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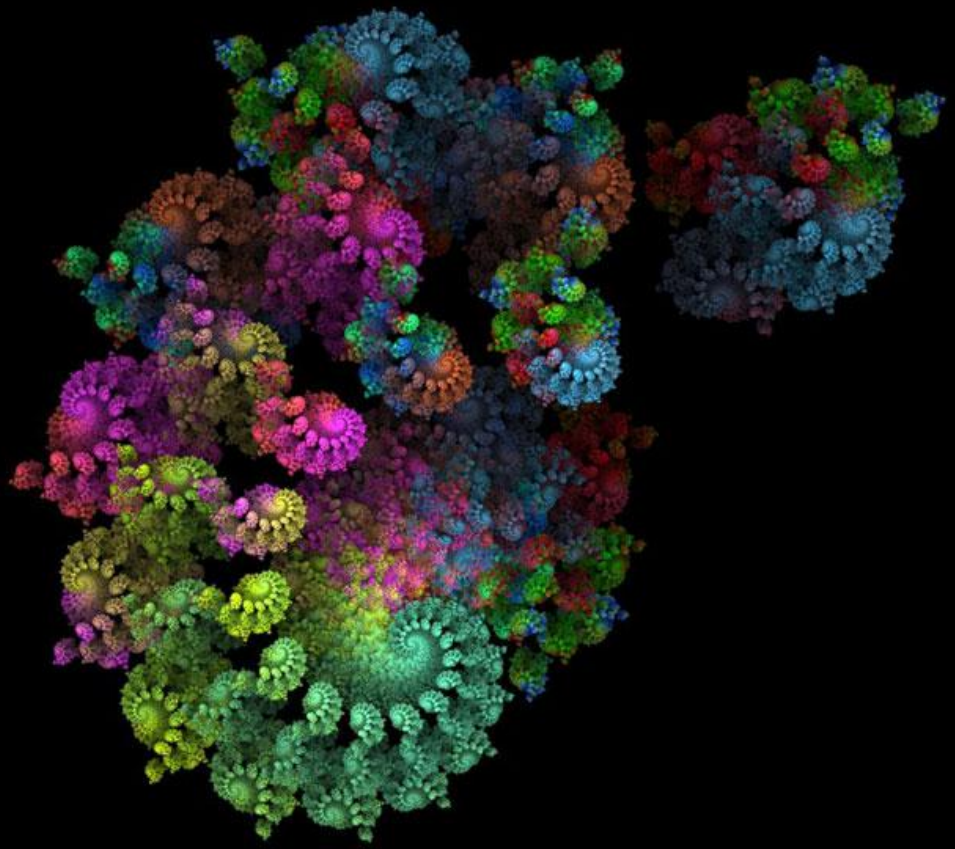
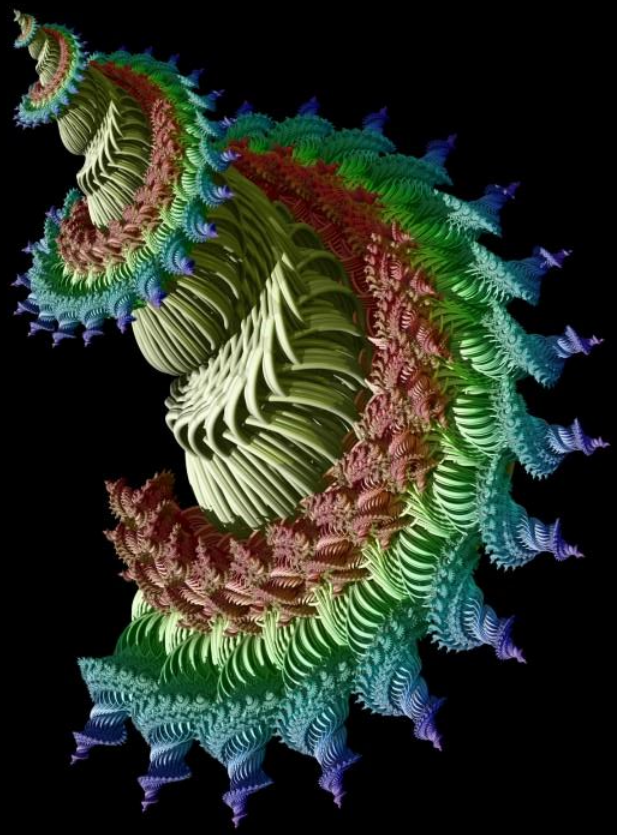
http://artstheanswer.blogspot.com/2011/04/fractalsmath-and-art-in-harmony.html





Coordinates and Transformations

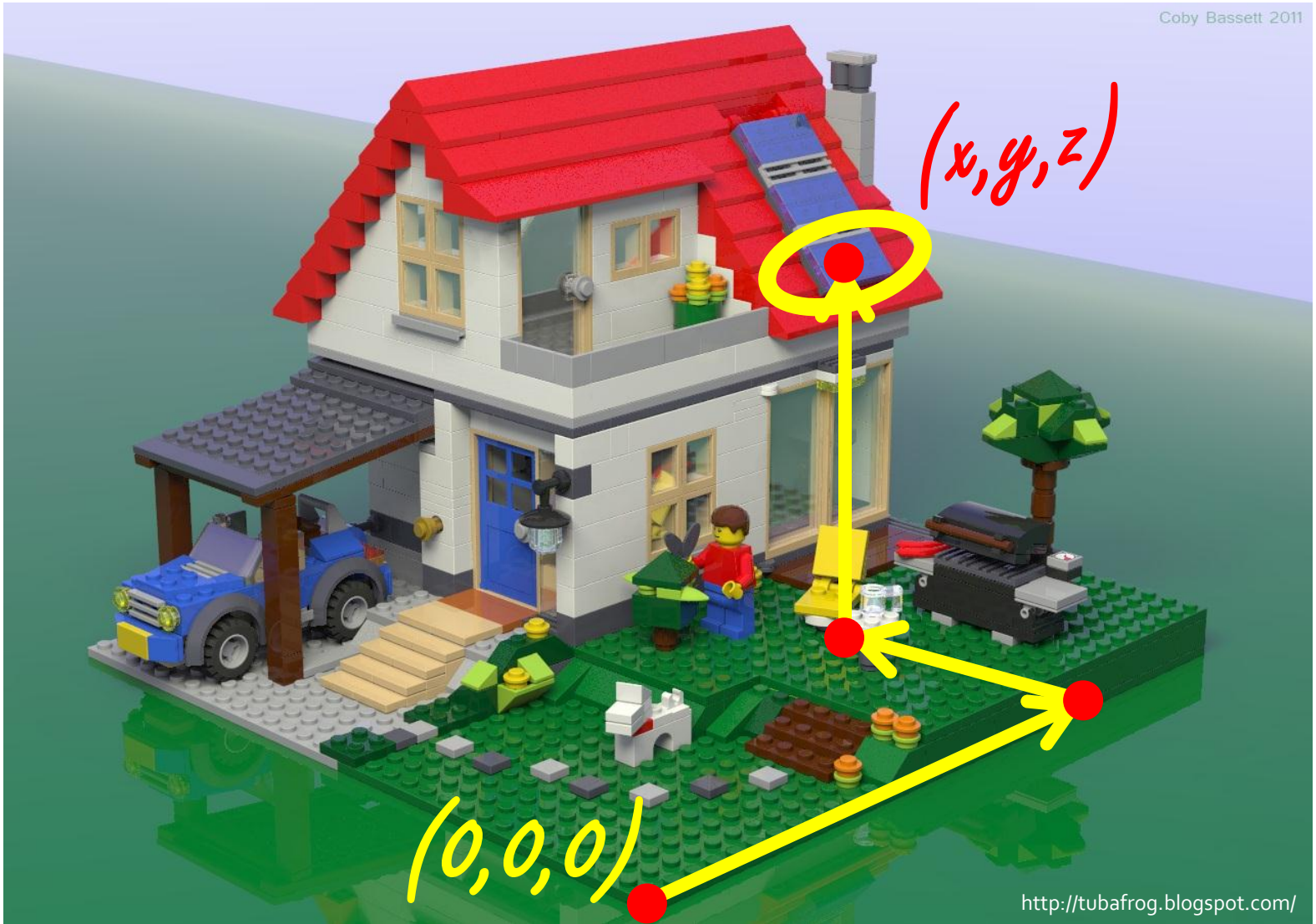
CS 148, Summer 2012
Introduction to Computer Graphics and Imaging

