Optics I: lenses and apertures

CS 178, Spring 2012

Begun 4/10/12, finished 4/12

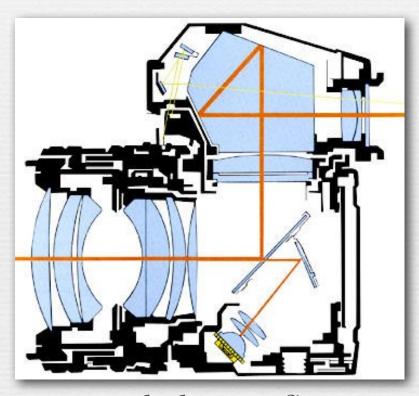


Marc Levoy
Computer Science Department
Stanford University

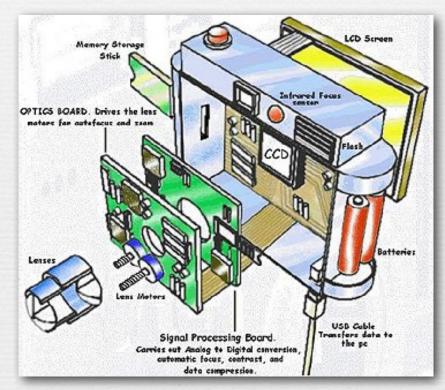
Outline

- why study lenses?
- ♦ thin lenses
 - graphical constructions, algebraic formulae
- thick lenses
 - center of perspective, lens as $3D \rightarrow 3D$ transformation
- depth of field
- → aberrations & distortion
- vignetting, glare, and other lens artifacts
- diffraction and lens quality
- → special lenses
 - telephoto, zoom

Cameras and their lenses

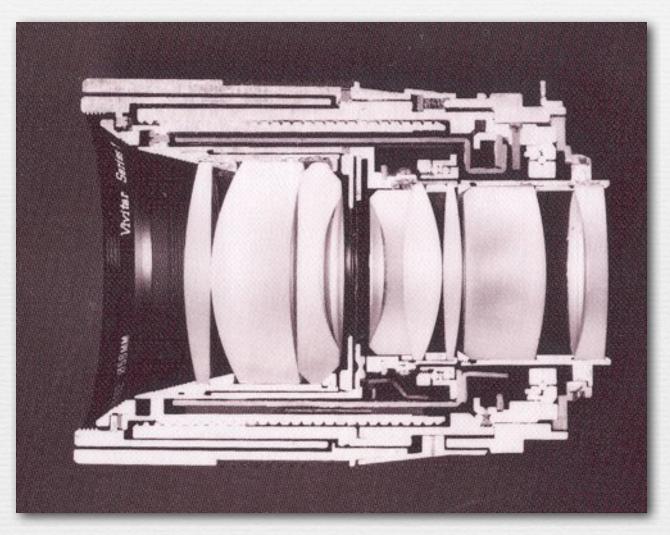


single lens reflex (SLR) camera



digital still camera (DSC), i.e. point-and-shoot

Cutaway view of a real lens



Vivitar Series 1 90mm f/2.5 Cover photo, Kingslake, *Optics in Photography*

Lens quality varies

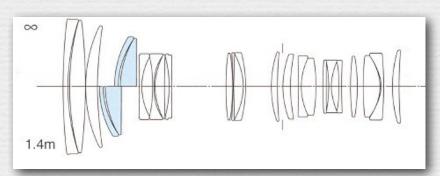
- ♦ Why is this toy so expensive?
 - EF 70-200mm f/2.8L IS USM
 - \$1700



- ♦ Why is it better than this toy?
 - EF 70-300mm f/4-5.6 IS USM
 - \$550



◆ And why is it so complicated?





