Color I: trichromatic theory

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Outline

- spectral power distributions
- color response in animals and humans
- 3D colorspace of the human visual system
  - and color filter arrays in cameras
- reproducing colors using three primaries
- additive versus subtractive color mixing
- cylindrical color systems used by artists (and Photoshop)
- chromaticity diagrams
  - color temperature and white balancing
  - standardized color spaces and gamut mapping
Newton’s Experimentum Crucis

- sunlight can be divided into colors using a prism
- these colors cannot be further divided using a 2nd prism
- experiment performed 1665, drawing made in 1672
Newton’s Experimentum Crucis

- alternatively, the divided colors can be recombined using a lens and 2nd prism into a new beam that has exactly the same properties as the original
The visible light spectrum

- wavelengths between 400nm and 700 nm (0.4µ - 0.7µ)
- exactly the colors in a rainbow
The visible light spectrum

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Spectral power distribution (SPD)

- units of power are watts (joules per second)
- shown here are spectra of common illumination sources
- plots above are relative amounts (%) of each wavelength

(LampTech)
Interaction of light with matter

- spectrum of illumination is multiplied wavelength-by-wavelength by reflectance spectrum of object
  - cause is absorption by the material
  - so the spectrum you see depends on the illumination
- transmittance operates the same way

\[
\text{illumination} \times \text{reflectance} = \text{stimulus that enters your eye}
\]
Example

Low Pressure Sodium (SOX)

my old van

= black
Examples of reflectance spectra

- two different spectra may appear alike to us
  - white petal and white flower (above left)
  - these are called metamers
- Newton observed this, but could not explain it

Questions?

- two reflectance spectra that match (i.e. are metamers) under one illuminant may not match under another
- clothes that match in the store may not match outdoors
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1. organisms having only one kind of retinal receptor cannot distinguish changes in intensity from changes in wavelength, hence they have no color discrimination
   - for example a unit amount of $\lambda_1$ versus $\lambda_2$ above
   - or a unit amount of $\lambda_1$ versus half as much of $\lambda_3$ (assuming the sensitivity to $\lambda_3$ is twice the response to $\lambda_1$)
   - example: horseshoe crab
2. this organism can discriminate a response in the range wavelengths covered by A versus B, but cannot discriminate within those ranges

3. this organism has color discrimination over the range of wavelengths shown
   - for each wavelength within this range, the ratio of responses of receptors A and B is unique; hence the organism can identify which wavelength (e.g. $\lambda_1$ or $\lambda_2$) it’s looking at

4. this organism has a larger range of color vision
   - example: dog, horse
5. humans can discriminate wavelengths from 400nm to 700nm
   - we can also discriminate mixtures of wavelengths that
dichromats cannot; this will become clearer later

❖ at the retinal level, our response to light is linear

a. if the response to a unit stimulus at $\lambda_1$ of is $(\rho_1, \gamma_1, \beta_1)$, and to
   a unit stimulus at $\lambda_2$ is $(\rho_2, \gamma_2, \beta_2)$, then the response to a
   superposition of stimuli $\lambda_1$ and $\lambda_2$ is $(\rho_1 + \rho_2, \gamma_1 + \gamma_2, \beta_1 + \beta_2)$
b. the response to $n$ units of a stimulus at $\lambda_1$ is $(n \rho_1, n \gamma_1, n \beta_1)$
c. a system that obeys superposition (a) and scaling (b) is linear

Trichromats
(contents of whiteboard)
Human response to an arbitrary stimulus

- spectrum of stimulus arriving in one small area on retina
  \[ \times \]
  - spectral sensitivity of each type of cone (L,M,S)
  \[ = \]
  - multiply wavelength-by-wavelength to get response spectra
    \[ \int \]
    - integrate over wavelengths to get total response for that type of cone

- output is three numbers (\( \rho, \gamma, \beta \)) per area on retina
Human response to an arbitrary stimulus

- stated algebraically, given a stimulus spectrum $L_e(\lambda)$, the human response to it ($\rho$, $\gamma$, $\beta$) are the integrals over all visible wavelengths of our responses
  