Justin Solomon

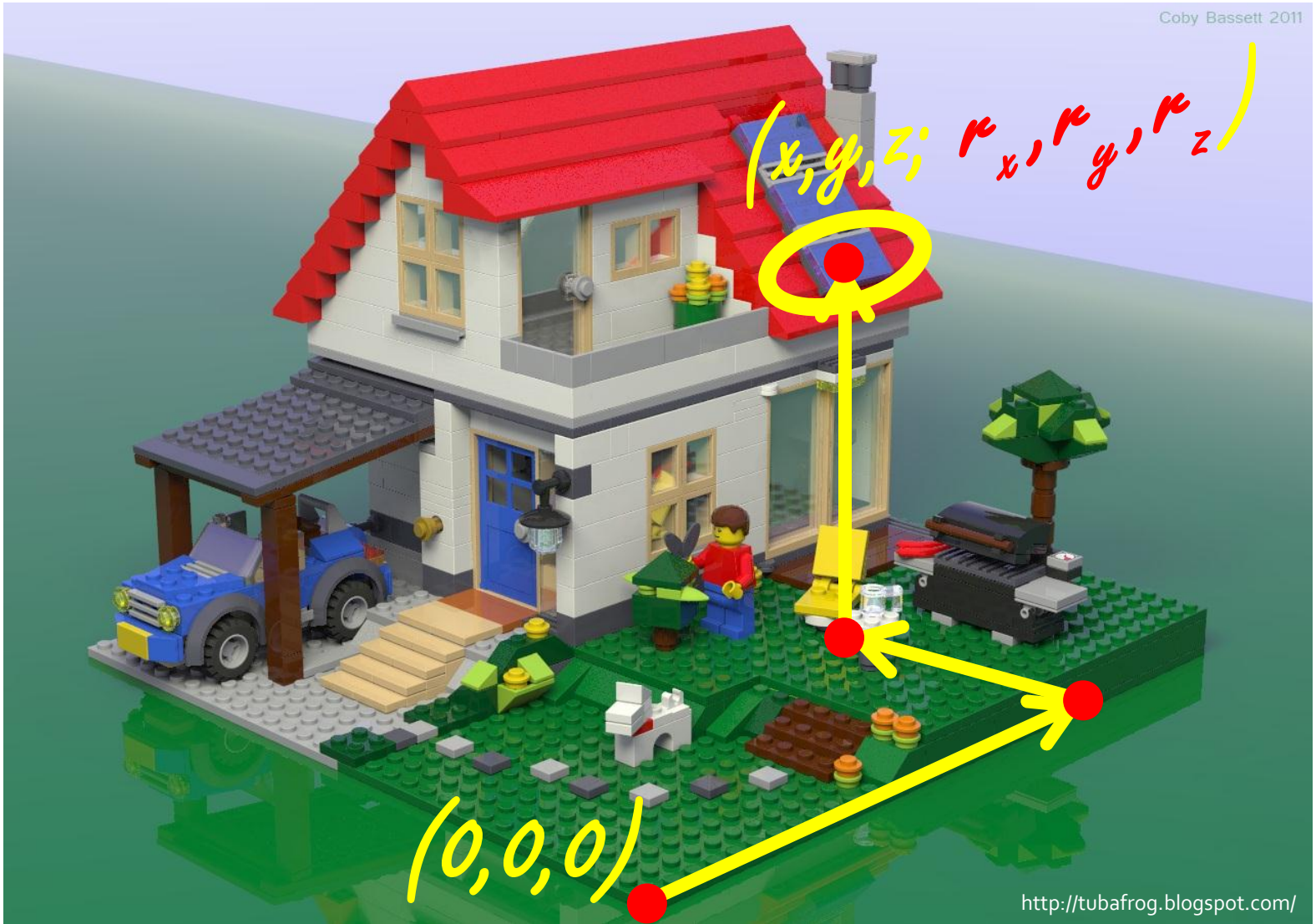
Specifying Complex Scenes

Specifying Complex Scenes

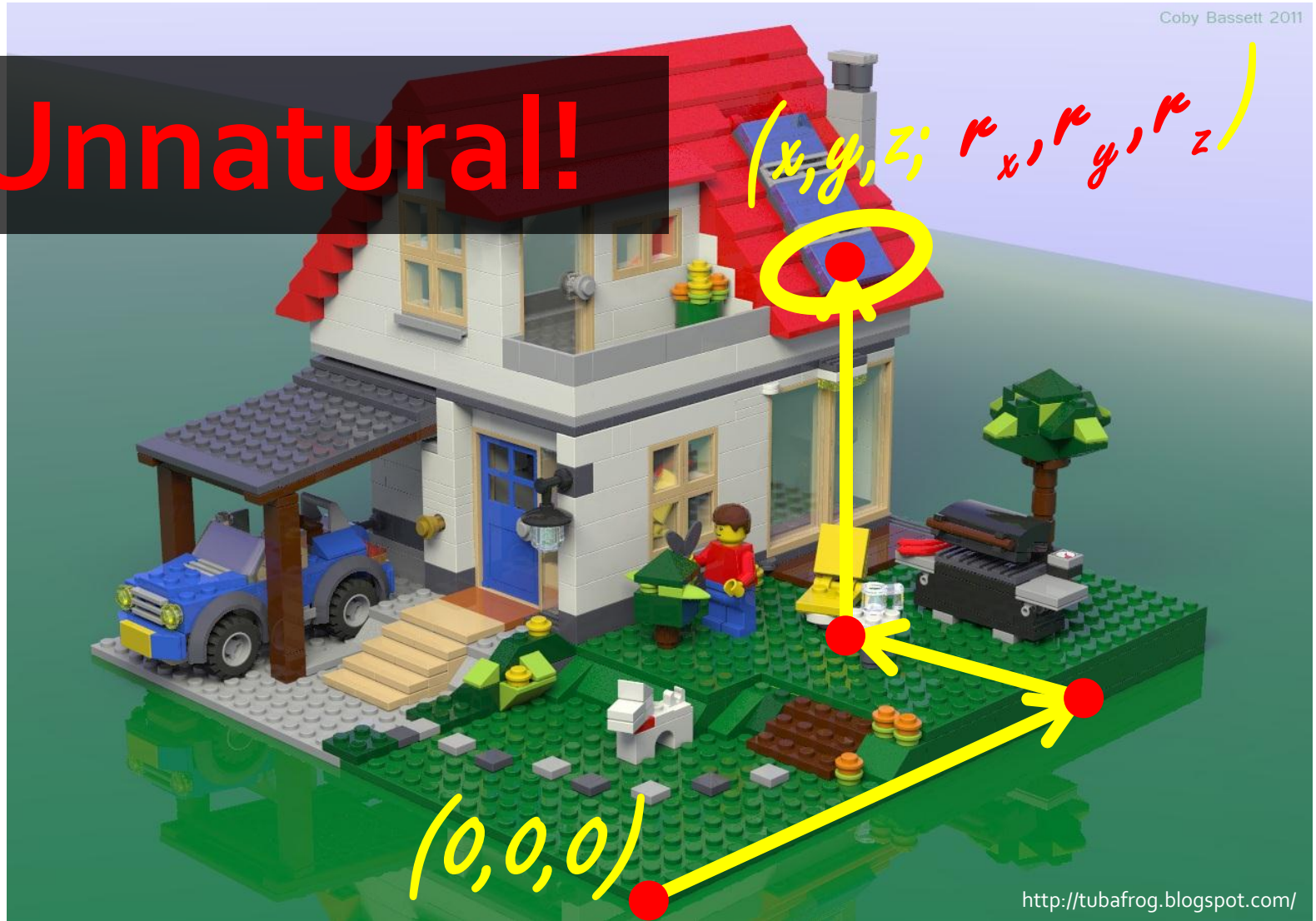
Specifying Complex Scenes



Specifying Complex Scenes



Specifying Complex Scenes



Specifying Complex Scenes

A Philosophical Point

Where is (0,0,0)?

A Philosophical Point

Where is (0,0,0)?

A Philosophical Point

Did it move?

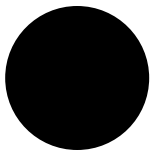
Observation

Rarely do we use "absolute" displacements in real life.

Conclusion

Work with what's convenient and combine later.

Preliminaries



Point.

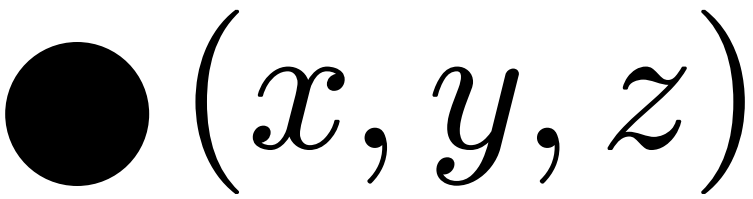
Preliminaries

● Point.

(

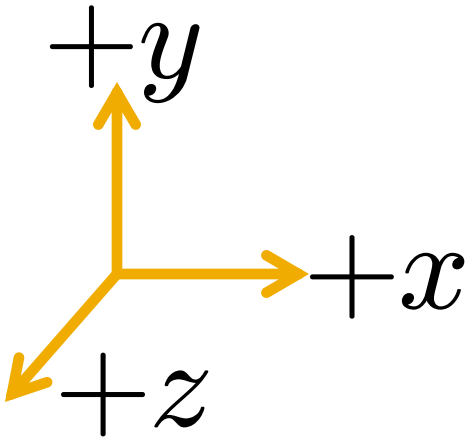
)

Preliminaries



Preliminaries

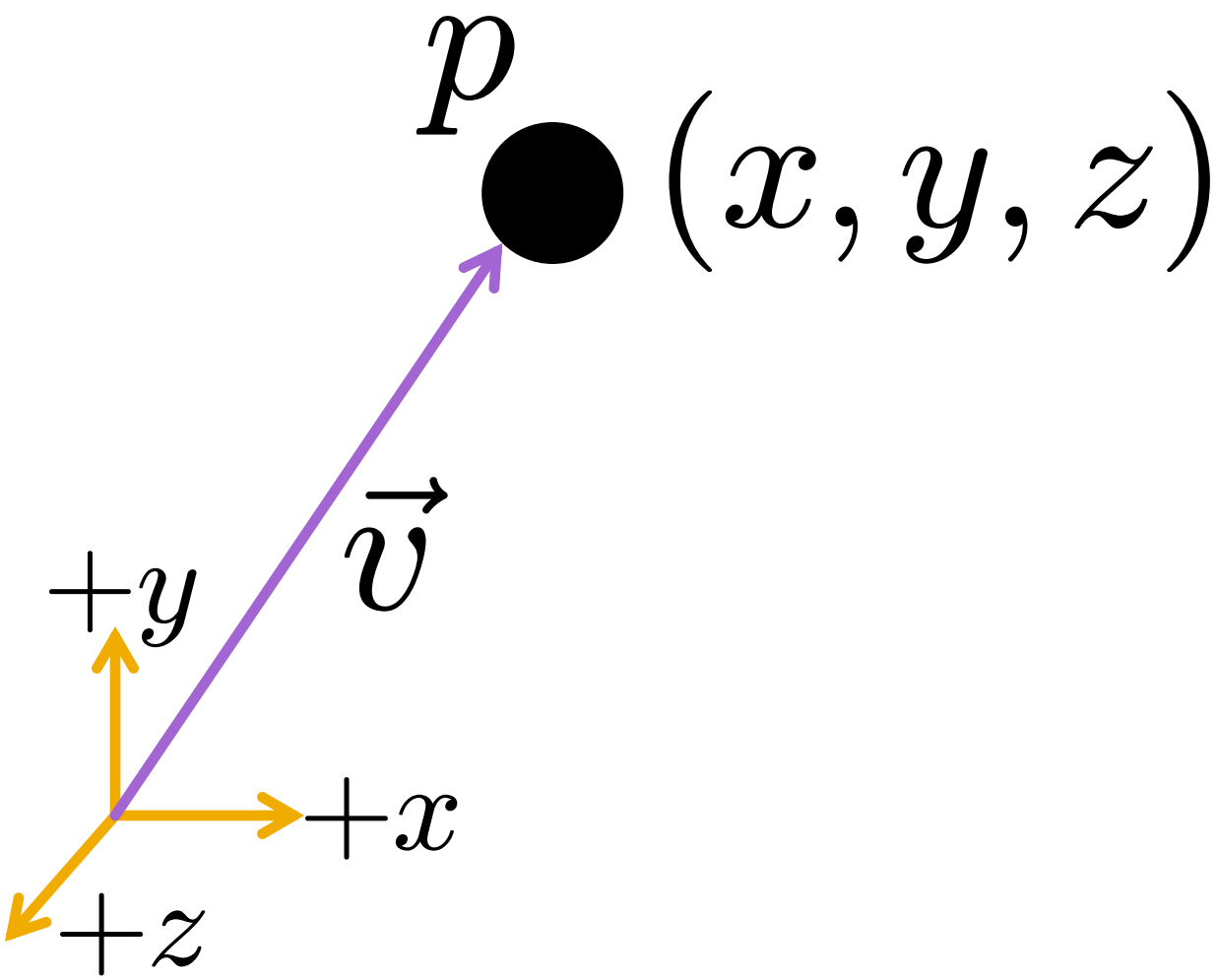
(x, y, z)



Origin.

