Orthogonal Projection Derivative Filters on the BCC Lattice

Definitions of the Building Block Functions

■ **Helper functions and definitions of 1D B-splines**

```
R = 1 / 2 * {{1, -1, -1, 1}, {-1, 1, -1, 1}, {-1, -1, 1, 1}, {1, 1, 1, 1}};
```

$$\text{Sbs}[n\_, x\_] := \text{InverseFourierTransform}\left[\left(\frac{1 - \text{Exp}[-I*w]}{I*w}\right)^{n+1}, w, x, \text{FourierParameters} \rightarrow \{1, -1\}\right];$$

```
Box0[x_] = FullSimplify[Sbs[0, x]];
Tent[x_] = FullSimplify[Sbs[1, x]];
Quad[x_] = FullSimplify[Sbs[2, x]];
Cubic[x_] = FullSimplify[Sbs[3, x]];
Quartic[x_] = FullSimplify[Sbs[4, x]];
Tentprime[x_] = FullSimplify[Box0[x] - Box0[x - 1]];
Quadprime[x_] = FullSimplify[Tent[x] - Tent[x - 1]];
Cubicprime[x_] = FullSimplify[Quad[x] - Quad[x - 1]];
Quarticprime[x_] = FullSimplify[Cubic[x] - Cubic[x - 1]];
```

■ **Linear, Quintic, Nonic, Tredecic and Septendecic Box Splines**

$$\text{Lbs}[x\_, y\_, z\_] := 4*\text{Integrate}\left[\frac{1}{16}\text{Product}\left[\text{Box0}\left[\frac{1}{2}*R[[k]].\{x, y, z, w\}\right], \{k, 1, 4\}\right], \{w, 0, 4\}\right];$$

$$\text{Qbs}[x\_, y\_, z\_] := 4*\text{Integrate}\left[\frac{1}{16}\text{Product}\left[\text{Tent}\left[\frac{1}{2}*R[[k]].\{x, y, z, w\}\right], \{k, 1, 4\}\right], \{w, 0, 8\}\right];$$

$$\text{Nbs}[x\_, y\_, z\_] :=$$
$$4*\text{Integrate}\left[\frac{1}{16}\text{Product}\left[\text{Quad}\left[\frac{1}{2}*R[[k]].\{x, y, z, w\}\right], \{k, 1, 4\}\right], \{w, 0, 12\}\right];$$

$$\text{Tdbs}[x\_, y\_, z\_] :=$$
$$4*\text{Integrate}\left[\frac{1}{16}\text{Product}\left[\text{Cubic}\left[\frac{1}{2}*R[[k]].\{x, y, z, w\}\right], \{k, 1, 4\}\right], \{w, 0, 16\}\right];$$

$$\text{Sdbs}[x\_, y\_, z\_] :=$$
$$4*\text{Integrate}\left[\frac{1}{16}\text{Product}\left[\text{Quartic}\left[\frac{1}{2}*R[[k]].\{x, y, z, w\}\right], \{k, 1, 4\}\right], \{w, 0, 20\}\right];$$

A function that generates BCC points within the bounding box $[-n, n]^3$

```
BCCpoints[n_] :=
  Select[Tuples[Range[-n, n], 3], (EvenQ[#[[1]]] && EvenQ[#[[2]]] && EvenQ[#[[3]]] ) ||
    (OddQ[#[[1]]] && OddQ[#[[2]]] && OddQ[#[[3]]] ) &];
```

Derivative Filters

The following filters are implemented according to equation (31) in the manuscript.

■ **The Derivative of a Linear Box Spline orthogonally projected onto a Linear Box Spline (*LL*)**

This is tantamount to taking the derivative of a Quintic Box - Spline,

```
CdLL[x_, y_, z_, i_] :=
   4
  ── * Integrate[Sum[Tentprime[1 * R[[j]].{x, y, z, w}] * 1 R[[j]][[i]] * Product[
  16                        ─                           ─
                           2                           2

       Tent[1 * R[[k]].{x, y, z, w}], {k, Select[{1, 2, 3, 4}, # ≠ j &]}], {j, 1, 4}], {w, 0, 8}];
            ─
           2
```

and sampling it at the BCC Lattice sites.

```
CdLLx = Select[{#, CdLL[#[[1]], #[[2]], #[[3]], 1]} & /@ BCCpoints[4], #[[2]] ≠ 0 &]
```

$$\left\{\left\{\{-2, 0, 0\}, \frac{1}{12}\right\}, \left\{\{-1, -1, -1\}, \frac{1}{12}\right\}, \left\{\{-1, -1, 1\}, \frac{1}{12}\right\}, \left\{\{-1, 1, -1\}, \frac{1}{12}\right\}, \left\{\{-1, 1, 1\}, \frac{1}{12}\right\},\right.$$
$$\left\{\{1, -1, -1\}, -\frac{1}{12}\right\}, \left\{\{1, -1, 1\}, -\frac{1}{12}\right\}, \left\{\{1, 1, -1\}, -\frac{1}{12}\right\}, \left\{\{1, 1, 1\}, -\frac{1}{12}\right\}, \left\{\{2, 0, 0\}, -\frac{1}{12}\right\}\right\}$$

■ **The Derivative of a Quintic Box Spline orthogonally projected onto a Linear Box Spline (*QL*)**

This is tantamount to taking the derivative of a Nonic (9th order polynomials, generated by the projection of 4D tensor product Quadratic B-Splines) Box Spline

```
CdQL[x_, y_, z_, i_] :=
   4
  ── * Integrate[Sum[Quadprime[1 * R[[j]].{x, y, z, w}] * 1 R[[j]][[i]] * Product[
  16                         ─                           ─
                            2                           2

       Quad[1 * R[[k]].{x, y, z, w}], {k, Select[{1, 2, 3, 4}, # ≠ j &]}], {j, 1, 4}], {w, 0, 12}];
           ─
          2
```

```
CdQLx = Select[{#, CdQL[#[[1]], #[[2]], #[[3]], 1]} & /@ BCCpoints[6], #[[2]] ≠ 0 &]
```

$$\Big\{\Big\{\{-4, 0, 0\}, \frac{1}{2240}\Big\}, \Big\{\{-3, -1, -1\}, \frac{1}{336}\Big\}, \Big\{\{-3, -1, 1\}, \frac{1}{336}\Big\}, \Big\{\{-3, 1, -1\}, \frac{1}{336}\Big\},$$

$$\Big\{\{-3, 1, 1\}, \frac{1}{336}\Big\}, \Big\{\{-2, -2, -2\}, \frac{1}{1344}\Big\}, \Big\{\{-2, -2, 0\}, \frac{23}{2240}\Big\}, \Big\{\{-2, -2, 2\}, \frac{1}{1344}\Big\},$$

$$\Big\{\{-2, 0, -2\}, \frac{23}{2240}\Big\}, \Big\{\{-2, 0, 0\}, \frac{79}{1120}\Big\}, \Big\{\{-2, 0, 2\}, \frac{23}{2240}\Big\}, \Big\{\{-2, 2, -2\}, \frac{1}{1344}\Big\},$$

$$\Big\{\{-2, 2, 0\}, \frac{23}{2240}\Big\}, \Big\{\{-2, 2, 2\}, \frac{1}{1344}\Big\}, \Big\{\{-1, -3, -1\}, \frac{3}{2240}\Big\}, \Big\{\{-1, -3, 1\}, \frac{3}{2240}\Big\},$$

$$\Big\{\{-1, -1, -3\}, \frac{3}{2240}\Big\}, \Big\{\{-1, -1, -1\}, \frac{187}{3360}\Big\}, \Big\{\{-1, -1, 1\}, \frac{187}{3360}\Big\}, \Big\{\{-1, -1, 3\}, \frac{3}{2240}\Big\},$$

$$\Big\{\{-1, 1, -3\}, \frac{3}{2240}\Big\}, \Big\{\{-1, 1, -1\}, \frac{187}{3360}\Big\}, \Big\{\{-1, 1, 1\}, \frac{187}{3360}\Big\}, \Big\{\{-1, 1, 3\}, \frac{3}{2240}\Big\},$$

$$\Big\{\{-1, 3, -1\}, \frac{3}{2240}\Big\}, \Big\{\{-1, 3, 1\}, \frac{3}{2240}\Big\}, \Big\{\{1, -3, -1\}, -\frac{3}{2240}\Big\}, \Big\{\{1, -3, 1\}, -\frac{3}{2240}\Big\},$$

$$\Big\{\{1, -1, -3\}, -\frac{3}{2240}\Big\}, \Big\{\{1, -1, -1\}, -\frac{187}{3360}\Big\}, \Big\{\{1, -1, 1\}, -\frac{187}{3360}\Big\}, \Big\{\{1, -1, 3\}, -\frac{3}{2240}\Big\},$$

$$\Big\{\{1, 1, -3\}, -\frac{3}{2240}\Big\}, \Big\{\{1, 1, -1\}, -\frac{187}{3360}\Big\}, \Big\{\{1, 1, 1\}, -\frac{187}{3360}\Big\}, \Big\{\{1, 1, 3\}, -\frac{3}{2240}\Big\},$$

$$\Big\{\{1, 3, -1\}, -\frac{3}{2240}\Big\}, \Big\{\{1, 3, 1\}, -\frac{3}{2240}\Big\}, \Big\{\{2, -2, -2\}, -\frac{1}{1344}\Big\}, \Big\{\{2, -2, 0\}, -\frac{23}{2240}\Big\},$$

$$\Big\{\{2, -2, 2\}, -\frac{1}{1344}\Big\}, \Big\{\{2, 0, -2\}, -\frac{23}{2240}\Big\}, \Big\{\{2, 0, 0\}, -\frac{79}{1120}\Big\}, \Big\{\{2, 0, 2\}, -\frac{23}{2240}\Big\},$$

$$\Big\{\{2, 2, -2\}, -\frac{1}{1344}\Big\}, \Big\{\{2, 2, 0\}, -\frac{23}{2240}\Big\}, \Big\{\{2, 2, 2\}, -\frac{1}{1344}\Big\}, \Big\{\{3, -1, -1\}, -\frac{1}{336}\Big\},$$

$$\Big\{\{3, -1, 1\}, -\frac{1}{336}\Big\}, \Big\{\{3, 1, -1\}, -\frac{1}{336}\Big\}, \Big\{\{3, 1, 1\}, -\frac{1}{336}\Big\}, \Big\{\{4, 0, 0\}, -\frac{1}{2240}\Big\}\Big\}$$

