Announcements (from whiteboard)

Reading for this week:
- London 3 - lens
- London 15 - view camera
- Hecht, Optics, 5.1 - 5.3 (in reader)

Assignment #2 - sports & action - due Sunday eve
Outline

- why study lenses?
- geometrical optics
- depth of field
- aberrations
- vignetting, glare, and other lens artifacts
- diffraction
- measuring lens quality
Cameras and their lenses

- **single lens reflex (SLR) camera**
- **digital still camera (DSC)**, i.e. point-and-shoot
Lens quality varies

- Why is this toy so expensive?
  - EF 70-200mm f/2.8L IS USM
  - $1700

- Why is it better than this toy?
  - EF 70-300mm f/4-5.6 IS USM
  - $550

- Why is it so complicated?
Cutaway view of a real lens

Vivitar Series 1 90mm f/2.5
Cover photo, Kingslake, *Optics in Photography*
Image quality varies
Zoom lens versus prime lens

Canon 100-400mm f/3.5-5.6L zoom @ f/5.6

Canon 400mm f/5.6L @ f/5.6
Parameters of lenses

- zoom versus prime
- focal length (field of view)
- maximum aperture (minimum F-number, like f/2.8)
  - varies with focal length in a zoom lens
- image stabilization, faster autofocus, etc.
- minimum focusing distance
- other quality issues
- special-purpose lenses
  - fisheye
  - macro (1:1)
  - perspective control (a.k.a. tilt-shift)
Physical versus geometrical optics

✦ light can be modeled as traveling waves
✦ the perpendiculartes to waves can be drawn as rays
✦ diffraction causes these rays to bend, e.g. at a slit
✦ geometrical optics assumes
  • $\lambda \to 0$
  • no diffraction
  • straight rays in free space (a.k.a. rectilinear propagation)
Physical versus geometrical optics

physical optics

geometrical optics
Some definitions

- **object space** on the left; **image space** on the right
- if rays leaving a point arrive at another point (as shown), the optical system is called *stigmatic* for these two points
- $S$ and $P$ are called *conjugate points*
Snell’s law of refraction

This ratio is flipped. The correct form is $nt / ni$. This error triggered all the errors that followed in class. Each one is denoted in these notes with a comment box like this one. Sorry about that!
Snell’s law of refraction

- as waves change speed at an interface, they also change direction
- index of refraction is defined as the ratio between the speed of light in a vacuum / speed in some medium

\[
\frac{x_i}{x_t} = \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}
\]

I had this ratio flipped in class. It is corrected here.
Typical refractive indices ($n$)

- air = 1.0
- water = 1.33
- glass = 1.5 - 1.8
- microscope immersion oil = 1.52

- when transiting from air to glass, light bends towards the normal
- when transiting from glass to air, light bends away from the normal
- light striking a surface perpendicularly does not bend
Q. What shape should an interface be to make parallel rays converge to a point?

A. a hyperbola

✦ so lenses should be hyperbolic!
Spherical lenses

- two roughly fitting curved surfaces ground together will eventually become spherical
- spheres don’t bring parallel rays to a point
  - this is called spherical aberration
  - nearly axial rays behave best
The paraxial approximation

\[ e \approx 0 \]
\[ \cos \phi \approx 1 \]
\[ \sin \phi \approx \phi \]
\[ \tan \phi \approx \sin \phi \approx \phi \]

Paraxial approximation

\[ f = 50 \text{mm} \]
\[ N = \frac{F}{2.0} \]
\[ A = \frac{f}{N} = 25 \text{mm} \]
\[ \phi = \alpha \tan \left( \frac{3.5}{2 \times 50} \right) = 14^\circ \]
\[ 2\phi = 28^\circ \]
\[ \sin 14^\circ = 0.2419 \]
\[ \tan 14^\circ = 0.2493 \]
\[ h = 43.3 \text{mm diag} \]
\[ \text{FOV} = 2 \tan \left( \frac{h}{2f} \right) = 47^\circ \]
The paraxial approximation is a.k.a. first-order optics

- assume first term of \( \sin \phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots \)
  - i.e. \( \sin \phi \approx \phi \)

