CS164: Surface Reconstruction From Density Data

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Overview

- Surface Representations
  - Explicit Surfaces
  - Implicit Surfaces
- Marching Cubes
  - Hermite Data/Extended Marching Cubes
  - Dual Contouring
  - Topological Guarantees

Surface Representations

Measurements,
Simulation

Modeling

Marching Cubes

Volume Samples

Explicit Surface

Signed Distance Transform

Easy

Sample/Evaluate

Interpolate

Today

Wednesday

Triangulate

Sampling, e.g. LDC

Raytracing

Implicit Surface

Surface Samples

Define, e.g. MLS surface

Analytic Description,
Simulation

Measurements

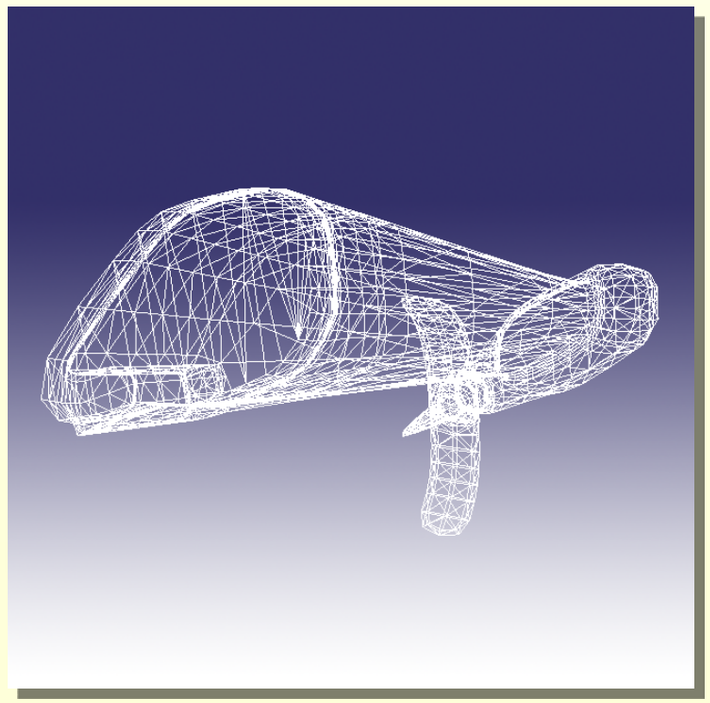
Explicit (Parametric) Surfaces

- "The surface consists of these points: ..."

$$\{f(\mathbf{u})|\mathbf{u} \in \mathbb{R}^2\}$$

- Splines (treated earlier)
- Piecewise-linear surfaces (polygonal meshes)

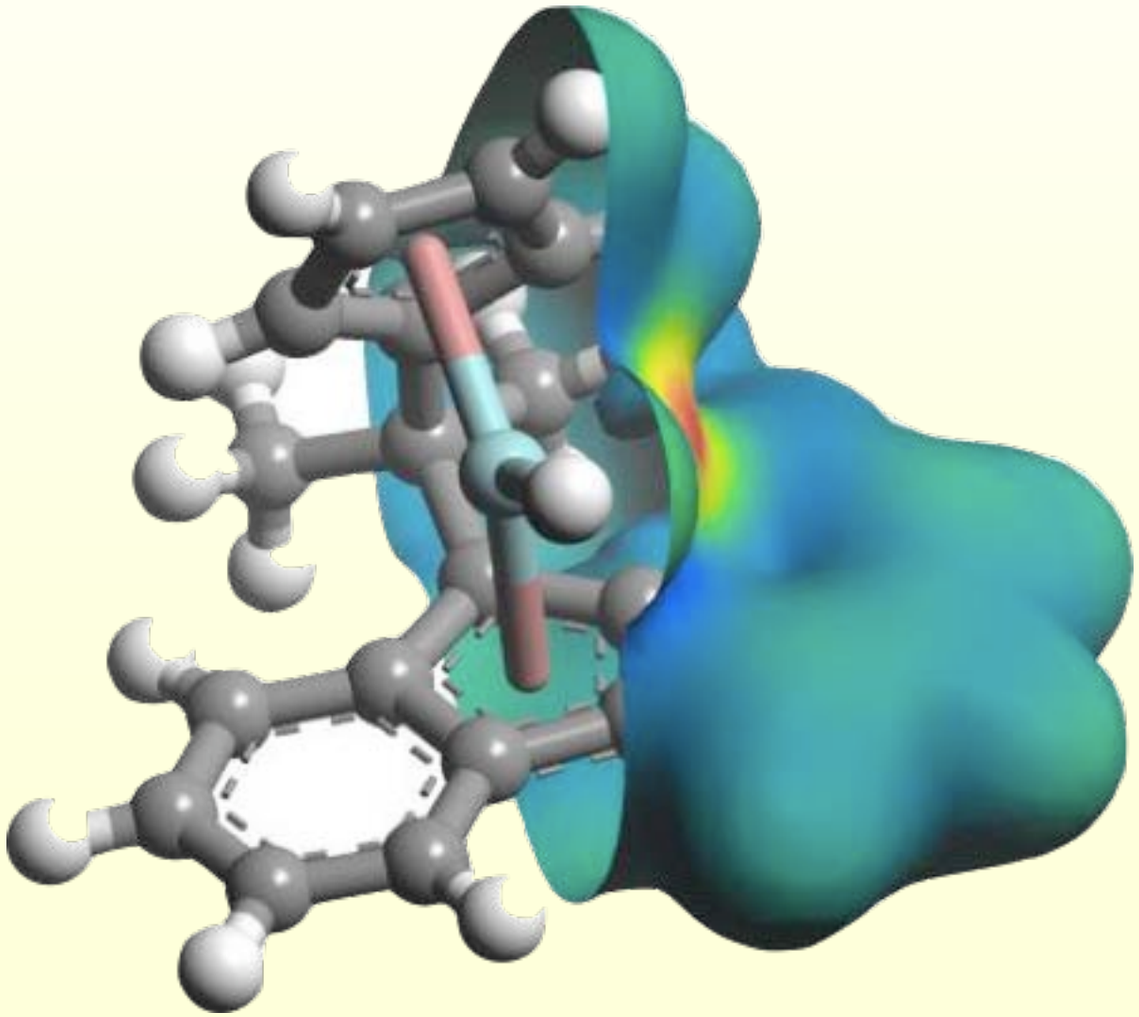
- Most common: triangle meshes



Implicit Surfaces

- “The surface consists of all points, which...”

$$\{\mathbf{x}|f(\mathbf{x})=0\}$$



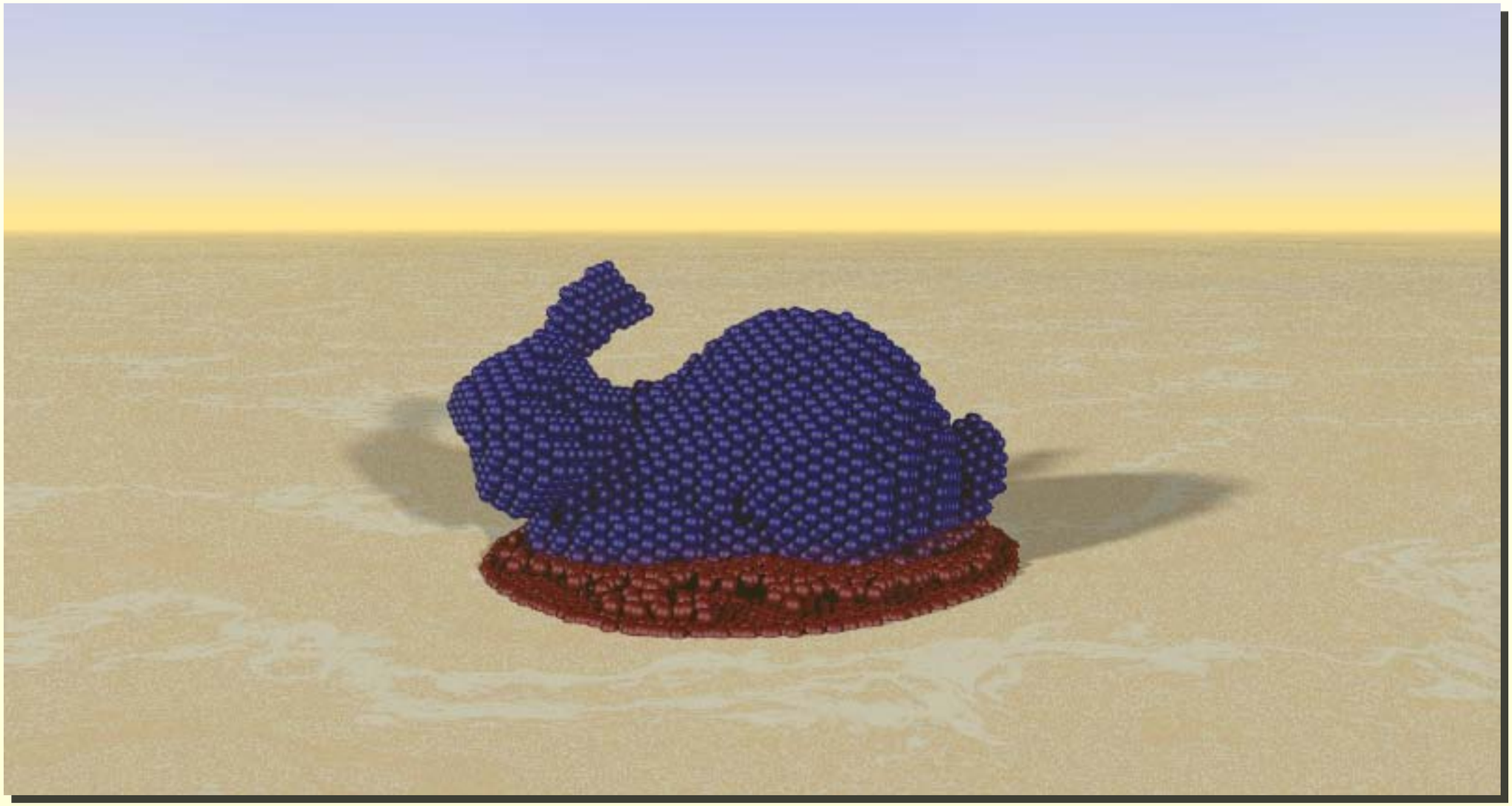
Isosurface around Zirconocene molecule [Accelrys]

Implicit vs. Explicit

- Different sources
- Explicit: $\{f(\mathbf{u})|\mathbf{u} \in \mathbb{R}^2\}$
  - Image of a function from a parameter domain
  - Easy to name and enumerate points
  - Hard to check whether a given point is on the surface
- Implicit: $\{\mathbf{x}|f(\mathbf{x}) = 0\}$
  - Kernel of a function
  - Hard to name and enumerate points
  - Easy to check whether a given point is on the surface

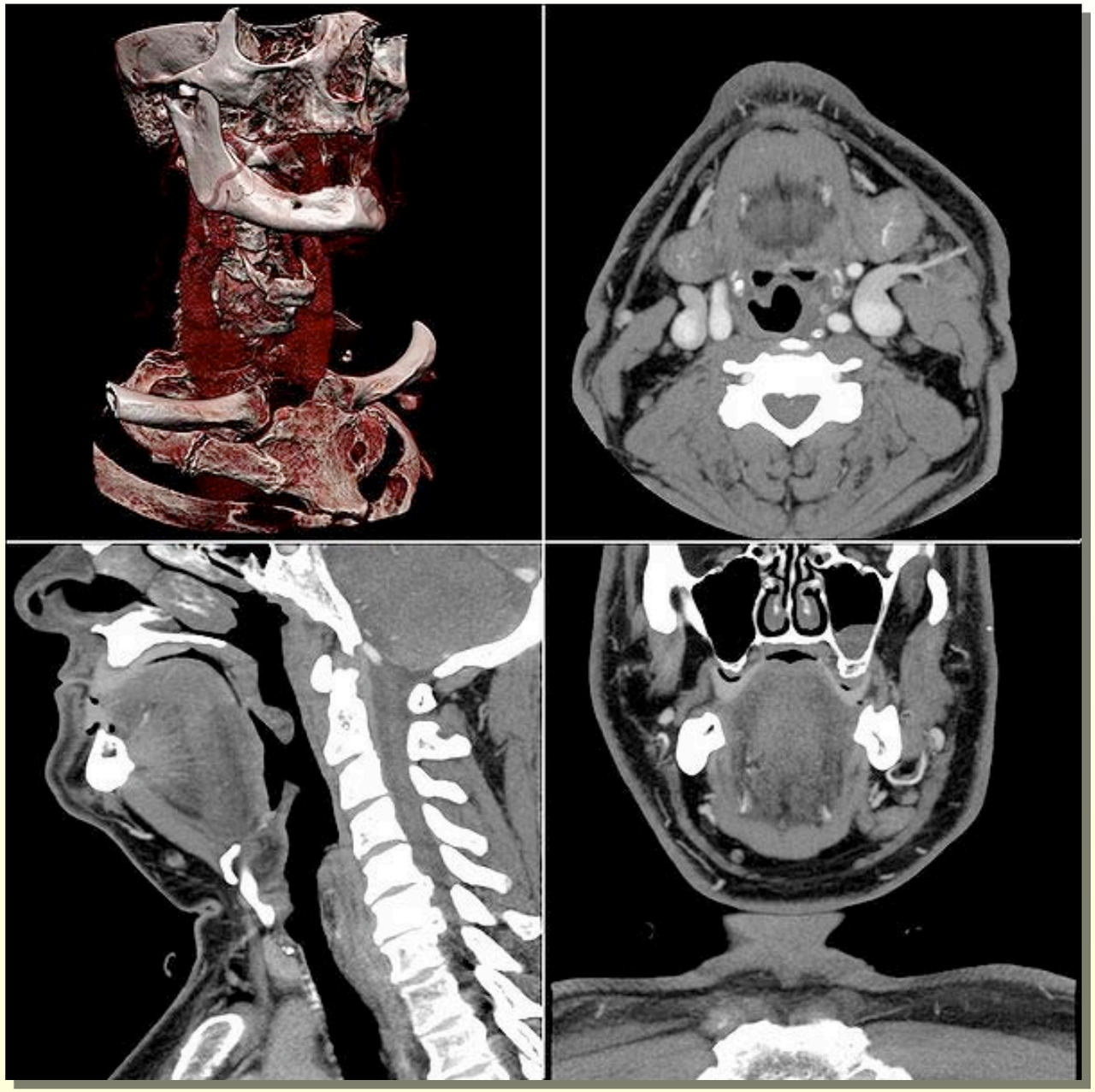
Iso-Surface of a Discrete Density Field

- Example: $f(\mathbf{x}) = \sum_i w(\mathbf{x}, \mathbf{x}_i) - \rho_0$

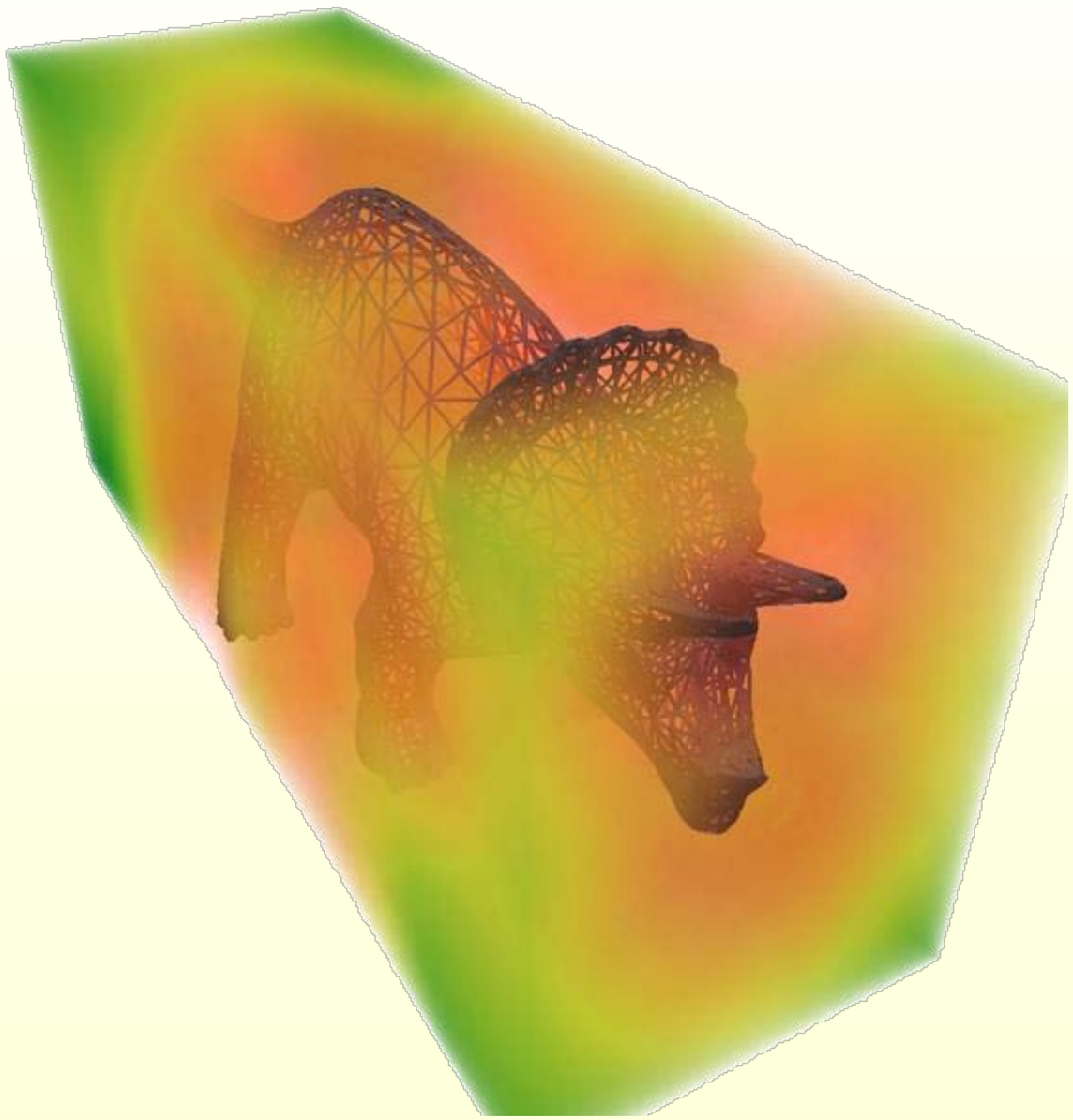


Iso-Surface in a CAT Scan

- $f(\mathbf{x}) = I(\mathbf{x}) - q$
- Samples in a regular grid
- Trilinear within voxels
- How can we make an isosurface explicit?