■ The Derivative of a Quintic Box Spline orthogonally projected onto a Quintic Box Spline (*QQ*)

This is tantamount to sampling the derivative of a Tredecic Box Spline (generated by Cubic B - Splines).

```
CdQQ[x_, y_, z_, i_] :=
   4
   ── * Integrate[Sum[Cubicprime[ 1/2 * R[[j]].{x, y, z, w}] * 1/2 R[[j]][[i]] * Product[Cubic[
   16

      1/2 * R[[k]].{x, y, z, w}], {k, Select[{1, 2, 3, 4}, # ≠ j &]}], {j, 1, 4}], {w, 0, 16}];

CdQQx = Select[{#, CdQQ[#[[1]], #[[2]], #[[3]], 1]} & /@ BCCpoints[8], #[[2]] ≠ 0 &]
```

$$\left\{\left\{\{-6,0,0\},\frac{1}{2\,395\,008}\right\},\left\{\{-5,-1,-1\},\frac{17}{1\,496\,880}\right\},\left\{\{-5,-1,1\},\frac{17}{1\,496\,880}\right\},\right.$$

$$\left\{\{-5,1,-1\},\frac{17}{1\,496\,880}\right\},\left\{\{-5,1,1\},\frac{17}{1\,496\,880}\right\},\left\{\{-4,-2,-2\},\frac{5}{342\,144}\right\},$$

$$\left\{\{-4,-2,0\},\frac{173}{997\,920}\right\},\left\{\{-4,-2,2\},\frac{5}{342\,144}\right\},\left\{\{-4,0,-2\},\frac{173}{997\,920}\right\},\left\{\{-4,0,0\},\frac{1}{660}\right\},$$

$$\left\{\{-4,0,2\},\frac{173}{997\,920}\right\},\left\{\{-4,2,-2\},\frac{5}{342\,144}\right\},\left\{\{-4,2,0\},\frac{173}{997\,920}\right\},\left\{\{-4,2,2\},\frac{5}{342\,144}\right\},$$

$$\left\{\{-3,-3,-3\},\frac{1}{855\,360}\right\},\left\{\{-3,-3,-1\},\frac{647}{3\,991\,680}\right\},\left\{\{-3,-3,1\},\frac{647}{3\,991\,680}\right\},$$

$$\left\{\{-3,-3,3\},\frac{1}{855\,360}\right\},\left\{\{-3,-1,-3\},\frac{647}{3\,991\,680}\right\},\left\{\{-3,-1,-1\},\frac{919}{142\,560}\right\},$$

$$\left\{\{-3,-1,1\},\frac{919}{142\,560}\right\},\left\{\{-3,-1,3\},\frac{647}{3\,991\,680}\right\},\left\{\{-3,1,-3\},\frac{647}{3\,991\,680}\right\},$$

$$\left\{\{-3,1,-1\},\frac{919}{142\,560}\right\},\left\{\{-3,1,1\},\frac{919}{142\,560}\right\},\left\{\{-3,1,3\},\frac{647}{3\,991\,680}\right\},\left\{\{-3,3,-3\},\frac{1}{855\,360}\right\},$$

$$\left\{\{-3,3,-1\},\frac{647}{3\,991\,680}\right\},\left\{\{-3,3,1\},\frac{647}{3\,991\,680}\right\},\left\{\{-3,3,3\},\frac{1}{855\,360}\right\},$$

$$\left\{\{-2,-4,-2\},\frac{13}{1\,496\,880}\right\},\left\{\{-2,-4,0\},\frac{467}{3\,991\,680}\right\},\left\{\{-2,-4,2\},\frac{13}{1\,496\,880}\right\},$$

$$\left\{\{-2,-2,-4\},\frac{13}{1\,496\,880}\right\},\left\{\{-2,-2,-2\},\frac{503}{187\,110}\right\},\left\{\{-2,-2,0\},\frac{6623}{498\,960}\right\},$$

$$\left\{\{-2,-2,2\},\frac{503}{187\,110}\right\},\left\{\{-2,-2,4\},\frac{13}{1\,496\,880}\right\},\left\{\{-2,0,-4\},\frac{467}{3\,991\,680}\right\},$$

$$\left\{\{-2,0,-2\},\frac{6623}{498\,960}\right\},\left\{\{-2,0,0\},\frac{19\,477}{362\,880}\right\},\left\{\{-2,0,2\},\frac{6623}{498\,960}\right\},\left\{\{-2,0,4\},\frac{467}{3\,991\,680}\right\},$$

$$\left\{\{-2,2,-4\},\frac{13}{1\,496\,880}\right\},\left\{\{-2,2,-2\},\frac{503}{187\,110}\right\},\left\{\{-2,2,0\},\frac{6623}{498\,960}\right\},\left\{\{-2,2,2\},\frac{503}{187\,110}\right\},$$

$$\left\{\{-2,2,4\},\frac{13}{1\,496\,880}\right\},\left\{\{-2,4,-2\},\frac{13}{1\,496\,880}\right\},\left\{\{-2,4,0\},\frac{467}{3\,991\,680}\right\},$$

$$\left\{\{-2,4,2\},\frac{13}{1\,496\,880}\right\},\left\{\{-1,-5,-1\},\frac{47}{11\,975\,040}\right\},\left\{\{-1,-5,1\},\frac{47}{11\,975\,040}\right\},$$

$$\left\{\{-1,-3,-3\},\frac{53}{997\,920}\right\},\left\{\{-1,-3,-1\},\frac{5011}{1\,995\,840}\right\},\left\{\{-1,-3,1\},\frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{-1,-3,3\},\frac{53}{997\,920}\right\},\left\{\{-1,-1,-5\},\frac{47}{11\,975\,040}\right\},\left\{\{-1,-1,-3\},\frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{-1, -1, -1\}, \frac{45\,803}{1\,197\,504}\right\}, \left\{\{-1, -1, 1\}, \frac{45\,803}{1\,197\,504}\right\}, \left\{\{-1, -1, 3\}, \frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{-1, -1, 5\}, \frac{47}{11\,975\,040}\right\}, \left\{\{-1, 1, -5\}, \frac{47}{11\,975\,040}\right\}, \left\{\{-1, 1, -3\}, \frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{-1, 1, -1\}, \frac{45\,803}{1\,197\,504}\right\}, \left\{\{-1, 1, 1\}, \frac{45\,803}{1\,197\,504}\right\}, \left\{\{-1, 1, 3\}, \frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{-1, 1, 5\}, \frac{47}{11\,975\,040}\right\}, \left\{\{-1, 3, -3\}, \frac{53}{997\,920}\right\}, \left\{\{-1, 3, -1\}, \frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{-1, 3, 1\}, \frac{5011}{1\,995\,840}\right\}, \left\{\{-1, 3, 3\}, \frac{53}{997\,920}\right\}, \left\{\{-1, 5, -1\}, \frac{47}{11\,975\,040}\right\},$$

$$\left\{\{-1, 5, 1\}, \frac{47}{11\,975\,040}\right\}, \left\{\{1, -5, -1\}, -\frac{47}{11\,975\,040}\right\}, \left\{\{1, -5, 1\}, -\frac{47}{11\,975\,040}\right\},$$

$$\left\{\{1, -3, -3\}, -\frac{53}{997\,920}\right\}, \left\{\{1, -3, -1\}, -\frac{5011}{1\,995\,840}\right\}, \left\{\{1, -3, 1\}, -\frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{1, -3, 3\}, -\frac{53}{997\,920}\right\}, \left\{\{1, -1, -5\}, -\frac{47}{11\,975\,040}\right\}, \left\{\{1, -1, -3\}, -\frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{1, -1, -1\}, -\frac{45\,803}{1\,197\,504}\right\}, \left\{\{1, -1, 1\}, -\frac{45\,803}{1\,197\,504}\right\}, \left\{\{1, -1, 3\}, -\frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{1, -1, 5\}, -\frac{47}{11\,975\,040}\right\}, \left\{\{1, 1, -5\}, -\frac{47}{11\,975\,040}\right\}, \left\{\{1, 1, -3\}, -\frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{1, 1, -1\}, -\frac{45\,803}{1\,197\,504}\right\}, \left\{\{1, 1, 1\}, -\frac{45\,803}{1\,197\,504}\right\}, \left\{\{1, 1, 3\}, -\frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{1, 1, 5\}, -\frac{47}{11\,975\,040}\right\}, \left\{\{1, 3, -3\}, -\frac{53}{997\,920}\right\}, \left\{\{1, 3, -1\}, -\frac{5011}{1\,995\,840}\right\},$$

$$\left\{\{1, 3, 1\}, -\frac{5011}{1\,995\,840}\right\}, \left\{\{1, 3, 3\}, -\frac{53}{997\,920}\right\}, \left\{\{1, 5, -1\}, -\frac{47}{11\,975\,040}\right\},$$

$$\left\{\{1, 5, 1\}, -\frac{47}{11\,975\,040}\right\}, \left\{\{2, -4, -2\}, -\frac{13}{1\,496\,880}\right\}, \left\{\{2, -4, 0\}, -\frac{467}{3\,991\,680}\right\},$$

$$\left\{\{2, -4, 2\}, -\frac{13}{1\,496\,880}\right\}, \left\{\{2, -2, -4\}, -\frac{13}{1\,496\,880}\right\}, \left\{\{2, -2, -2\}, -\frac{503}{187\,110}\right\},$$

$$\left\{\{2, -2, 0\}, -\frac{6623}{498\,960}\right\}, \left\{\{2, -2, 2\}, -\frac{503}{187\,110}\right\}, \left\{\{2, -2, 4\}, -\frac{13}{1\,496\,880}\right\},$$