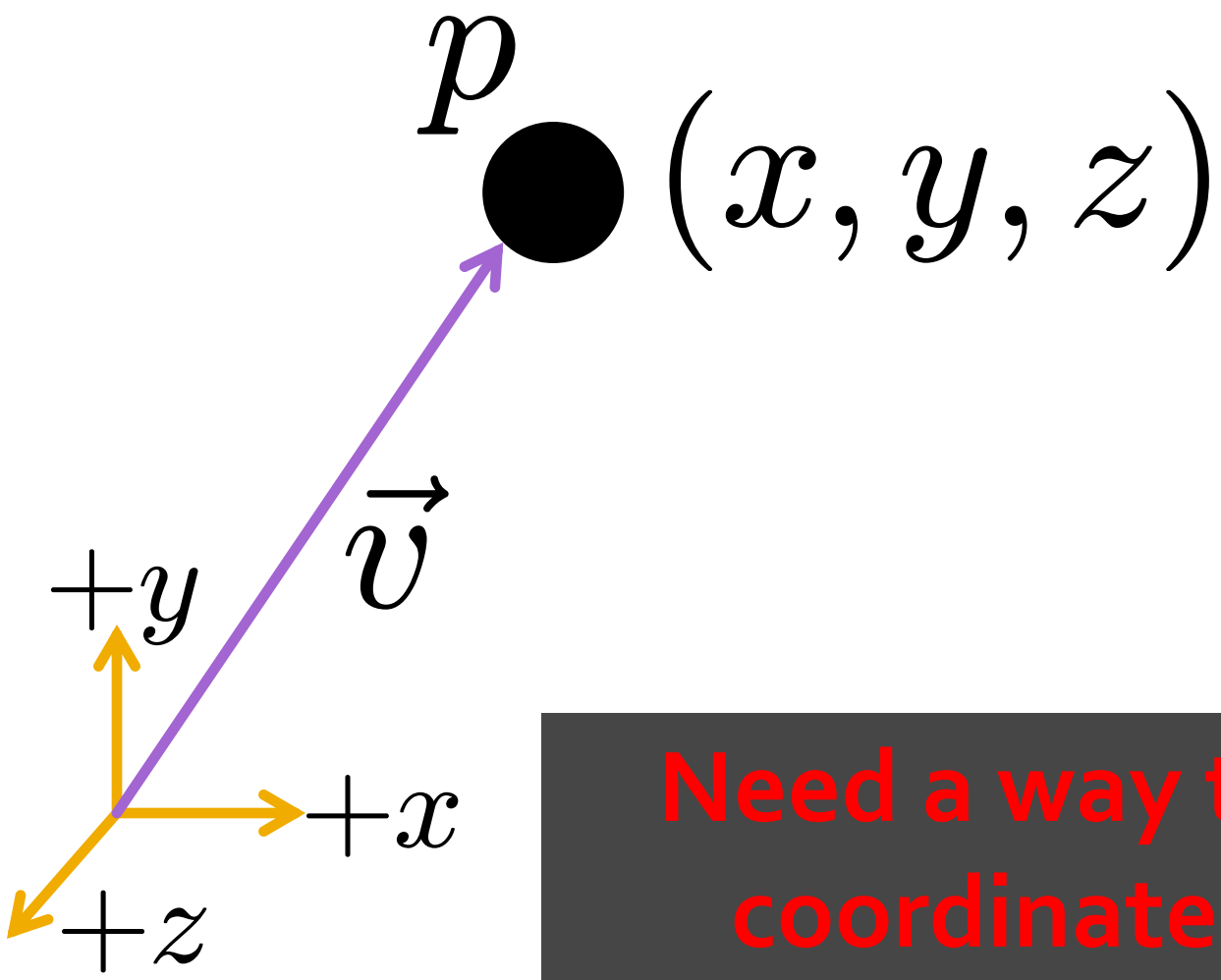
(because I said so)

Preliminaries



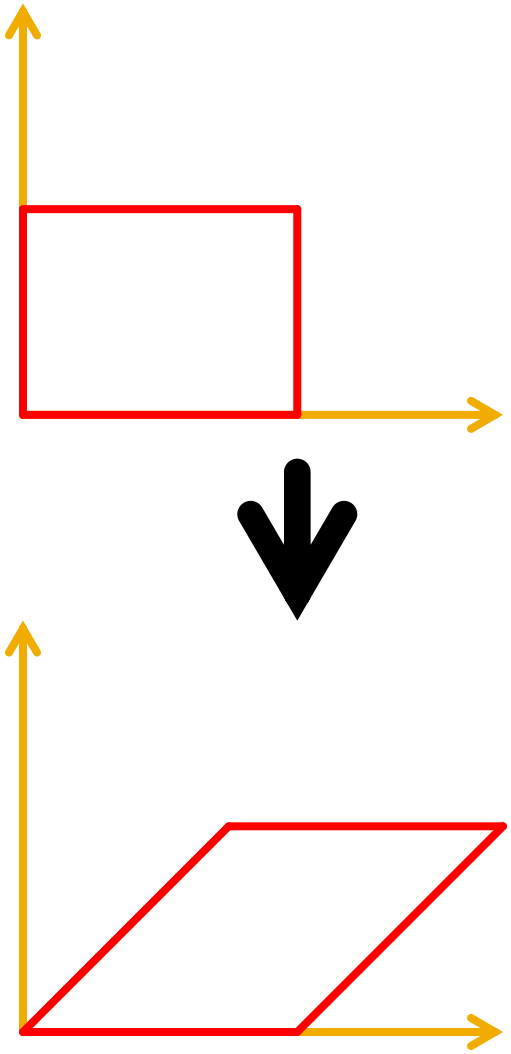
WARNING
IDENTIFYING POINTS AND VECTORS

Preliminaries



Need a way to combine coordinate systems.

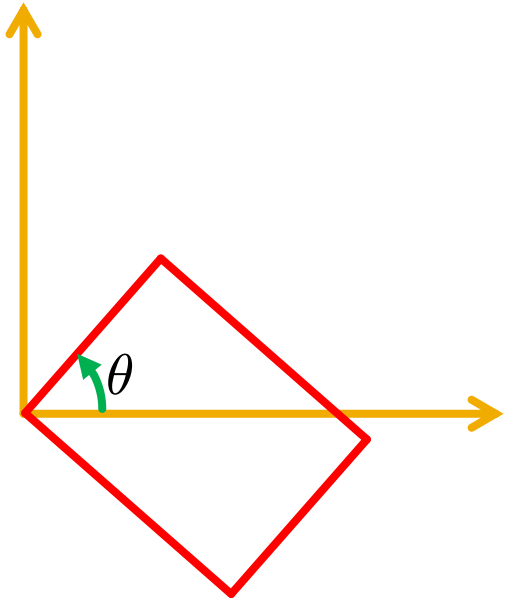
Linear Transformations



$$M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

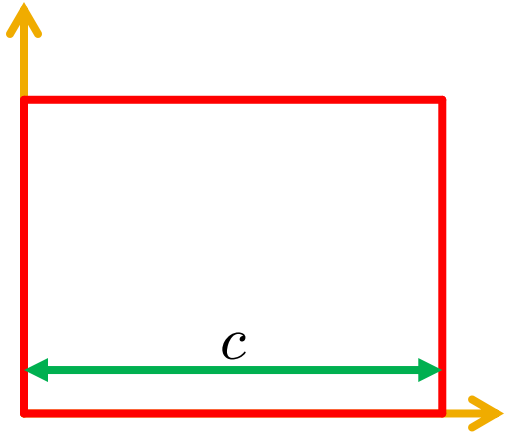
Classes of Linear Transformations

Rotate



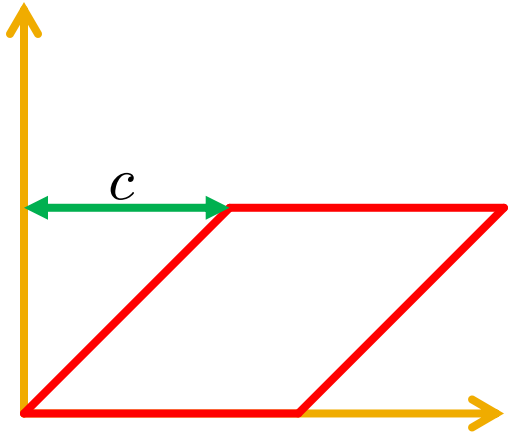
$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Scale



$$\begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix}$$

Shear



$$\begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix}$$

Composing Matrices

"First rotate by 90 degrees, then shear."

