Noise and ISO

CS 178, Spring 2011

Marc Levoy
Computer Science Department
Stanford University
Outline

- examples of camera sensor noise
  - don’t confuse it with JPEG compression artifacts
- probability, mean, variance, signal-to-noise ratio (SNR)
- laundry list of noise sources
  - photon shot noise, dark current, hot pixels, fixed pattern noise, read noise
- SNR (again), dynamic range (DR), bits per pixel
- ISO

- denoising
  - by aligning and averaging multiple shots
  - by image processing will be covered in a later lecture
Nokia N95 cell phone at dusk

- 8×8 blocks are JPEG compression
- Unwanted sinusoidal patterns within each block are JPEG’s attempt to compress noisy pixels
Canon 5D II at noon

- ISO 200
- f/13.0
- 1/320 sec
- RAW w/o denoising

chromatic aberration!
Canon 5D II at noon

- ISO 200
- f/13.0
- 1/320 sec
- RAW w/o denoising

post-processed using Canon software
Canon 5D II at dusk

- ISO 6400
- f/4.0
- 1/13 sec
- RAW w/o denoising
Canon 5D II at dusk
Canon 5D II at dusk

- ISO 6400
- f/4.0
- 1/13 sec
Photon shot noise

- the number of photons arriving during an exposure varies from exposure to exposure and from pixel to pixel
- this number is governed by the Poisson distribution
Poisson distribution

- expresses the probability that a certain number of events will occur during an interval of time

- applicable to rare events that occur
  - with a known average rate, and
  - independently of the time since the last event

- if on average $\lambda$ events occur in an interval of time, the probability $p$ that $k$ events occur instead is

\[
p(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}
\]
Mean and variance

- the mean of a probability density function $p(x)$ is
  \[ \mu = \int x p(x) \, dx \]
- the variance of a probability density function $p(x)$ is
  \[ \sigma^2 = \int (x - \mu)^2 p(x) \, dx \]
- the mean and variance of the Poisson distribution are
  \[ \mu = \lambda \]
  \[ \sigma^2 = \lambda \]
- the standard deviation is
  \[ \sigma = \sqrt{\lambda} \]

Deviation grows slower than the average.
Signal-to-noise ratio (SNR)

\[ SNR = \frac{\text{mean pixel value}}{\text{standard deviation of pixel value}} = \frac{\mu}{\sigma} \]

\[ SNR \ (dB) = 20 \log_{10} \left( \frac{\mu}{\sigma} \right) \]

example

• if SNR improves from 100:1 to 200:1, it improves \(20 \log_{10}(200) - 20 \log_{10}(100) = +6 \text{ dB}\)
Photon shot noise (again)

- photons arrive in a Poisson distribution
  \[ \mu = \lambda \]
  \[ \sigma = \sqrt{\lambda} \]
  so
  \[ SNR = \frac{\mu}{\sigma} = \sqrt{\lambda} \]

- shot noise scales as square root of number of photons

- examples
  - doubling the width and height of a pixel increases its area by 4x, hence # of photons by 4x, hence SNR by 2x or +6 dB
  - opening the aperture by 1 f/stop increases the # of photons by 2x, hence SNR by \( \sqrt{2} \) or +3 dB

I could sense from the questions in class that it seemed surprising to many students that SNR could rise as a scene gets brighter (a good thing) even though noise is rising at the same time (a bad thing).

Here’s a simple example. If on average 9 photons arrive at a pixel during an exposure, the standard deviation of this (according to the Poisson distribution) is \( \sqrt{9} = 3 \) photons. This means that \( SNR = \frac{mean}{stddev} = \frac{9}{3} = 3:1 \). Now suppose instead that 100 photons arrive at the pixel, either because the scene got brighter or we increased the exposure time or we switched to a camera with bigger pixels. Now the stddev is \( \sqrt{100} = 10 \), and \( SNR = \frac{100}{10} = 10:1 \). The noise got worse (stddev of 10 photons versus 3 photons), but the SNR got better (10:1 versus 3:1). The apparent image quality will be better in the second case.
Empirical example