$$\left\{\{2, 0, -4\}, -\frac{467}{3\,991\,680}\right\}, \left\{\{2, 0, -2\}, -\frac{6623}{498\,960}\right\}, \left\{\{2, 0, 0\}, -\frac{19\,477}{362\,880}\right\}, \left\{\{2, 0, 2\}, -\frac{6623}{498\,960}\right\},$$

$$\left\{\{2, 0, 4\}, -\frac{467}{3\,991\,680}\right\}, \left\{\{2, 2, -4\}, -\frac{13}{1\,496\,880}\right\}, \left\{\{2, 2, -2\}, -\frac{503}{187\,110}\right\},$$

$$\left\{\{2, 2, 0\}, -\frac{6623}{498\,960}\right\}, \left\{\{2, 2, 2\}, -\frac{503}{187\,110}\right\}, \left\{\{2, 2, 4\}, -\frac{13}{1\,496\,880}\right\}, \left\{\{2, 4, -2\}, -\frac{13}{1\,496\,880}\right\},$$

$$\left\{\{2, 4, 0\}, -\frac{467}{3\,991\,680}\right\}, \left\{\{2, 4, 2\}, -\frac{13}{1\,496\,880}\right\}, \left\{\{3, -3, -3\}, -\frac{1}{855\,360}\right\},$$

$$\left\{\{3, -3, -1\}, -\frac{647}{3\,991\,680}\right\}, \left\{\{3, -3, 1\}, -\frac{647}{3\,991\,680}\right\}, \left\{\{3, -3, 3\}, -\frac{1}{855\,360}\right\},$$

$$\left\{\{3, -1, -3\}, -\frac{647}{3\,991\,680}\right\}, \left\{\{3, -1, -1\}, -\frac{919}{142\,560}\right\}, \left\{\{3, -1, 1\}, -\frac{919}{142\,560}\right\},$$

$$\left\{\{3, -1, 3\}, -\frac{647}{3\,991\,680}\right\}, \left\{\{3, 1, -3\}, -\frac{647}{3\,991\,680}\right\}, \left\{\{3, 1, -1\}, -\frac{919}{142\,560}\right\},$$

$$\left\{\{3, 1, 1\}, -\frac{919}{142\,560}\right\}, \left\{\{3, 1, 3\}, -\frac{647}{3\,991\,680}\right\}, \left\{\{3, 3, -3\}, -\frac{1}{855\,360}\right\}, \left\{\{3, 3, -1\}, -\frac{647}{3\,991\,680}\right\},$$

$$\left\{\{3, 3, 1\}, -\frac{647}{3\,991\,680}\right\}, \left\{\{3, 3, 3\}, -\frac{1}{855\,360}\right\}, \left\{\{4, -2, -2\}, -\frac{5}{342\,144}\right\},$$

$$\left\{\{4, -2, 0\}, -\frac{173}{997\,920}\right\}, \left\{\{4, -2, 2\}, -\frac{5}{342\,144}\right\}, \left\{\{4, 0, -2\}, -\frac{173}{997\,920}\right\},$$

$$\left\{\{4, 0, 0\}, -\frac{1}{660}\right\}, \left\{\{4, 0, 2\}, -\frac{173}{997\,920}\right\}, \left\{\{4, 2, -2\}, -\frac{5}{342\,144}\right\}, \left\{\{4, 2, 0\}, -\frac{173}{997\,920}\right\},$$

$$\left\{\{4, 2, 2\}, -\frac{5}{342\,144}\right\}, \left\{\{5, -1, -1\}, -\frac{17}{1\,496\,880}\right\}, \left\{\{5, -1, 1\}, -\frac{17}{1\,496\,880}\right\},$$

$$\left\{\{5, 1, -1\}, -\frac{17}{1\,496\,880}\right\}, \left\{\{5, 1, 1\}, -\frac{17}{1\,496\,880}\right\}, \left\{\{6, 0, 0\}, -\frac{1}{2\,395\,008}\right\}\right\}$$

■ **The derivative of a Nonic Box Spline orthogonally projected onto a Linear Box Spline (*NL*)**

This is also tantamount to taking the derivative of a Tredecic Box - Spline

```
CdNLx = CdQQx;
```

■ **The Derivative of a Nonic Box Spline orthogonally projected onto a Quintic Box Spline (*NQ*)**

This amounts to taking the derivative of a Septendecic (17th order polynomial, generated by 4D tensor product Quartic B-Splines) box spline.

```
CdNQ[x_, y_, z_, i_] :=
  4
 ── * Integrate[Sum[Quarticprime[─ * R[[j]].{x, y, z, w}] * ─ R[[j]][[i]] Product[Quartic[
 16                               2                          2

       1
       ─ * R[[k]].{x, y, z, w}], {k, Select[{1, 2, 3, 4}, # ≠ j &]}], {j, 1, 4}], {w, 0, 20}];
       2

CdNQx = Select[{#, CdNQ[#[[1]], #[[2]], #[[3]], 1]} & /@ BCCpoints[10], #[[2]] ≠ 0 &]
```

$$\left\{\left\{\{-8,\,0,\,0\},\,\frac{1}{8\,539\,914\,240}\right\},\,\left\{\{-7,\,-1,\,-1\},\,\frac{67}{5\,977\,939\,968}\right\},\right.$$

$$\left\{\{-7,\,-1,\,1\},\,\frac{67}{5\,977\,939\,968}\right\},\,\left\{\{-7,\,1,\,-1\},\,\frac{67}{5\,977\,939\,968}\right\},\,\left\{\{-7,\,1,\,1\},\,\frac{67}{5\,977\,939\,968}\right\},$$

$$\left\{\{-6,\,-2,\,-2\},\,\frac{317}{7\,472\,424\,960}\right\},\,\left\{\{-6,\,-2,\,0\},\,\frac{34\,037}{59\,779\,399\,680}\right\},\,\left\{\{-6,\,-2,\,2\},\,\frac{317}{7\,472\,424\,960}\right\},$$

$$\left\{\{-6,\,0,\,-2\},\,\frac{34\,037}{59\,779\,399\,680}\right\},\,\left\{\{-6,\,0,\,0\},\,\frac{74\,167}{9\,963\,233\,280}\right\},\,\left\{\{-6,\,0,\,2\},\,\frac{34\,037}{59\,779\,399\,680}\right\},$$

$$\left\{\{-6,\,2,\,-2\},\,\frac{317}{7\,472\,424\,960}\right\},\,\left\{\{-6,\,2,\,0\},\,\frac{34\,037}{59\,779\,399\,680}\right\},\,\left\{\{-6,\,2,\,2\},\,\frac{317}{7\,472\,424\,960}\right\},$$

$$\left\{\{-5,\,-3,\,-3\},\,\frac{17}{905\,748\,480}\right\},\,\left\{\{-5,\,-3,\,-1\},\,\frac{99\,427}{59\,779\,399\,680}\right\},\,\left\{\{-5,\,-3,\,1\},\,\frac{99\,427}{59\,779\,399\,680}\right\},$$

$$\left\{\{-5,\,-3,\,3\},\,\frac{17}{905\,748\,480}\right\},\,\left\{\{-5,\,-1,\,-3\},\,\frac{99\,427}{59\,779\,399\,680}\right\},\,\left\{\{-5,\,-1,\,-1\},\,\frac{2\,812\,801}{29\,889\,699\,840}\right\},$$

$$\left\{\{-5,\,-1,\,1\},\,\frac{2\,812\,801}{29\,889\,699\,840}\right\},\,\left\{\{-5,\,-1,\,3\},\,\frac{99\,427}{59\,779\,399\,680}\right\},\,\left\{\{-5,\,1,\,-3\},\,\frac{99\,427}{59\,779\,399\,680}\right\},$$

$$\left\{\{-5,\,1,\,-1\},\,\frac{2\,812\,801}{29\,889\,699\,840}\right\},\,\left\{\{-5,\,1,\,1\},\,\frac{2\,812\,801}{29\,889\,699\,840}\right\},\,\left\{\{-5,\,1,\,3\},\,\frac{99\,427}{59\,779\,399\,680}\right\},$$

$$\left\{\{-5,\,3,\,-3\},\,\frac{17}{905\,748\,480}\right\},\,\left\{\{-5,\,3,\,-1\},\,\frac{99\,427}{59\,779\,399\,680}\right\},\,\left\{\{-5,\,3,\,1\},\,\frac{99\,427}{59\,779\,399\,680}\right\},$$

$$\left\{\{-5,\,3,\,3\},\,\frac{17}{905\,748\,480}\right\},\,\left\{\{-4,\,-4,\,-4\},\,\frac{1}{1\,811\,496\,960}\right\},\,\left\{\{-4,\,-4,\,-2\},\,\frac{335}{569\,327\,616}\right\},$$

$$\left\{\{-4,\,-4,\,0\},\,\frac{83\,819}{19\,926\,466\,560}\right\},\,\left\{\{-4,\,-4,\,2\},\,\frac{335}{569\,327\,616}\right\},\,\left\{\{-4,\,-4,\,4\},\,\frac{1}{1\,811\,496\,960}\right\},$$

$$\left\{\{-4,\,-2,\,-4\},\,\frac{335}{569\,327\,616}\right\},\,\left\{\{-4,\,-2,\,-2\},\,\frac{75\,541}{543\,449\,088}\right\},\,\left\{\{-4,\,-2,\,0\},\,\frac{1\,788\,509}{2\,717\,245\,440}\right\},$$

$$\left\{\{-4,\,-2,\,2\},\,\frac{75\,541}{543\,449\,088}\right\},\,\left\{\{-4,\,-2,\,4\},\,\frac{335}{569\,327\,616}\right\},\,\left\{\{-4,\,0,\,-4\},\,\frac{83\,819}{19\,926\,466\,560}\right\},$$

$$\left\{\{-4,\,0,\,-2\},\,\frac{1\,788\,509}{2\,717\,245\,440}\right\},\,\left\{\{-4,\,0,\,0\},\,\frac{27\,077\,587}{9\,963\,233\,280}\right\},\,\left\{\{-4,\,0,\,2\},\,\frac{1\,788\,509}{2\,717\,245\,440}\right\},$$

$$\left\{\{-4,\,0,\,4\},\,\frac{83\,819}{19\,926\,466\,560}\right\},\,\left\{\{-4,\,2,\,-4\},\,\frac{335}{569\,327\,616}\right\},\,\left\{\{-4,\,2,\,-2\},\,\frac{75\,541}{543\,449\,088}\right\},$$

$$\left\{\{-4,\,2,\,0\},\,\frac{1\,788\,509}{2\,717\,245\,440}\right\},\,\left\{\{-4,\,2,\,2\},\,\frac{75\,541}{543\,449\,088}\right\},\,\left\{\{-4,\,2,\,4\},\,\frac{335}{569\,327\,616}\right\},$$

$$\left\{\{-4,\,4,\,-4\},\,\frac{1}{1\,811\,496\,960}\right\},\,\left\{\{-4,\,4,\,-2\},\,\frac{335}{569\,327\,616}\right\},\,\left\{\{-4,\,4,\,0\},\,\frac{83\,819}{19\,926\,466\,560}\right\},$$