$$M_1 = \text{rotate } 90°$$

$$M_2 = \text{shear}$$

Composing Matrices

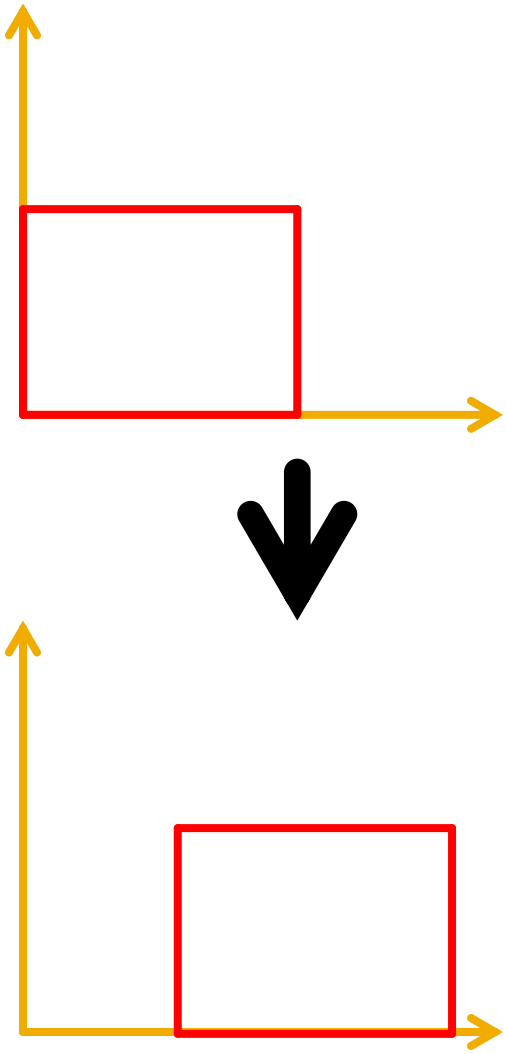
"First rotate by 90 degrees, then shear."

$$M_1 = \text{rotate } 90°$$

$$M_2 = \text{shear}$$

$$\boxed{M_2 M_1}$$

Problem!



???

Homogeneous Coordinates in R^2

All points of the form

$$(x, y; w)$$

where we identify

$$(x, y; w) \equiv (cx, cy; cw)$$

for all nonzero *c*.

Some Exercises

- Convert (x,y) to homogeneous coordinates.

- Convert the following points from homogeneous coordinates:
  - $(1,5;1)$ →
  - $(12,-6;3)$ →
  - $(5,15;-5)$ →

Some Exercises

- Convert (*x*,*y*) to homogeneous coordinates.

$$(x, y; 1)$$

- Convert the following points from homogeneous coordinates:
  - (1,5;1) →
  - (12,-6;3) →
  - (5,15;-5) →

Some Exercises

- Convert (*x*,*y*) to homogeneous coordinates.

$$(x, y; 1) \text{ or } (-7x, -7y; -7) \ldots$$

- Convert the following points from homogeneous coordinates:
  - (1,5;1) →
  - (12,-6;3) →
  - (5,15;-5) →

Some Exercises

- Convert (*x*,*y*) to homogeneous coordinates.

$$(x, y; 1)$$

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Some Exercises

- Convert (*x*,*y*) to homogeneous coordinates.

$$(x, y; 1)$$

- Convert the following points from homogeneous coordinates:
  - (1,5;1) → (1,5)
  - (12,-6;3) → (4,-2)
  - (5,15;-5) →

Some Exercises

- Convert (*x*,*y*) to homogeneous coordinates.

$$(x, y; 1)$$

- Convert the following points from homogeneous coordinates:
  - (1,5;1) → (1,5)
  - (12,-6;3) → (4,-2)
  - (5,15;-5) → (-1,-3)

Conversion Formula

$$(x, y; w)$$

Conversion Formula

$$(x, y; w) \equiv \left(\frac{x}{w}, \frac{y}{w}; 1\right)$$

Conversion Formula

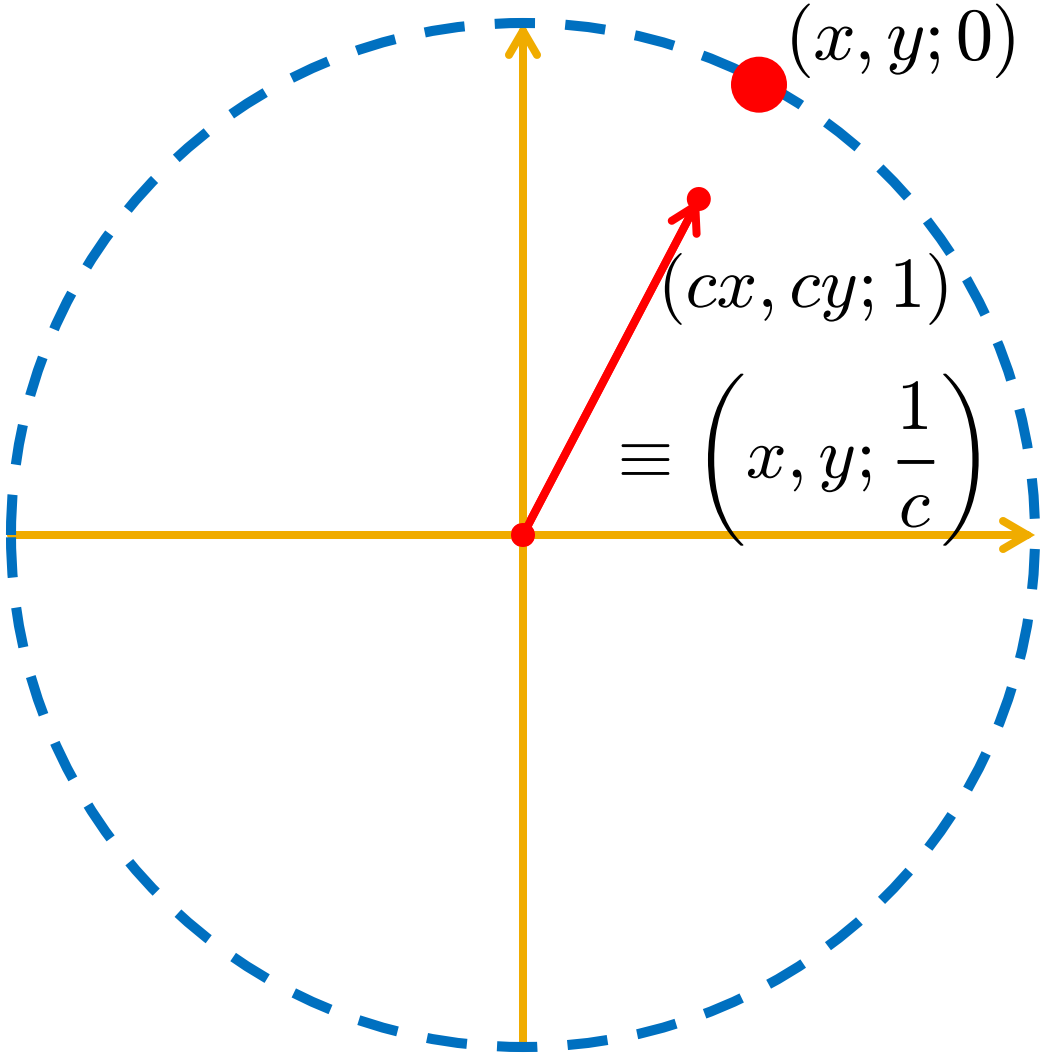
$$(x, y; w) \equiv \left(\frac{x}{w}, \frac{y}{w}; 1\right) \rightarrow \left(\frac{x}{w}, \frac{y}{w}\right)$$

Conversion Formula

$$(x, y; w) \equiv \left(\frac{x}{w}, \frac{y}{w}; 1\right) \rightarrow \left(\frac{x}{w}, \frac{y}{w}\right)$$

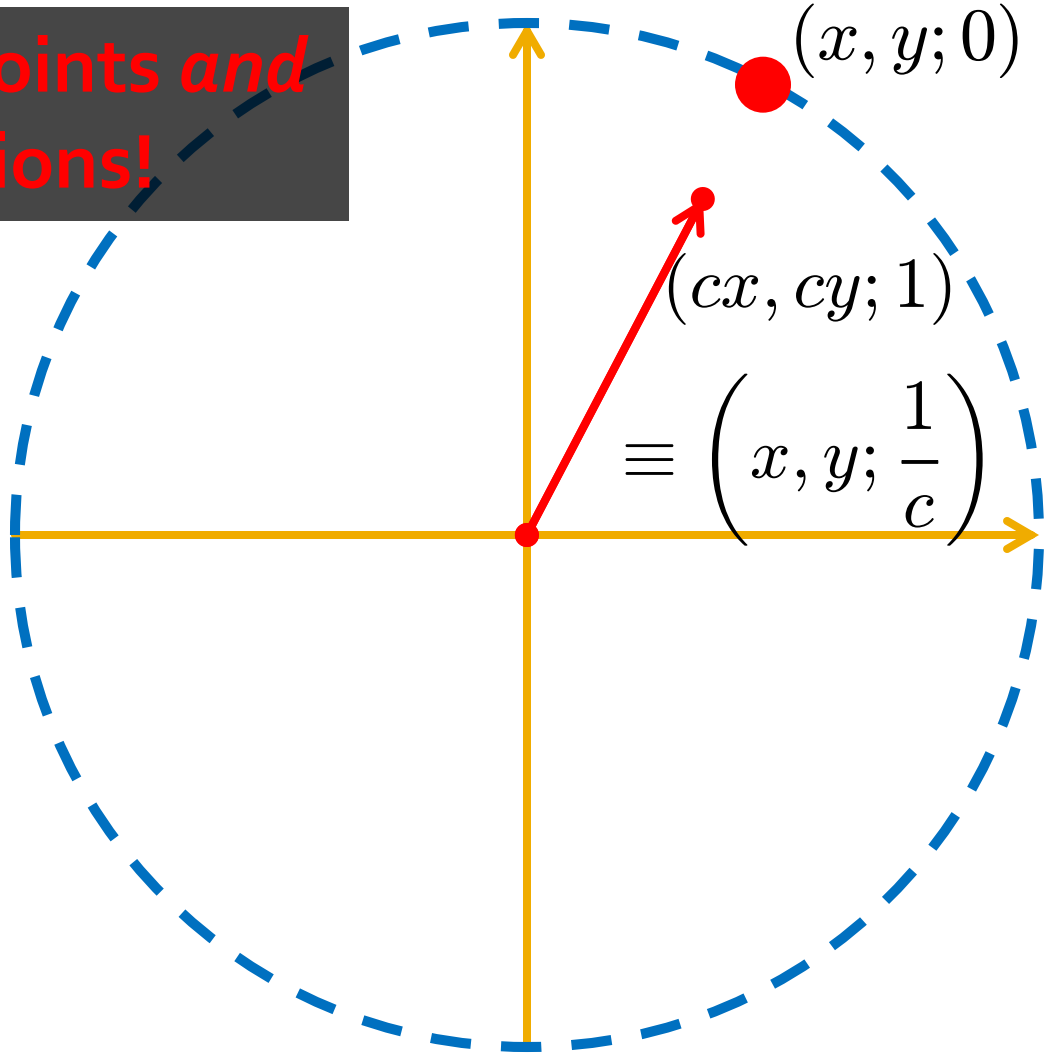
What if w is zero?