- Kodak Q14 test chart

- Canon 10D, ISO 1600, crop from JPEG image

- noise grows as $\sqrt{\text{signal}}$
- better SNR in light tile than in dark tile
- after gamma transform, you see noise only in the dark tile

(http://www.imatest.com/docs/noise.html)
Dark current

- electrons dislodged by random thermal activity
- increases linearly with exposure time
- increases exponentially with temperature
- varies across sensor, and includes its own shot noise

Canon 20D, 612 sec exposure

As I mentioned in class, “shot noise” is a vague term referring to random fluctuations that arise when counting numbers of particles (photons, electrons). What’s more important is to remember the difference between “photon shot noise” and other sources of random fluctuations that affect photographs, like dark current shot noise as described on this slide.

Don’t confuse with photon shot noise.

(http://theory.uchicago.edu/~ejm/pix/20d/tests/noise/)
Hot pixels

- electrons leaking into well due to manufacturing defects
- increases linearly with exposure time
- increases with temperature, but hard to model
- changes over time, and every camera has them

Canon 20D, 15 sec and 30 sec exposures
Fixing dark current and hot pixels

例：
- Aptina MT9P031（在诺基亚N95手机）
  - 有效满阱容量 = ~8500电子/像素
  - 黑色电流 = 25电子/像素/秒

解法#1：冷却传感器
- Retiga 4000R生物成像相机
  - Peltier冷却25°C低于环境
  - 有效满阱容量 = 40,000电子/像素
  - 黑色电流 = 1.64电子/像素/秒

解法#2：暗帧减法
- 可以在高端SLR上使用
  - 补偿平均黑色电流
  - 也补偿热像素和FPN
Fixed pattern noise (FPN)

- manufacturing variations across pixels, columns, blocks
- mainly in CMOS sensors
- doesn’t change over time, so read once and subtract
Read noise

- thermal noise in readout circuitry
- again, mainly in CMOS sensors
- not fixed pattern, so only solution is cooling

Canon 1Ds Mark III, cropped

this image tainted by JPEG artifacts?
Recap

- **photon shot noise**
  - unavoidable randomness in number of photons arriving
  - grows as the square root of the number of photons, so brighter lighting and longer exposures will be less noisy

- **dark current noise**
  - grows with exposure time and sensor temperature
  - minimal for most exposure times used in photography
  - correct by subtraction, but only corrects for average dark current

- **hot pixels, fixed pattern noise**
  - caused by manufacturing defects, correct by subtraction

- **read noise**
  - electronic noise when reading pixels, unavoidable

Questions?
Signal-to-noise ratio
(with more detailed noise model)

\[
SNR = \frac{\text{mean pixel value}}{\text{standard deviation of pixel value}} = \frac{\mu}{\sigma}
\]

\[
= \frac{P Q_e t}{\sqrt{P Q_e t + D t + N_r^2}}
\]

where

\(P\) = incident photon flux (photons/pixel/sec)
\(Q_e\) = quantum efficiency
\(t\) = exposure time (sec)
\(D\) = dark current (electrons/pixel/sec), including hot pixels
\(N_r\) = read noise (rms electrons/pixel), including fixed pattern noise

SNR changes with scene brightness, aperture, and exposure time.

(formula from http://learn.hamamatsu.com/articles/ccdsnr.html)
Signal-to-noise ratio
(with more detailed noise model)

\[
SNR = \frac{\text{mean pixel value}}{\text{standard deviation of pixel value}} = \frac{\mu}{\sigma}
\]

\[
= \frac{P Q_e t}{\sqrt{P Q_e t + D t + N_r^2}}
\]

✨ examples

- Retiga 4000R = \( (1000 \times 55\%) / \sqrt{(1000 \times 55\% + 1.64 + 12^2)} \)
  = 20.8:1 assuming 1000 photons/pixel/sec for 1 second

- Aptina MT9P031 = \( (1000 \div 11 \times 69\%) / \sqrt{(1000 \div 11 \times 69\% + 25 + 2.6^2)} \)
  = 6.5:1 assuming pixels are 1/11 as large as Retiga’s