$\left\{\{-4, 4, 2\}, \dfrac{335}{569\,327\,616}\right\}, \left\{\{-4, 4, 4\}, \dfrac{1}{1\,811\,496\,960}\right\}, \left\{\{-3, -5, -3\}, \dfrac{731}{59\,779\,399\,680}\right\},$

$\left\{\{-3, -5, -1\}, \dfrac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{-3, -5, 1\}, \dfrac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{-3, -5, 3\}, \dfrac{731}{59\,779\,399\,680}\right\},$

$\left\{\{-3, -3, -5\}, \dfrac{731}{59\,779\,399\,680}\right\}, \left\{\{-3, -3, -3\}, \dfrac{9901}{355\,829\,760}\right\}, \left\{\{-3, -3, -1\}, \dfrac{85\,321}{135\,862\,272}\right\},$

$\left\{\{-3, -3, 1\}, \dfrac{85\,321}{135\,862\,272}\right\}, \left\{\{-3, -3, 3\}, \dfrac{9901}{355\,829\,760}\right\}, \left\{\{-3, -3, 5\}, \dfrac{731}{59\,779\,399\,680}\right\},$

$\left\{\{-3, -1, -5\}, \dfrac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{-3, -1, -3\}, \dfrac{85\,321}{135\,862\,272}\right\}, \left\{\{-3, -1, -1\}, \dfrac{11\,806\,073}{1\,423\,319\,040}\right\},$

$\left\{\{-3, -1, 1\}, \dfrac{11\,806\,073}{1\,423\,319\,040}\right\}, \left\{\{-3, -1, 3\}, \dfrac{85\,321}{135\,862\,272}\right\}, \left\{\{-3, -1, 5\}, \dfrac{15\,373}{11\,955\,879\,936}\right\},$

$\left\{\{-3, 1, -5\}, \dfrac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{-3, 1, -3\}, \dfrac{85\,321}{135\,862\,272}\right\}, \left\{\{-3, 1, -1\}, \dfrac{11\,806\,073}{1\,423\,319\,040}\right\},$

$\left\{\{-3, 1, 1\}, \dfrac{11\,806\,073}{1\,423\,319\,040}\right\}, \left\{\{-3, 1, 3\}, \dfrac{85\,321}{135\,862\,272}\right\}, \left\{\{-3, 1, 5\}, \dfrac{15\,373}{11\,955\,879\,936}\right\},$

$\left\{\{-3, 3, -5\}, \dfrac{731}{59\,779\,399\,680}\right\}, \left\{\{-3, 3, -3\}, \dfrac{9901}{355\,829\,760}\right\}, \left\{\{-3, 3, -1\}, \dfrac{85\,321}{135\,862\,272}\right\},$

$\left\{\{-3, 3, 1\}, \dfrac{85\,321}{135\,862\,272}\right\}, \left\{\{-3, 3, 3\}, \dfrac{9901}{355\,829\,760}\right\}, \left\{\{-3, 3, 5\}, \dfrac{731}{59\,779\,399\,680}\right\},$

$\left\{\{-3, 5, -3\}, \dfrac{731}{59\,779\,399\,680}\right\}, \left\{\{-3, 5, -1\}, \dfrac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{-3, 5, 1\}, \dfrac{15\,373}{11\,955\,879\,936}\right\},$

$\left\{\{-3, 5, 3\}, \dfrac{731}{59\,779\,399\,680}\right\}, \left\{\{-2, -6, -2\}, \dfrac{173}{8\,539\,914\,240}\right\}, \left\{\{-2, -6, 0\}, \dfrac{19\,853}{59\,779\,399\,680}\right\},$

$\left\{\{-2, -6, 2\}, \dfrac{173}{8\,539\,914\,240}\right\}, \left\{\{-2, -4, -4\}, \dfrac{281}{1\,067\,489\,280}\right\}, \left\{\{-2, -4, -2\}, \dfrac{432\,799}{5\,434\,490\,880}\right\},$

$\left\{\{-2, -4, 0\}, \dfrac{11\,607\,821}{29\,889\,699\,840}\right\}, \left\{\{-2, -4, 2\}, \dfrac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{-2, -4, 4\}, \dfrac{281}{1\,067\,489\,280}\right\},$

$\left\{\{-2, -2, -6\}, \dfrac{173}{8\,539\,914\,240}\right\}, \left\{\{-2, -2, -4\}, \dfrac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{-2, -2, -2\}, \dfrac{10\,927\,349}{2\,717\,245\,440}\right\},$

$\left\{\{-2, -2, 0\}, \dfrac{30\,805\,517}{2\,299\,207\,680}\right\}, \left\{\{-2, -2, 2\}, \dfrac{10\,927\,349}{2\,717\,245\,440}\right\}, \left\{\{-2, -2, 4\}, \dfrac{432\,799}{5\,434\,490\,880}\right\},$

$\left\{\{-2, -2, 6\}, \dfrac{173}{8\,539\,914\,240}\right\}, \left\{\{-2, 0, -6\}, \dfrac{19\,853}{59\,779\,399\,680}\right\}, \left\{\{-2, 0, -4\}, \dfrac{11\,607\,821}{29\,889\,699\,840}\right\},$

$\left\{\{-2, 0, -2\}, \dfrac{30\,805\,517}{2\,299\,207\,680}\right\}, \left\{\{-2, 0, 0\}, \dfrac{173\,942\,723}{4\,269\,957\,120}\right\}, \left\{\{-2, 0, 2\}, \dfrac{30\,805\,517}{2\,299\,207\,680}\right\},$

$\left\{\{-2, 0, 4\}, \frac{11\,607\,821}{29\,889\,699\,840}\right\}, \left\{\{-2, 0, 6\}, \frac{19\,853}{59\,779\,399\,680}\right\}, \left\{\{-2, 2, -6\}, \frac{173}{8\,539\,914\,240}\right\},$

$\left\{\{-2, 2, -4\}, \frac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{-2, 2, -2\}, \frac{10\,927\,349}{2\,717\,245\,440}\right\}, \left\{\{-2, 2, 0\}, \frac{30\,805\,517}{2\,299\,207\,680}\right\},$

$\left\{\{-2, 2, 2\}, \frac{10\,927\,349}{2\,717\,245\,440}\right\}, \left\{\{-2, 2, 4\}, \frac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{-2, 2, 6\}, \frac{173}{8\,539\,914\,240}\right\},$

$\left\{\{-2, 4, -4\}, \frac{281}{1\,067\,489\,280}\right\}, \left\{\{-2, 4, -2\}, \frac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{-2, 4, 0\}, \frac{11\,607\,821}{29\,889\,699\,840}\right\},$

$\left\{\{-2, 4, 2\}, \frac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{-2, 4, 4\}, \frac{281}{1\,067\,489\,280}\right\}, \left\{\{-2, 6, -2\}, \frac{173}{8\,539\,914\,240}\right\},$

$\left\{\{-2, 6, 0\}, \frac{19\,853}{59\,779\,399\,680}\right\}, \left\{\{-2, 6, 2\}, \frac{173}{8\,539\,914\,240}\right\}, \left\{\{-1, -7, -1\}, \frac{19}{5\,434\,490\,880}\right\},$

$\left\{\{-1, -7, 1\}, \frac{19}{5\,434\,490\,880}\right\}, \left\{\{-1, -5, -3\}, \frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{-1, -5, -1\}, \frac{74\,801}{2\,846\,638\,080}\right\},$

$\left\{\{-1, -5, 1\}, \frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{-1, -5, 3\}, \frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{-1, -3, -5\}, \frac{10\,279}{29\,889\,699\,840}\right\},$

$\left\{\{-1, -3, -3\}, \frac{3\,336\,451}{14\,944\,849\,920}\right\}, \left\{\{-1, -3, -1\}, \frac{182\,225\,761}{59\,779\,399\,680}\right\},$

$\left\{\{-1, -3, 1\}, \frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{-1, -3, 3\}, \frac{3\,336\,451}{14\,944\,849\,920}\right\}, \left\{\{-1, -3, 5\}, \frac{10\,279}{29\,889\,699\,840}\right\},$

$\left\{\{-1, -1, -7\}, \frac{19}{5\,434\,490\,880}\right\}, \left\{\{-1, -1, -5\}, \frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{-1, -1, -3\}, \frac{182\,225\,761}{59\,779\,399\,680}\right\},$

$\left\{\{-1, -1, -1\}, \frac{202\,867\,141}{7\,472\,424\,960}\right\}, \left\{\{-1, -1, 1\}, \frac{202\,867\,141}{7\,472\,424\,960}\right\}, \left\{\{-1, -1, 3\}, \frac{182\,225\,761}{59\,779\,399\,680}\right\},$

$\left\{\{-1, -1, 5\}, \frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{-1, -1, 7\}, \frac{19}{5\,434\,490\,880}\right\}, \left\{\{-1, 1, -7\}, \frac{19}{5\,434\,490\,880}\right\},$

$\left\{\{-1, 1, -5\}, \frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{-1, 1, -3\}, \frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{-1, 1, -1\}, \frac{202\,867\,141}{7\,472\,424\,960}\right\},$

$\left\{\{-1, 1, 1\}, \frac{202\,867\,141}{7\,472\,424\,960}\right\}, \left\{\{-1, 1, 3\}, \frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{-1, 1, 5\}, \frac{74\,801}{2\,846\,638\,080}\right\},$