Points at Infinity



Points at Infinity

Encodes points *and* directions!



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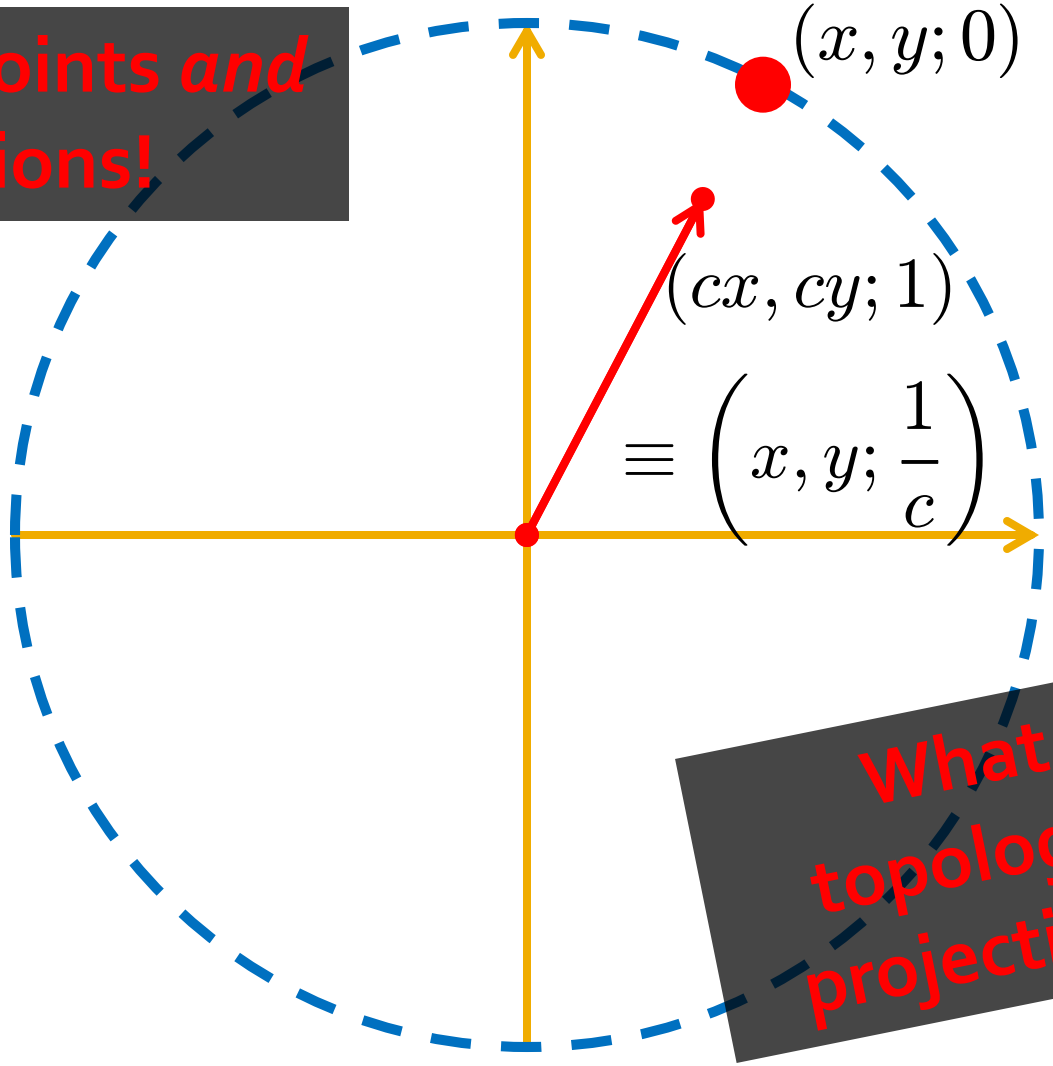


What is the topology of the projective plane?

Projective Transformations

$$\begin{pmatrix} 1 & 0 & u \\ 0 & 1 & v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} u+x \\ v+y \\ 1 \end{pmatrix}$$

Includes translation!

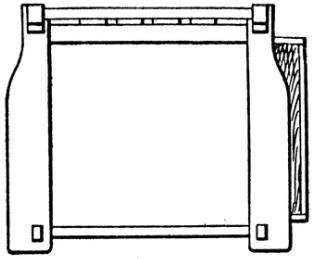
Challenge

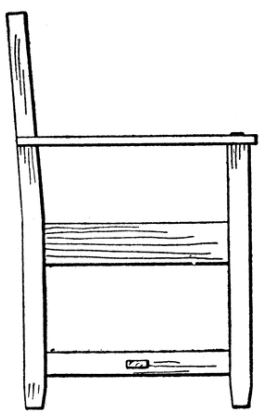
What do eigenvectors of projective transformation matrices represent?

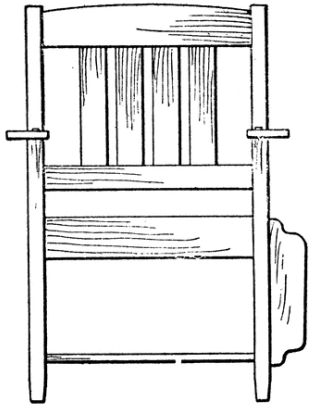
The Perspective Problem

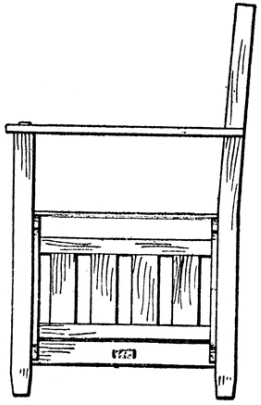
The world is 3D.
Your screen is flat.