Distance Fields



- The implicit function is the distance to the surface
  - chosen with a positive sign outside and a negative sign inside
- Allow us to perform easily Constructive Solid Geometry (CSG) operations
- Multiscale (oct-tree based) representations are common

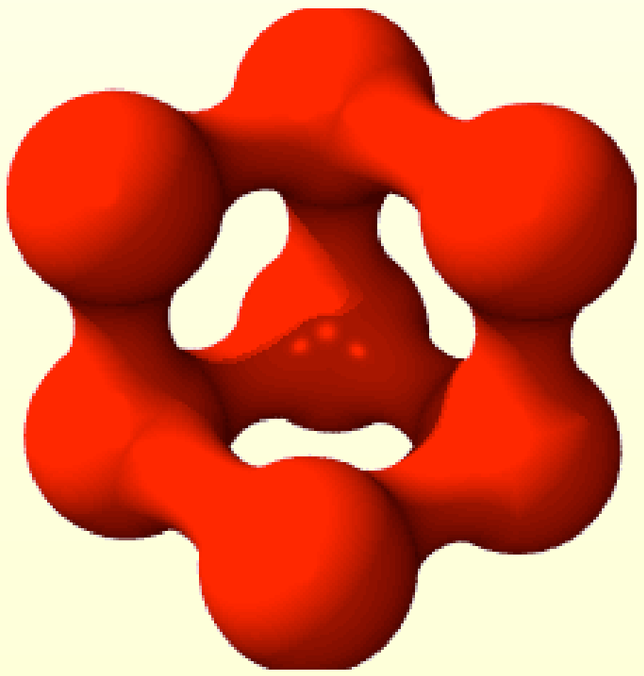
Marching Cubes
(and variants)

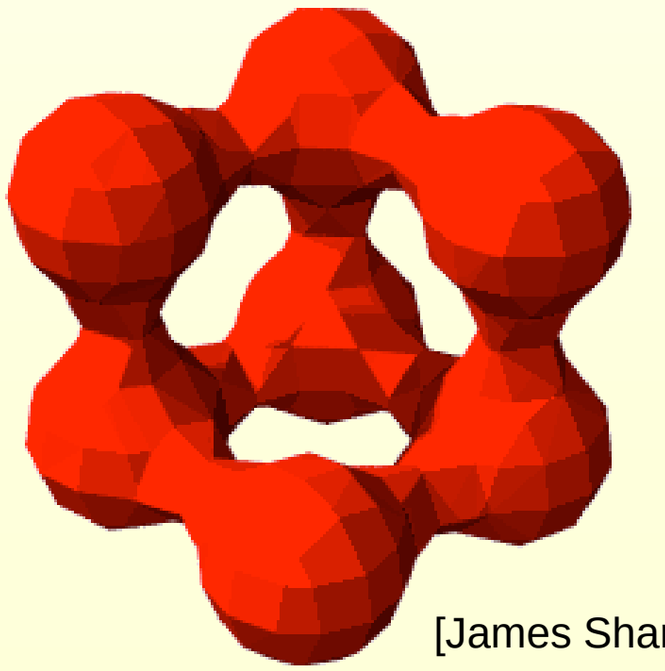
Problem Statement

Given a function $I(\mathbf{x})$ defining an implicit surface

$$\mathcal{S} = \{\mathbf{x} | f(\mathbf{x}) = I(\mathbf{x}) - q = 0\},$$

create a triangle mesh that approximates the surface S.

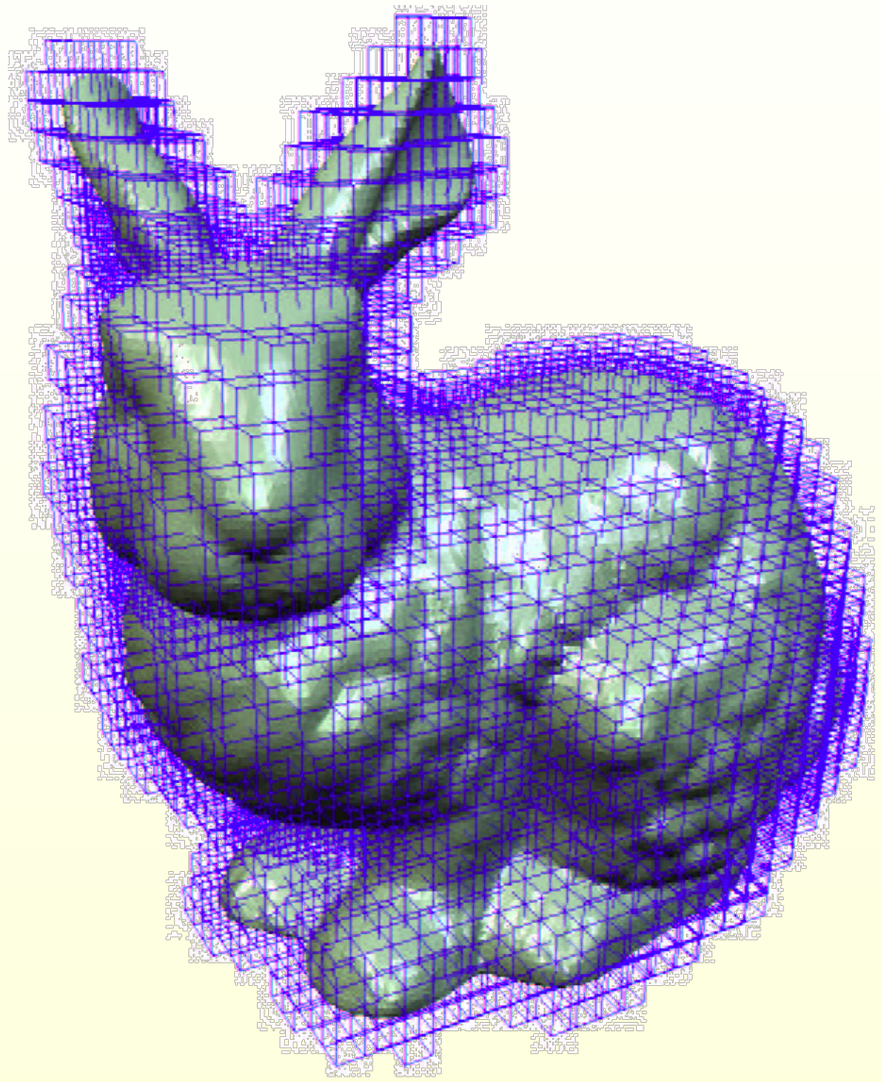




[James Sharman]

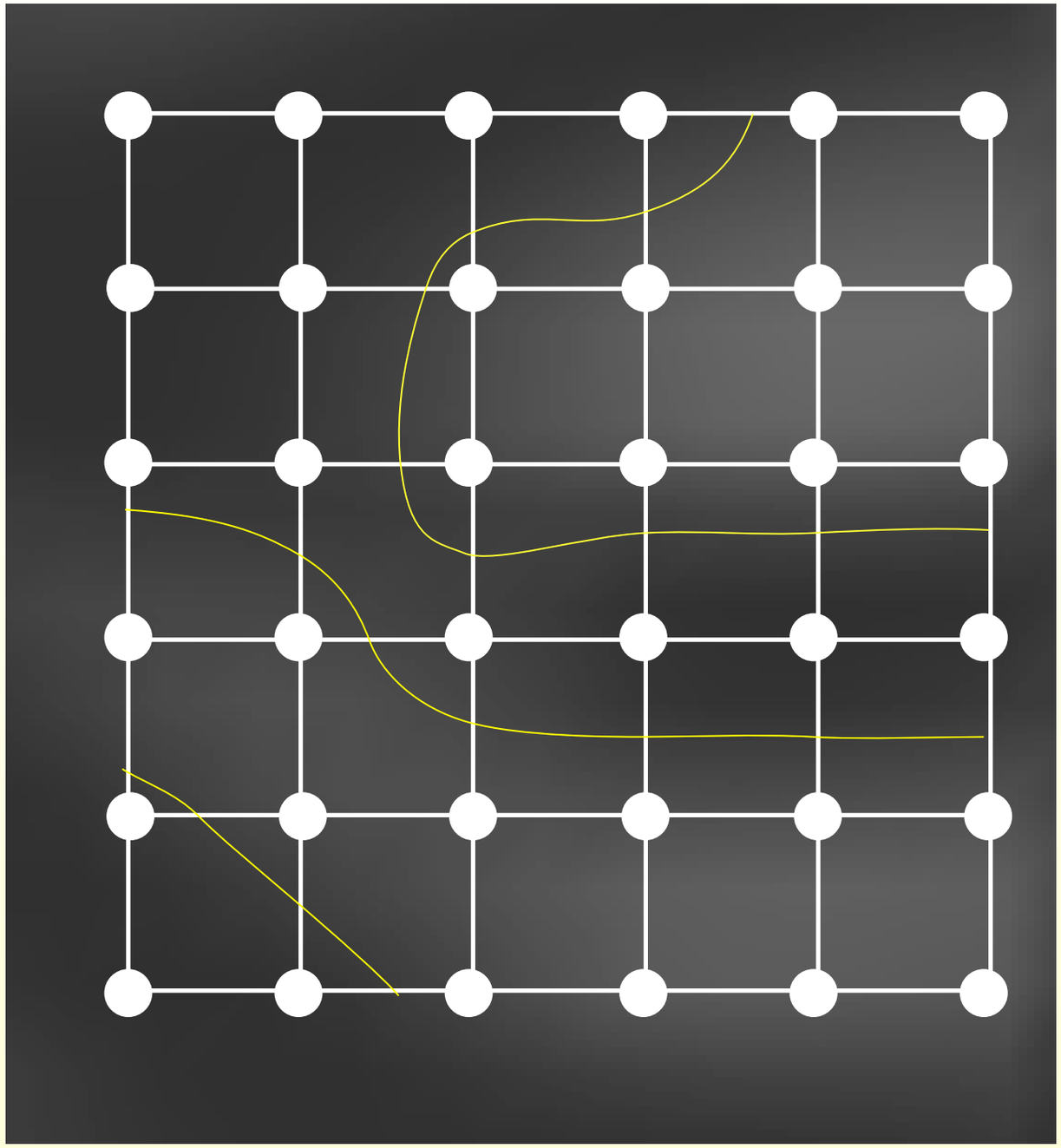
Overview

- Marching Cubes
  - 2D case: Marching Squares
  - 3D case: Marching Cubes
  - Marching Tetrahedra
- Extended Marching Cubes
- Dual Contouring



Marching Squares

- Given $I(\mathbf{x})$
  - $I(\mathbf{x}) < q$: $\mathbf{x}$ outside
  - $I(\mathbf{x}) > q$: $\mathbf{x}$ inside
- Discretize space
- Evaluate $f(\mathbf{x})$ on the grid



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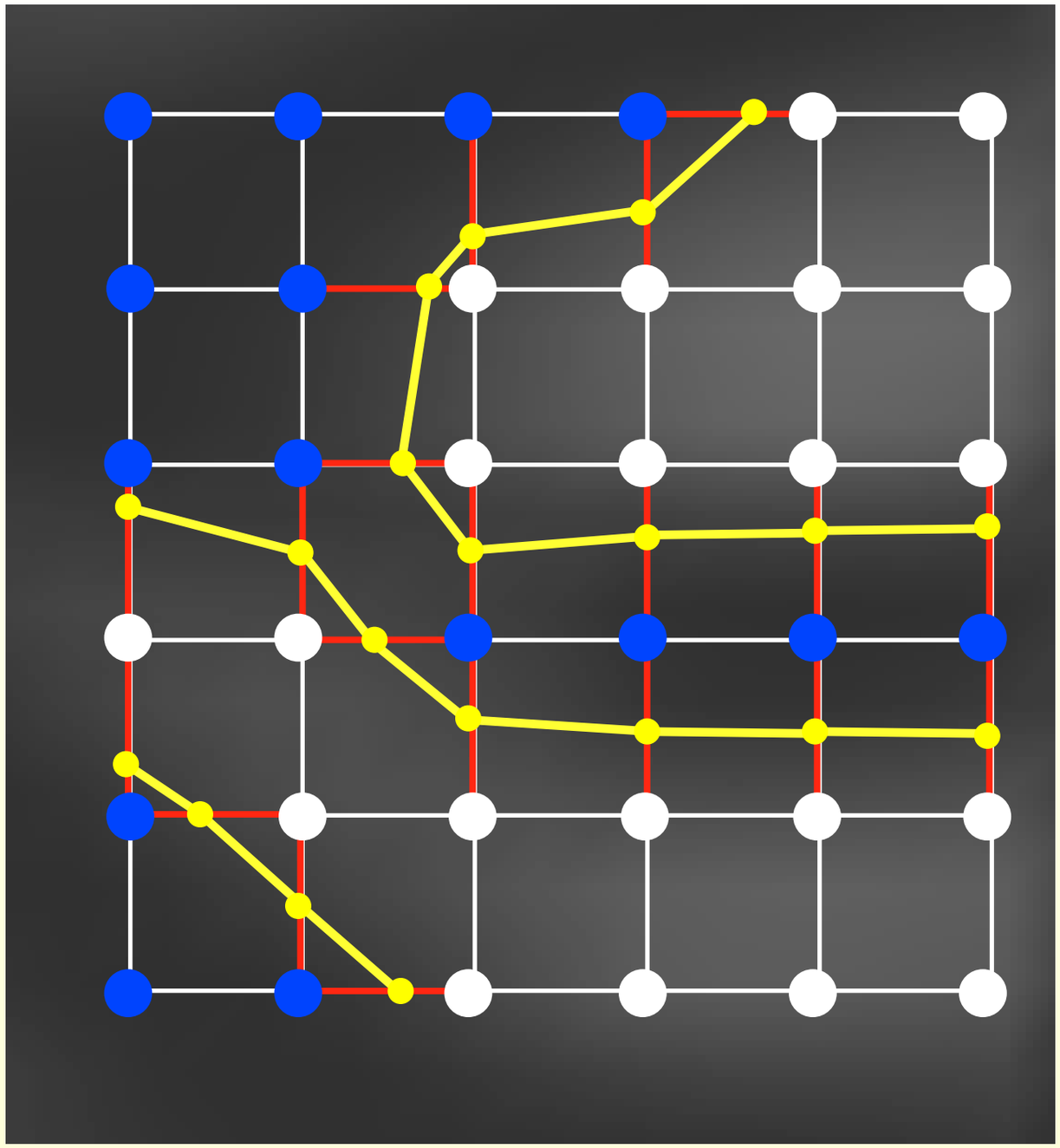
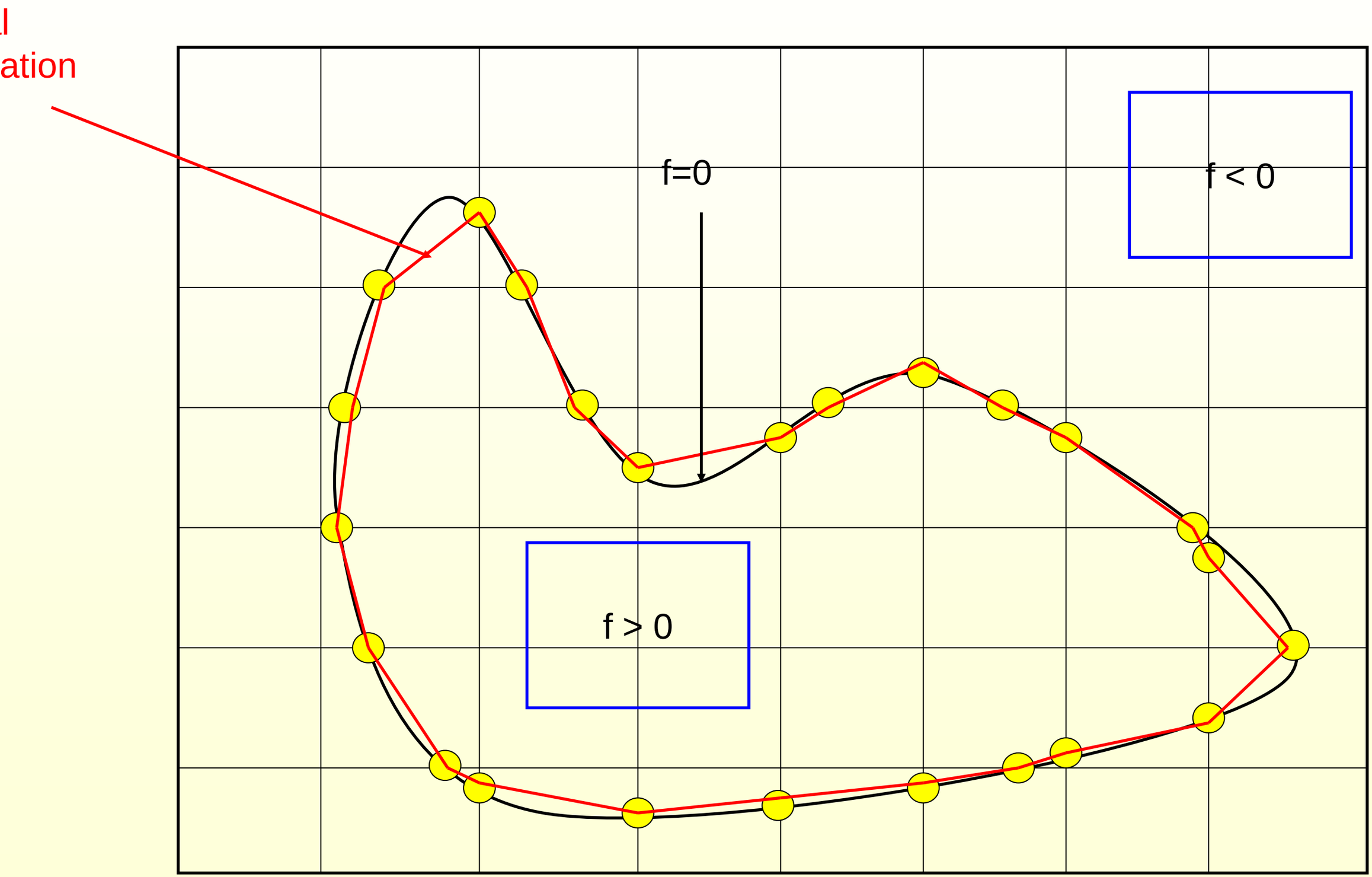
- Evaluate $f(\mathbf{x})$ on the grid
- Classify grid points
- Classify grid edges
- Compute intersections
- Connect intersections