$\left\{\{-1, 1, 7\}, \frac{19}{5\,434\,490\,880}\right\}, \left\{\{-1, 3, -5\}, \frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{-1, 3, -3\}, \frac{3\,336\,451}{14\,944\,849\,920}\right\},$

$\left\{\{-1, 3, -1\}, \frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{-1, 3, 1\}, \frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{-1, 3, 3\}, \frac{3\,336\,451}{14\,944\,849\,920}\right\},$

$\left\{\{-1, 3, 5\}, \frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{-1, 5, -3\}, \frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{-1, 5, -1\}, \frac{74\,801}{2\,846\,638\,080}\right\},$

$$\left\{\{-1, 5, 1\}, \frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{-1, 5, 3\}, \frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{-1, 7, -1\}, \frac{19}{5\,434\,490\,880}\right\},$$

$$\left\{\{-1, 7, 1\}, \frac{19}{5\,434\,490\,880}\right\}, \left\{\{1, -7, -1\}, -\frac{19}{5\,434\,490\,880}\right\}, \left\{\{1, -7, 1\}, -\frac{19}{5\,434\,490\,880}\right\},$$

$$\left\{\{1, -5, -3\}, -\frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{1, -5, -1\}, -\frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{1, -5, 1\}, -\frac{74\,801}{2\,846\,638\,080}\right\},$$

$$\left\{\{1, -5, 3\}, -\frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{1, -3, -5\}, -\frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{1, -3, -3\}, -\frac{3\,336\,451}{14\,944\,849\,920}\right\},$$

$$\left\{\{1, -3, -1\}, -\frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{1, -3, 1\}, -\frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{1, -3, 3\}, -\frac{3\,336\,451}{14\,944\,849\,920}\right\},$$

$$\left\{\{1, -3, 5\}, -\frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{1, -1, -7\}, -\frac{19}{5\,434\,490\,880}\right\}, \left\{\{1, -1, -5\}, -\frac{74\,801}{2\,846\,638\,080}\right\},$$

$$\left\{\{1, -1, -3\}, -\frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{1, -1, -1\}, -\frac{202\,867\,141}{7\,472\,424\,960}\right\}, \left\{\{1, -1, 1\}, -\frac{202\,867\,141}{7\,472\,424\,960}\right\},$$

$$\left\{\{1, -1, 3\}, -\frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{1, -1, 5\}, -\frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{1, -1, 7\}, -\frac{19}{5\,434\,490\,880}\right\},$$

$$\left\{\{1, 1, -7\}, -\frac{19}{5\,434\,490\,880}\right\}, \left\{\{1, 1, -5\}, -\frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{1, 1, -3\}, -\frac{182\,225\,761}{59\,779\,399\,680}\right\},$$

$$\left\{\{1, 1, -1\}, -\frac{202\,867\,141}{7\,472\,424\,960}\right\}, \left\{\{1, 1, 1\}, -\frac{202\,867\,141}{7\,472\,424\,960}\right\}, \left\{\{1, 1, 3\}, -\frac{182\,225\,761}{59\,779\,399\,680}\right\},$$

$$\left\{\{1, 1, 5\}, -\frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{1, 1, 7\}, -\frac{19}{5\,434\,490\,880}\right\}, \left\{\{1, 3, -5\}, -\frac{10\,279}{29\,889\,699\,840}\right\},$$

$$\left\{\{1, 3, -3\}, -\frac{3\,336\,451}{14\,944\,849\,920}\right\}, \left\{\{1, 3, -1\}, -\frac{182\,225\,761}{59\,779\,399\,680}\right\}, \left\{\{1, 3, 1\}, -\frac{182\,225\,761}{59\,779\,399\,680}\right\},$$

$$\left\{\{1, 3, 3\}, -\frac{3\,336\,451}{14\,944\,849\,920}\right\}, \left\{\{1, 3, 5\}, -\frac{10\,279}{29\,889\,699\,840}\right\}, \left\{\{1, 5, -3\}, -\frac{10\,279}{29\,889\,699\,840}\right\},$$

$$\left\{\{1, 5, -1\}, -\frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{1, 5, 1\}, -\frac{74\,801}{2\,846\,638\,080}\right\}, \left\{\{1, 5, 3\}, -\frac{10\,279}{29\,889\,699\,840}\right\},$$

$$\left\{\{1, 7, -1\}, -\frac{19}{5\,434\,490\,880}\right\}, \left\{\{1, 7, 1\}, -\frac{19}{5\,434\,490\,880}\right\}, \left\{\{2, -6, -2\}, -\frac{173}{8\,539\,914\,240}\right\},$$

$$\left\{\{2, -6, 0\}, -\frac{19\,853}{59\,779\,399\,680}\right\}, \left\{\{2, -6, 2\}, -\frac{173}{8\,539\,914\,240}\right\}, \left\{\{2, -4, -4\}, -\frac{281}{1\,067\,489\,280}\right\},$$

$$\left\{\{2, -4, -2\}, -\frac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{2, -4, 0\}, -\frac{11\,607\,821}{29\,889\,699\,840}\right\}, \left\{\{2, -4, 2\}, -\frac{432\,799}{5\,434\,490\,880}\right\},$$

$$\left\{\{2, -4, 4\}, -\frac{281}{1\,067\,489\,280}\right\}, \left\{\{2, -2, -6\}, -\frac{173}{8\,539\,914\,240}\right\}, \left\{\{2, -2, -4\}, -\frac{432\,799}{5\,434\,490\,880}\right\},$$

$$\left\{\{2, -2, -2\}, -\frac{10\,927\,349}{2\,717\,245\,440}\right\}, \left\{\{2, -2, 0\}, -\frac{30\,805\,517}{2\,299\,207\,680}\right\}, \left\{\{2, -2, 2\}, -\frac{10\,927\,349}{2\,717\,245\,440}\right\},$$

$$\left\{\{2, -2, 4\}, -\frac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{2, -2, 6\}, -\frac{173}{8\,539\,914\,240}\right\}, \left\{\{2, 0, -6\}, -\frac{19\,853}{59\,779\,399\,680}\right\},$$

$$\left\{\{2, 0, -4\}, -\frac{11\,607\,821}{29\,889\,699\,840}\right\}, \left\{\{2, 0, -2\}, -\frac{30\,805\,517}{2\,299\,207\,680}\right\}, \left\{\{2, 0, 0\}, -\frac{173\,942\,723}{4\,269\,957\,120}\right\},$$

$$\left\{\{2, 0, 2\}, -\frac{30\,805\,517}{2\,299\,207\,680}\right\}, \left\{\{2, 0, 4\}, -\frac{11\,607\,821}{29\,889\,699\,840}\right\}, \left\{\{2, 0, 6\}, -\frac{19\,853}{59\,779\,399\,680}\right\},$$

$$\left\{\{2, 2, -6\}, -\frac{173}{8\,539\,914\,240}\right\}, \left\{\{2, 2, -4\}, -\frac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{2, 2, -2\}, -\frac{10\,927\,349}{2\,717\,245\,440}\right\},$$

$$\left\{\{2, 2, 0\}, -\frac{30\,805\,517}{2\,299\,207\,680}\right\}, \left\{\{2, 2, 2\}, -\frac{10\,927\,349}{2\,717\,245\,440}\right\}, \left\{\{2, 2, 4\}, -\frac{432\,799}{5\,434\,490\,880}\right\},$$

$$\left\{\{2, 2, 6\}, -\frac{173}{8\,539\,914\,240}\right\}, \left\{\{2, 4, -4\}, -\frac{281}{1\,067\,489\,280}\right\}, \left\{\{2, 4, -2\}, -\frac{432\,799}{5\,434\,490\,880}\right\},$$

$$\left\{\{2, 4, 0\}, -\frac{11\,607\,821}{29\,889\,699\,840}\right\}, \left\{\{2, 4, 2\}, -\frac{432\,799}{5\,434\,490\,880}\right\}, \left\{\{2, 4, 4\}, -\frac{281}{1\,067\,489\,280}\right\},$$

$$\left\{\{2, 6, -2\}, -\frac{173}{8\,539\,914\,240}\right\}, \left\{\{2, 6, 0\}, -\frac{19\,853}{59\,779\,399\,680}\right\}, \left\{\{2, 6, 2\}, -\frac{173}{8\,539\,914\,240}\right\},$$

$$\left\{\{3, -5, -3\}, -\frac{731}{59\,779\,399\,680}\right\}, \left\{\{3, -5, -1\}, -\frac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{3, -5, 1\}, -\frac{15\,373}{11\,955\,879\,936}\right\},$$

$$\left\{\{3, -5, 3\}, -\frac{731}{59\,779\,399\,680}\right\}, \left\{\{3, -3, -5\}, -\frac{731}{59\,779\,399\,680}\right\}, \left\{\{3, -3, -3\}, -\frac{9901}{355\,829\,760}\right\},$$

$$\left\{\{3, -3, -1\}, -\frac{85\,321}{135\,862\,272}\right\}, \left\{\{3, -3, 1\}, -\frac{85\,321}{135\,862\,272}\right\}, \left\{\{3, -3, 3\}, -\frac{9901}{355\,829\,760}\right\},$$

$$\left\{\{3, -3, 5\}, -\frac{731}{59\,779\,399\,680}\right\}, \left\{\{3, -1, -5\}, -\frac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{3, -1, -3\}, -\frac{85\,321}{135\,862\,272}\right\},$$

$$\left\{\{3, -1, -1\}, -\frac{11\,806\,073}{1\,423\,319\,040}\right\}, \left\{\{3, -1, 1\}, -\frac{11\,806\,073}{1\,423\,319\,040}\right\}, \left\{\{3, -1, 3\}, -\frac{85\,321}{135\,862\,272}\right\},$$

$$\left\{\{3, -1, 5\}, -\frac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{3, 1, -5\}, -\frac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{3, 1, -3\}, -\frac{85\,321}{135\,862\,272}\right\},$$

$$\left\{\{3, 1, -1\}, -\frac{11\,806\,073}{1\,423\,319\,040}\right\}, \left\{\{3, 1, 1\}, -\frac{11\,806\,073}{1\,423\,319\,040}\right\}, \left\{\{3, 1, 3\}, -\frac{85\,321}{135\,862\,272}\right\},$$

$$\left\{\{3, 1, 5\}, -\frac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{3, 3, -5\}, -\frac{731}{59\,779\,399\,680}\right\}, \left\{\{3, 3, -3\}, -\frac{9901}{355\,829\,760}\right\},$$

$$\left\{\{3, 3, -1\}, -\frac{85\,321}{135\,862\,272}\right\}, \left\{\{3, 3, 1\}, -\frac{85\,321}{135\,862\,272}\right\}, \left\{\{3, 3, 3\}, -\frac{9901}{355\,829\,760}\right\},$$