Stanford Big Dish Panasonic GF1 Panasonic 45-200/4-5.6 zoom, at 200mm f/4.6 \$300

Leica 90mm/2.8 Elmarit-M prime, at f/4 \$2000

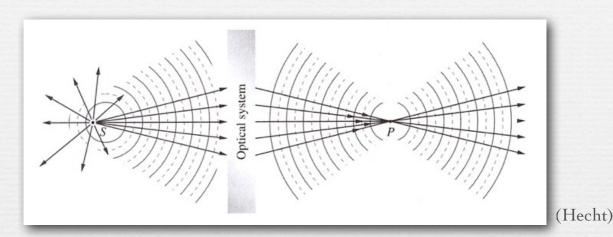
Zoom lens versus prime lens



Canon 100-400mm/4.5-5.6 zoom, at 300mm and f/5.6 \$1600

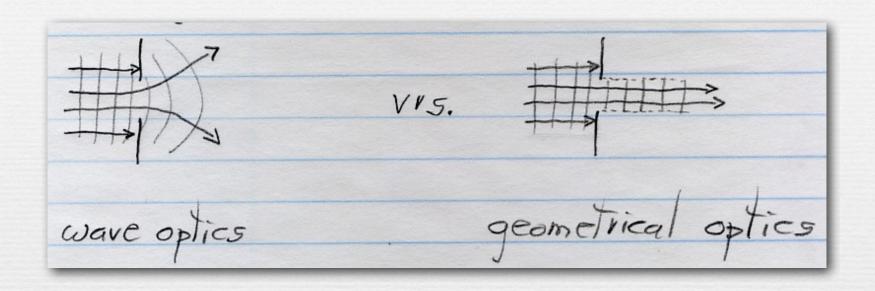
Canon 300mm/2.8 prime, at f/5.6 \$4300

Physical versus geometrical optics



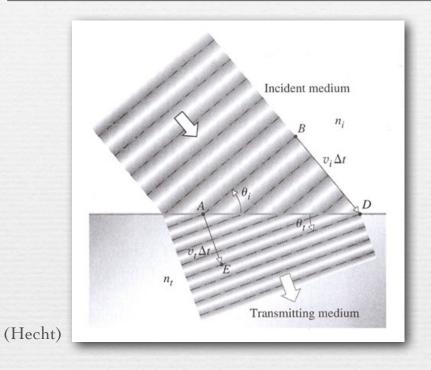
- light can be modeled as traveling waves
- ◆ the perpendiculars to these waves can be drawn as rays
- ♦ diffraction causes these rays to bend, e.g. at a slit
- → geometrical optics assumes
 - $\lambda \rightarrow 0$
 - no diffraction
 - in free space, rays are straight (a.k.a. rectilinear propagation)

Physical versus geometrical optics (contents of whiteboard)

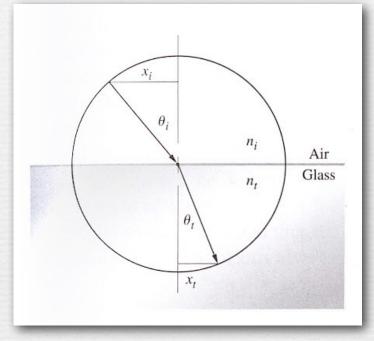


- in geometrical optics, we assume that rays do not bend as they pass through a narrow slit
- \star this assumption is valid if the slit is much larger than the wavelength, represented on the previous slide by the limit $\lambda \to 0$
- physical optics is a.k.a. wave optics

Snell's law of refraction



- as waves change speed at an interface, they also change direction
- \bullet index of refraction n_t is defined as



$$\frac{x_i}{x_t} = \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}$$

 $\frac{\text{speed of light in a vacuum}}{\text{speed of light in medium } t}$

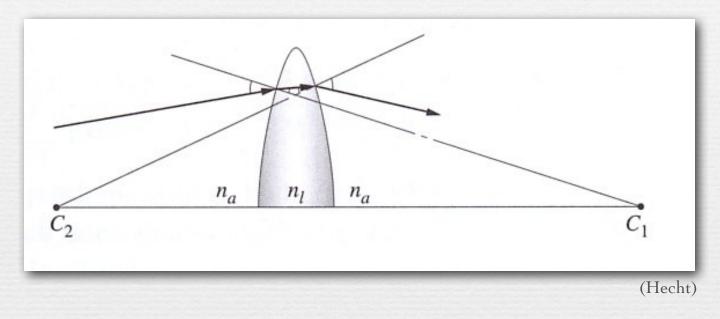
Typical refractive indices (n)