A Closed Curve



Marching Squares

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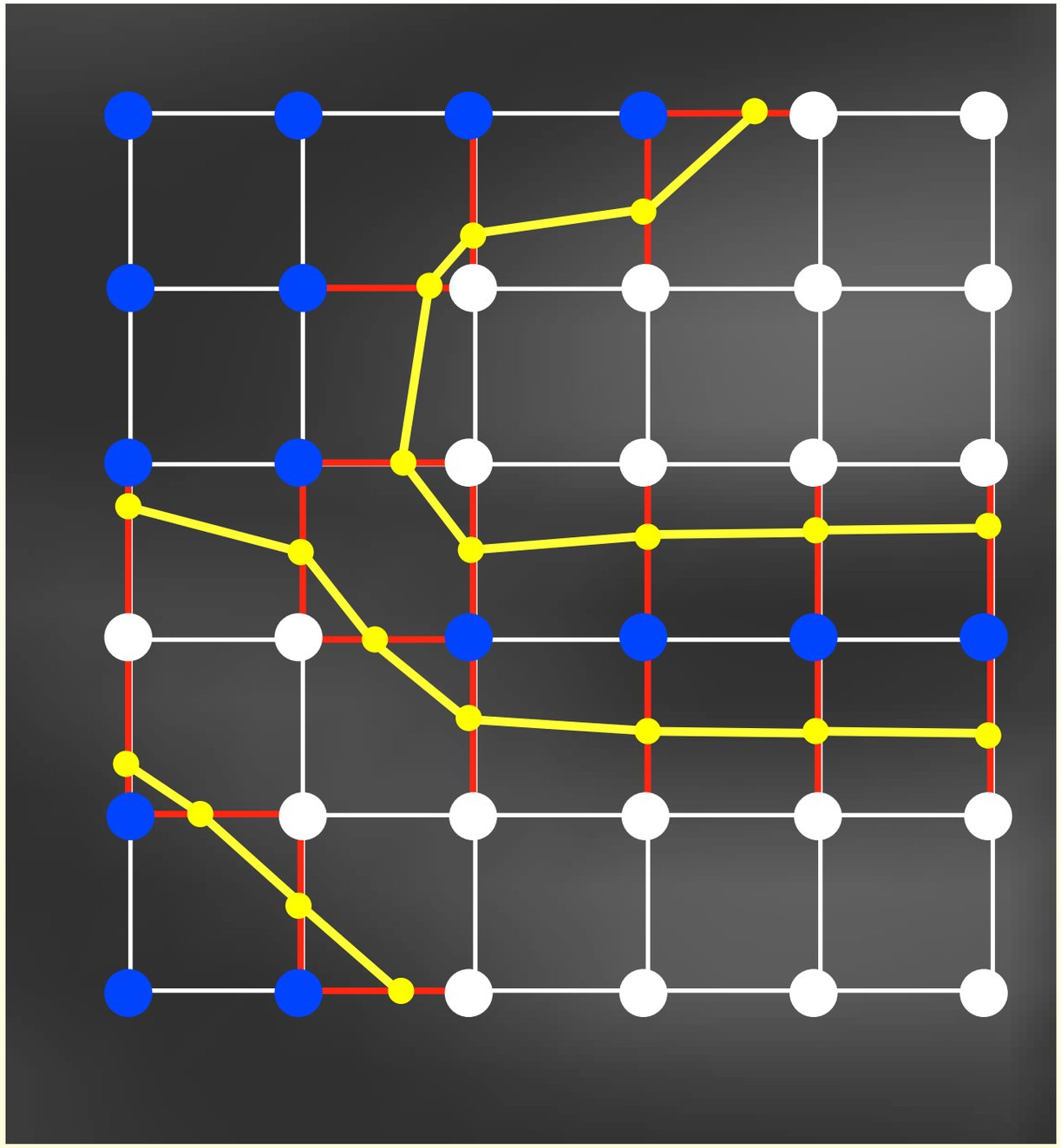


Computing Intersections

- Edges with a sign switch contain intersections

$$f(\mathbf{x}_1) < 0 \text{ and } f(\mathbf{x}_2) \geq 0$$

$$\Rightarrow \quad f(\mathbf{x}_1 + t(\mathbf{x}_2 - \mathbf{x}_1)) = 0 \text{ for some } 0 < t \leq 1$$

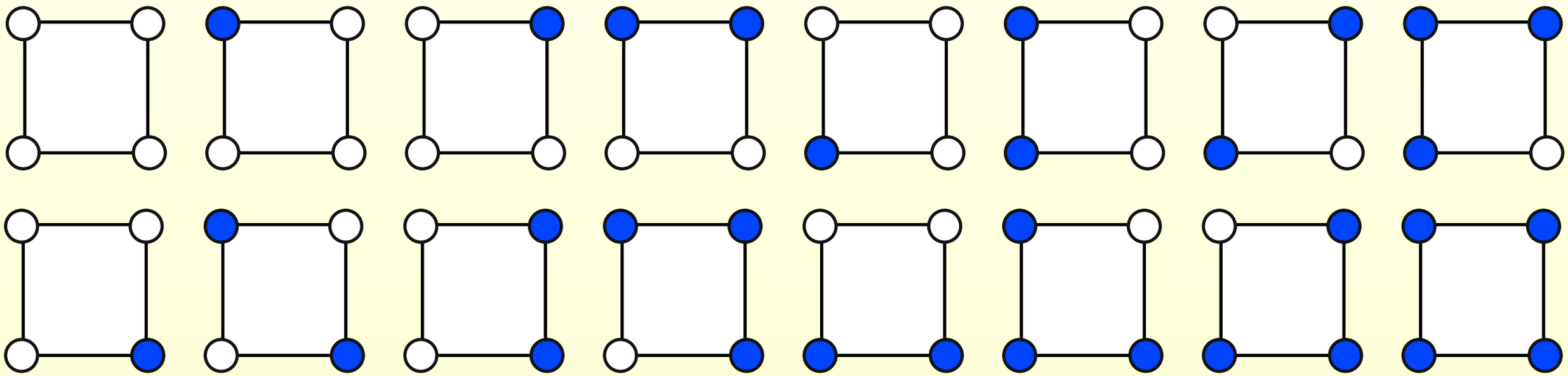
$$f(\mathbf{x}_1 + t(\mathbf{x}_2 - \mathbf{x}_1)) = 0$$

- Nonlinear equation, use raycasting to find root
- Sampled data
  - f is trilinear $\quad f(\mathbf{x}_1) + t(f(\mathbf{x}_2) - f(\mathbf{x}_1)) = 0$
  - f is linear along $\mathbf{x}_2 - \mathbf{x}_1$

$$t = -f(\mathbf{x}_1)/(f(\mathbf{x}_2) - f(\mathbf{x}_1))$$

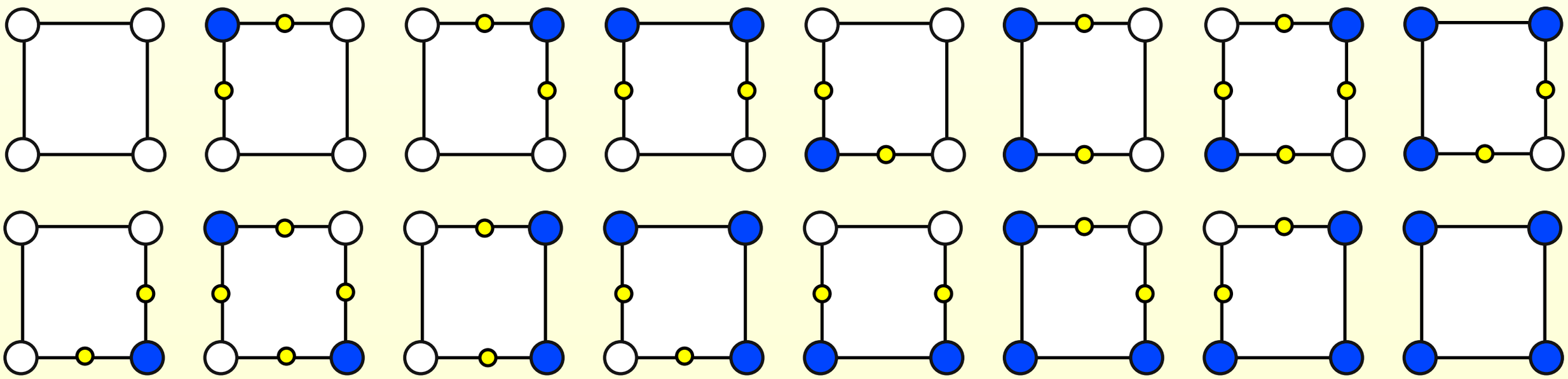
Connecting Intersections

- Treat each cell separately
- Enumerate all possible inside/outside combinations



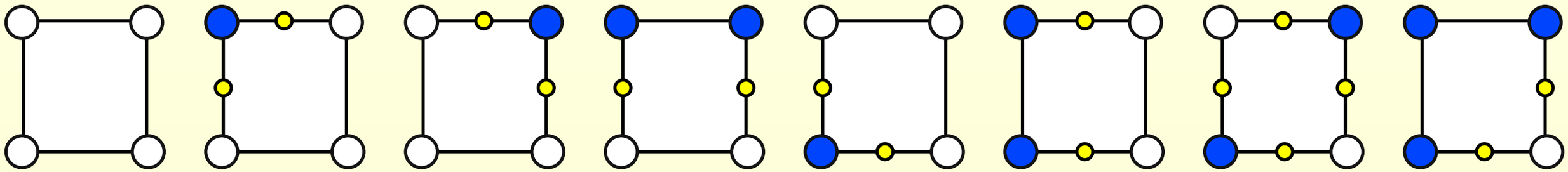
Connecting Intersections

- Treat each cell separately
- Enumerate all possible inside/outside combinations
- Group those leading to the same intersections



Connecting Intersections

- Treat each cell separately
- Enumerate all possible inside/outside combinations
- Group those leading to the same intersections
- Group those equivalent after rotations



Connecting Intersections

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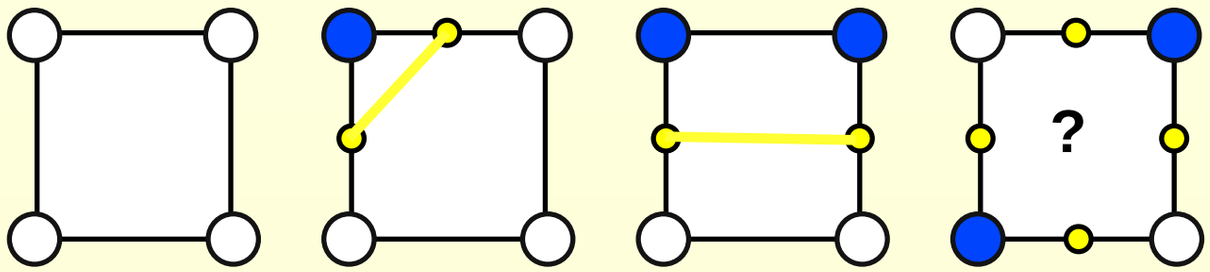
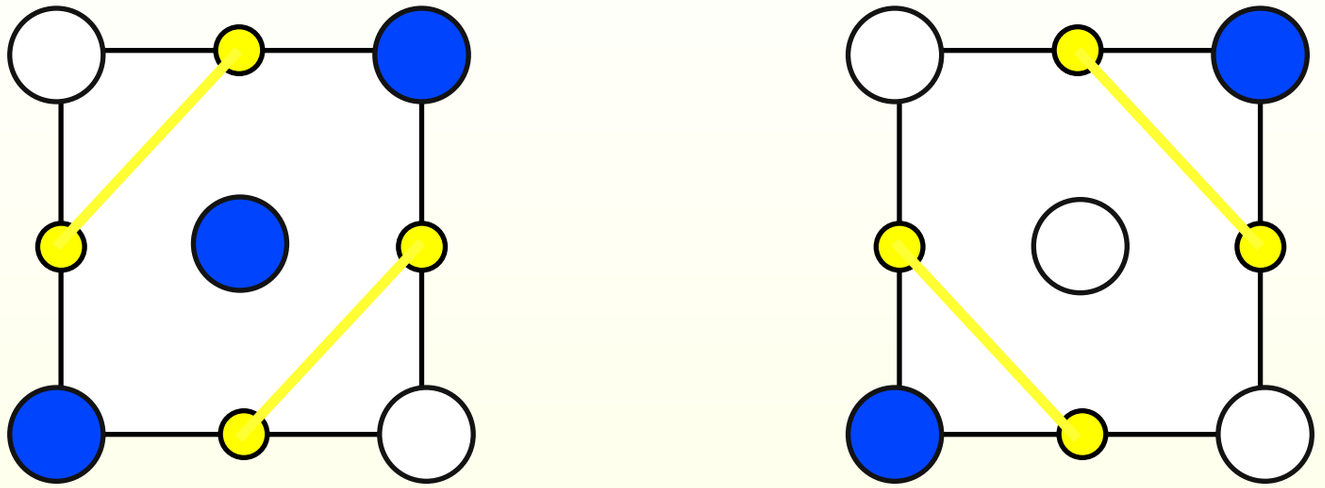
- Connect intersections

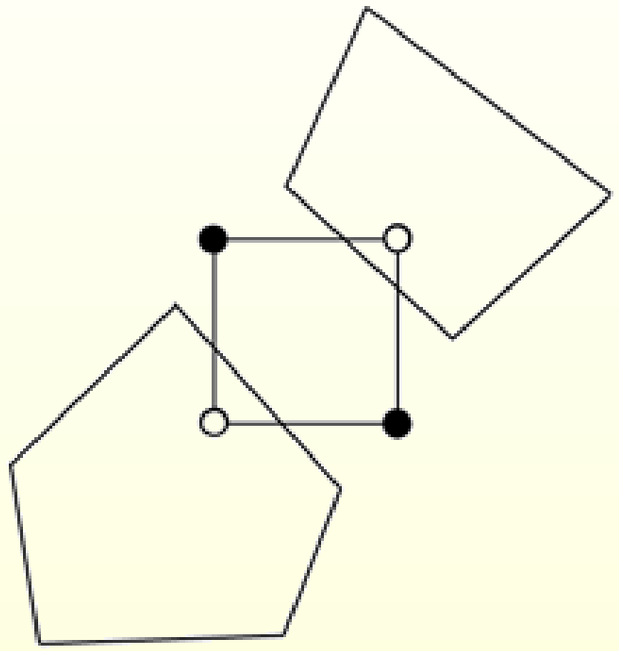


Ambiguous Case

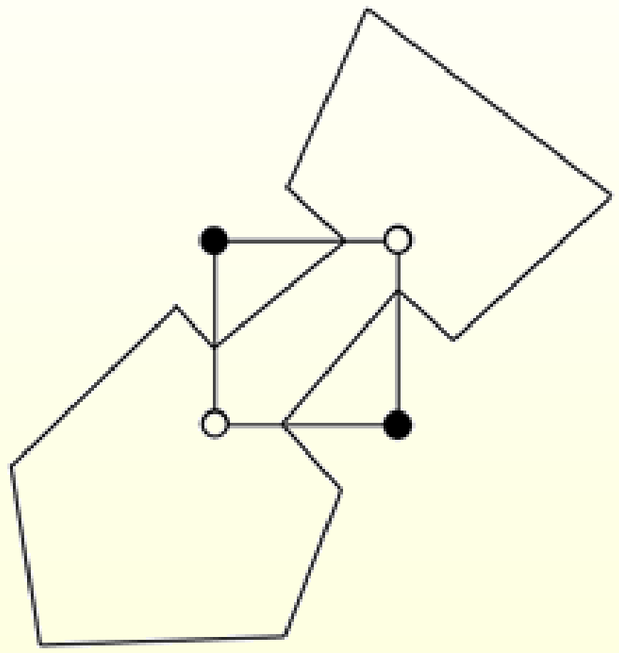


- No way to decide without further sampling
- No more samples available: Just choose one option