$\left\{\{3, 3, 5\}, -\dfrac{731}{59\,779\,399\,680}\right\}, \left\{\{3, 5, -3\}, -\dfrac{731}{59\,779\,399\,680}\right\}, \left\{\{3, 5, -1\}, -\dfrac{15\,373}{11\,955\,879\,936}\right\},$

$\left\{\{3, 5, 1\}, -\dfrac{15\,373}{11\,955\,879\,936}\right\}, \left\{\{3, 5, 3\}, -\dfrac{731}{59\,779\,399\,680}\right\}, \left\{\{4, -4, -4\}, -\dfrac{1}{1\,811\,496\,960}\right\},$

$\left\{\{4, -4, -2\}, -\dfrac{335}{569\,327\,616}\right\}, \left\{\{4, -4, 0\}, -\dfrac{83\,819}{19\,926\,466\,560}\right\}, \left\{\{4, -4, 2\}, -\dfrac{335}{569\,327\,616}\right\},$

$\left\{\{4, -4, 4\}, -\dfrac{1}{1\,811\,496\,960}\right\}, \left\{\{4, -2, -4\}, -\dfrac{335}{569\,327\,616}\right\}, \left\{\{4, -2, -2\}, -\dfrac{75\,541}{543\,449\,088}\right\},$

$\left\{\{4, -2, 0\}, -\dfrac{1\,788\,509}{2\,717\,245\,440}\right\}, \left\{\{4, -2, 2\}, -\dfrac{75\,541}{543\,449\,088}\right\}, \left\{\{4, -2, 4\}, -\dfrac{335}{569\,327\,616}\right\},$

$\left\{\{4, 0, -4\}, -\dfrac{83\,819}{19\,926\,466\,560}\right\}, \left\{\{4, 0, -2\}, -\dfrac{1\,788\,509}{2\,717\,245\,440}\right\}, \left\{\{4, 0, 0\}, -\dfrac{27\,077\,587}{9\,963\,233\,280}\right\},$

$\left\{\{4, 0, 2\}, -\dfrac{1\,788\,509}{2\,717\,245\,440}\right\}, \left\{\{4, 0, 4\}, -\dfrac{83\,819}{19\,926\,466\,560}\right\}, \left\{\{4, 2, -4\}, -\dfrac{335}{569\,327\,616}\right\},$

$\left\{\{4, 2, -2\}, -\dfrac{75\,541}{543\,449\,088}\right\}, \left\{\{4, 2, 0\}, -\dfrac{1\,788\,509}{2\,717\,245\,440}\right\}, \left\{\{4, 2, 2\}, -\dfrac{75\,541}{543\,449\,088}\right\},$

$\left\{\{4, 2, 4\}, -\dfrac{335}{569\,327\,616}\right\}, \left\{\{4, 4, -4\}, -\dfrac{1}{1\,811\,496\,960}\right\}, \left\{\{4, 4, -2\}, -\dfrac{335}{569\,327\,616}\right\},$

$\left\{\{4, 4, 0\}, -\dfrac{83\,819}{19\,926\,466\,560}\right\}, \left\{\{4, 4, 2\}, -\dfrac{335}{569\,327\,616}\right\}, \left\{\{4, 4, 4\}, -\dfrac{1}{1\,811\,496\,960}\right\},$

$\left\{\{5, -3, -3\}, -\dfrac{17}{905\,748\,480}\right\}, \left\{\{5, -3, -1\}, -\dfrac{99\,427}{59\,779\,399\,680}\right\}, \left\{\{5, -3, 1\}, -\dfrac{99\,427}{59\,779\,399\,680}\right\},$

$\left\{\{5, -3, 3\}, -\dfrac{17}{905\,748\,480}\right\}, \left\{\{5, -1, -3\}, -\dfrac{99\,427}{59\,779\,399\,680}\right\}, \left\{\{5, -1, -1\}, -\dfrac{2\,812\,801}{29\,889\,699\,840}\right\},$

$\left\{\{5, -1, 1\}, -\dfrac{2\,812\,801}{29\,889\,699\,840}\right\}, \left\{\{5, -1, 3\}, -\dfrac{99\,427}{59\,779\,399\,680}\right\}, \left\{\{5, 1, -3\}, -\dfrac{99\,427}{59\,779\,399\,680}\right\},$

$\left\{\{5, 1, -1\}, -\dfrac{2\,812\,801}{29\,889\,699\,840}\right\}, \left\{\{5, 1, 1\}, -\dfrac{2\,812\,801}{29\,889\,699\,840}\right\}, \left\{\{5, 1, 3\}, -\dfrac{99\,427}{59\,779\,399\,680}\right\},$

$\left\{\{5, 3, -3\}, -\dfrac{17}{905\,748\,480}\right\}, \left\{\{5, 3, -1\}, -\dfrac{99\,427}{59\,779\,399\,680}\right\}, \left\{\{5, 3, 1\}, -\dfrac{99\,427}{59\,779\,399\,680}\right\},$

$\left\{\{5, 3, 3\}, -\dfrac{17}{905\,748\,480}\right\}, \left\{\{6, -2, -2\}, -\dfrac{317}{7\,472\,424\,960}\right\}, \left\{\{6, -2, 0\}, -\dfrac{34\,037}{59\,779\,399\,680}\right\},$

$\left\{\{6, -2, 2\}, -\dfrac{317}{7\,472\,424\,960}\right\}, \left\{\{6, 0, -2\}, -\dfrac{34\,037}{59\,779\,399\,680}\right\}, \left\{\{6, 0, 0\}, -\dfrac{74\,167}{9\,963\,233\,280}\right\},$

$$\left\{\{6, 0, 2\}, -\frac{34\,037}{59\,779\,399\,680}\right\}, \left\{\{6, 2, -2\}, -\frac{317}{7\,472\,424\,960}\right\}, \left\{\{6, 2, 0\}, -\frac{34\,037}{59\,779\,399\,680}\right\},$$

$$\left\{\{6, 2, 2\}, -\frac{317}{7\,472\,424\,960}\right\}, \left\{\{7, -1, -1\}, -\frac{67}{5\,977\,939\,968}\right\}, \left\{\{7, -1, 1\}, -\frac{67}{5\,977\,939\,968}\right\},$$

$$\left\{\{7, 1, -1\}, -\frac{67}{5\,977\,939\,968}\right\}, \left\{\{7, 1, 1\}, -\frac{67}{5\,977\,939\,968}\right\}, \left\{\{8, 0, 0\}, -\frac{1}{8\,539\,914\,240}\right\}\right\}$$

Autocorrelation Sequences

▪ Linear Box Spline

The autocorrelation sequence of the linear box spline is obtained by sampling the quintic box spline at the BCC lattice points that are within its support

```
ACLbs = Select[{#, Qbs[#[[1]], #[[2]], #[[3]]]} & /@ BCCpoints[4], #[[2]] ≠ 0 &]
```

$$\left\{\left\{\{-2, 0, 0\}, \frac{1}{30}\right\}, \left\{\{-1, -1, -1\}, \frac{1}{20}\right\}, \left\{\{-1, -1, 1\}, \frac{1}{20}\right\}, \left\{\{-1, 1, -1\}, \frac{1}{20}\right\}, \left\{\{-1, 1, 1\}, \frac{1}{20}\right\},\right.$$

$$\left\{\{0, -2, 0\}, \frac{1}{30}\right\}, \left\{\{0, 0, -2\}, \frac{1}{30}\right\}, \left\{\{0, 0, 0\}, \frac{2}{5}\right\}, \left\{\{0, 0, 2\}, \frac{1}{30}\right\}, \left\{\{0, 2, 0\}, \frac{1}{30}\right\},$$

$$\left.\left\{\{1, -1, -1\}, \frac{1}{20}\right\}, \left\{\{1, -1, 1\}, \frac{1}{20}\right\}, \left\{\{1, 1, -1\}, \frac{1}{20}\right\}, \left\{\{1, 1, 1\}, \frac{1}{20}\right\}, \left\{\{2, 0, 0\}, \frac{1}{30}\right\}\right\}$$

▪ Quintic Box Spline

The autocorrelation of the Quintic box spline is obtained by sampling the Tredecic box spline

```
ACQbs = Select[{#, Tdbs[#[[1]], #[[2]], #[[3]]]} & /@ BCCpoints[8], #[[2]] ≠ 0 &]
```

$$\left\{\left\{\{-6, 0, 0\}, \frac{1}{15\,567\,552}\right\}, \left\{\{-5, -1, -1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{-5, -1, 1\}, \frac{167}{77\,837\,760}\right\},\right.$$

$$\left\{\{-5, 1, -1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{-5, 1, 1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{-4, -2, -2\}, \frac{59}{19\,459\,440}\right\},$$

$$\left\{\{-4, -2, 0\}, \frac{659}{15\,567\,552}\right\}, \left\{\{-4, -2, 2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{-4, 0, -2\}, \frac{659}{15\,567\,552}\right\},$$

$$\left.\left\{\{-4, 0, 0\}, \frac{2383}{4\,864\,860}\right\}, \left\{\{-4, 0, 2\}, \frac{659}{15\,567\,552}\right\}, \left\{\{-4, 2, -2\}, \frac{59}{19\,459\,440}\right\},\right.$$

$\left\{\{-4, 2, 0\}, \frac{659}{15\,567\,552}\right\}, \left\{\{-4, 2, 2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{-3, -3, -3\}, \frac{1}{3\,706\,560}\right\},$

