

# Orthogonal Projection Derivative Filters on the CC Lattice

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## Building Block Functions

```
In[1]:= mu[x_] := If[x < 0, 0, 1]
B[n_, x_] :=
  Sum[(-1)^j / (Factorial[n]) Binomial[n+1, j] (x + (n+1)/2 - j)^n * mu[x + (n+1)/2 - j], {j, 0, n+1}]

In[3]:= Bprime[n_, x_] := B[n-1, x + 1/2] - B[n-1, x - 1/2];
```

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## Derivative Filters

The derivative filters only need to be computed in 1 D as the 3 D versions can be obtained via a straightforward tensor product with a sampled B-spline (given below). Please see Equation (28) in the manuscript for details.

### ■ Linear - Linear (*ll*)

This is given by the derivative of a cubic B - Spline

```
In[4]:= Table[Bprime[3, t], {t, -10, 10}]

Out[4]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/2, 0, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

### ■ Cubic - Linear (*cl*)

This is given by the derivative of a Quintic B - Spline

```
In[5]:= Table[Bprime[5, t], {t, -10, 10}]

Out[5]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 1/24, 5/12, 0, -5/12, -1/24, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

### ■ Cubic - Cubic and Quintic - Linear (*cc* and *ql*)

These are given by the derivative of a heptic B - Spline

```
In[6]:= Table[Bprime[7, t], {t, -10, 10}]

Out[6]= {0, 0, 0, 0, 0, 0, 0, 0, 1/720, 7/90, 49/144, 0, -49/144, -7/90, -1/720, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

### ■ Quintic - Cubic (qc)

This is given by the derivative of a Nonic B - Spline

In[7]:= **Table[Bprime[9, t], {t, -10, 10}]**

Out[7]=  $\left\{0, 0, 0, 0, 0, 0, \frac{1}{40320}, \frac{41}{6720}, \frac{289}{2880}, \frac{809}{2880}, 0, -\frac{809}{2880}, -\frac{289}{2880}, -\frac{41}{6720}, -\frac{1}{40320}, 0, 0, 0, 0, 0, 0\right\}$

## Autocorrelation Sequences for the Duals

Linear is given by sampling the Cubic B - Spline and Cubic is given by sampling the Heptic B - Spline

In[8]:= **Table[B[3, t], {t, -10, 10}]**  
**Table[B[7, t], {t, -10, 10}]**

Out[8]=  $\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{6}, \frac{2}{3}, \frac{1}{6}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}$

Out[9]=  $\left\{0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{5040}, \frac{1}{42}, \frac{397}{1680}, \frac{151}{315}, \frac{397}{1680}, \frac{1}{42}, \frac{1}{5040}, 0, 0, 0, 0, 0, 0, 0\right\}$

## Sampled Sequences for Pre - Filtering and for Tensor Multiplication

### ■ Cubic

This is the same as the autocorrelation sequence of the linear B - spline.

### ■ Quintic

In[10]:= **Table[B[5, t], {t, -10, 10}]**

Out[10]=  $\left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{120}, \frac{13}{60}, \frac{11}{20}, \frac{13}{60}, \frac{1}{120}, 0, 0, 0, 0, 0, 0, 0\right\}$

### ■ Heptic

This is the same as the autocorrelation sequence of the cubic B - spline.

### ■ Nonic

In[11]:= **Table[B[9, t], {t, -10, 10}]**

Out[11]=  $\left\{0, 0, 0, 0, 0, 0, \frac{1}{362880}, \frac{251}{181440}, \frac{913}{22680}, \frac{44117}{181440}, \frac{15619}{36288}, \frac{44117}{181440}, \frac{913}{22680}, \frac{251}{181440}, \frac{1}{362880}, 0, 0, 0, 0, 0, 0\right\}$