But the Choice Could be Wrong



Break contour

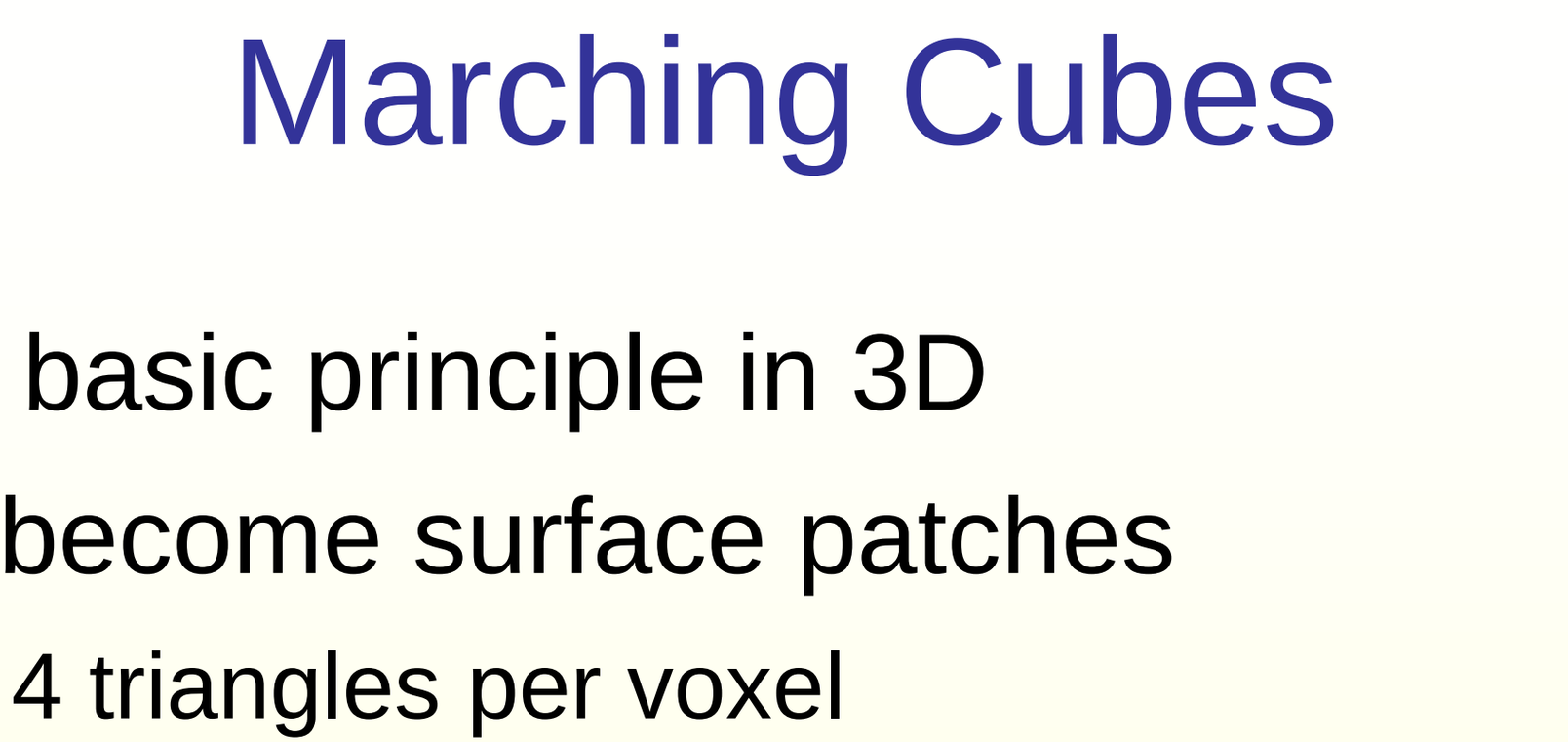


Join contour

Understanding the context matters …

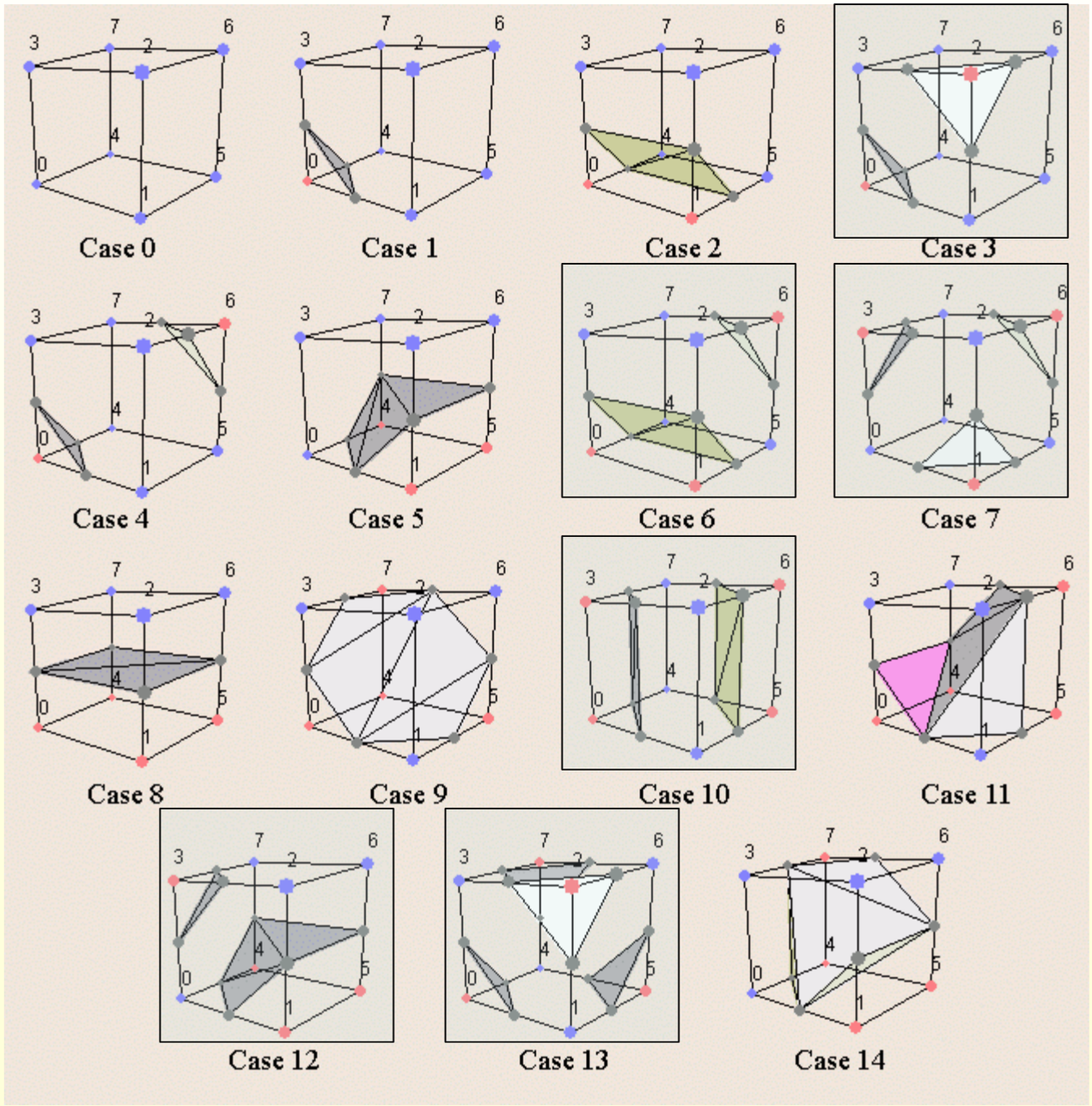
Marching Cubes

- Same basic principle in 3D
- Lines become surface patches
  - Up to 4 triangles per voxel
- 256 different cases, 15 after symmetries

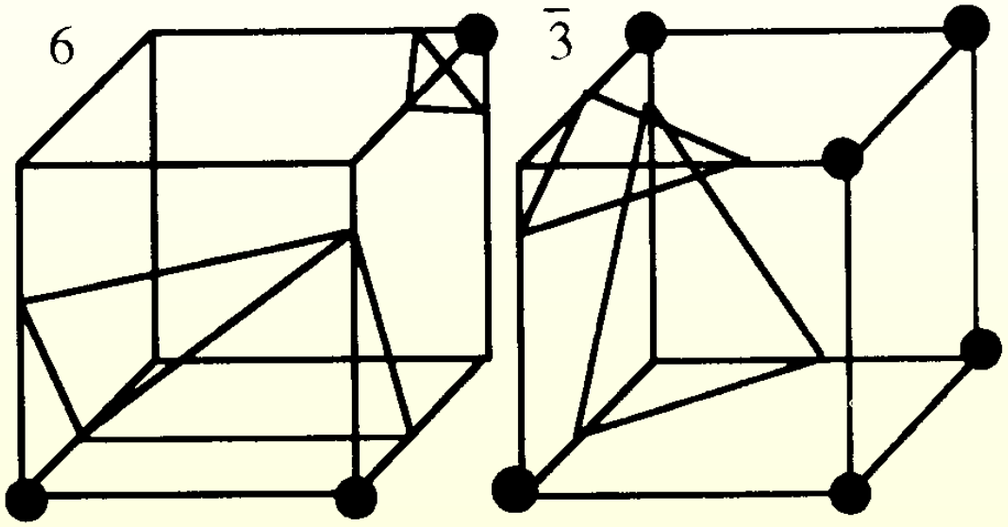


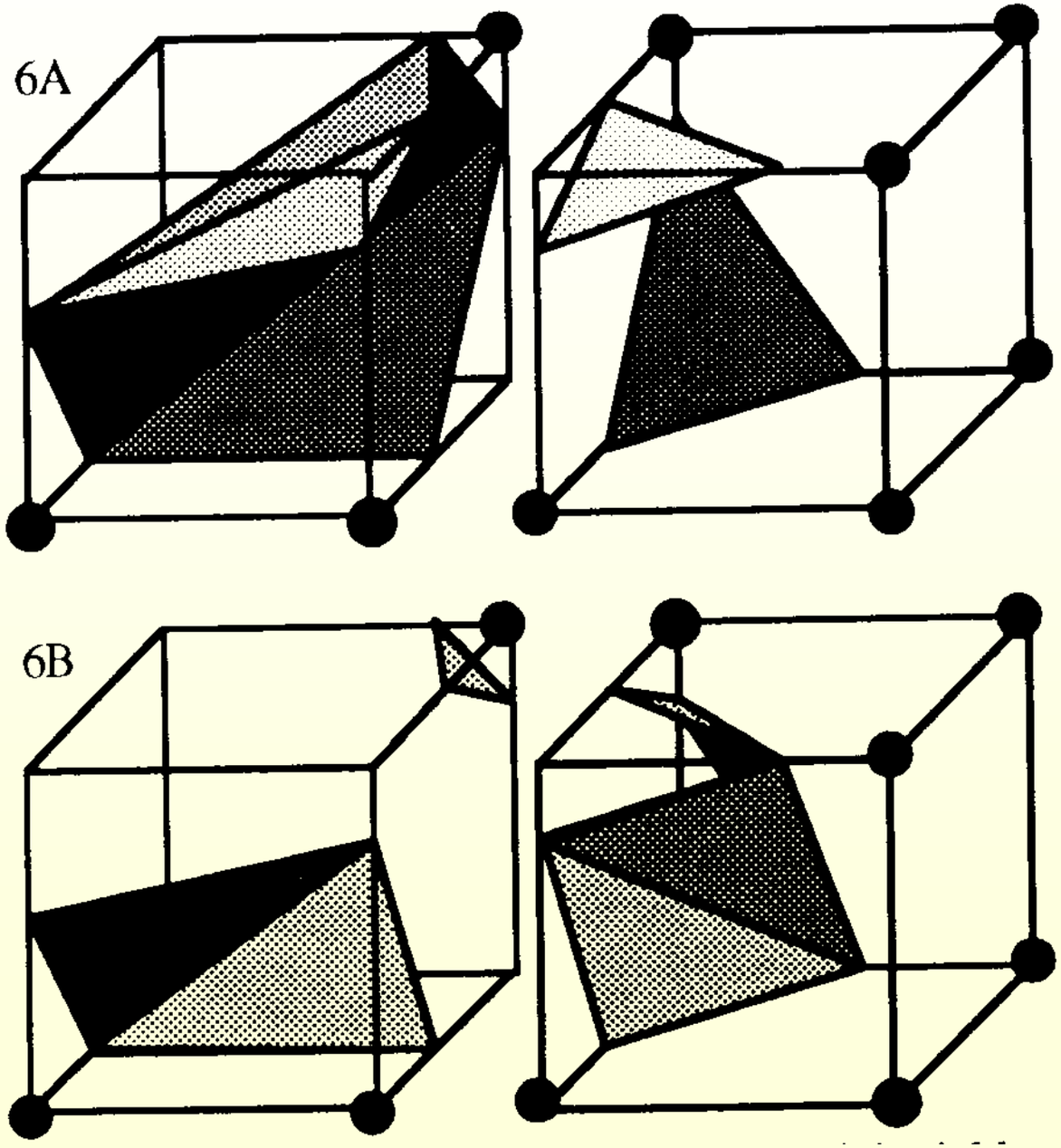
Ambiguous Cases

Ambiguous



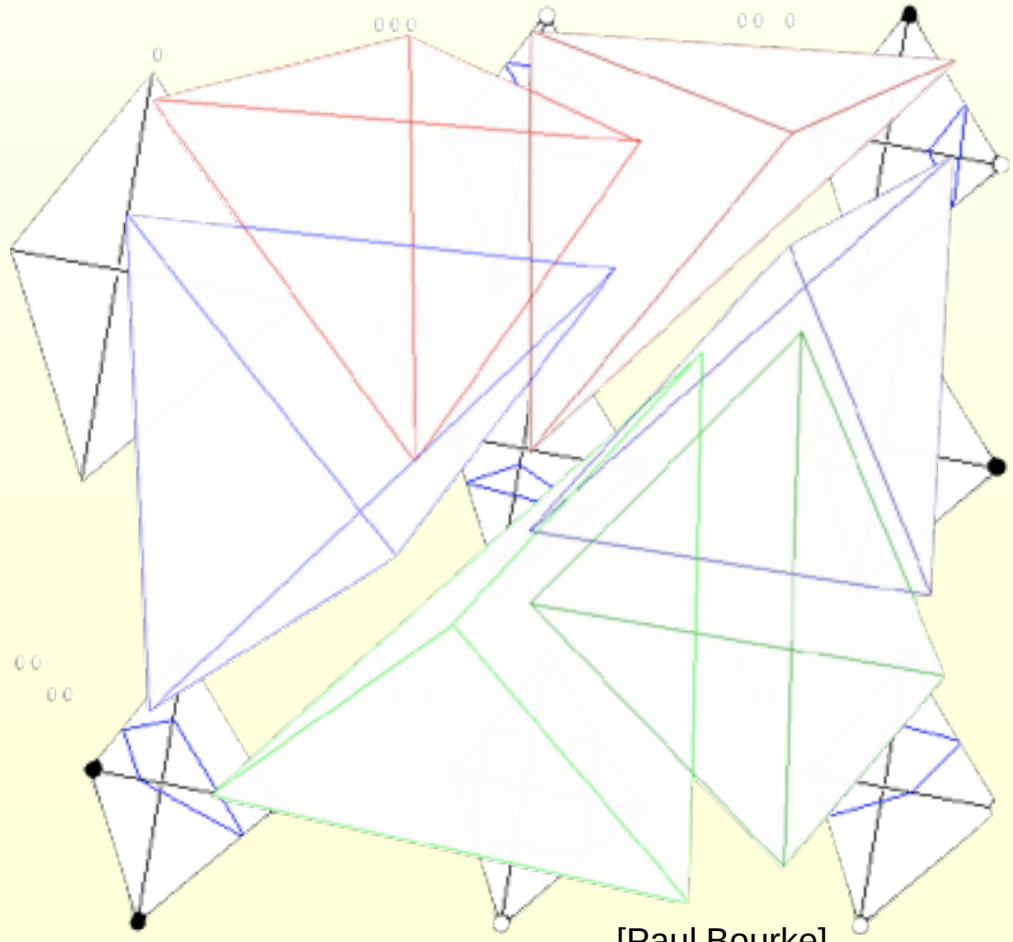
Inconsistent Triangulations





Marching Tetrahedra

- Different discretization: Tetrahedra
  - 6 tetrahedra per voxel (if we start from cubes)
  - 16 cases, 8 after symmetry
  - Up to 2 triangles per tet
  - No ambiguities
- Used when input data discretized as tetrahedra



[Paul Bourke]

Implementation

- Big lookup tables

```
int edgeTable[256]={0x0  , 0x109, 0x203, 0x30a, 0x406,
0x50f, 0x605, 0x70c,0x80c, 0x905, 0xa0f, 0xb06, 0xc0a,
0xd03, 0xe09, 0xf00,0x190, 0x99 , 0x393, 0x29a, 0x596,
0x49f, 0x795, 0x69c,0x99c, 0x895, 0xb9f, 0xa96, 0xd9a,
0xc93, 0xf99, 0xe90,0x230, 0x339, 0x33 , 0x13a, 0x636,
0x73f, 0x435, 0x53c,0xa3c, 0xb35, 0x83f, 0x936, 0xe3a,
0xf33, 0xc39, 0xd30,0x3a0, 0x2a9, 0x1a3, 0xaa , 0x7a6,
0x6af, 0x5a5, 0x4ac,0xbac, 0xaa5, 0x9af, 0x8a6, 0xfaa,
0xea3, 0xda9, 0xca0,0x460, 0x569, 0x663, 0x76a, 0x66 ,
0x16f, 0x265, 0x36c,0xc6c, 0xd65, 0xe6f, 0xf66, 0x86a,
0x963, 0xa69, 0xb60,0x5f0, 0x4f9, 0x7f3, 0x6fa, 0x1f6, 0xff ,
0x3f5, 0x2fc,0xdfc, 0xcf5, 0xfff, 0xef6, 0x9fa, 0x8f3, 0xbf9,
0xaf0,0x650, 0x759, 0x453, 0x55a, 0x256, 0x35f, 0x55 ,
0x15c,0xe5c, 0xf55, 0xc5f, 0xd56, 0xa5a, 0xb53, 0x859,
0x950,0x7c0, 0x6c9, 0x5c3, 0x4ca, 0x3c6, 0x2cf, 0x1c5,
0xcc ,0xfcc, 0xec5, 0xdcf, 0xcc6, 0xbca, 0xac3, 0x9c9,
0x8c0,0x8c0, 0x9c9, 0xac3, 0xbca, 0xcc6, 0xdcf, 0xec5,
0xfcc,0xcc , 0x1c5, 0x2cf, 0x3c6, 0x4ca, 0x5c3, 0x6c9,
0x7c0,0x950, 0x859, 0xb53, 0xa5a, 0xd56, 0xc5f, 0xf55,
0xe5c,0x15c, 0x55 , 0x35f, 0x256, 0x55a, 0x453, 0x759,
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0x5f0,0xb60, 0xa69, 0x963, 0x86a, 0xf66, 0xe6f, 0xd65,
0xc6c,0x36c, 0x265, 0x16f, 0x66 , 0x76a, 0x663, 0x569,
0x460,0xca0, 0xda9, 0xea3, 0xfaa, 0x8a6, 0x9af, 0xaa5,
0xbac,0x4ac, 0x5a5, 0x6af, 0x7a6, 0xaa , 0x1a3, 0x2a9,
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0x230,0xe90, 0xf99, 0xc93, 0xd9a, 0xa96, 0xb9f, 0x895,
0x99c,0x69c, 0x795, 0x49f, 0x596, 0x29a, 0x393, 0x99 ,
0x190,0xf00, 0xe09, 0xd03, 0xc0a, 0xb06, 0xa0f, 0x905,
0x80c,0x70c, 0x605, 0x50f, 0x406, 0x30a, 0x203, 0x109,
0x0   };
```

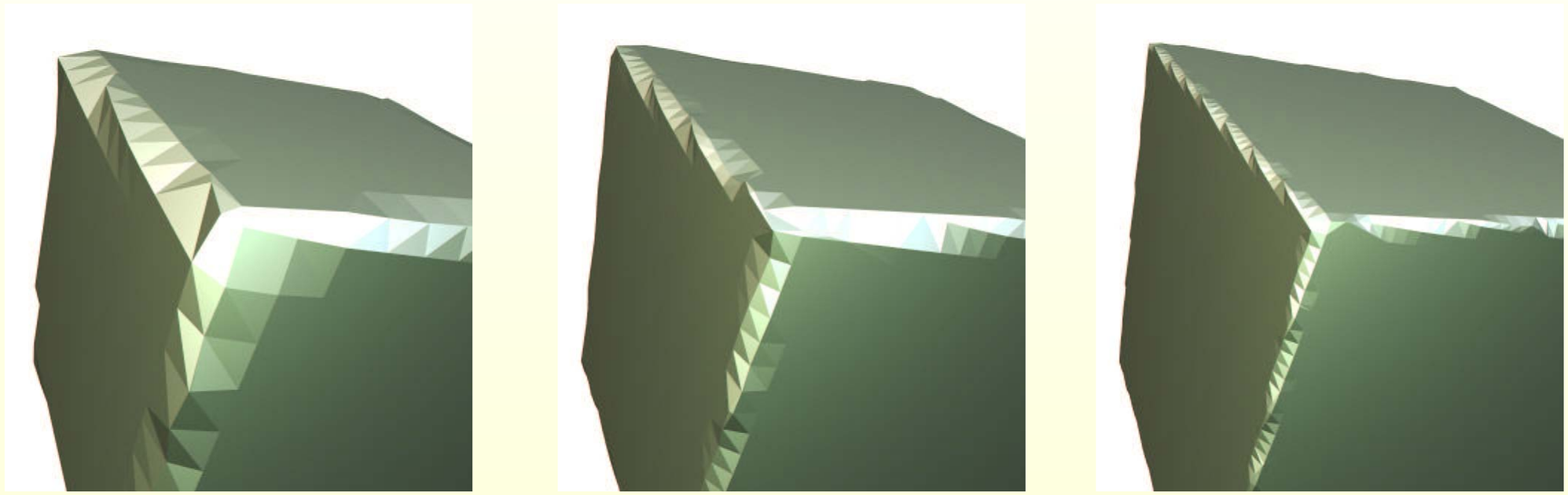
```
int triTable[256][16] ={{-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1},{0, 8, 3, -1, -1, -1, -1, -1, -1, -1,
```

[Paul Bourke]