Lots of Options



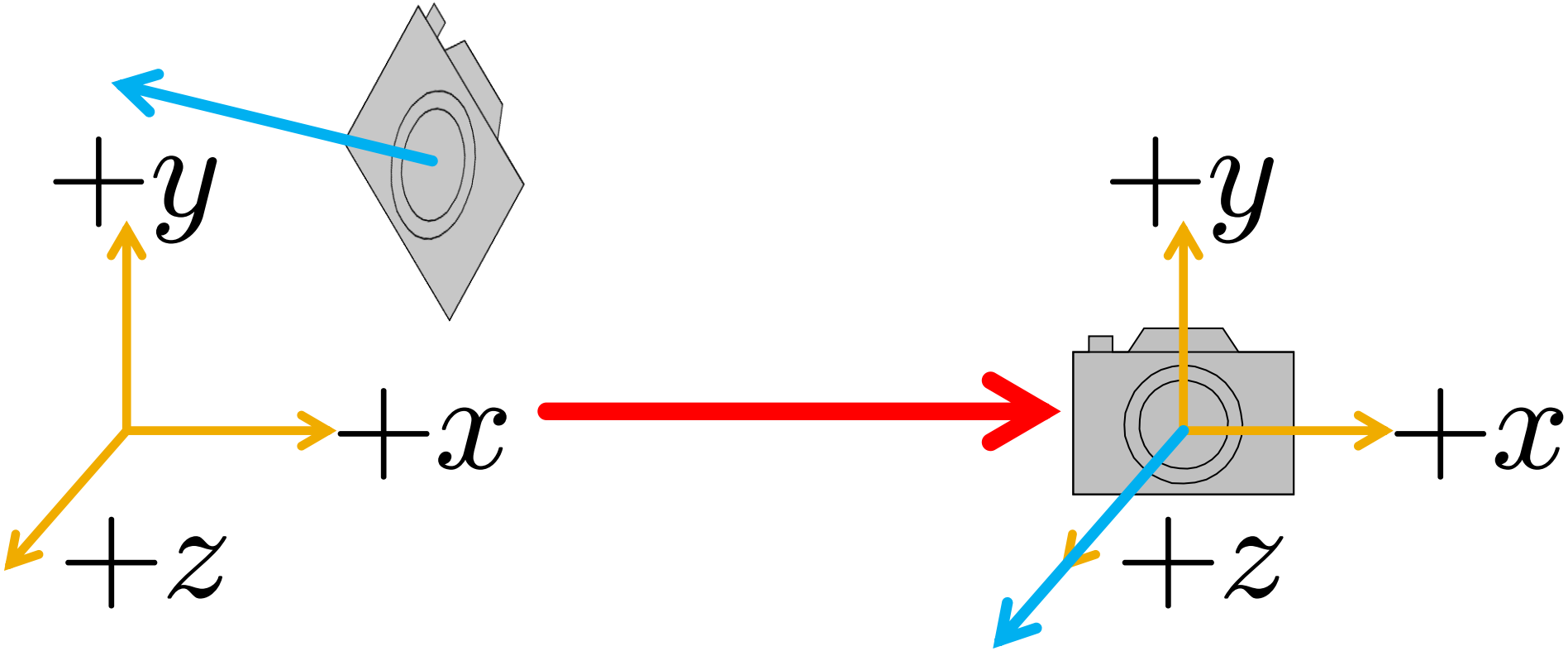






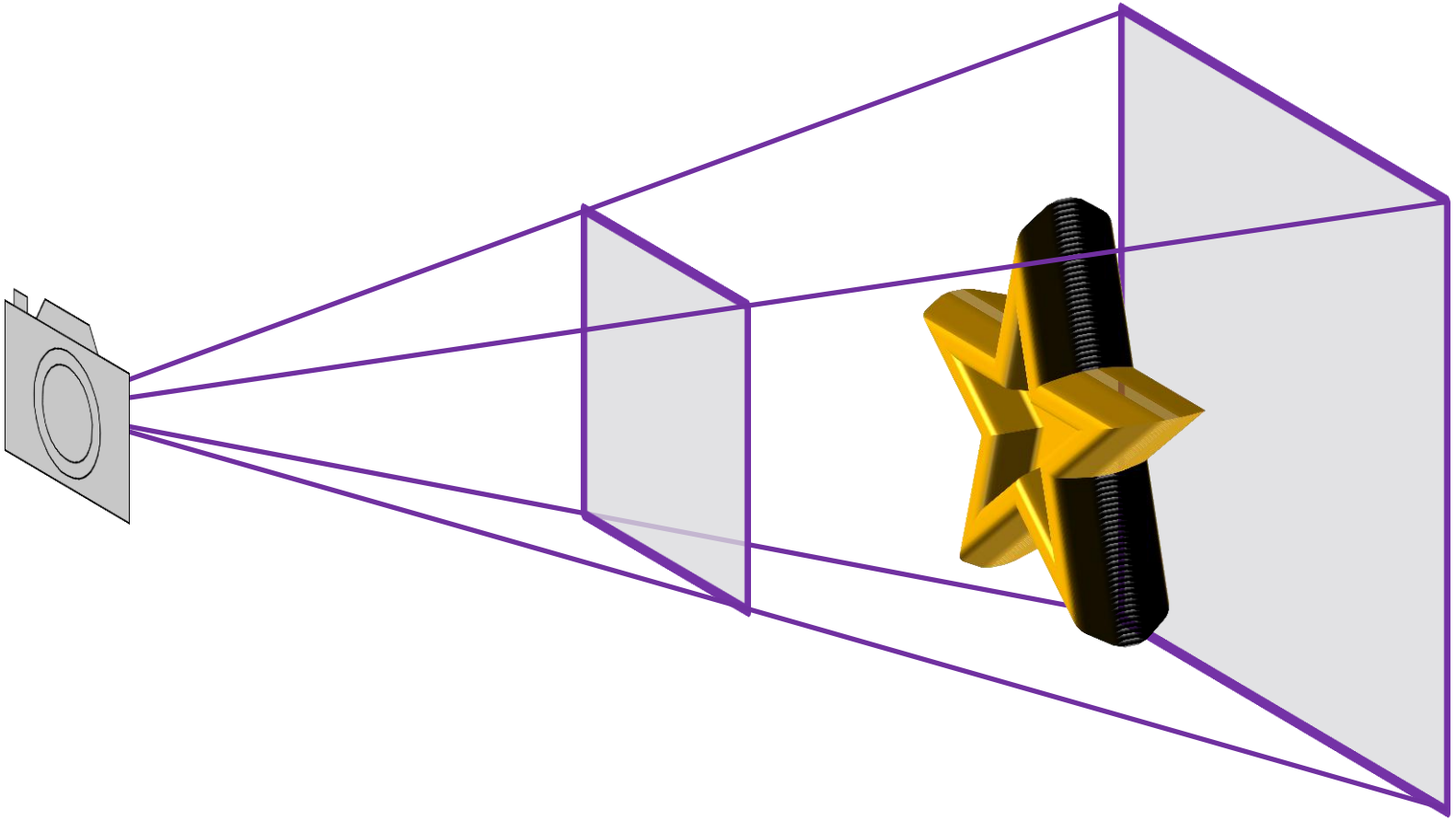
http://www.nikonweb.com/fisheye/
http://etc.usf.edu/clipart/52100/52103/52103_chair_0-p.htm
http://en.wikipedia.org/wiki/File:One_point_perspective.jpg

Camera Transformation Stack



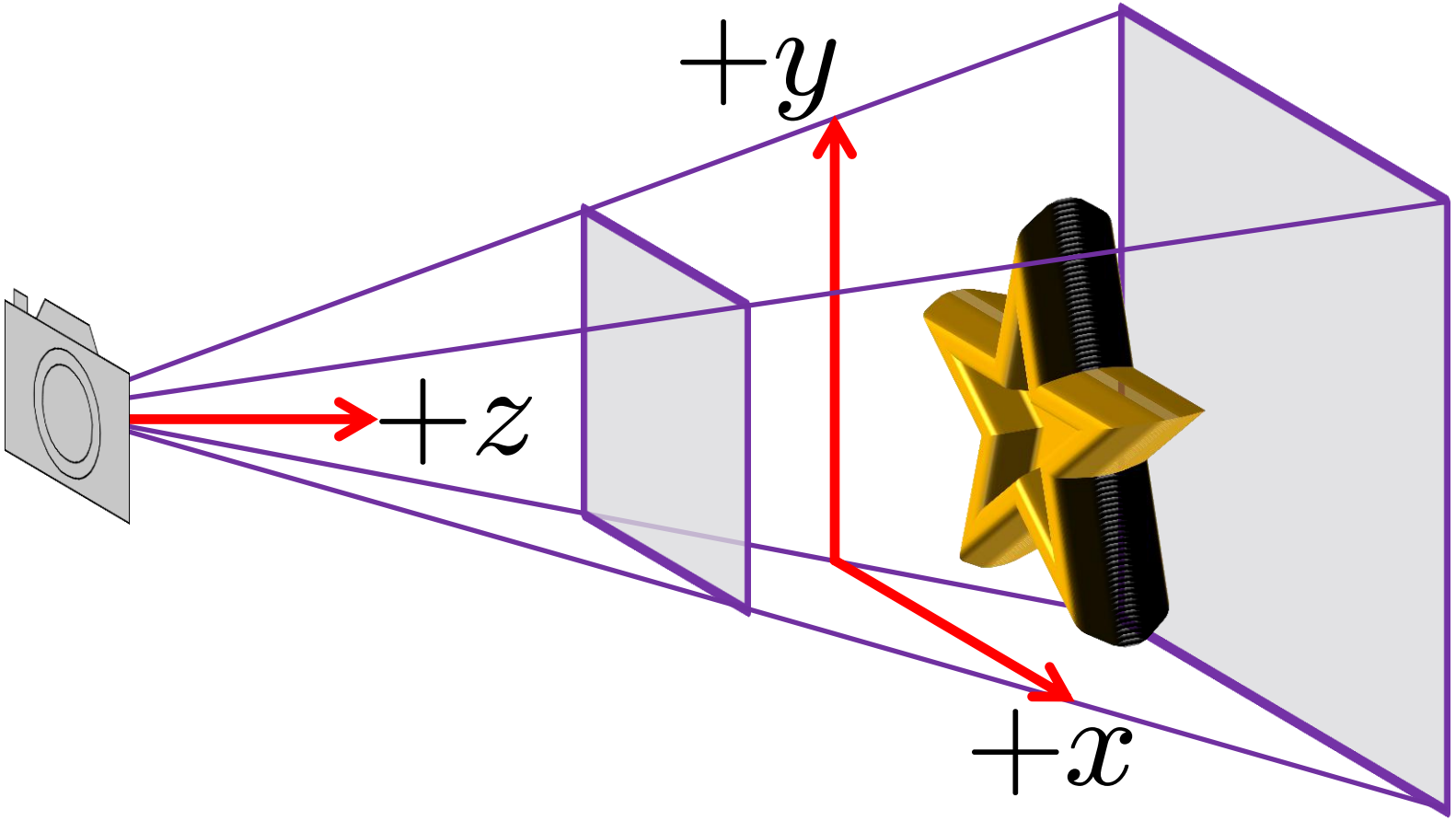
1. Camera (eye) transformation

Camera Transformation Stack



2. Projection transformation

Camera Transformation Stack



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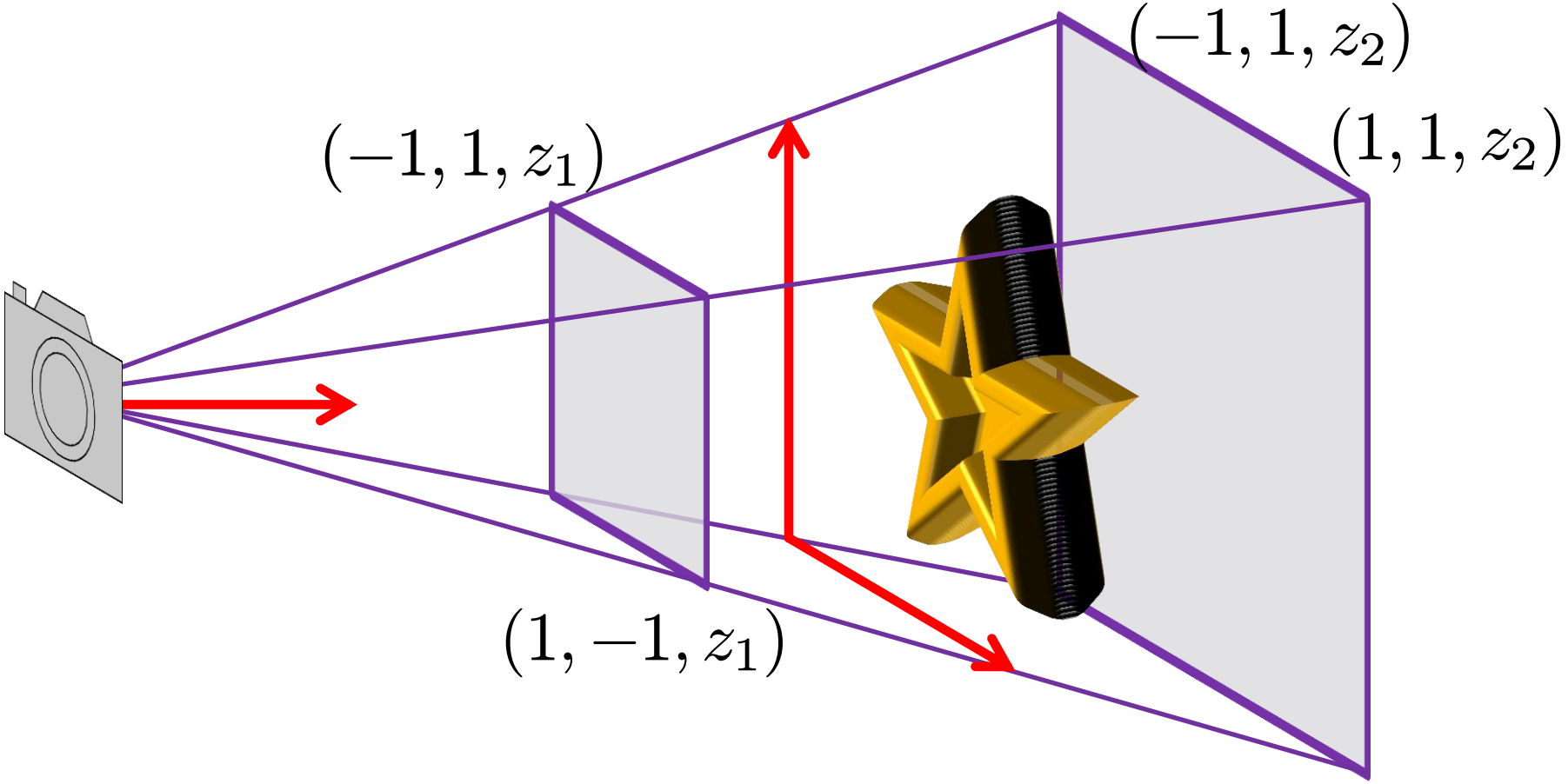