- \star air = ~ 1.0
- → water = 1.33
- glass = 1.5 1.8



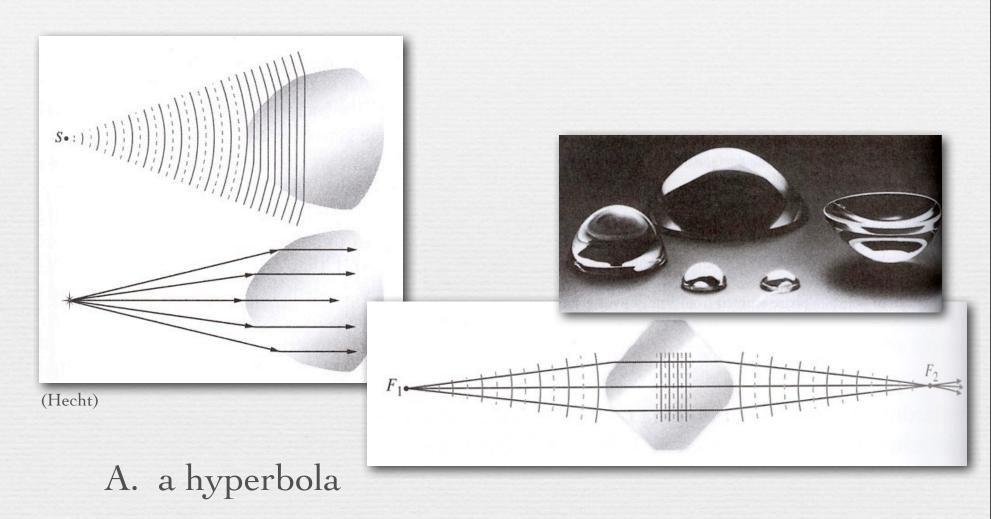
mirage due to changes in the index of refraction of air with temperature

Refraction in glass lenses



- when transiting from air to glass,
 light bends towards the normal
- when transiting from glass to air, light bends away from the normal
- ◆ light striking a surface perpendicularly does not bend

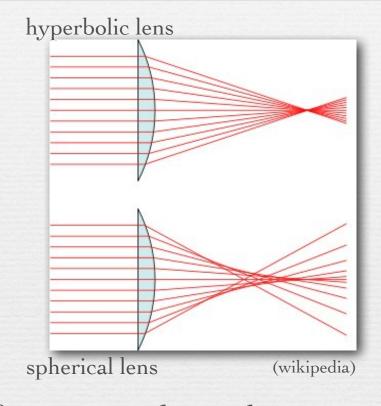
Q. What shape should an interface be to make parallel rays converge to a point?



→ so lenses should be hyperbolic!

Spherical lenses





- two roughly fitting curved surfaces ground together will eventually become spherical
- * spheres don't bring parallel rays to a point
 - this is called *spherical aberration*
 - nearly axial rays (paraxial rays) behave best

Examples of spherical aberration





Canon 135mm soft focus lens

(gtmerideth)

(Canon)