$\left\{\{-3, -3, -1\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-3, -3, 1\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-3, -3, 3\}, \frac{1}{3\,706\,560}\right\},$

$\left\{\{-3, -1, -3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-3, -1, -1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-3, -1, 1\}, \frac{9743}{3\,538\,080}\right\},$

$\left\{\{-3, -1, 3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-3, 1, -3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-3, 1, -1\}, \frac{9743}{3\,538\,080}\right\},$

$\left\{\{-3, 1, 1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-3, 1, 3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-3, 3, -3\}, \frac{1}{3\,706\,560}\right\},$

$\left\{\{-3, 3, -1\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-3, 3, 1\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-3, 3, 3\}, \frac{1}{3\,706\,560}\right\},$

$\left\{\{-2, -4, -2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{-2, -4, 0\}, \frac{659}{15\,567\,552}\right\}, \left\{\{-2, -4, 2\}, \frac{59}{19\,459\,440}\right\},$

$\left\{\{-2, -2, -4\}, \frac{59}{19\,459\,440}\right\}, \left\{\{-2, -2, -2\}, \frac{1613}{1\,081\,080}\right\}, \left\{\{-2, -2, 0\}, \frac{54\,889}{6\,486\,480}\right\},$

$\left\{\{-2, -2, 2\}, \frac{1613}{1\,081\,080}\right\}, \left\{\{-2, -2, 4\}, \frac{59}{19\,459\,440}\right\}, \left\{\{-2, 0, -4\}, \frac{659}{15\,567\,552}\right\},$

$\left\{\{-2, 0, -2\}, \frac{54\,889}{6\,486\,480}\right\}, \left\{\{-2, 0, 0\}, \frac{3\,029\,911}{77\,837\,760}\right\}, \left\{\{-2, 0, 2\}, \frac{54\,889}{6\,486\,480}\right\},$

$\left\{\{-2, 0, 4\}, \frac{659}{15\,567\,552}\right\}, \left\{\{-2, 2, -4\}, \frac{59}{19\,459\,440}\right\}, \left\{\{-2, 2, -2\}, \frac{1613}{1\,081\,080}\right\},$

$\left\{\{-2, 2, 0\}, \frac{54\,889}{6\,486\,480}\right\}, \left\{\{-2, 2, 2\}, \frac{1613}{1\,081\,080}\right\}, \left\{\{-2, 2, 4\}, \frac{59}{19\,459\,440}\right\},$

$\left\{\{-2, 4, -2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{-2, 4, 0\}, \frac{659}{15\,567\,552}\right\}, \left\{\{-2, 4, 2\}, \frac{59}{19\,459\,440}\right\},$

$\left\{\{-1, -5, -1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{-1, -5, 1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{-1, -3, -3\}, \frac{719}{15\,567\,552}\right\},$

$\left\{\{-1, -3, -1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-1, -3, 1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-1, -3, 3\}, \frac{719}{15\,567\,552}\right\},$

$\left\{\{-1, -1, -5\}, \frac{167}{77\,837\,760}\right\}, \left\{\{-1, -1, -3\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-1, -1, -1\}, \frac{1\,396\,301}{25\,945\,920}\right\},$

$\left\{\{-1, -1, 1\}, \frac{1\,396\,301}{25\,945\,920}\right\}, \left\{\{-1, -1, 3\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-1, -1, 5\}, \frac{167}{77\,837\,760}\right\},$

$\left\{\{-1, 1, -5\}, \frac{167}{77\,837\,760}\right\}, \left\{\{-1, 1, -3\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-1, 1, -1\}, \frac{1\,396\,301}{25\,945\,920}\right\},$

$$\left\{\{-1, 1, 1\}, \frac{1\,396\,301}{25\,945\,920}\right\}, \left\{\{-1, 1, 3\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-1, 1, 5\}, \frac{167}{77\,837\,760}\right\},$$

$$\left\{\{-1, 3, -3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-1, 3, -1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{-1, 3, 1\}, \frac{9743}{3\,538\,080}\right\},$$

$$\left\{\{-1, 3, 3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{-1, 5, -1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{-1, 5, 1\}, \frac{167}{77\,837\,760}\right\},$$

$$\left\{\{0, -6, 0\}, \frac{1}{15\,567\,552}\right\}, \left\{\{0, -4, -2\}, \frac{659}{15\,567\,552}\right\}, \left\{\{0, -4, 0\}, \frac{2383}{4\,864\,860}\right\},$$

$$\left\{\{0, -4, 2\}, \frac{659}{15\,567\,552}\right\}, \left\{\{0, -2, -4\}, \frac{659}{15\,567\,552}\right\}, \left\{\{0, -2, -2\}, \frac{54\,889}{6\,486\,480}\right\},$$

$$\left\{\{0, -2, 0\}, \frac{3\,029\,911}{77\,837\,760}\right\}, \left\{\{0, -2, 2\}, \frac{54\,889}{6\,486\,480}\right\}, \left\{\{0, -2, 4\}, \frac{659}{15\,567\,552}\right\},$$

$$\left\{\{0, 0, -6\}, \frac{1}{15\,567\,552}\right\}, \left\{\{0, 0, -4\}, \frac{2383}{4\,864\,860}\right\}, \left\{\{0, 0, -2\}, \frac{3\,029\,911}{77\,837\,760}\right\}, \left\{\{0, 0, 0\}, \frac{40\,853}{270\,270}\right\},$$

$$\left\{\{0, 0, 2\}, \frac{3\,029\,911}{77\,837\,760}\right\}, \left\{\{0, 0, 4\}, \frac{2383}{4\,864\,860}\right\}, \left\{\{0, 0, 6\}, \frac{1}{15\,567\,552}\right\}, \left\{\{0, 2, -4\}, \frac{659}{15\,567\,552}\right\},$$

$$\left\{\{0, 2, -2\}, \frac{54\,889}{6\,486\,480}\right\}, \left\{\{0, 2, 0\}, \frac{3\,029\,911}{77\,837\,760}\right\}, \left\{\{0, 2, 2\}, \frac{54\,889}{6\,486\,480}\right\}, \left\{\{0, 2, 4\}, \frac{659}{15\,567\,552}\right\},$$

$$\left\{\{0, 4, -2\}, \frac{659}{15\,567\,552}\right\}, \left\{\{0, 4, 0\}, \frac{2383}{4\,864\,860}\right\}, \left\{\{0, 4, 2\}, \frac{659}{15\,567\,552}\right\}, \left\{\{0, 6, 0\}, \frac{1}{15\,567\,552}\right\},$$

$$\left\{\{1, -5, -1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{1, -5, 1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{1, -3, -3\}, \frac{719}{15\,567\,552}\right\},$$

$$\left\{\{1, -3, -1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{1, -3, 1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{1, -3, 3\}, \frac{719}{15\,567\,552}\right\},$$

$$\left\{\{1, -1, -5\}, \frac{167}{77\,837\,760}\right\}, \left\{\{1, -1, -3\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{1, -1, -1\}, \frac{1\,396\,301}{25\,945\,920}\right\},$$

$$\left\{\{1, -1, 1\}, \frac{1\,396\,301}{25\,945\,920}\right\}, \left\{\{1, -1, 3\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{1, -1, 5\}, \frac{167}{77\,837\,760}\right\},$$

$$\left\{\{1, 1, -5\}, \frac{167}{77\,837\,760}\right\}, \left\{\{1, 1, -3\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{1, 1, -1\}, \frac{1\,396\,301}{25\,945\,920}\right\},$$

$$\left\{\{1, 1, 1\}, \frac{1\,396\,301}{25\,945\,920}\right\}, \left\{\{1, 1, 3\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{1, 1, 5\}, \frac{167}{77\,837\,760}\right\}, \left\{\{1, 3, -3\}, \frac{719}{15\,567\,552}\right\},$$

$$\left\{\{1, 3, -1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{1, 3, 1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{1, 3, 3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{1, 5, -1\}, \frac{167}{77\,837\,760}\right\},$$

$$\left\{\{1, 5, 1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{2, -4, -2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{2, -4, 0\}, \frac{659}{15\,567\,552}\right\},$$

$$\left\{\{2, -4, 2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{2, -2, -4\}, \frac{59}{19\,459\,440}\right\}, \left\{\{2, -2, -2\}, \frac{1613}{1\,081\,080}\right\},$$

$$\left\{\{2, -2, 0\}, \frac{54\,889}{6\,486\,480}\right\}, \left\{\{2, -2, 2\}, \frac{1613}{1\,081\,080}\right\}, \left\{\{2, -2, 4\}, \frac{59}{19\,459\,440}\right\},$$

$$\left\{\{2, 0, -4\}, \frac{659}{15\,567\,552}\right\}, \left\{\{2, 0, -2\}, \frac{54\,889}{6\,486\,480}\right\}, \left\{\{2, 0, 0\}, \frac{3\,029\,911}{77\,837\,760}\right\}, \left\{\{2, 0, 2\}, \frac{54\,889}{6\,486\,480}\right\},$$

$$\left\{\{2, 0, 4\}, \frac{659}{15\,567\,552}\right\}, \left\{\{2, 2, -4\}, \frac{59}{19\,459\,440}\right\}, \left\{\{2, 2, -2\}, \frac{1613}{1\,081\,080}\right\},$$

$$\left\{\{2, 2, 0\}, \frac{54\,889}{6\,486\,480}\right\}, \left\{\{2, 2, 2\}, \frac{1613}{1\,081\,080}\right\}, \left\{\{2, 2, 4\}, \frac{59}{19\,459\,440}\right\}, \left\{\{2, 4, -2\}, \frac{59}{19\,459\,440}\right\},$$

$$\left\{\{2, 4, 0\}, \frac{659}{15\,567\,552}\right\}, \left\{\{2, 4, 2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{3, -3, -3\}, \frac{1}{3\,706\,560}\right\},$$

$$\left\{\{3, -3, -1\}, \frac{719}{15\,567\,552}\right\}, \left\{\{3, -3, 1\}, \frac{719}{15\,567\,552}\right\}, \left\{\{3, -3, 3\}, \frac{1}{3\,706\,560}\right\},$$