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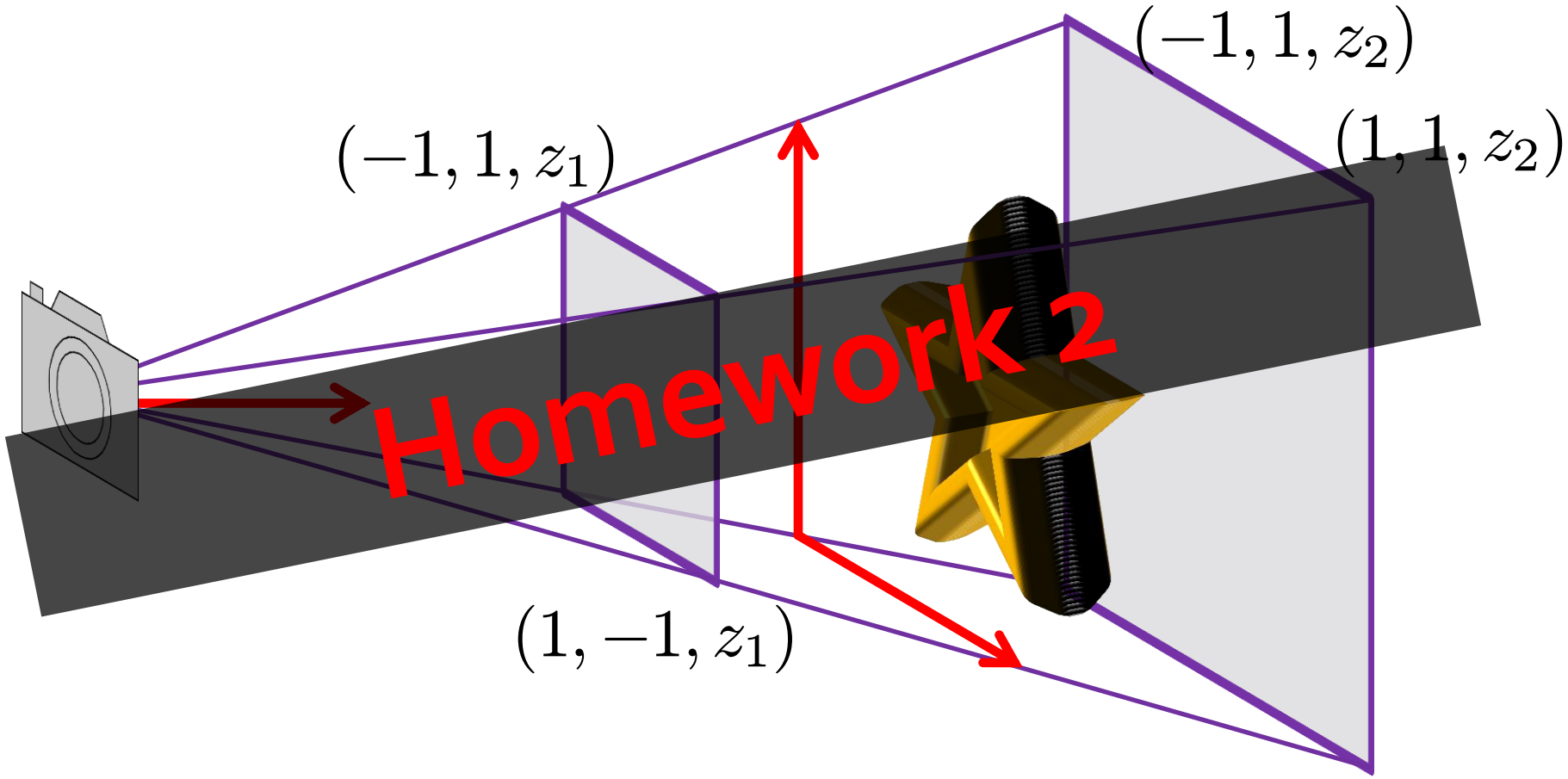