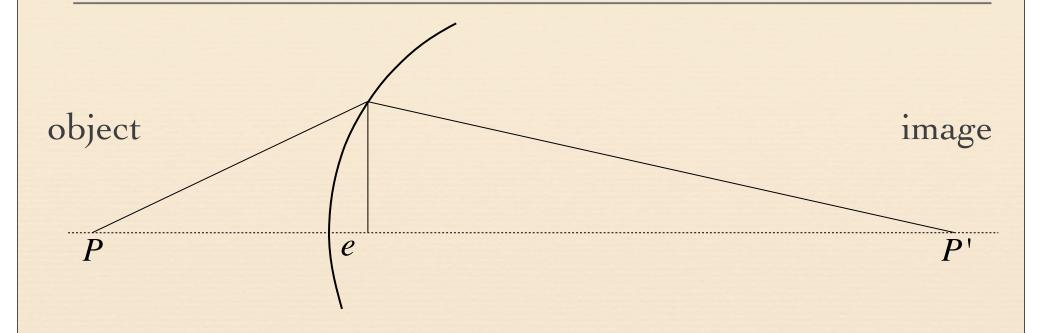








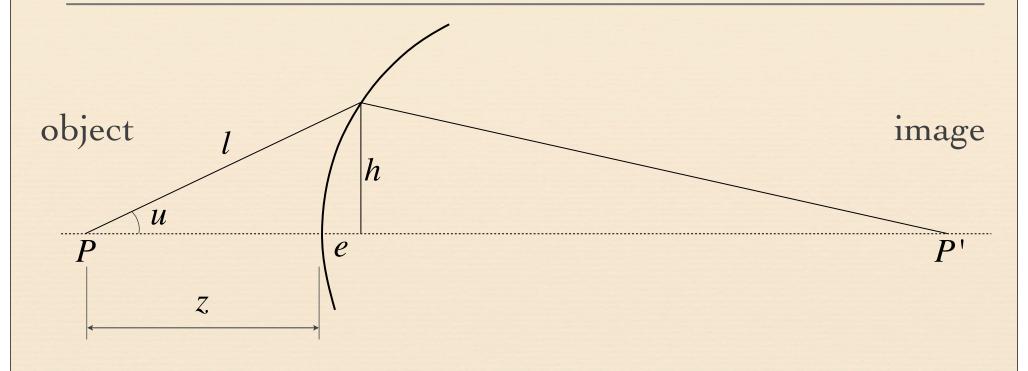
Paraxial approximation



 \bullet assume $e \approx 0$

Not responsible on exams for orange-tinted slides

Paraxial approximation



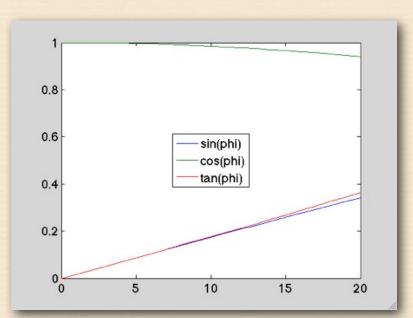
- \bullet assume $e \approx 0$
- → assume $sin u = h/l \approx u$ (for u in radians)
- \bullet assume $\cos u \approx z/l \approx 1$
- ♦ assume $tan u \approx sin u \approx u$

The paraxial approximation is a.k.a. first-order optics

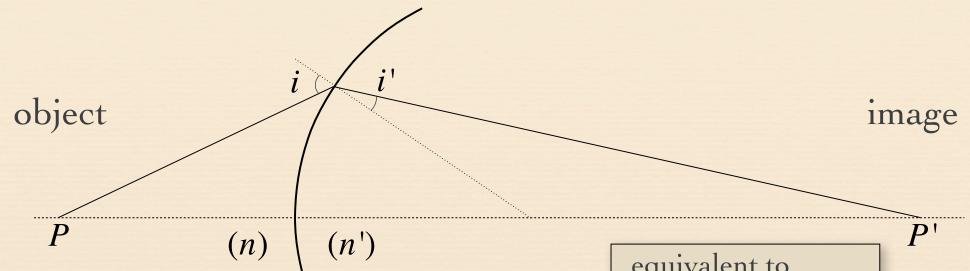
- * assume first term of $\sin \phi = \phi \frac{\phi^3}{3!} + \frac{\phi^5}{5!} \frac{\phi^7}{7!} + \dots$ • i.e. $\sin \phi \approx \phi$
- * assume first term of $\cos \phi = 1 \frac{\phi^2}{2!} + \frac{\phi^4}{4!} \frac{\phi^6}{6!} + \dots$ • i.e. $\cos \phi \approx 1$
 - so $tan \phi \approx sin \phi \approx \phi$

these are the Taylor series for $\sin \phi$ and $\cos \phi$

(phi in degrees)



Paraxial focusing



Snell's law:

$$n \sin i = n' \sin i'$$

paraxial approximation:

