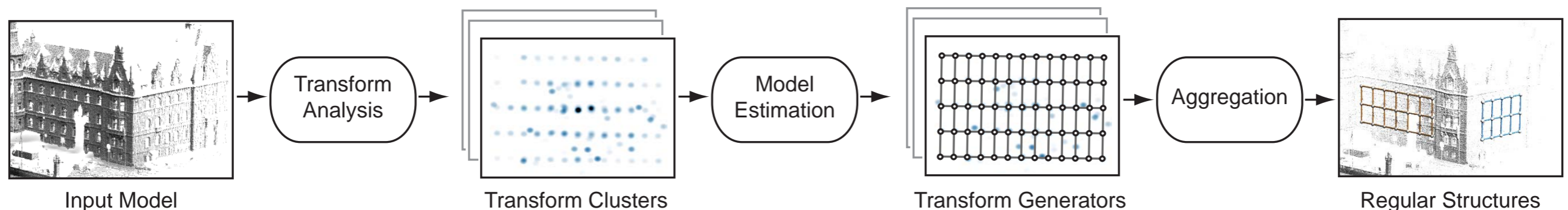
# Structure Discovery

- Algorithm is fully automatic
- Requires no prior information on size, shape, or location of repetitive elements



# Structure Discovery

- Algorithm is fully automatic
- Requires no prior information on size, shape, or location of repetitive elements
- Robust, efficient, independent of dimension  
→ *general tool for scientific data analysis*



# Acknowledgements

---



# Acknowledgements

- **Funding Agencies:**  
Austrian Science Fund (FWF)  
Darpa HR0011-05-1-0007  
NIH GM-072970  
NSF FRG-0354543  
TCS

# Acknowledgements

- **Funding Agencies:**

Austrian Science Fund (FWF)

Darpa HR0011-05-1-0007

NIH GM-072970

NSF FRG-0354543

TCS

- **Data Source:**

Institute of Cartography and Geoinformatics,  
Leibniz University, Germany



# Acknowledgements

- **Funding Agencies:**

Austrian Science Fund (FWF)

Darpa HR0011-05-1-0007

NIH GM-072970

NSF FRG-0354543

TCS

- **Data Source:**

Institute of Cartography and Geoinformatics,  
Leibniz University, Germany

- **Scanning, code snippets:**

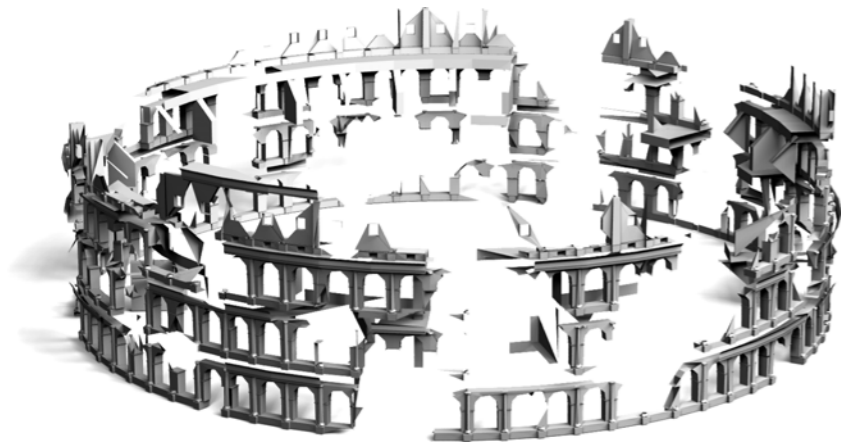
Michael Eigensatz

Balint Miklos

Heinz Schmiedhofer



# Thank You



# Thank You