Problems & Solutions

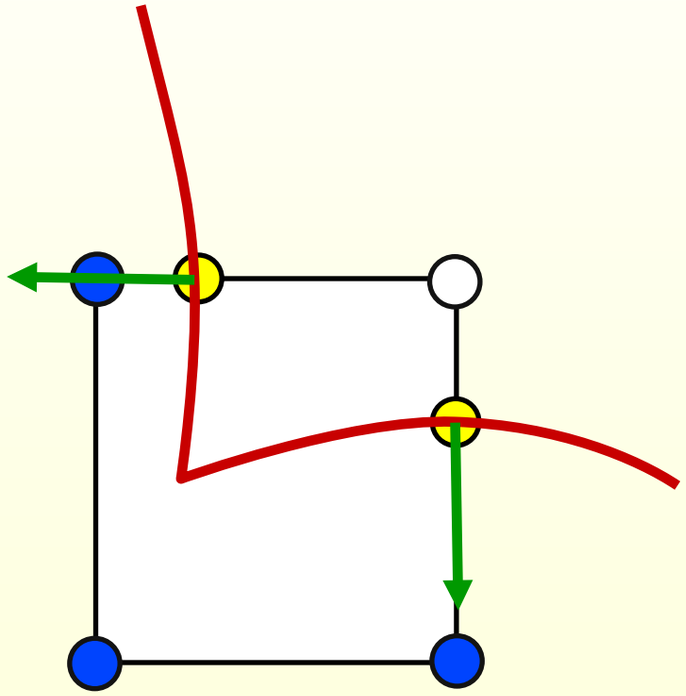
No Sharp Features

- Increasing grid resolution does not help
- Normals do not converge



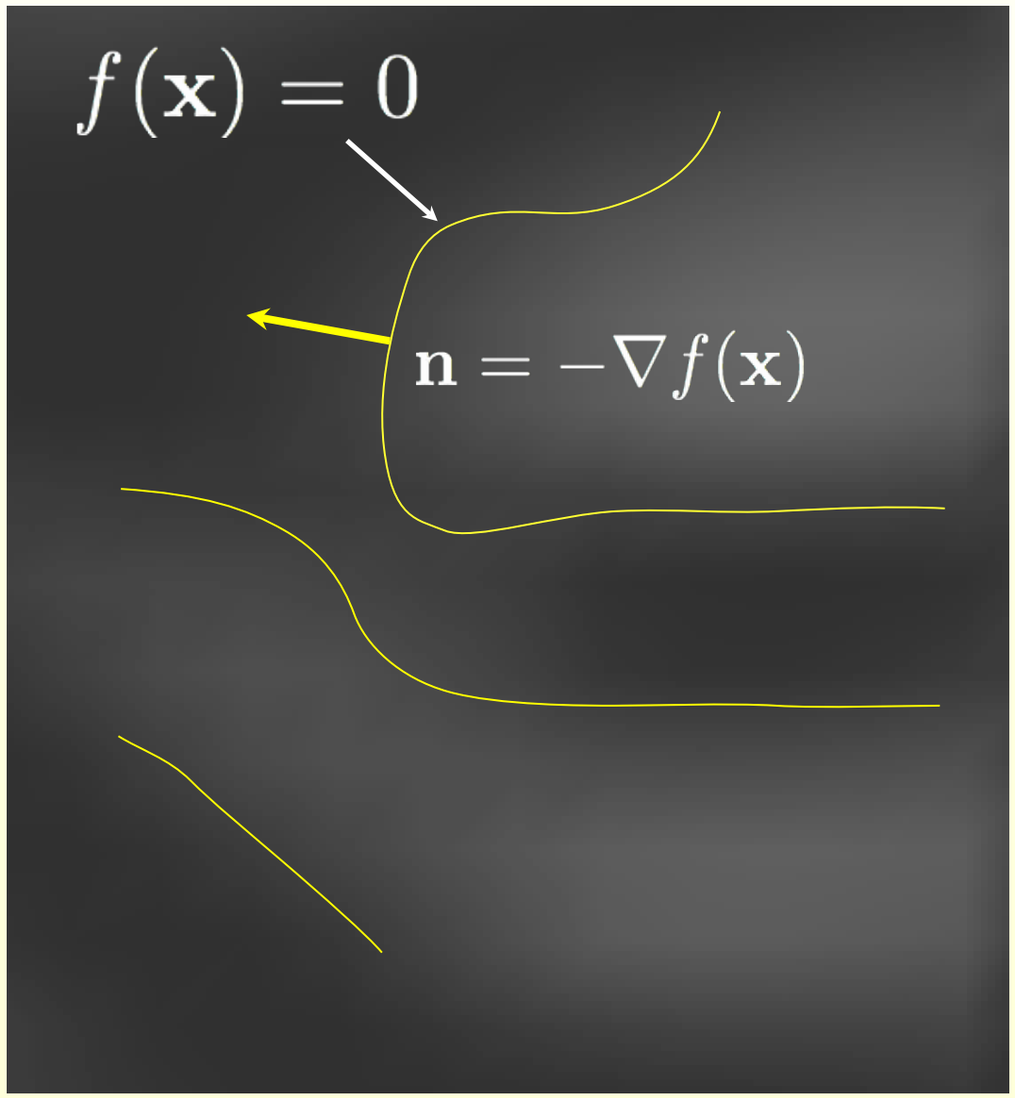
- Use normal information to find edges and corners

Using Hermite Interpolation Data



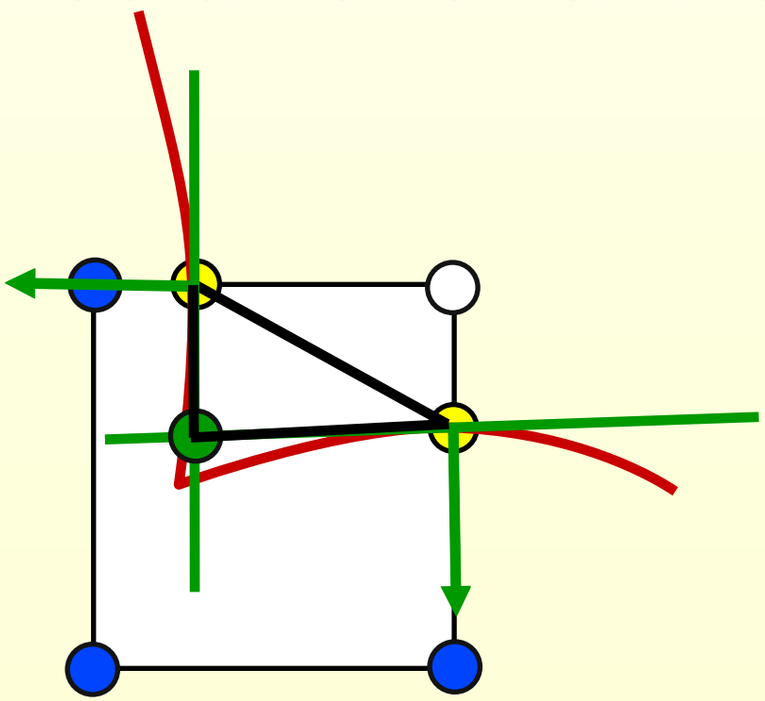
Where Do Normals Come From?

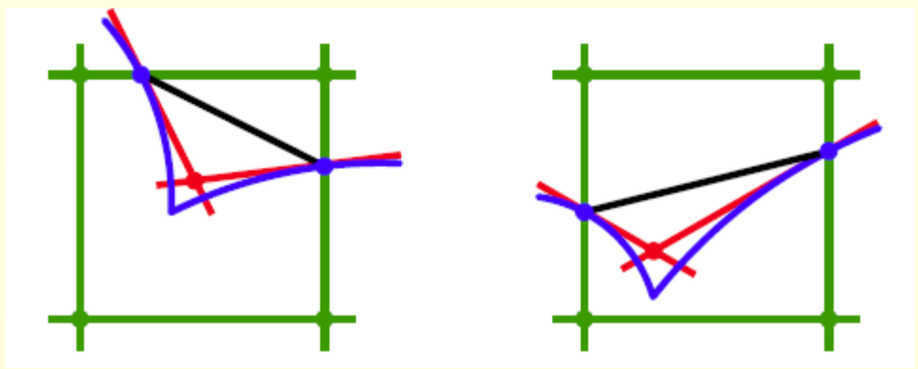
- Gradient of the function $f(\mathbf{x})$ defining the surface



Extended Marching Cubes

- Sharp features are not well approximated
- Use normal information
- Special treatment for corners and edges
  - Corner/edge if large angle between normals
  - Add a vertex at the intersection of tangent planes

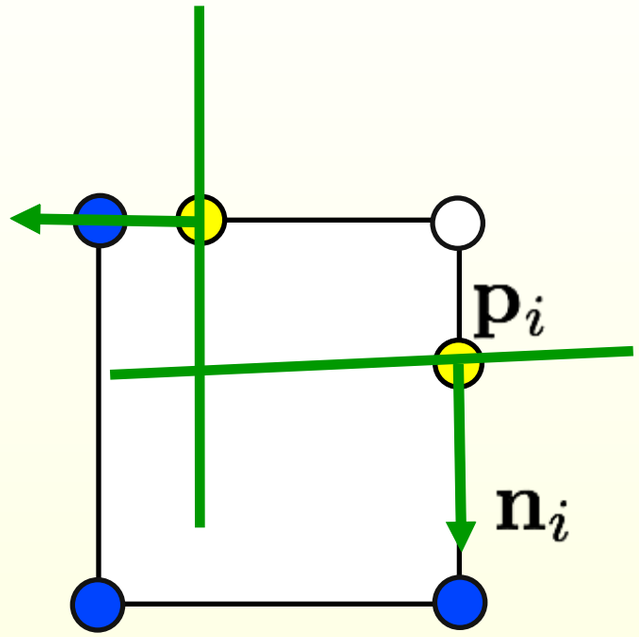




Extended Marching Cubes

- Computing the intersection

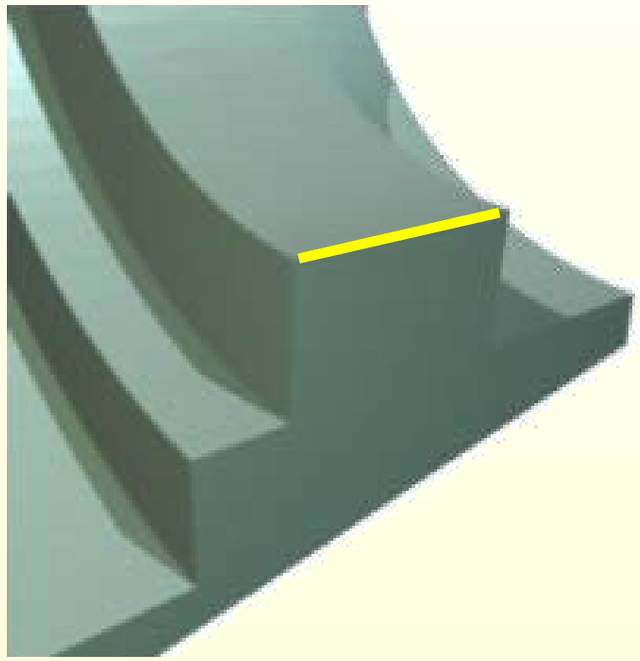
$$\mathbf{Ax} = \begin{bmatrix} \mathbf{n}_1^T \\ \vdots \\ \mathbf{n}_k^T \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{n}_1^T \mathbf{p}_1 \\ \vdots \\ \mathbf{n}_k^T \mathbf{p}_k \end{bmatrix} = \mathbf{b}$$

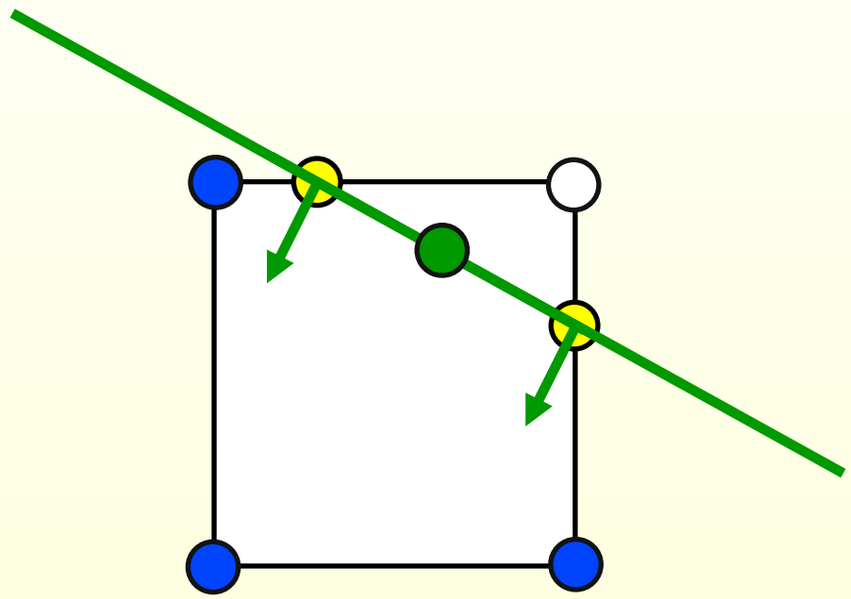


- Sometimes underdetermined
  - When planes are (almost) parallel
- Sometimes overdetermined
  - When too many planes

Underdetermined

- Many solutions to $\mathbf{Ax} = \mathbf{b}$
- Standard edge case in 3D





- Choose the solution closest to the center of gravity of all intersections

Overdetermined

- Find least-squares solution: minimize $\|\mathbf{Ax} - \mathbf{b}\|^2$
- Compute the pseudo-inverse $\mathbf{A}^+$ using SVD

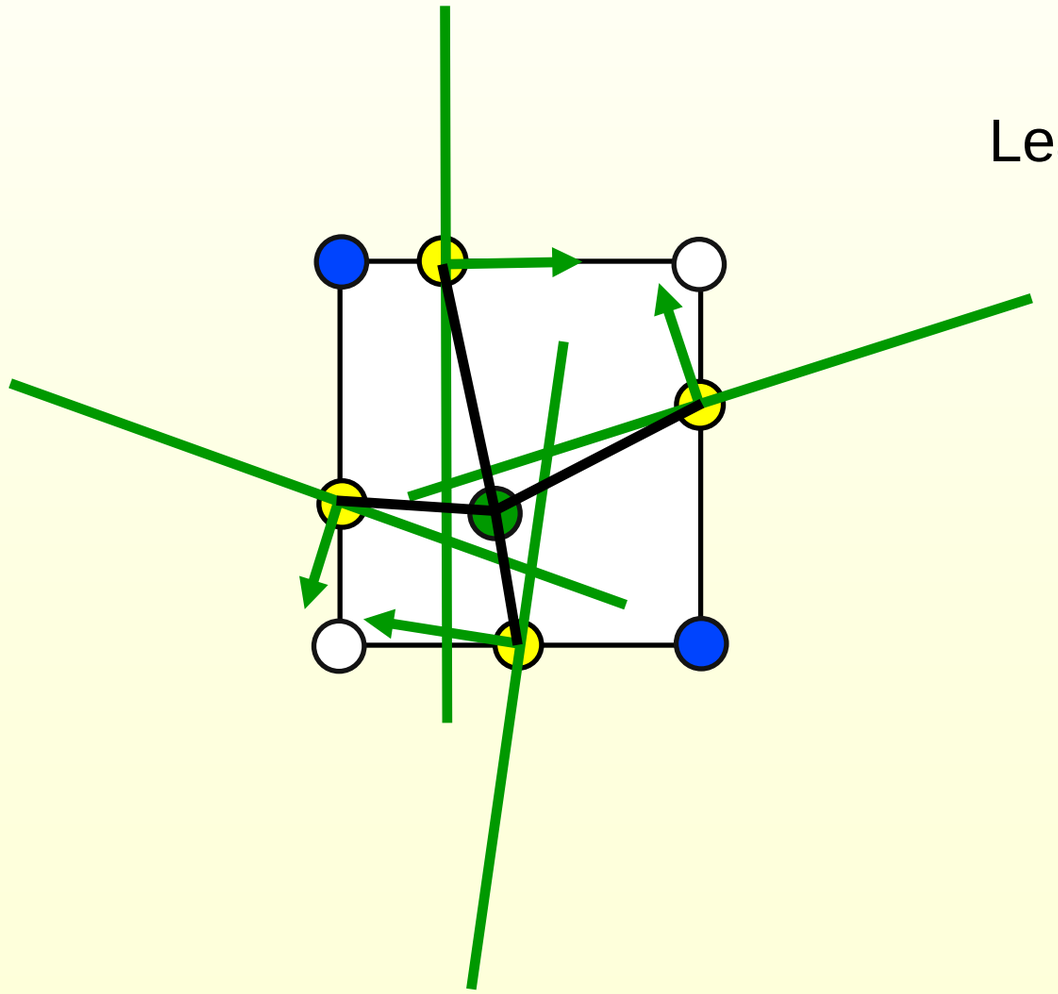
$$\mathbf{x} = \mathbf{A}^+\mathbf{b} = \mathbf{A}^+[\mathbf{n}_1^T\mathbf{p}_1 \ldots \mathbf{n}_k^T\mathbf{p}_k]^T$$

- Solution for over-determined system

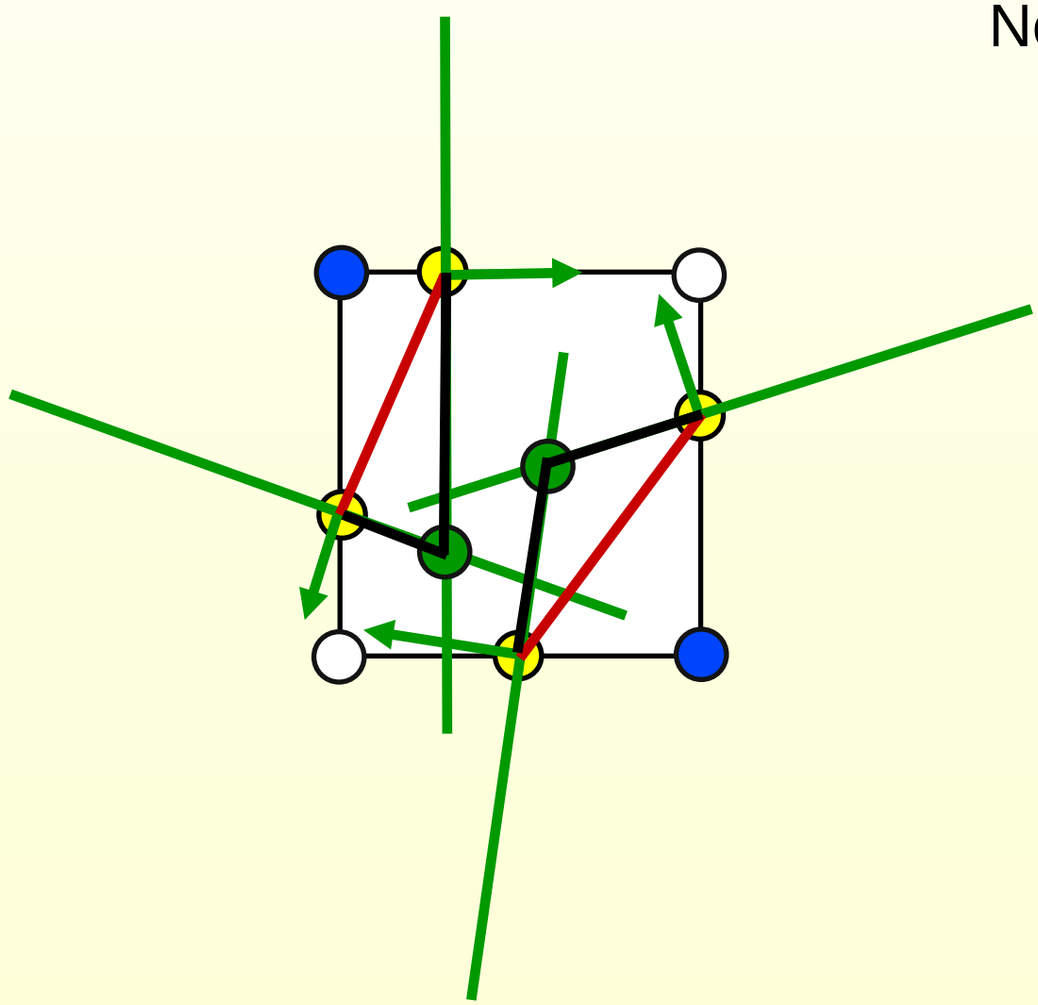
$$\mathbf{Ay} = [\ldots \mathbf{n}_i^T(\mathbf{p}_i - \sum_i \mathbf{p}_i/k) \ldots]$$

$$\mathbf{x} = \mathbf{A}^+([\ldots \mathbf{n}_i(\mathbf{p}_i - \sum_i \mathbf{p}_i/k) \ldots]) + \sum_i \mathbf{p}_i/k$$