  $L_e(\lambda) \rho(\lambda)$,
  $L_e(\lambda) \gamma(\lambda)$,
  $L_e(\lambda) \beta(\lambda)$

  to each constituent wavelength $\lambda$, i.e.

  $$ (\rho, \gamma, \beta) = \left( \int_{400 \text{ nm}}^{700 \text{ nm}} L_e(\lambda) \rho(\lambda) \, d\lambda, \int_{400 \text{ nm}}^{700 \text{ nm}} L_e(\lambda) \gamma(\lambda) \, d\lambda, \int_{400 \text{ nm}}^{700 \text{ nm}} L_e(\lambda) \beta(\lambda) \, d\lambda \right) $$

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Human 3D colorspace

- the three types of cones in our retina (Long, Medium, Short wavelength) define the axes of a three-dimensional space
- our response to any stimulus spectrum can be summarized by three numbers $(\rho, \gamma, \beta)$ and plotted as a point in this space
- our responses to all visible single-wavelength spectra (a.k.a. pure wavelengths $\lambda$, i.e. positions along the rainbow), if connected together, form a curve in this space, called the locus of spectral colors; the sequence of $(\rho, \gamma, \beta)$ numbers form the tristimulus sensitivity functions $\rho(\lambda)$, $\gamma(\lambda)$, and $\beta(\lambda)$

http://graphics.stanford.edu/courses/cs178/applets/locus.html
1. our response to any mixture ($\Sigma = 1$) of two pure wavelengths falls on a line connecting the responses to each wavelength

2. our response to any mixture ($\Sigma = 1$) of three pure wavelengths falls on a triangle connecting the responses to each wavelength; our response to any mixture or scaling ($\Sigma \leq 1$) of three pure wavelengths falls in a tetrahedron defined by this triangle and the origin

3. our responses to all possible mixtures or scalings ($\Sigma \leq 1$) of all visible wavelengths forms an irregular volume called the *gamut of perceivable colors*, equal to the convex hull of the spectral locus
4. to a deuteranope - a color-blind person who is missing their medium-wavelength receptor, i.e. their gamma receptor - this diagram is squashed into the rectangle shown above on the rho-beta plane

- as a result, spectra whose \((\rho, \gamma, \beta)\) responses lie along the dotted lines cannot be distinguished; they will appear as the same color, i.e. as metamers
- by a similar argument, many spectra distinguishable to pentachromats (e.g. Mallard ducks) are indistinguishable to trichromats (humans)
Color blindness

- protanopia (1% of males)
- deuteranopia (1% of males)
- tritanopia (< 1% of both genders)

(wikipedia)

- protanomaly (1% of males)
- deuteranomaly (6% of males)
- tritanomaly (< 1% of both genders)
The advantage of being color blind

- the maze (at left) is recreated (at right) using subtle intensity differences, but overridden by stronger red-green color differences
- only a deuteranope can see the maze at right
Canon 30D color filters

- you want the camera’s R, G, and B color filters to have the same spectral sensitivities as our L, M, and S cones
  - you don’t want objects in the real world to be metamers to one system and not the other
  - otherwise, colored patterns the camera sees might be invisible to a person (bad), or patterns you see might be invisible to a camera (also bad)

http://graphics.stanford.edu/courses/cs178/applets/locus.html

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Maxwell’s color matching experiment

- Maxwell actually used a slightly different procedure
  - see http://www.handprint.com/HP/WCL/color6.html for details
  - the procedure below is used in modern versions of the experiment

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/colormatching.html
Maxwell’s color matching experiment
(summary of live demo)

1. given a stimulus wavelength, the amount of each primary required to match it is given by three numbers \((r, g, b)\)

2. some stimuli cannot be matched unless first desaturated by adding a primary to it before matching; the amount added is denoted by negative values of \(r\), \(g\), or \(b\)