$$ni \approx n'i'$$

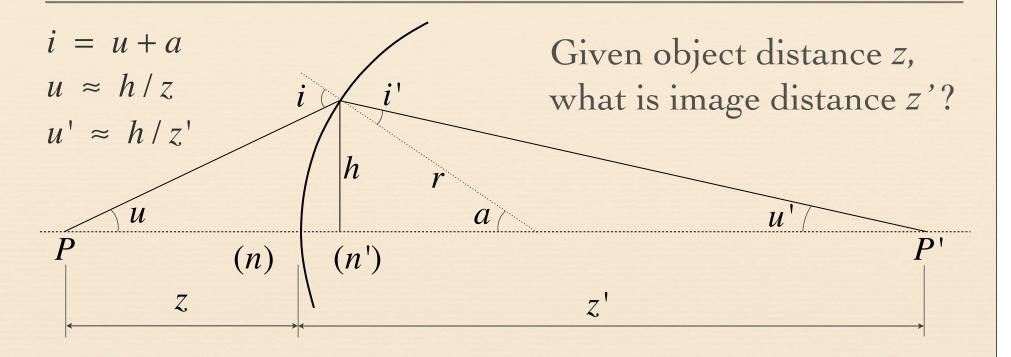
equivalent to

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i}$$

with

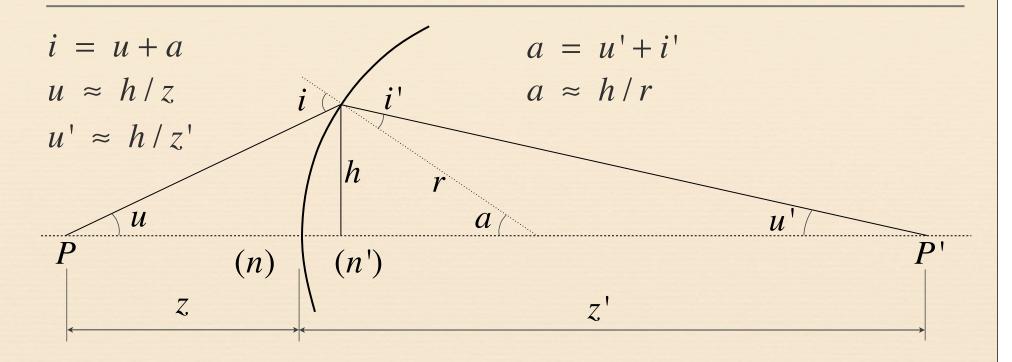
$$n = n_i$$
 for air
 $n' = n_t$ for glass
 i, i' in radians
 θ_i , θ_t in degrees

Paraxial focusing



$$ni \approx n'i'$$

Paraxial focusing



$$n(u+a) \approx n'(a-u')$$

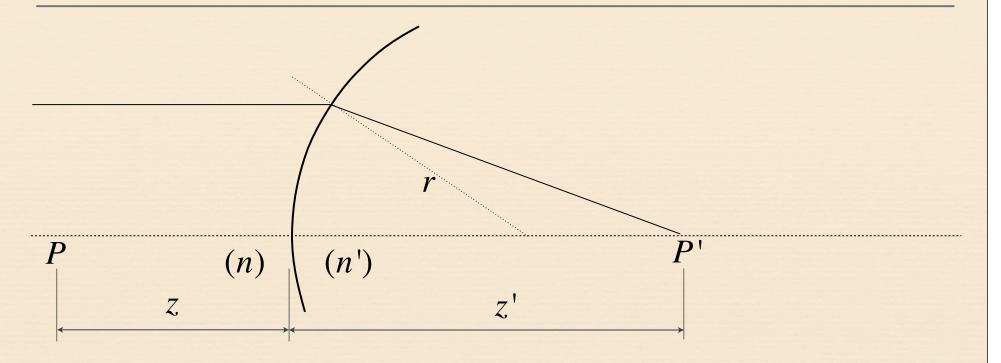
$$n(h/z+h/r) \approx n'(h/r-h/z')$$

$$n/z+n/r \approx n'/r-n'/z'$$

 $ni \approx n'i'$

 \star *h* has canceled out, so any ray from *P* will focus to *P*'

Focal length



What happens if z is ∞ ?

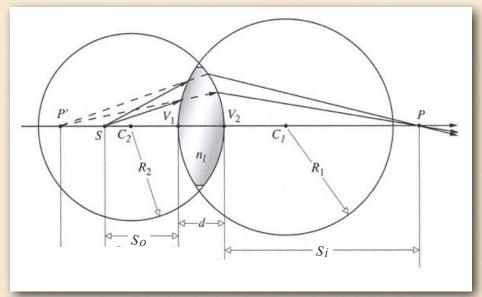
$$n/z + n/r \approx n'/r - n'/z'$$

$$n/r \approx n'/r - n'/z'$$

$$z' \approx (r n')/(n'-n)$$

Lensmaker's formula

 using similar derivations, one can extend these results to two spherical interfaces forming a lens in air



(Hecht, edited)

• as $d \rightarrow 0$ (thin lens approximation), we obtain the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Gaussian lens formula