Extended Marching Cubes

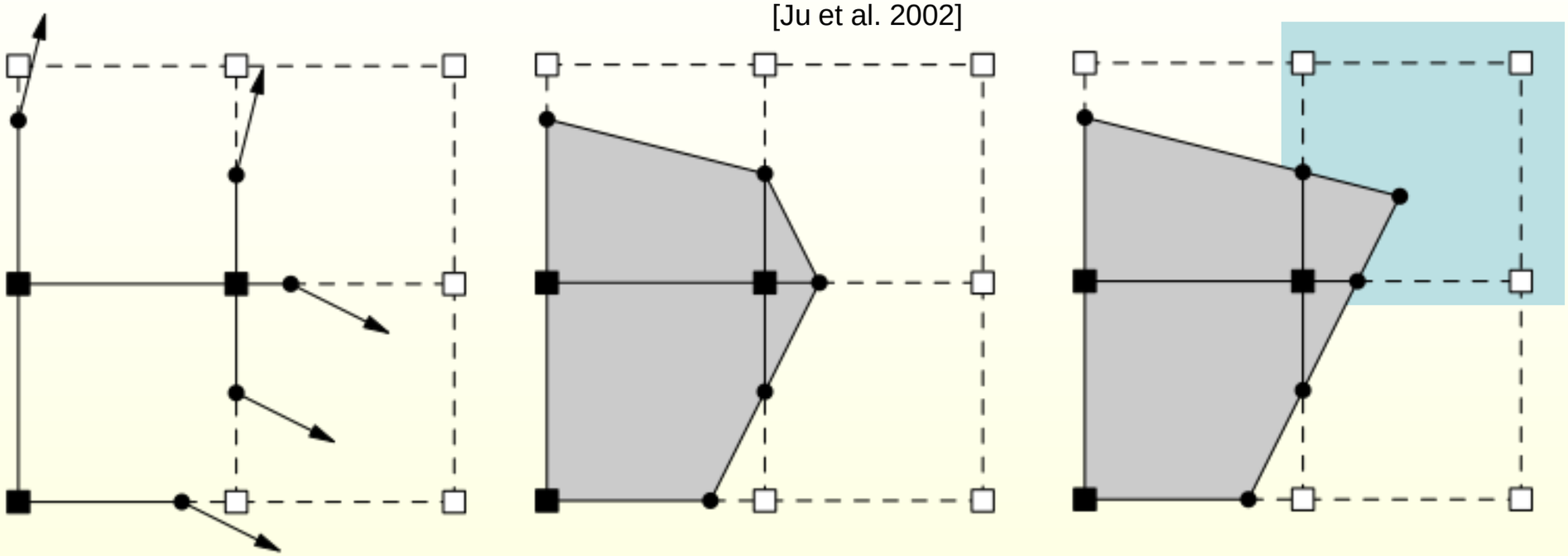


Extended Marching Cubes



Comparison

[Ju et al. 2002]

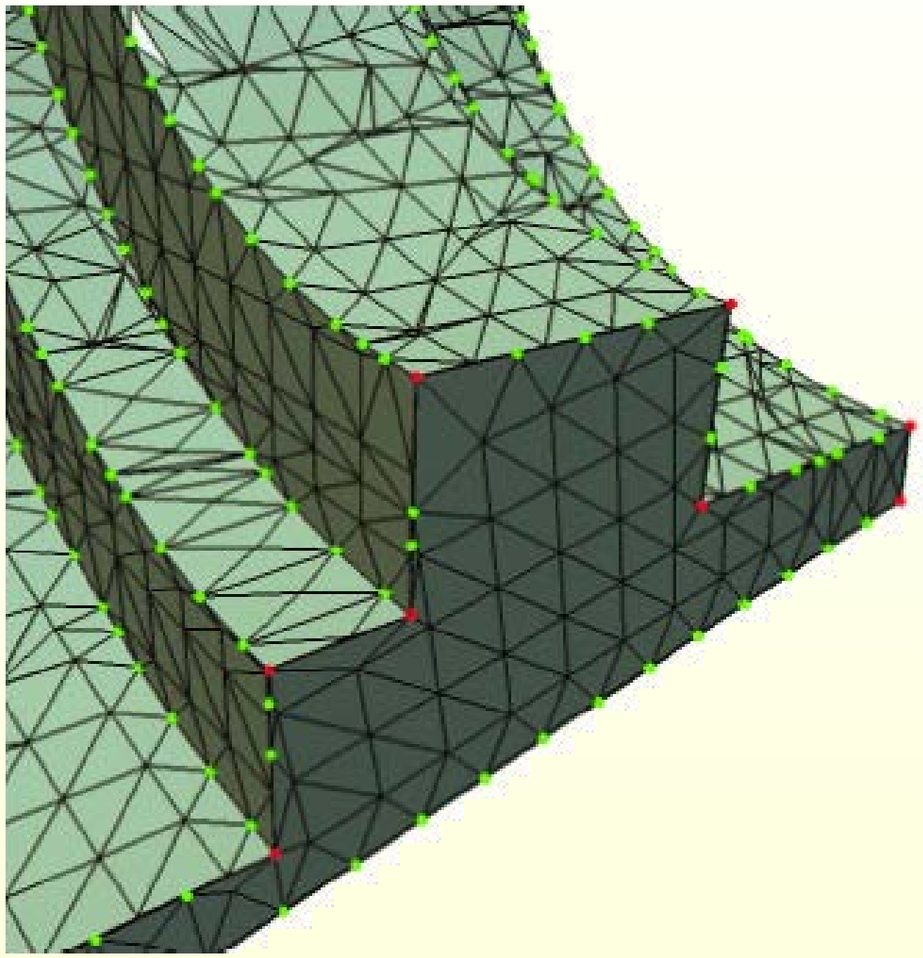


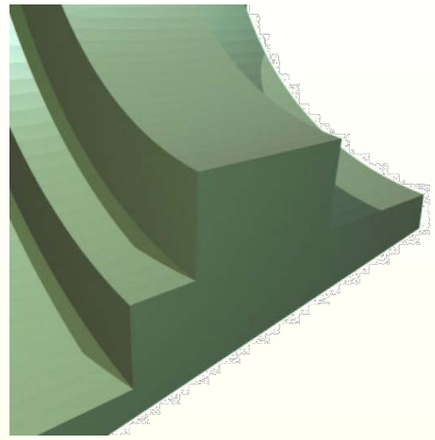
Input data

Marching Cubes

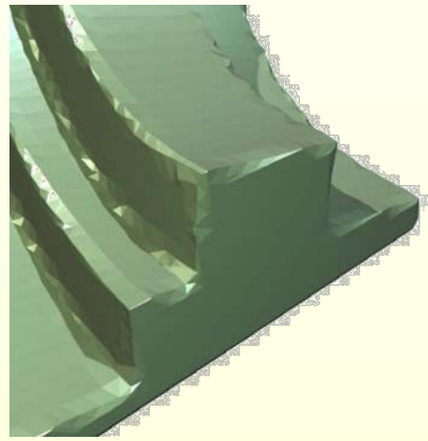
Extended Marching Cubes

Edge and Corner Finding

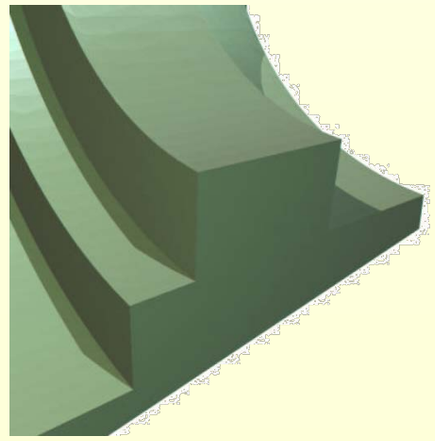




Original
CAD model



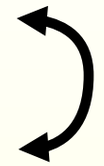
MC reconstruction



Extended MC
reconstruction

Dual Methods

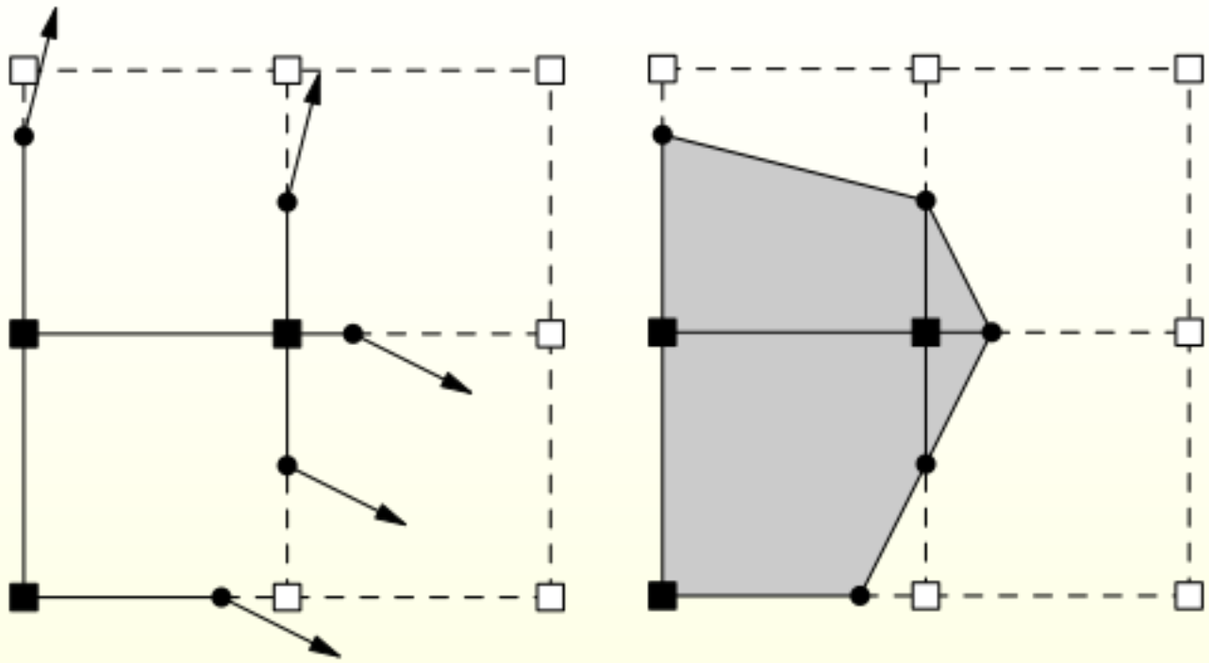
- Marching Cubes (Primal)
  - 1 point per edge
  - Connecting primitives (triangles) per voxel



- Dual Contouring (Dual)
  - 1 point per voxel
  - Connecting primitives (quads) per edge

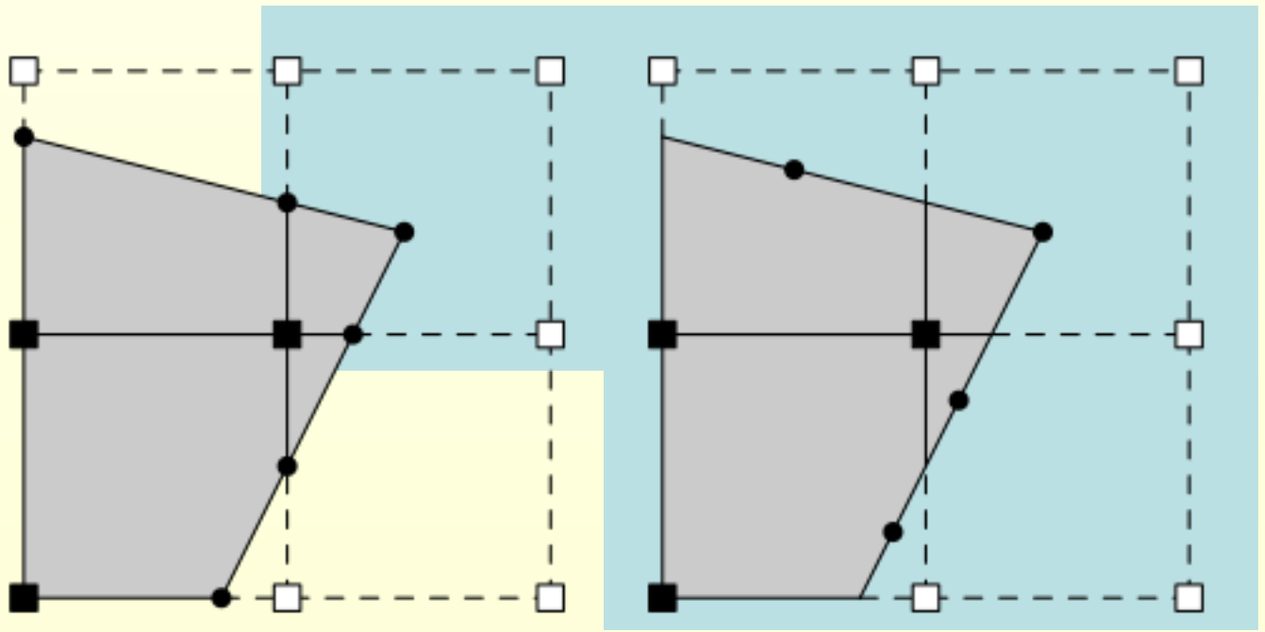
Comparison

Input data



Marching Cubes

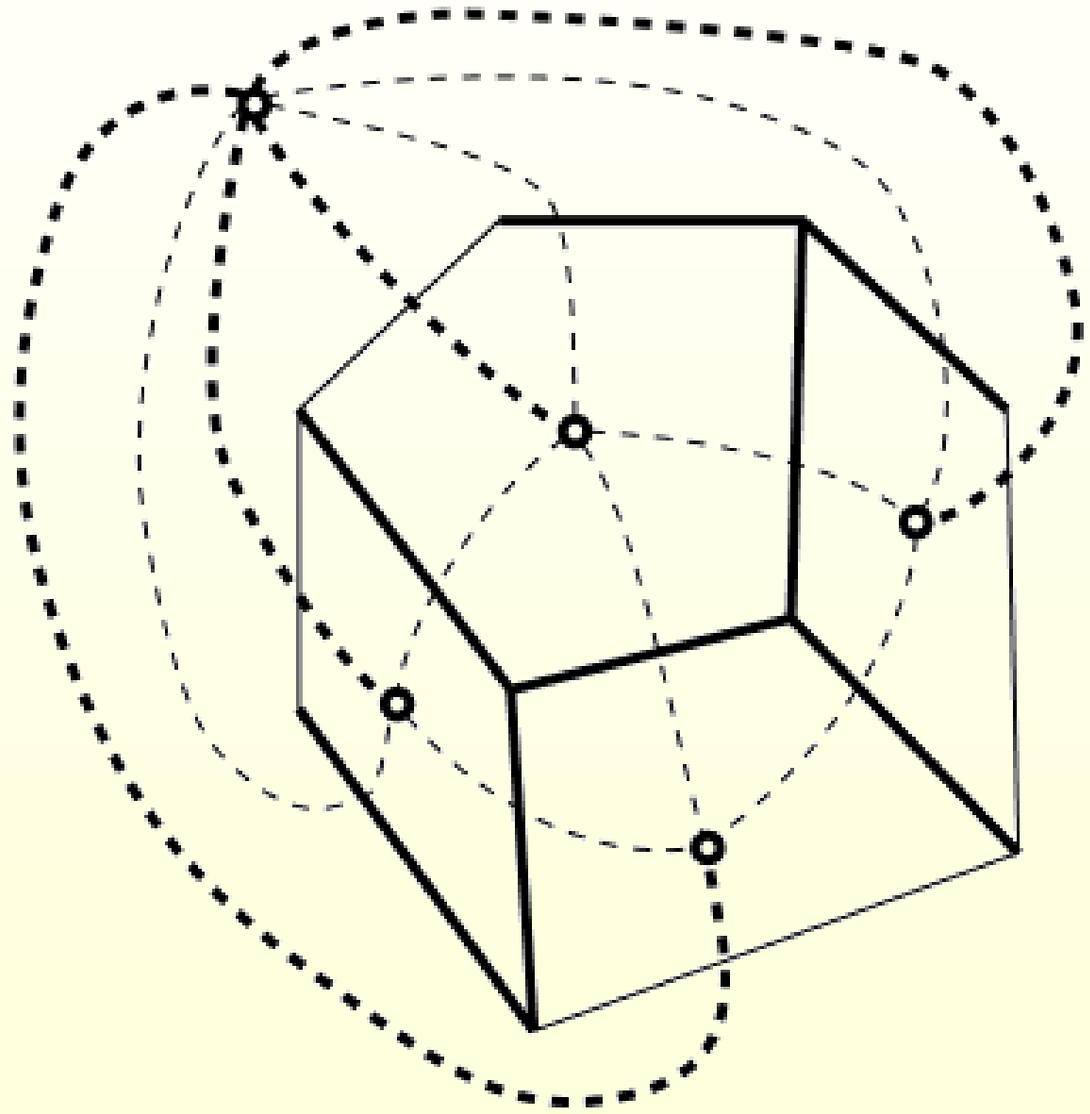
Extended Marching Cubes



Dual Contouring

[Ju et al. 2002]