✨ for 10 photons/pixel/sec for 100 seconds

- Retiga = 18.7:1
- Aptina = 1.2:1

Don’t use your cell phone for astrophotography!
Dynamic range

\[ DR = \frac{\text{max output swing}}{\text{noise in the dark}} = \frac{\text{saturation level} - D t}{\sqrt{D t + N_r^2}} \]

- examples
  - Retiga 4000R = \(\frac{40,000 - 1.64}{\sqrt{1.64 + 12^2}}\) = 3,313:1 (11.7 bits) for a 1 second exposure
  - Aptina MT9P031 = \(\frac{8500 - 25}{\sqrt{25 + 2.6^2}}\) = 1500:1 (10.5 bits) for a 1 second exposure

- determines precision required in ADC, and useful # of bits in RAW image
- any less than ~10 bits would be < 8 bits after gamma transform for JPEG encoding, and you would see quantization artifacts
Low-light cameras

\[ DR = \frac{\text{max output swing}}{\text{noise in the dark}} \]

\[ DR = \frac{\text{saturation level} - D t}{\sqrt{D t + N_r^2}} \]

- Andor iXon+888 back-illuminated CCD
  - $40,000
- performance
  - \( DR = \frac{(80,000 - 0.001)}{\sqrt{(0.001 + 6^2)}} \approx 13,333:1 \) (13.7 bits) for a 1 second exposure
  - “electron multiplication” mode
    - \( DR = \frac{(80,000 - 0.001)}{\sqrt{(0.001 + <1^2)}} \approx 80,000:1 \) (16.2 bits)
    - “can see a black cat in a coal mine”
- if cooled to -75º C
- compare to 10.5 bits for Aptina
- don’t use your cell phone for fluorescence microscopy!
amplifies signal before quantization by ADC

- if you quantize a low signal, then brighten it in Photoshop, you will see quantization artifacts (contouring)
- amplification also reduces the impact of read noise, since amplification occurs early in the reading process
- so raising the ISO improves SNR

- doubling ISO doubles the signal, which is linear with light
  - so effect on signal is the same as $2 \times$ exposure time, or $-1$ f/stop
  - maximum ISO on Canon 5D II is 6400; higher ISOs implemented using multiplication after ADC?

- but raising exposure time typically improves SNR faster
  - thus, maximize exposure time to the limits imposed by object motion, camera shake, or sensor saturation, then maximize ISO to the limit imposed by ADC saturation
SNR and ISO over the years

- SNR has been improving with better sensor designs
- but total # of megapixels has risen to offset these improvements, making pixels smaller, so SNR in a pixel has remained static
- display resolutions have not risen as fast as megapixels, so we’re increasingly downsizing our images for display
- if you average 4 camera pixels to produce 1 for display, SNR doubles, so for the same display area, SNR has been improving
- this allows higher ISOs to be used in everyday photography

(http://www.dxomark.com/index.php/eng/Insights/SNR-evolution-over-time)
Nikon D3S, ISO 25,600, denoised in Lightroom 3, photograph by Fredo Durand
Nikon D3S, ISO 25,600, denoised in Lightroom 3, photograph by Fredo Durand
RAW image from camera, before denoising in Lightroom
Fredo says it was nearly too dark to read the menu, so it really looked like this (darkened)
or maybe it looked like this? (tone mapped to approximate human dark adaptation)
Recap

- **signal-to-noise ratio (SNR)** is mean/stddev of pixel value
  - rises with $\sqrt{\text{brightness and/or exposure time}}$
  - depends also on dark current and read noise
  - poor for short exposures and very long exposures

- **dynamic range (DR)** is max swing / noise in the dark
  - fixed for a particular sensor and exposure time
  - determines # of useful bits in RAW image

- **ISO** is amplification of signal before conversion to digital
  - maximize exposure time until camera or object blurs, then maximize ISO, making sure not to saturate
  - can combine multiple short-exposure high-ISO pictures

Questions?
Slide credits

- Eddy Talvala