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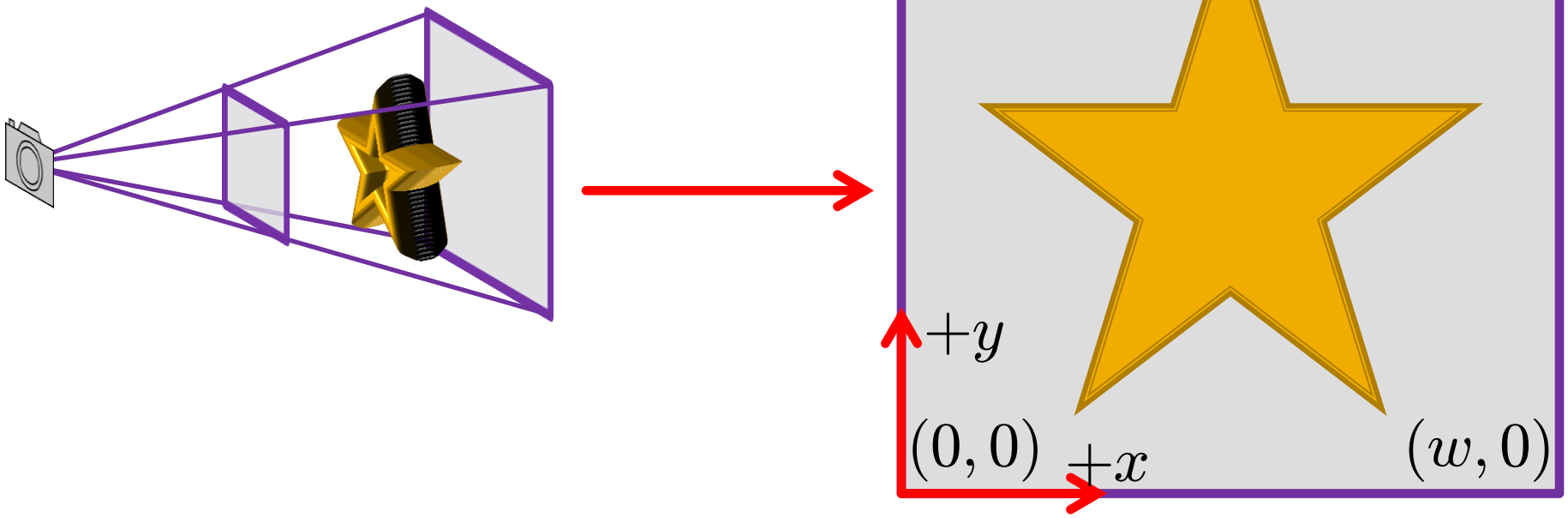
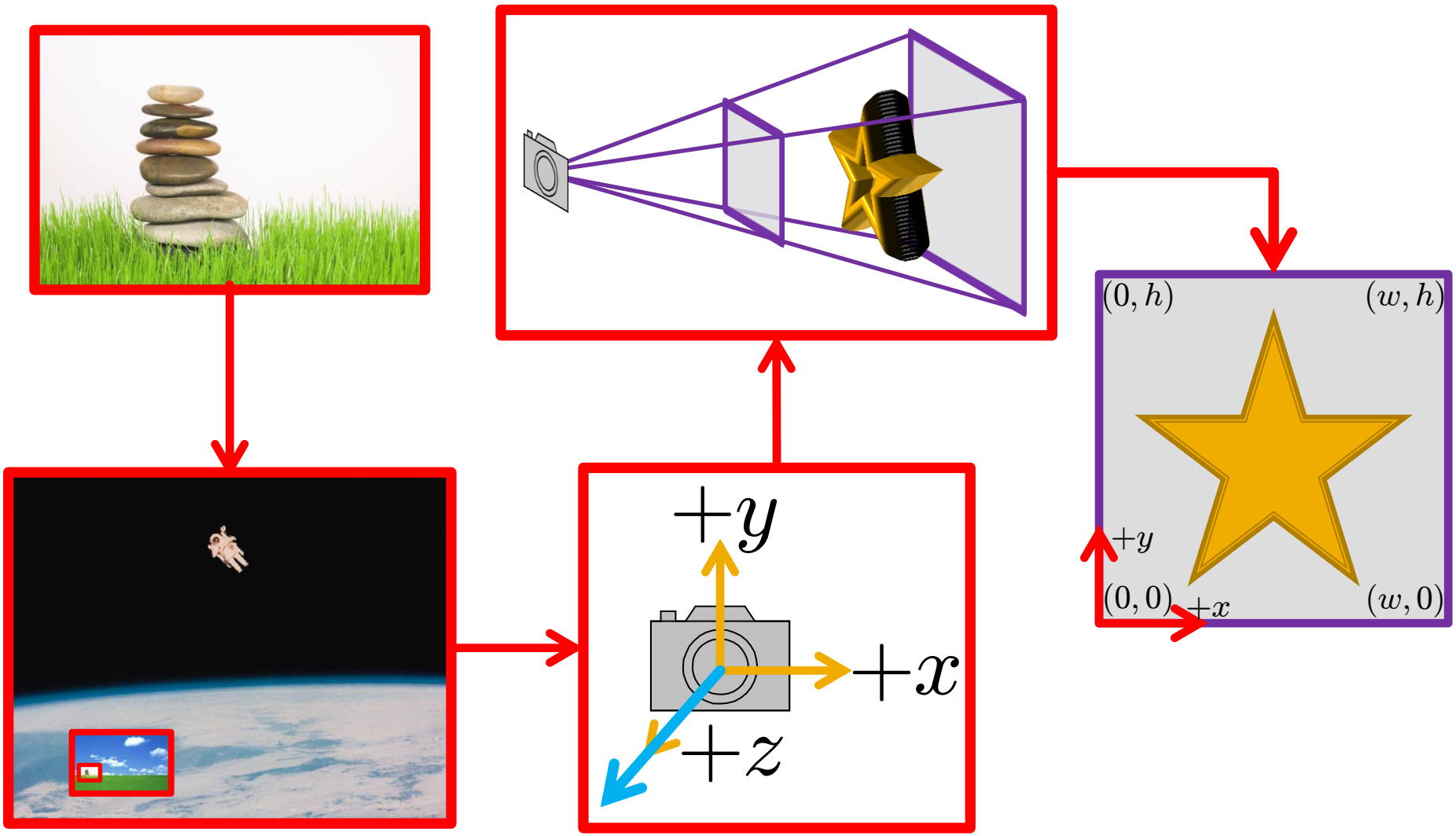
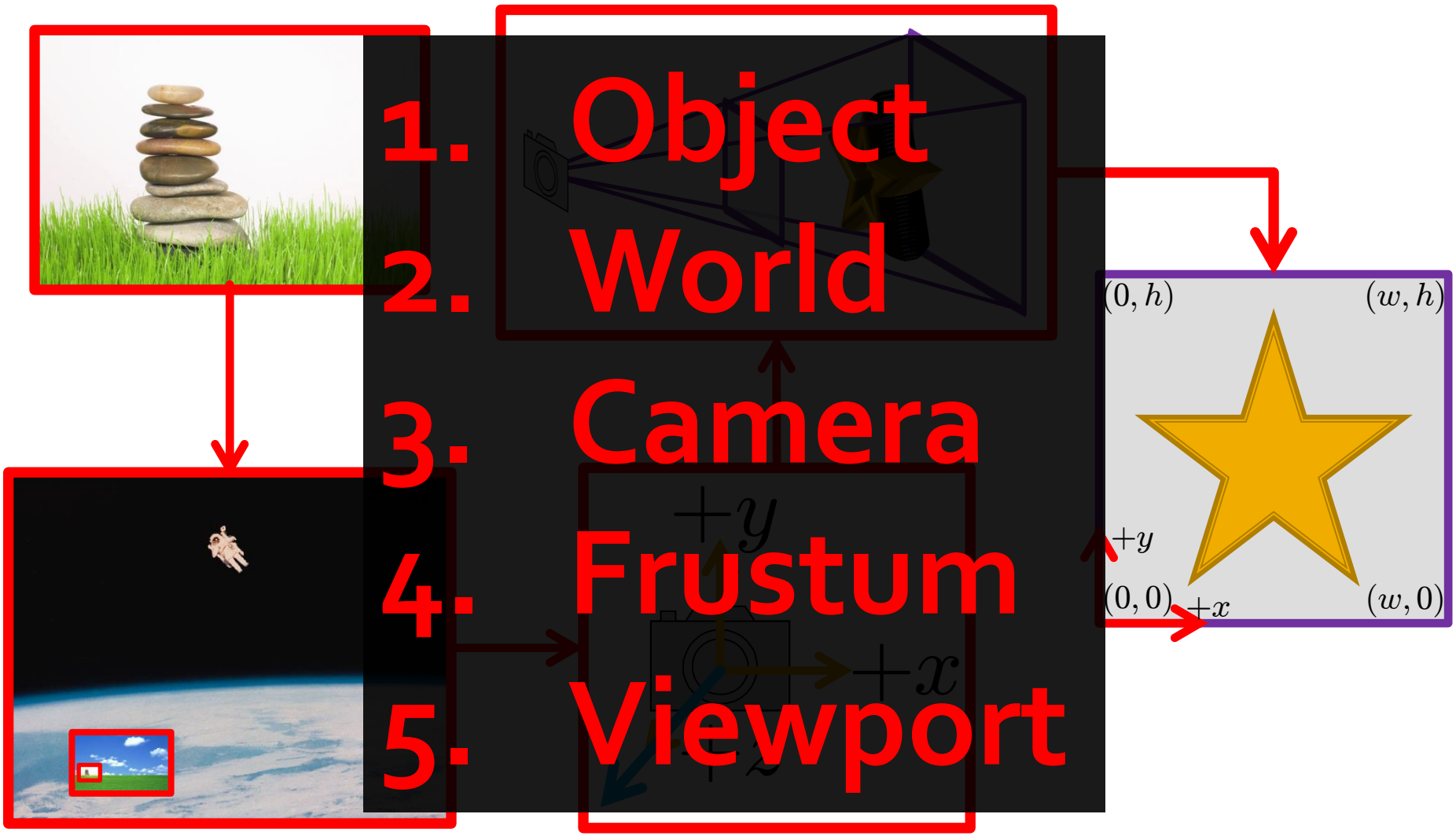


3. Viewport transformation

Summary of Transformations



Summary of Transformations



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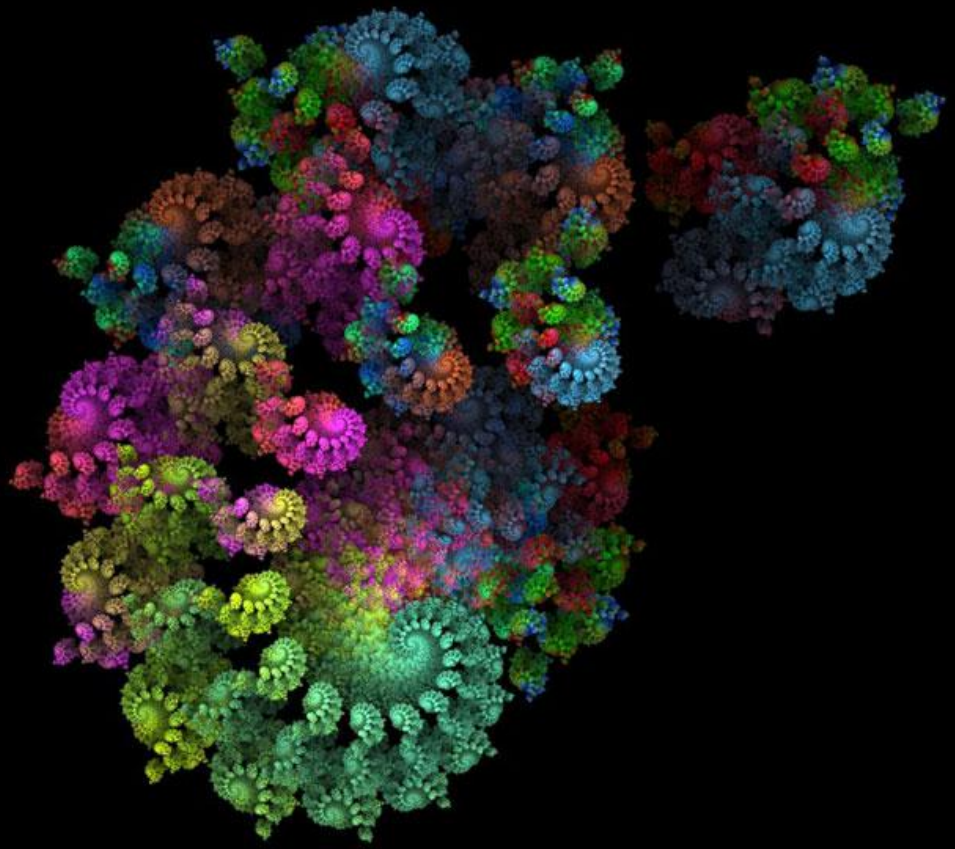

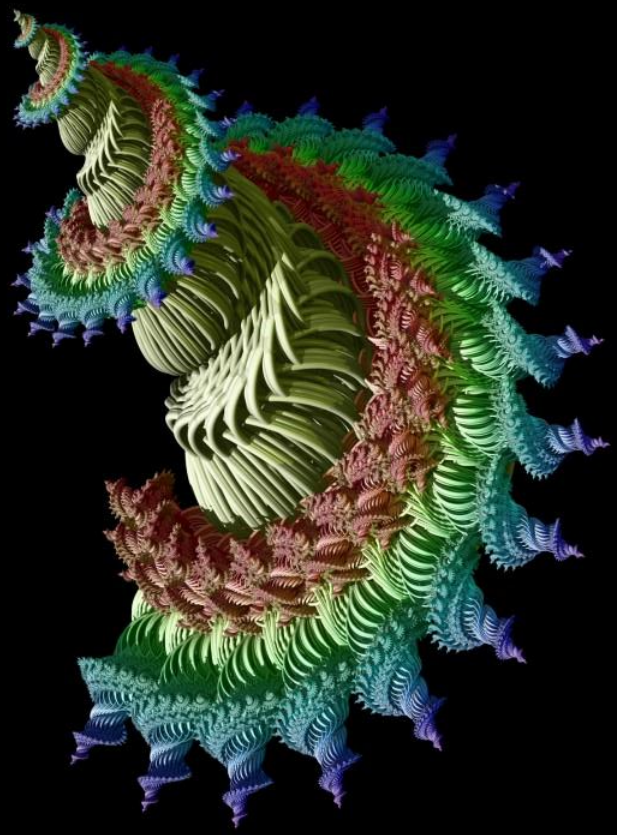

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