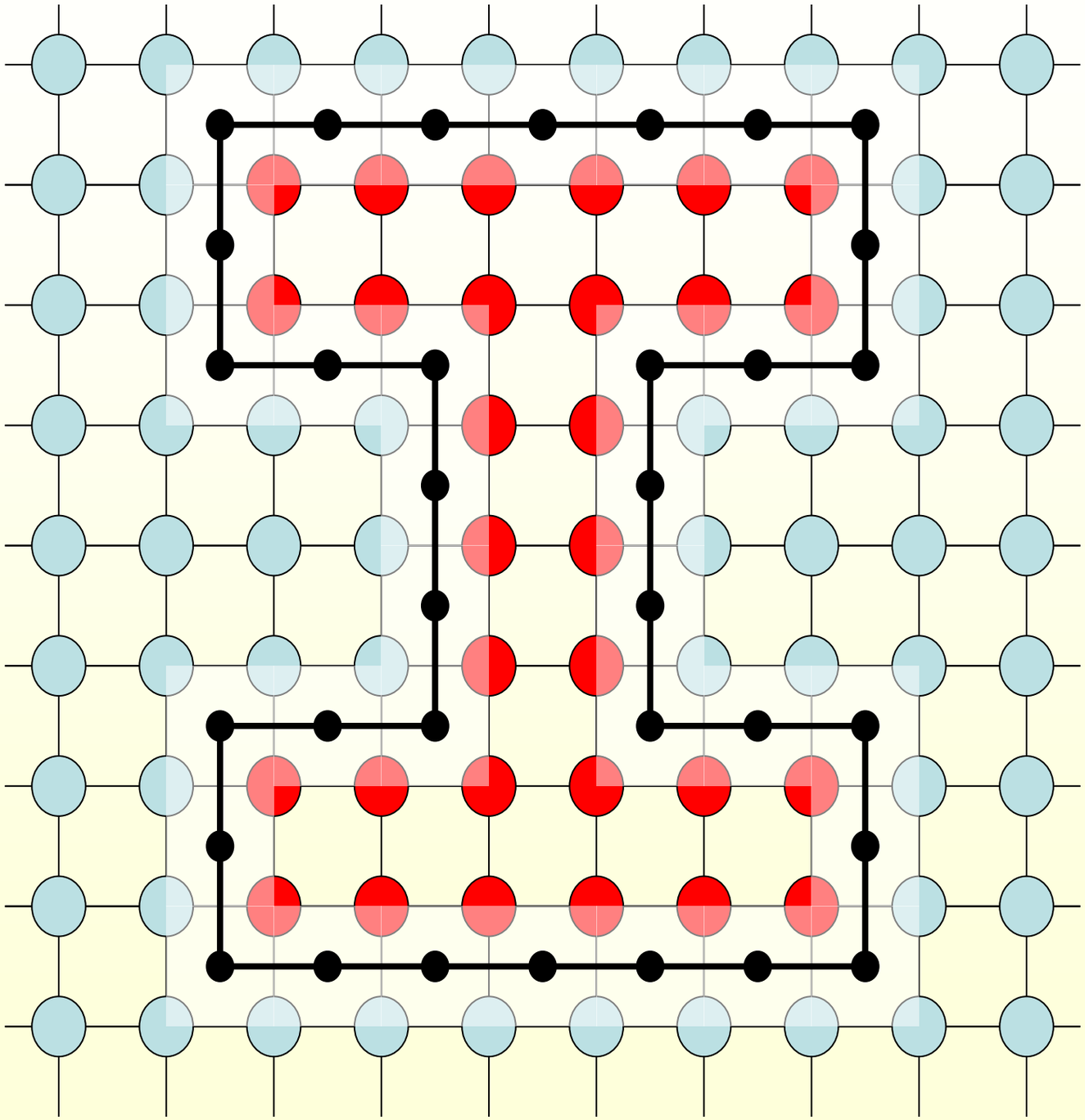
Planar Graph Duals



Primal

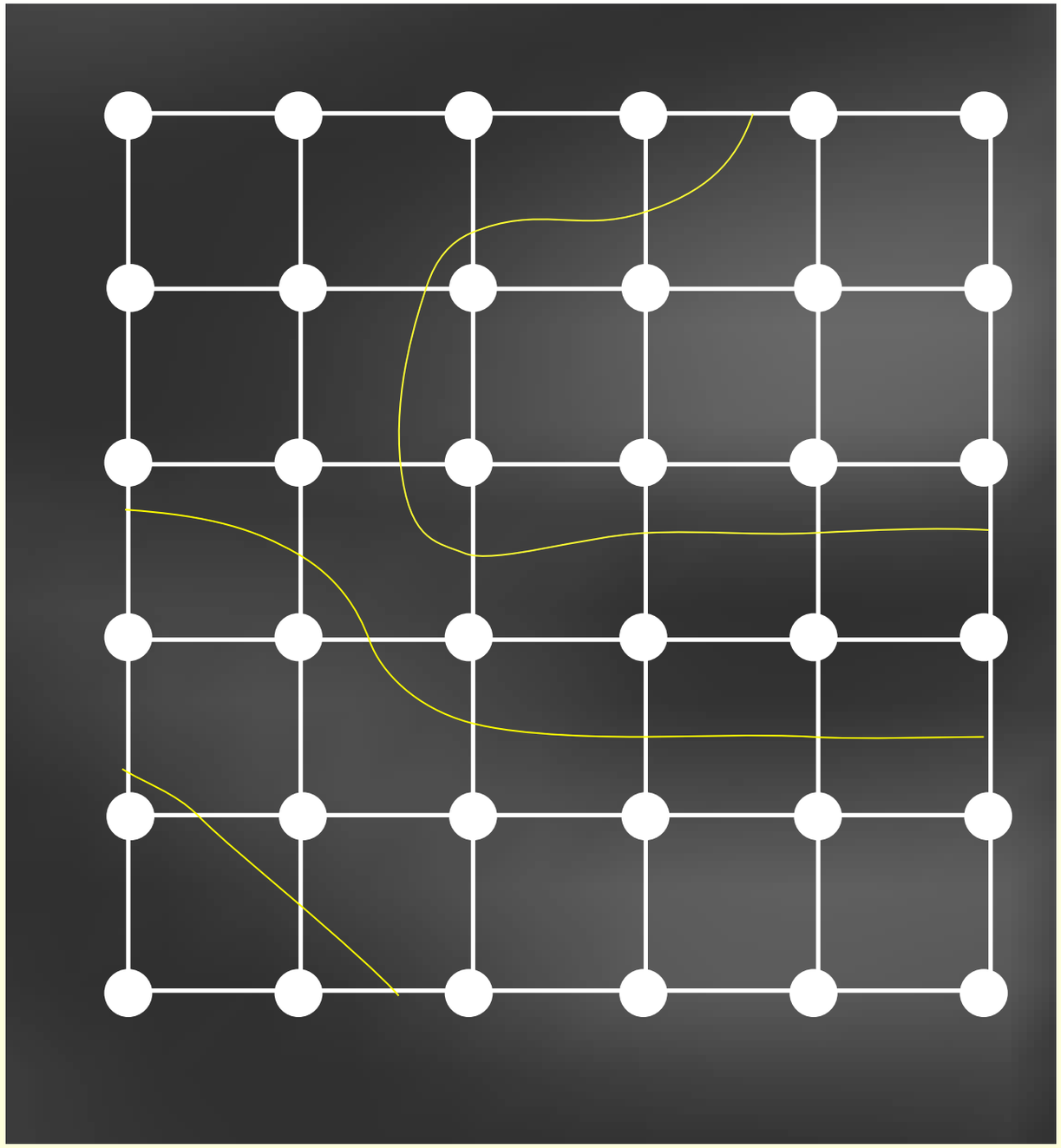
Dual

Primal vs. Dual Contouring



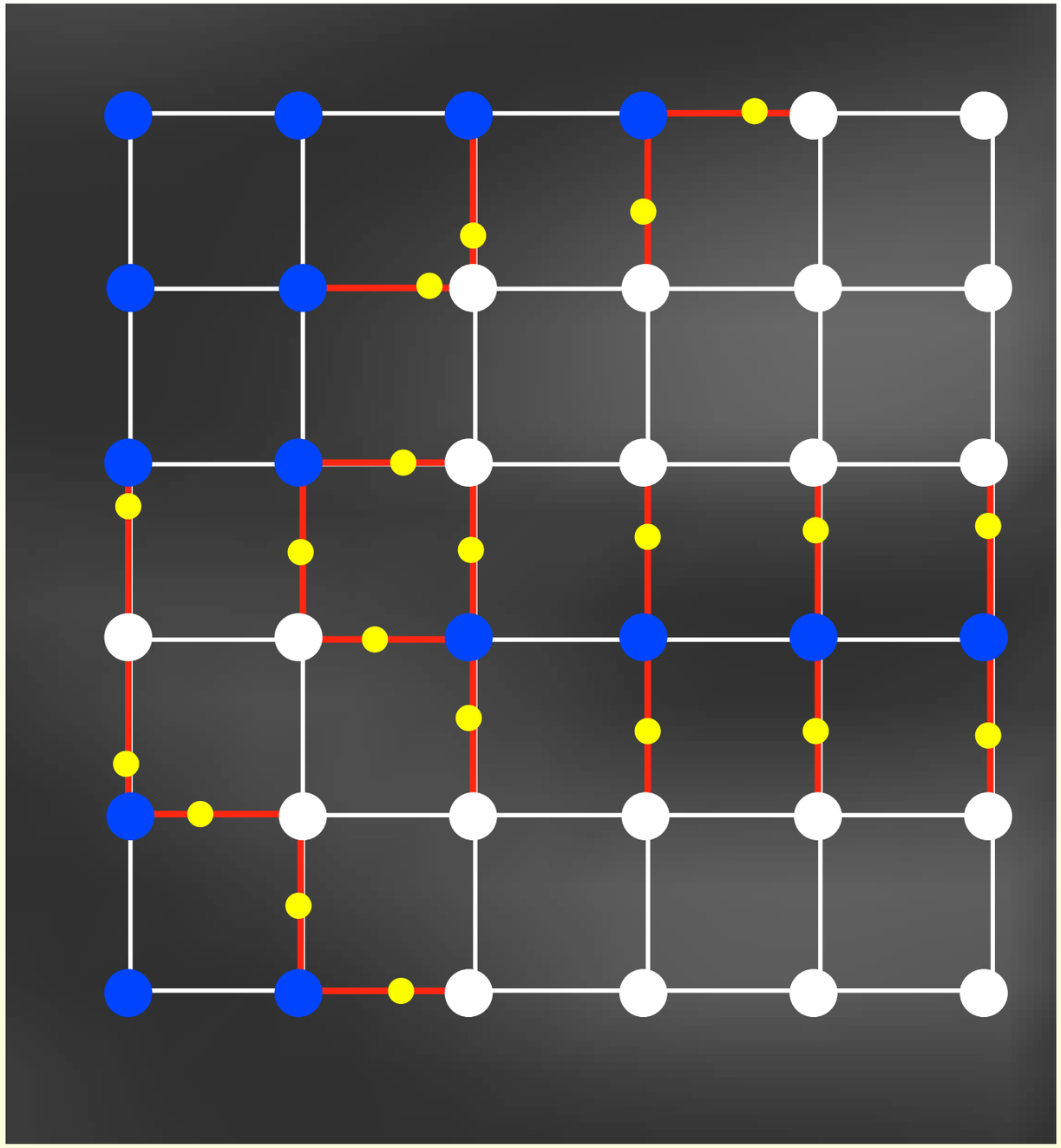
Marching Squares

- Given $I(\mathbf{x})$
  - $I(\mathbf{x}) < q$: $\mathbf{x}$ outside
  - $I(\mathbf{x}) > q$: $\mathbf{x}$ inside
- Discretize space
- Evaluate $f(\mathbf{x})$ on the grid



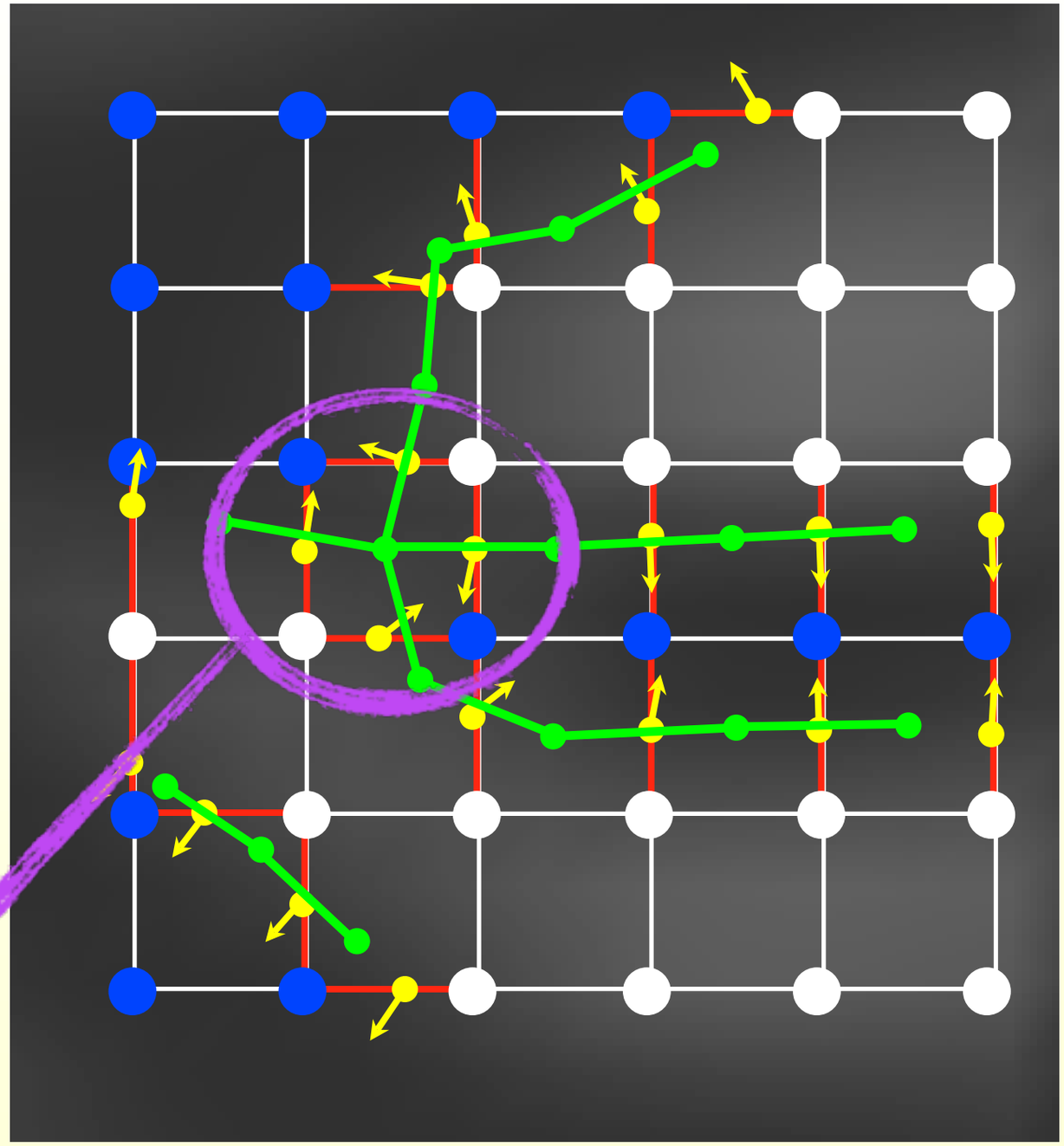
2D Dual Contouring

- Given $I(\mathbf{x})$
  - $I(\mathbf{x}) < q$: $\mathbf{x}$ outside
  - $I(\mathbf{x}) > q$: $\mathbf{x}$ inside
- Discretize space
- Evaluate $f(\mathbf{x})$ on the grid
- Classify grid points
- Classify grid edges
- Compute intersections



2D Dual Contouring

- Compute intersections
- Compute normals
- Compute dual points
  - for each voxel with sign change
- Connect dual points
  - across each red edge
- We could reintroduce ambiguity

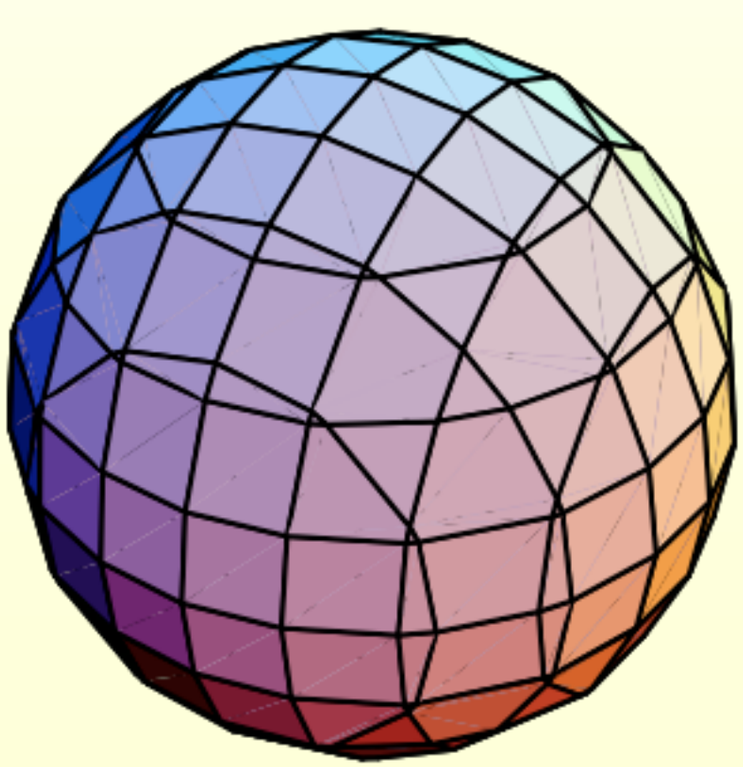


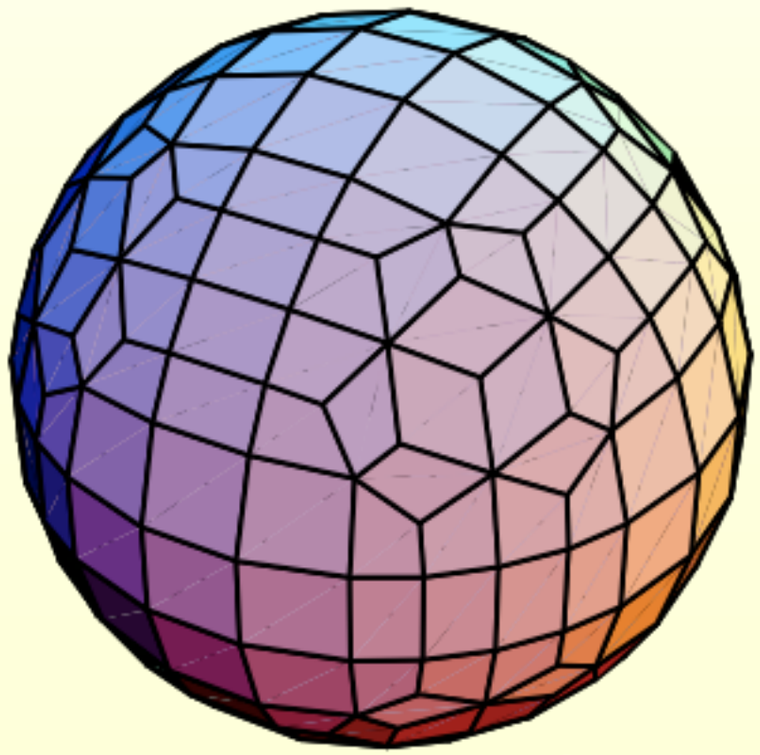
Dual Contouring

- No ambiguities
- Does not interpolate known surface points/normals
- No special cases for features
- No lookup tables
- Some trickiness in matrix pseudo-inverse
- Naturally produces quads, not triangles (in 3D)
  - What would dual marching tetrahedra produce?