- assume first term of \( \cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \ldots \)
  - i.e. \( \cos \phi \approx 1 \)
  - so \( \tan \phi \approx \sin \phi \approx \phi \)
Paraxial refraction and focusing

- this derivation uses classical paraxial notation (letters for angles, instead of Greek symbols)
- Hecht’s derivation uses Fermat’s principle instead of Snell’s law, but the result is the same
Paraxial refraction and focusing:

Paraxial approx:
\[ \frac{i}{i'} = \frac{h}{h'} \]

Snell:
\[ \frac{\sin i}{\sin i'} = \frac{n}{n'} \]

Paraxial refraction and focusing:
\[ n \approx \frac{n'}{z} \]
\[ n(u+a) = n'(u'-a) \]
\[ n(h/z + h/r) = n'(h/z' - h/r) \]
\[ \frac{n}{z} + \frac{n'}{2z'} \approx \frac{n' - h}{r} \]

This is flipped too. It should be \( \frac{n'}{n} \). Then the paraxial approximation should read \( \frac{i}{i'} = \frac{n}{n'} \), or \( ni = n'i' \) (as I had originally written). The rest of the derivation (below) is correct. Ignore the question mark I wrote on the whiteboard (below).
Paraxial refraction and focusing

\[ \frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \]

(Hecht)
Paraxial refraction and focusing

- The case where $s_i$ is at infinity...

\[ s_0 = \frac{n_4 R}{n_2 - n_1} \]

\[ s_i = 0 \]
Thin lens equation, a.k.a. lensmaker’s formula

✦ we just derived cases (a) and (b)
✦ for a thin lens in air, apply (c), then (a) with air and glass reversed, then set \( d = 0 \)
Thin lens equation, a.k.a. lensmaker’s formula

- we just derived cases (a) and (b)
- for a thin lens in air, apply (c), then (a) with air and glass reversed, then set $d = 0$
Thin lens equation, a.k.a. lensmaker’s formula

\[ \frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]  

(Hecht, eqn 5.15)
Thin lens equation, a.k.a. lensmaker’s formula

\[ \frac{1}{S_o} + \frac{1}{S_i} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]  

(Hecht, eqn 5.15)
Gaussian lens formula

✧ a geometrical derivation
✧ begins with Gauss’ ray diagram
Gaussian lens formula

- positive $s_i$ is rightward, positive $s_o$ is leftward
- positive $y$ is upward
Gaussian lens formula

\[
\frac{|y_i|}{y_o} = \frac{s_i}{s_o}
\]
Gaussian lens formula

\[ \frac{|y_i|}{y_o} = \frac{s_i}{s_o} \quad \text{and} \quad \frac{|y_i|}{y_o} = \frac{s_i - f}{f} \quad \ldots \quad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \]
Changing the focus distance (again)

- note that at $s_o = s_i = 2f$, we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

- in 1:1 imaging, if the sensor is 36mm wide, an object 36mm wide will fill the frame

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$
Thick lenses

- an optical system may contain many lenses, but can be characterized by a few numbers
Center of perspective

- In a thin lens, the *chief ray* traverses the lens (through its optical center) without changing direction.
- In a thick lens, the intersections of this ray with the optical axis are called the *nodal points*.
- For a lens in air, these coincide with the *principal points*.
- The first nodal point is the *center of perspective*.
Convex versus concave lenses

- positive focal length $f$ means parallel rays from the left converge to a point on the right
- negative focal length $f$ means parallel rays from the left converge to a point on the left (dashed lines above)

rays from a convex lens converge

rays from a concave lens diverge

(Hecht)
Convex versus concave lenses

Rays from a convex lens converge...producing a real image.