◆ Starting from the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right),$$
 (Hecht, eqn 5.15)

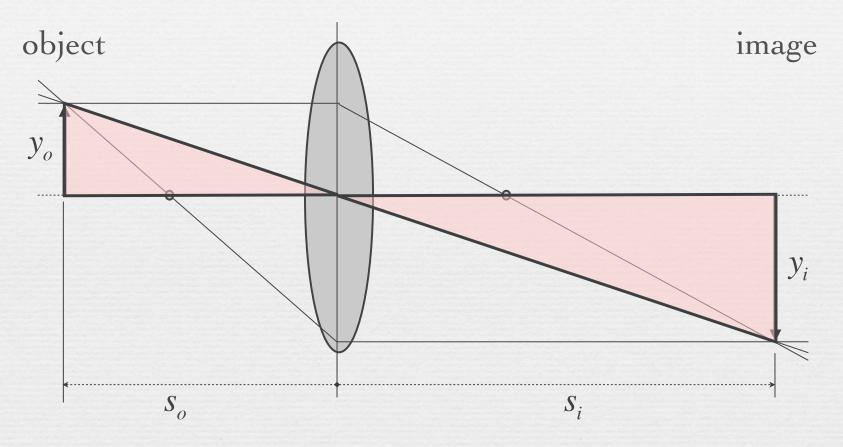
 \bullet and recalling that as object distance s_0 is moved to infinity, image distance s_i becomes focal length f_i , we get

$$\frac{1}{f_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$
 (Hecht, eqn 5.16)

◆ Equating these two, we get the Gaussian lens formula

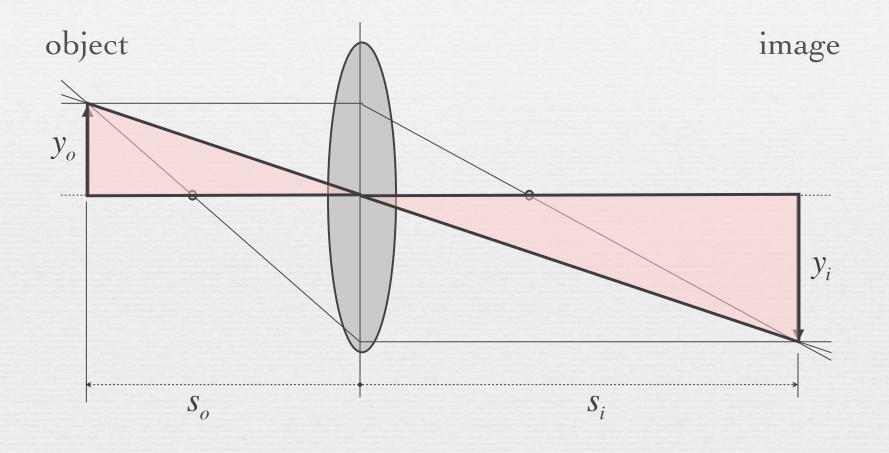
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_i}$$
. (Hecht, eqn 5.17)

From Gauss's ray construction to the Gaussian lens formula



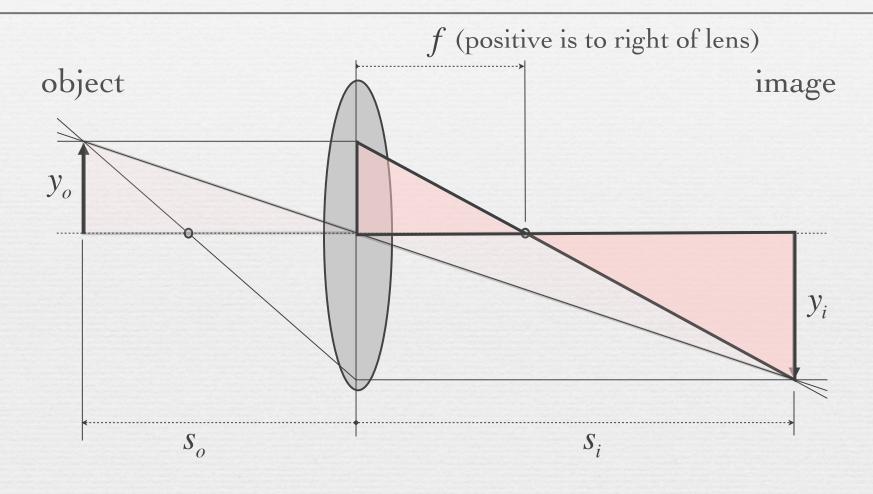
- \bullet positive s_i is rightward, positive s_o is leftward
- → positive y is upward