Mesh quality

- Dual contouring produces higher-quality meshes
  - Flat surfaces never have bad triangles/quads
  - Control over placement of samples

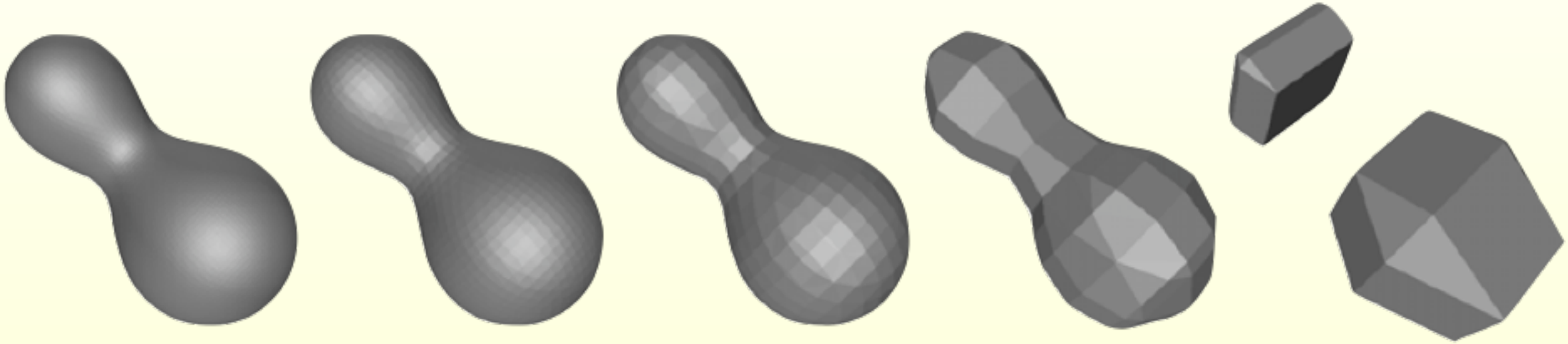




[Ju et al. 2002]

No Topological Guarantees

- Discrete Sampling: Expect Resolution Issues

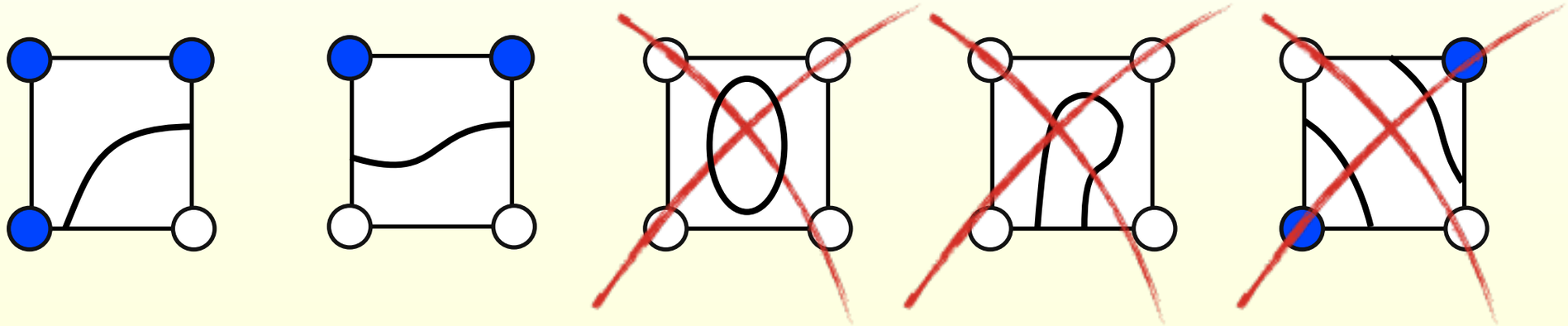


[Paul Bourke]

Isotopic Meshing

- Reduce mesh size until all features resolved
- Within each voxel that contains the surface:

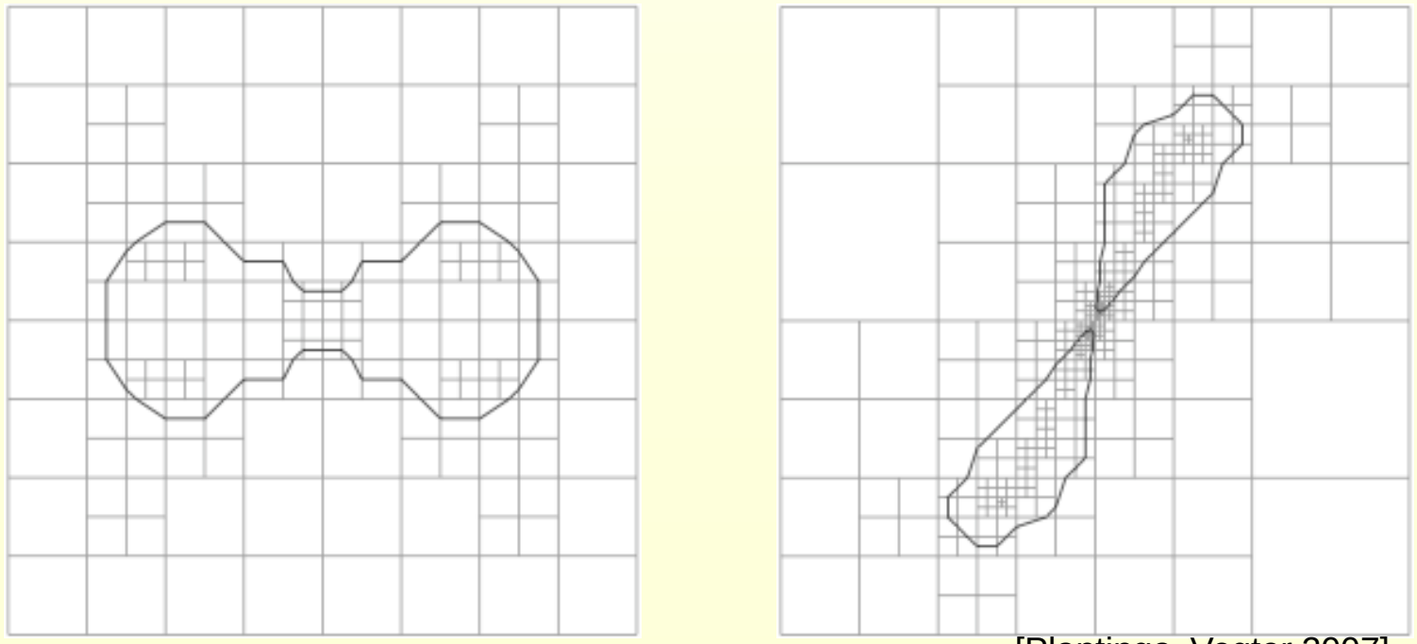
$$\nabla f(\mathbf{x}) \cdot \nabla f(\mathbf{y}) > 0 \quad \forall \mathbf{x}, \mathbf{y}$$



- Normals in a $\frac{\pi}{2}$ -cone

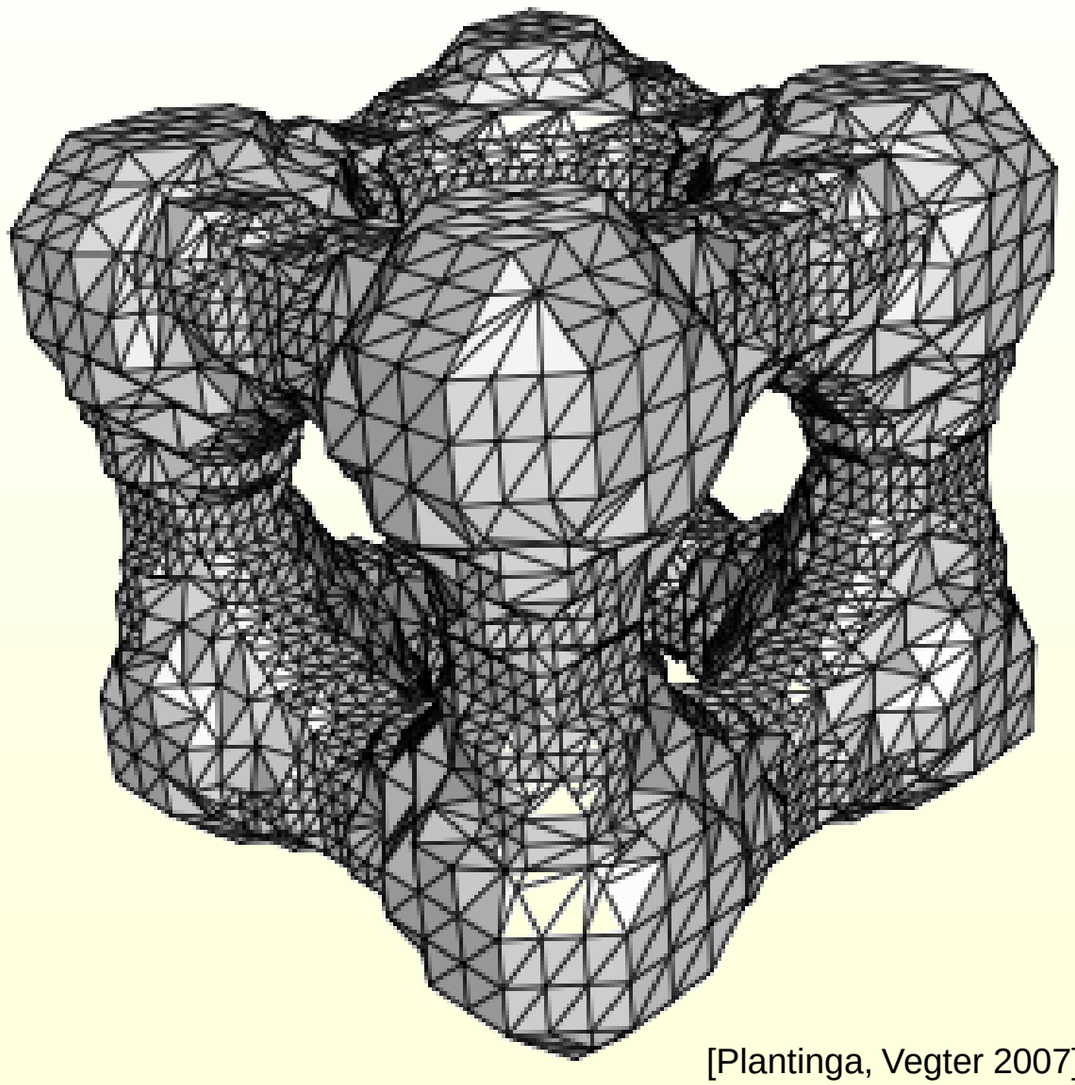
Isotopic Meshing

- Check the condition using interval arithmetic
  - Good for analytic surfaces
  - Expensive for sampled volume data
- Refine hierarchically: Use quadtree/octree
  - Less painful: Use tetrahedral subdivision



[Plantinga, Vegter 2007]

Isotopic Meshing



[Plantinga, Vegter 2007]

$$f(x,y,z) = x^4 - 5x^2 + y^4 - 5y^2 + z^4 - 5z^2 + 10$$

Summary

Measurements,
Simulation

Modeling

Marching Cubes

Volume Samples

Explicit Surface

Signed Distance Transform

Sample/Evaluate

Interpolate

Triangulate

Sampling, e.g. LDC

Raytracing

Implicit Surface

Surface Samples

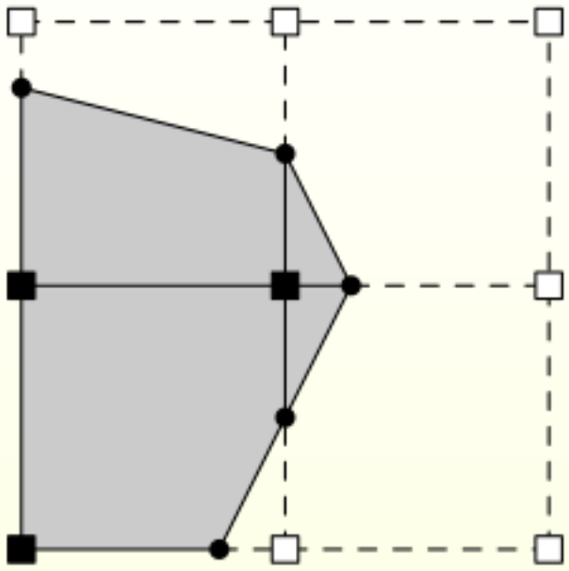
Define, e.g. MLS surface

Analytic Description,
Simulation

Measurements

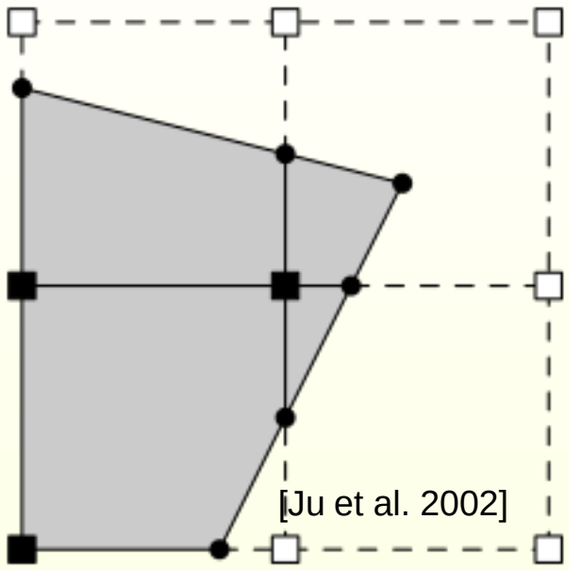
Marching Cubes Variants

Marching Cubes



- Lookup tables
- No sharp features

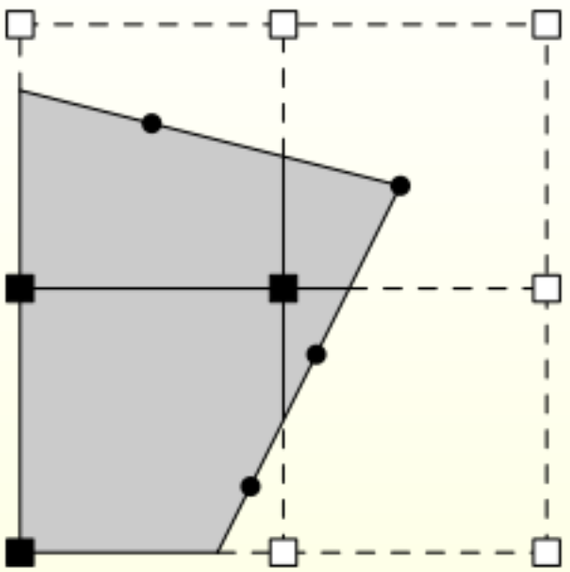
Extended Marching Cubes



[Ju et al. 2002]

- Hybrid method
- Edges/Corners are special cases
- Need feature threshold

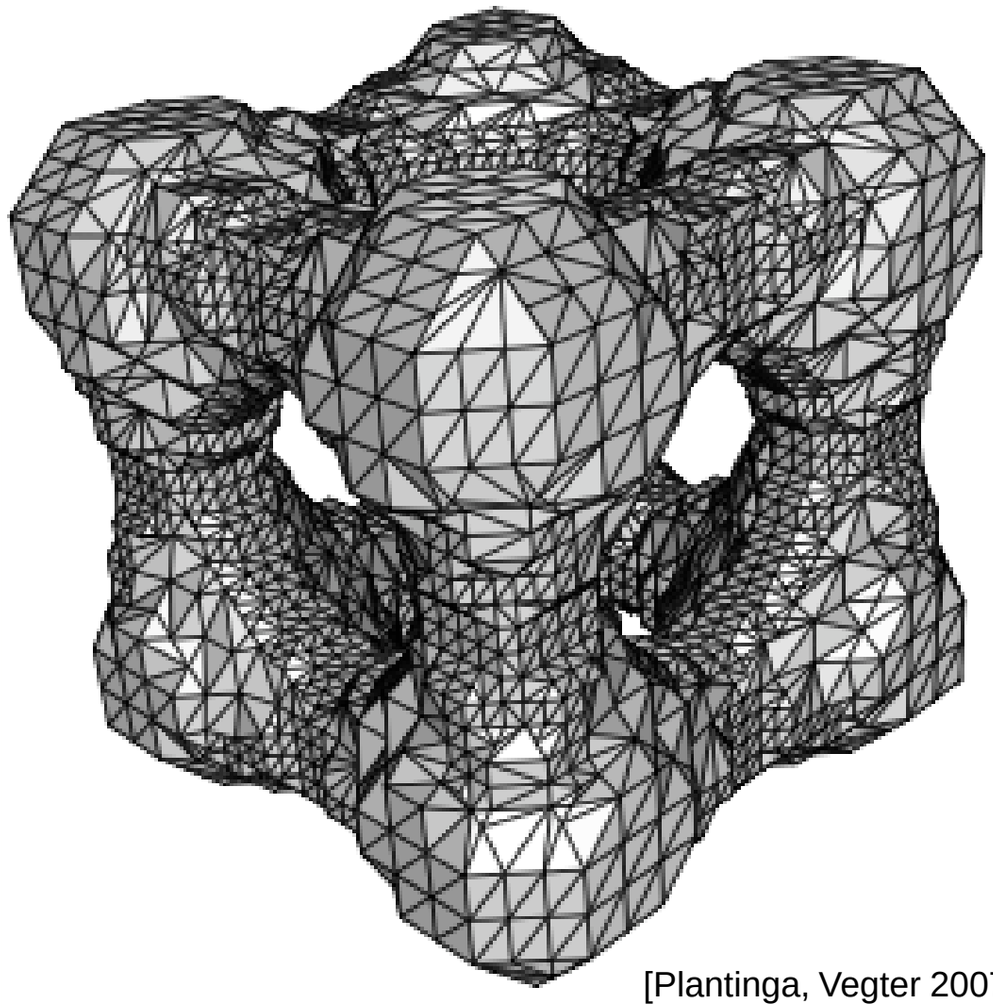
Dual Contouring



- Dual method
- No special cases
- No feature threshold

Topological Guarantees

- Enforced by subdivision
- Best for analytic surfaces
- Hierarchical refinement



[Plantinga, Vegter 2007]

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- Paul Bourke: http://local.wasp.uwa.edu.au/~pbourke/geometry/polygonise/