Rays from a concave lens diverge...producing a virtual image.
Convex versus concave lenses

...producing a real image

...producing a virtual image
A menagerie of lenses

Q. Given the lensmaker’s formula, how do you tell if parallel rays entering a lens will converge or diverge?

\[
\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}
\]
The power of a lens

\[ P = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f} \]

✦ units are meters\(^{-1}\)
✦ a.k.a. diopters

✦ my eyeglasses have the prescription
  • right eye: -0.75 diopters
  • left eye: -1.00 diopters

Q. What’s wrong with me?
A. Myopia
Newtonian form of the lens equation

\[ \frac{|y_i|}{y_o} = \frac{s_i}{s_o} \quad \text{and} \quad \frac{|y_i|}{y_o} = \frac{s_i - f}{f} \quad \text{imply} \quad x_o x_i = f^2 \quad \text{(Hecht, eqn 5.23)} \]
Magnification

- lateral magnification

\[ M_T \triangleq \frac{y_i}{y_o} = -\frac{s_i}{s_o} \]  
(Hecht, eqn 5.24)

- negative for a convex lens, because it inverts the image

- longitudinal magnification

\[ M_L = \frac{dx_i}{dx_o} = \frac{-f^2}{x_o^2} = -M_T^2 \]  
(Hecht, eqn 5.25)

- equal to the (negative) square of lateral magnification

In my lecture I showed this as \( dx_o/dx_i \). That is flipped. The correct formula is shown above.
Example: 100× microscope objective

- 1 micron laterally on specimen becomes 100 microns at a microscope’s camera sensor (about 15 pixels)
- 1 micron axially on specimen becomes 10,000 microns (10mm) at the sensor - well beyond the depth of focus
- depth of field of a 100× objective is less than 1 micron
Lenses perform a 3D perspective transform

- lenses transform a 3D object to a 3D image; the sensor extracts a 2D slice from that image
- as an object moves linearly toward the camera, its image moves non-proportionately
- as you move a sensor (or lens) linearly, the in-focus object plane moves non-proportionately
- as you refocus a camera, the image changes size!
Lenses perform a 3D perspective transform
Stops

- in photographic lenses, the aperture stop (A.S.) is typically in the middle of the lens system
- in a digital camera, the field stop (F.S.) is the edge of the sensor; no physical stop is needed
• the *entrance pupil* is the image of the aperture stop as seen from an axial point on the object
• the *exit pupil* is the image of the aperture stop as seen from an axial point on the image plane
• the center of the entrance pupil is the center of perspective
• you can find this point by following two lines of sight
Depth of field

- lower N means a wider aperture and less depth of field

\[ N = \frac{f}{A} \]
Circle of confusion (C)

- C depends on sensing medium, reproduction medium, viewing distance, human vision,...
  - for print from 35mm film, 0.02mm is typical
  - for high-end SLR, 6µ is typical (1 pixel)
  - less if downsizing for web, or lens is poor
Depth of field formula

\[ \frac{C}{M_T} \]

- DoF is asymmetrical around the in-focus object plane
- conjugate in object space is typically bigger than C

The shaded diamond on the right, in image space, is called the depth of focus. The shaded diamond on the left, in object space, is called the depth of field.
Depth of field formula

\[ \frac{C}{M_T} \approx \frac{CU}{f} \]

- DoF is asymmetrical around the in-focus object plane
- conjugate in object space is typically bigger than C
Depth of field formula

\[ \frac{C}{M_t} \approx \frac{CU}{f} \]

\[ \frac{D_1 f}{CU} = \frac{U - D_1}{f / N} \quad \cdots \quad D_1 = \frac{NCU^2}{f^2 + NCU} \]

\[ D_2 = \frac{NCU^2}{f^2 - NCU} \]
Depth of field formula

\[ D_{TOT} = D_1 + D_2 = \frac{2NCU^2 f^2}{f^4 - N^2C^2U^2} \]

- \( N^2C^2D^2 \) can be ignored when conjugate of circle of confusion is small relative to the aperture

\[ D_{TOT} \approx \frac{2NCU^2}{f^2} \]

- where
  - \( N \) is F-number of lens
  - \( C \) is circle of confusion (on image)
  - \( U \) is distance to in-focus plane (in object space)
  - \( f \) is focal length of lens
$D_{TOT} \approx \frac{2NCU^2}{f^2}$

- $N = f/4.1$
- $C = 2.5\mu$
- $U = 5.9m$ (19’)
- $f = 73mm$ (equiv to 362mm)
- $D_{TOT} = 132mm$

1 pixel on this video projector

- $C = 2.5\mu \times 2816 / 1024$ pixels
- $D_{EFF} = 363mm$
$N = f/6.3$
$C = 2.5 \mu$
$U = 17m (56')$
$f = 27mm$ (equiv to 135mm)
$D_{TOT} = 12.5m (41')$