$$\left\{\{3, -1, -3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{3, -1, -1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{3, -1, 1\}, \frac{9743}{3\,538\,080}\right\},$$

$$\left\{\{3, -1, 3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{3, 1, -3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{3, 1, -1\}, \frac{9743}{3\,538\,080}\right\},$$

$$\left\{\{3, 1, 1\}, \frac{9743}{3\,538\,080}\right\}, \left\{\{3, 1, 3\}, \frac{719}{15\,567\,552}\right\}, \left\{\{3, 3, -3\}, \frac{1}{3\,706\,560}\right\}, \left\{\{3, 3, -1\}, \frac{719}{15\,567\,552}\right\},$$

$$\left\{\{3, 3, 1\}, \frac{719}{15\,567\,552}\right\}, \left\{\{3, 3, 3\}, \frac{1}{3\,706\,560}\right\}, \left\{\{4, -2, -2\}, \frac{59}{19\,459\,440}\right\},$$

$$\left\{\{4, -2, 0\}, \frac{659}{15\,567\,552}\right\}, \left\{\{4, -2, 2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{4, 0, -2\}, \frac{659}{15\,567\,552}\right\},$$

$$\left\{\{4, 0, 0\}, \frac{2383}{4\,864\,860}\right\}, \left\{\{4, 0, 2\}, \frac{659}{15\,567\,552}\right\}, \left\{\{4, 2, -2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{4, 2, 0\}, \frac{659}{15\,567\,552}\right\},$$

$$\left\{\{4, 2, 2\}, \frac{59}{19\,459\,440}\right\}, \left\{\{5, -1, -1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{5, -1, 1\}, \frac{167}{77\,837\,760}\right\},$$

$$\left\{\{5, 1, -1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{5, 1, 1\}, \frac{167}{77\,837\,760}\right\}, \left\{\{6, 0, 0\}, \frac{1}{15\,567\,552}\right\}\right\}$$

Save The filters and the AC sequences

```
Export["CdLLx.mat", Flatten[#] & /@ CdLLx, "Mat"];
Export["CdQLx.mat", Flatten[#] & /@ CdQLx, "Mat"];
Export["CdQQx.mat", Flatten[#] & /@ CdQQx, "Mat"];
Export["CdNLx.mat", Flatten[#] & /@ CdNLx, "Mat"];
Export["CdNQx.mat", Flatten[#] & /@ CdNQx, "Mat"];

Export["ACLbs.mat", Flatten[#] & /@ ACLbs, "Mat"];
Export["ACQbs.mat", Flatten[#] & /@ ACQbs, "Mat"];
```

Sampled Sequences

These are useful when one wants to prefilter data so as to make the box splines interpolating

◾ **Sampled Quintic**

```
SSQbs = Select[{#, Qbs[#[[1]], #[[2]], #[[3]]]} & /@ BCCpoints[4], #[[2]] ≠ 0 &]
```

$$\left\{\left\{\{-2, 0, 0\}, \frac{1}{30}\right\}, \left\{\{-1, -1, -1\}, \frac{1}{20}\right\}, \left\{\{-1, -1, 1\}, \frac{1}{20}\right\}, \left\{\{-1, 1, -1\}, \frac{1}{20}\right\}, \left\{\{-1, 1, 1\}, \frac{1}{20}\right\},\right.$$

$$\left\{\{0, -2, 0\}, \frac{1}{30}\right\}, \left\{\{0, 0, -2\}, \frac{1}{30}\right\}, \left\{\{0, 0, 0\}, \frac{2}{5}\right\}, \left\{\{0, 0, 2\}, \frac{1}{30}\right\}, \left\{\{0, 2, 0\}, \frac{1}{30}\right\},$$

$$\left.\left\{\{1, -1, -1\}, \frac{1}{20}\right\}, \left\{\{1, -1, 1\}, \frac{1}{20}\right\}, \left\{\{1, 1, -1\}, \frac{1}{20}\right\}, \left\{\{1, 1, 1\}, \frac{1}{20}\right\}, \left\{\{2, 0, 0\}, \frac{1}{30}\right\}\right\}\right\}$$

▪ **Sampled nonic**

```
SSNbs = Select[{#, Nbs[#[[1]], #[[2]], #[[3]]]} & /@ BCCpoints[8], #[[2]] ≠ 0 &]
```

$$\left\{\left\{\{-4, 0, 0\}, \frac{1}{10\,080}\right\}, \left\{\{-3, -1, -1\}, \frac{17}{20\,160}\right\}, \left\{\{-3, -1, 1\}, \frac{17}{20\,160}\right\},\right.$$

$$\left\{\{-3, 1, -1\}, \frac{17}{20\,160}\right\}, \left\{\{-3, 1, 1\}, \frac{17}{20\,160}\right\}, \left\{\{-2, -2, -2\}, \frac{1}{4032}\right\}, \left\{\{-2, -2, 0\}, \frac{43}{10\,080}\right\},$$

$$\left\{\{-2, -2, 2\}, \frac{1}{4032}\right\}, \left\{\{-2, 0, -2\}, \frac{43}{10\,080}\right\}, \left\{\{-2, 0, 0\}, \frac{7}{180}\right\}, \left\{\{-2, 0, 2\}, \frac{43}{10\,080}\right\},$$

$$\left\{\{-2, 2, -2\}, \frac{1}{4032}\right\}, \left\{\{-2, 2, 0\}, \frac{43}{10\,080}\right\}, \left\{\{-2, 2, 2\}, \frac{1}{4032}\right\}, \left\{\{-1, -3, -1\}, \frac{17}{20\,160}\right\},$$

$$\left\{\{-1, -3, 1\}, \frac{17}{20\,160}\right\}, \left\{\{-1, -1, -3\}, \frac{17}{20\,160}\right\}, \left\{\{-1, -1, -1\}, \frac{1177}{20\,160}\right\},$$

$$\left\{\{-1, -1, 1\}, \frac{1177}{20\,160}\right\}, \left\{\{-1, -1, 3\}, \frac{17}{20\,160}\right\}, \left\{\{-1, 1, -3\}, \frac{17}{20\,160}\right\},$$

$$\left\{\{-1, 1, -1\}, \frac{1177}{20\,160}\right\}, \left\{\{-1, 1, 1\}, \frac{1177}{20\,160}\right\}, \left\{\{-1, 1, 3\}, \frac{17}{20\,160}\right\}, \left\{\{-1, 3, -1\}, \frac{17}{20\,160}\right\},$$

$$\left\{\{-1, 3, 1\}, \frac{17}{20\,160}\right\}, \left\{\{0, -4, 0\}, \frac{1}{10\,080}\right\}, \left\{\{0, -2, -2\}, \frac{43}{10\,080}\right\}, \left\{\{0, -2, 0\}, \frac{7}{180}\right\},$$

$$\left\{\{0, -2, 2\}, \frac{43}{10\,080}\right\}, \left\{\{0, 0, -4\}, \frac{1}{10\,080}\right\}, \left\{\{0, 0, -2\}, \frac{7}{180}\right\}, \left\{\{0, 0, 0\}, \frac{379}{1680}\right\},$$

$$\left\{\{0, 0, 2\}, \frac{7}{180}\right\}, \left\{\{0, 0, 4\}, \frac{1}{10\,080}\right\}, \left\{\{0, 2, -2\}, \frac{43}{10\,080}\right\}, \left\{\{0, 2, 0\}, \frac{7}{180}\right\},$$

$$\left\{\{0, 2, 2\}, \frac{43}{10\,080}\right\}, \left\{\{0, 4, 0\}, \frac{1}{10\,080}\right\}, \left\{\{1, -3, -1\}, \frac{17}{20\,160}\right\}, \left\{\{1, -3, 1\}, \frac{17}{20\,160}\right\},$$

$$\left\{\{1, -1, -3\}, \frac{17}{20\,160}\right\}, \left\{\{1, -1, -1\}, \frac{1177}{20\,160}\right\}, \left\{\{1, -1, 1\}, \frac{1177}{20\,160}\right\}, \left\{\{1, -1, 3\}, \frac{17}{20\,160}\right\},$$

$$\left\{\{1, 1, -3\}, \frac{17}{20\,160}\right\}, \left\{\{1, 1, -1\}, \frac{1177}{20\,160}\right\}, \left\{\{1, 1, 1\}, \frac{1177}{20\,160}\right\}, \left\{\{1, 1, 3\}, \frac{17}{20\,160}\right\},$$

$$\left\{\{1, 3, -1\}, \frac{17}{20\,160}\right\}, \left\{\{1, 3, 1\}, \frac{17}{20\,160}\right\}, \left\{\{2, -2, -2\}, \frac{1}{4032}\right\}, \left\{\{2, -2, 0\}, \frac{43}{10\,080}\right\},$$

$$\left\{\{2, -2, 2\}, \frac{1}{4032}\right\}, \left\{\{2, 0, -2\}, \frac{43}{10\,080}\right\}, \left\{\{2, 0, 0\}, \frac{7}{180}\right\}, \left\{\{2, 0, 2\}, \frac{43}{10\,080}\right\},$$

$$\left\{\{2, 2, -2\}, \frac{1}{4032}\right\}, \left\{\{2, 2, 0\}, \frac{43}{10\,080}\right\}, \left\{\{2, 2, 2\}, \frac{1}{4032}\right\}, \left\{\{3, -1, -1\}, \frac{17}{20\,160}\right\},$$

$$\left.\left\{\{3, -1, 1\}, \frac{17}{20\,160}\right\}, \left\{\{3, 1, -1\}, \frac{17}{20\,160}\right\}, \left\{\{3, 1, 1\}, \frac{17}{20\,160}\right\}, \left\{\{4, 0, 0\}, \frac{1}{10\,080}\right\}\right\}\right\}$$

▪ **Sampled Trdecic**

This is the same as sampling the autocorrelation sequence of a quintic box spline

```
SSTbs = ACQbs;
```

```
Export["SSQbs.mat", Flatten[#] & /@ SSQbs, "Mat"];
Export["SSNbs.mat", Flatten[#] & /@ SSNbs, "Mat"];
Export["SSTbs.mat", Flatten[#] & /@ SSTbs, "Mat"];
```