From Gauss's ray construction to the Gaussian lens formula



$$\frac{\left|y_{i}\right|}{y_{o}} = \frac{s_{i}}{s_{o}}$$

From Gauss's ray construction to the Gaussian lens formula

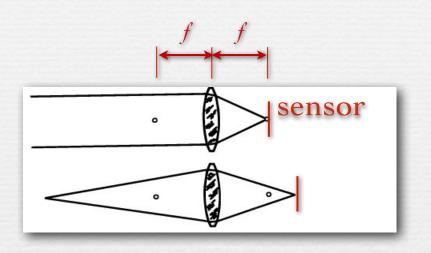


$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o}$$
 and $\frac{|y_i|}{y_o} = \frac{s_i - f}{f}$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

 to focus on objects at different distances, move sensor relative to lens



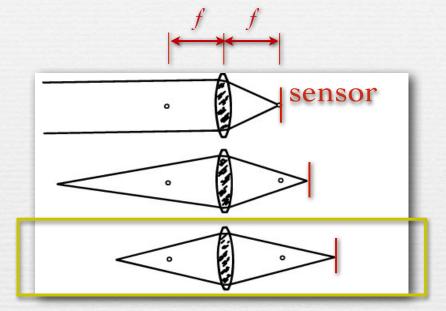
(FLASH DEMO)

http://graphics.stanford.edu/courses/ cs178/applets/gaussian.html

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

 to focus on objects at different distances, move sensor relative to lens



• at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

In 1:1 imaging, if the sensor is 36mm wide, an object 36mm wide will fill the frame.

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

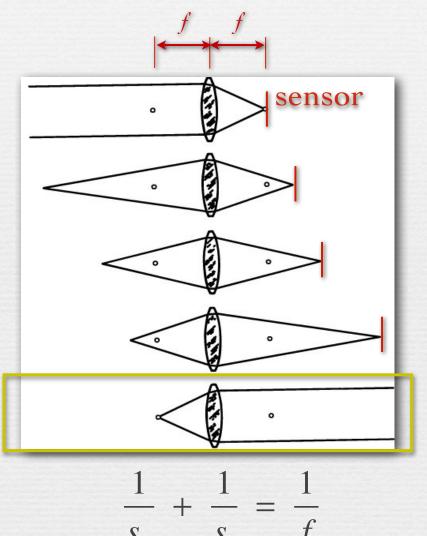
Changing the focus distance

to focus on objects at different distances, move sensor relative to lens

 \bullet at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

can't focus on objects closer to lens than its focal length *f*



Recap

- → approximations we sometimes make when analyzing lenses
 - geometrical optics instead of physical optics
 - spherical lenses instead of hyperbolic lenses
 - thin lens representation of thick optical systems
 - paraxial approximation of ray angles
- \star the Gaussian lens formula relates focal length f, object distance s_o , and image distance s_i
 - these settings, and sensor size, determine field of view
 - 1:1 imaging means $S_0 = S_i$ and both are $2 \times$ focal length
 - $S_0 = f$ is the minimum possible object distance for a lens

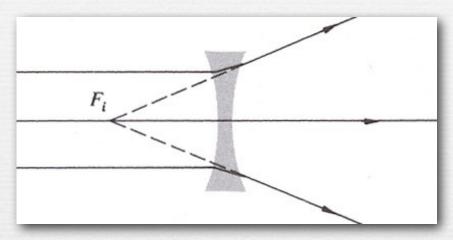


One must be careful applying this last rule in practice, because s_0 may not be measured to the front of the barrel of a complex lens assembly, but to a plane you can't see inside the assembly. Also, many lenses can't crank out to the s_i that would enable the theoretically minimum s_0 , because it is mechanically difficult or might produce poorly corrected (aberrant) images.