3. the sequence of \((r, g, b)\) values, some negative, required to match the locus of spectral colors across all \(\lambda\), form the trichromatic matching functions \(r(\lambda)\), \(g(\lambda)\), and \(b(\lambda)\) for a particular set of 3 primaries
Human response to an arbitrary stimulus (contents of whiteboard)

spectrum of stimulus

\[ \mathbf{r} \times \mathbf{g} \times \mathbf{b} \]

by the matching functions \( r(\lambda) \), \( g(\lambda) \), and \( b(\lambda) \)

for a particular set of 3 primaries

\[ \int \]

then integrate over wavelengths to

get the amount of that primary

required to reproduce that spectrum
spectra can be visually matched using mixtures of primary colors; such matches are called *metamers*.

due to the linearity of human retinal response, given a stimulus spectrum $L_e(\lambda)$, the amounts of each primary R, G, B required to match it, for any particular choice of 3 primaries, are the integrals over all visible wavelengths of the amounts $r(\lambda)$, $g(\lambda)$, and $b(\lambda)$ required to match each constituent wavelength $\lambda$, i.e.

$$ (R,G,B) = \begin{pmatrix} \int_{400 \text{nm}}^{700 \text{nm}} L_e(\lambda) \tilde{r}(\lambda) \, d\lambda, & \int_{400 \text{nm}}^{700 \text{nm}} L_e(\lambda) \tilde{g}(\lambda) \, d\lambda, & \int_{400 \text{nm}}^{700 \text{nm}} L_e(\lambda) \tilde{b}(\lambda) \, d\lambda \end{pmatrix} $$
Young-Helmholtz trichromatic theory
our response to varying amounts of a primary forms a vector in $(\rho, \gamma, \beta)$ space, rooted at the origin

to provide a normal range of color vision, three primaries are required, and their vectors must not lie on a plane

our responses to all possible mixtures and scales ($\Sigma \leq 1$) of three primaries form a tetrahedron called the *gamut of reproducible colors* for these primaries

[Flash demo](http://graphics.stanford.edu/courses/cs178/applets/locus.html)
3D interpretation of color matching

- the spectrum of each of the three primaries can be a pure wavelength (1) or a mixture of wavelengths (2)
- impure primaries have a smaller gamut in $(\rho, \gamma, \beta)$ space
- additional primaries can be added to increase the gamut

*Flash demo*:
http://graphics.stanford.edu/courses/cs178/applets/locus.html

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Additive versus subtractive mixing

• demo using color guns and filters
Additive versus subtractive mixing

- Superimposed colored lights or small adjacent dots combine *additively* - by adding their spectra wavelength-by-wavelength
- Layered dyes or sequenced color filters combine *subtractively* - by multiplying their transmittance spectra wavelength-by-wavelength

(Flash demo) http://graphics.stanford.edu/courses/cs178/applets/ColorMixing-narrowCMY.swf
Additive versus subtractive mixing

- superimposed colored lights or small adjacent dots combine *additively* - by adding their spectra wavelength-by-wavelength
- layered dyes or sequenced color filters combine *subtractively* - by multiplying their transmittance spectra wavelength-by-wavelength

http://graphics.stanford.edu/courses/cs178/applets/colormixing.html
Additive versus subtractive mixing

- narrow spectra, widely spaced in wavelength, are best for primaries to be combined additively
- wide spectra that overlap are best for primaries to be combined subtractively, but product of all three must be black
- the particular spectra chosen are flexible; additive primaries need not be R,G,B, nor subtractive primaries C,M,Y
- additional primaries may be added to either system, resulting in a larger gamut of reproducible colors; adding black to a subtractive system (called CMYK) ensures a deep black
- note: additive mixing can be interpreted as interpolation between points in rho-gamma-beta space, but subtractive mixing cannot, because the two spectra must be multiplied together, not added
patches of the 3 subtractive primaries (C,M,Y) overlap partially on the page, making patches of 8 meta-primaries (Wh,C,M,Y,MY,CY,CM,CMY), which combine additively in the eye when viewed from a distance

- \( M \times Y = R \), \( C \times Y = G \), \( C \times M = B \)
- these effects are modeled by the Neugebauer equations