1 pixel on this video projector
$C = 2.5 \mu \times 2816 / 1024$ pixels
$D_{EFF} = 34m (113')$
\[ N = f/5.6 \]
\[ C = 6.4\mu \]
\[ U = 0.7m \]
\[ f = 105mm \]
\[ D_{TOT} = 1.6mm \]

1 pixel on this video projector:
\[ C = 6.4\mu \times 5616 / 1024 \text{ pixels} \]
\[ D_{EFF} = 8.7mm \]
These numbers were replaced on 6/4/09, after a student pointed out that they didn't work out. I'm still not confident in them.

\[ N = f/2.8 \]
\[ C = 6.4\mu \]
\[ U = 31\text{mm} \]
\[ f = 65\text{mm} \]

(use \( N' = (1+MT)N \) at short conjugates (\( MT=5 \) here)) = f/16

\[ D_{TOT} = 0.048\text{mm}! \quad (48\mu) \]
Sidelight: macro lenses

\[
\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}
\]

Q. How can the Casio EX-F1 at 73mm and the Canon MP-E 65mm macro, which have similar \( f \)'s, have such different focusing distances?

✧ A. Because they are built to allow different \( s_i \)
  - this changes \( s_o \), which changes magnification \( M_T \triangleq -s_i / s_o \)
  - macro lenses are well corrected for aberrations at short \( s_o \)
Extension tube: converts a normal lens to a macro lens

- toilet paper tube, black construction paper, masking tape
- camera hack by Katie Dektar (CS 178, 2009)
DoF is linear with aperture

\[ D_{TOT} \approx \frac{2NCU^2}{f^2} \]

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178-09/applets/dof.swf

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(f/2.8)

(f/32)
DoF is quadratic with focusing distance

(we already know this, because $M_T$ scales with $U$, and $M_L$ goes as the square of $M_T$)

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178-09/applets/dof.swf
Hyperfocal distance

- the back depth of field

\[ D_2 = \frac{NCU^2}{f^2 - NCU} \]

- becomes infinite if

\[ U \geq \frac{f^2}{NC} \triangleq H \]

- In that case, the front depth of field becomes

\[ D_1 = \frac{H}{2} \]

- so if I had focused at 32m, everything from 16m to infinity would be in focus on an HD projector, including the men

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I incorrectly stated in class that when you plug \( U = \frac{f^2}{NC} \) into the formula for \( D_2 \) you get 0/0. You get \( H/0 \), which is infinity. And when you plug it into the formula for \( D_1 \) you get \( H/2 \), as shown on the slide.

This calculation earlier (and incorrectly) assumed an HD projector. The numbers actually work out for a normal projector, as shown. Note added 5/4/09.

http://graphics.stanford.edu/courses/cs178-09/applets/dof.swf

Last slide covered on Tuesday. The remaining slides were covered on Thursday.
DoF is inverse quadratic with focal length

\[
D_{\text{TOT}} \approx \frac{2NCU^2}{f^2}
\]

On a point and shoot camera, depth of field is always large. Why? After all, the N’s are comparable. But the focal lengths f’s are typically smaller, reflecting the physically smaller camera. Thus, depth of field is much larger.
DoF and the dolly-zoom

- if we zoom in (change $f$) and stand further back (change $U$) by the same factor
  
  \[ D_{TOT} \approx \frac{2NCU^2}{f^2} \]

- the depth of field stays the same!
  
  • useful for macro when you can’t get close enough

50mm f/4.8

200mm f/4.8, moved back 4× from subject
Parting thoughts on DoF: the zen of *bokeh*

- the appearance of sharp out-of-focus features in a photograph with shallow depth of field
  - determined by the shape of the aperture
  - people get religious about it
  - but not every picture with shallow DoF has evident bokeh...

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Bokeh is pronounced bow'-keh, with the emphasis on the 1st syllable. It's from Japanese. I pronounced it incorrectly in class.
Parting thoughts on DoF: seeing through occlusions

- depth of field is not a convolution of the image
  - i.e. not the same as blurring in Photoshop
  - DoF lets you eliminate occlusions, like a chain-link fence

(Fredo Durand)
Seeing through occlusions