Convex versus concave lenses

(Hecht)

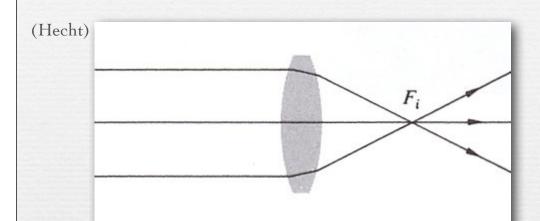
rays from a convex lens converge



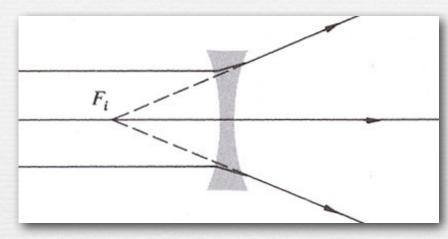
rays from a concave lens diverge

- ◆ positive focal length f means parallel rays from the left converge to a point on the right
- → negative focal length f means parallel rays from the left converge to a point on the left (dashed lines above)

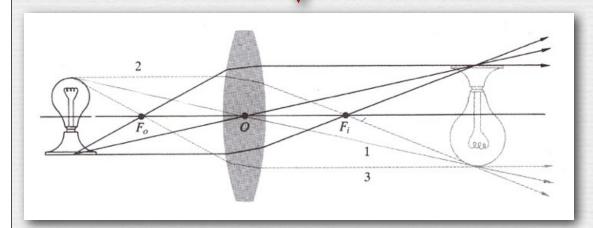
Convex versus concave lenses



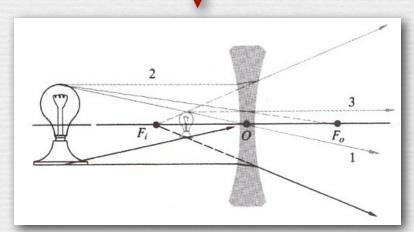
rays from a convex lens converge



rays from a concave lens diverge

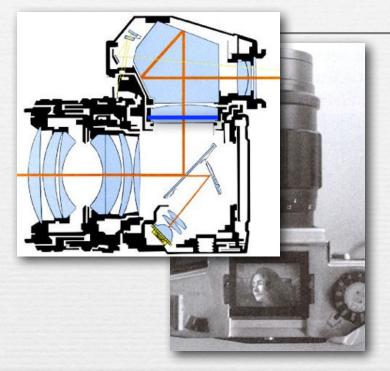


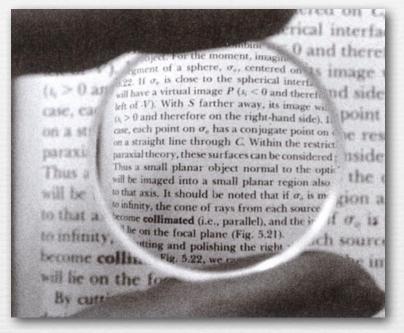
...producing a real image

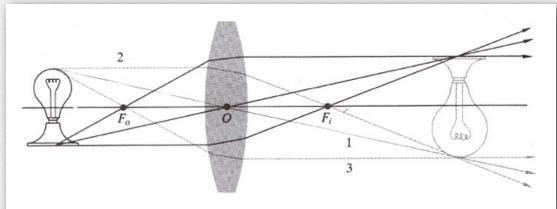


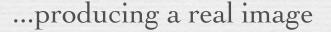
...producing a virtual image

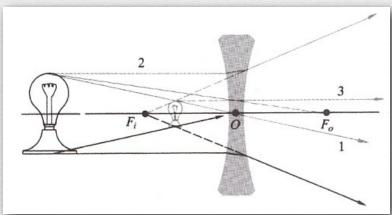
Convex versus concave lenses











...producing a virtual image

The power of a lens

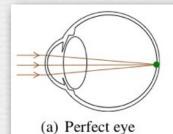
$$P = \frac{1}{f}$$

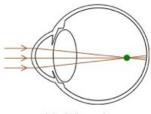
units are meters-1

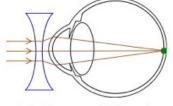
a.k.a. diopters

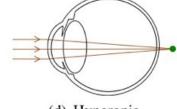
- my eyeglasses have the prescription
 - right eye: -0.75 diopters
 - left eye: -1.00 diopters
- Q. What's wrong with me?
- A. Myopia (nearsightedness)

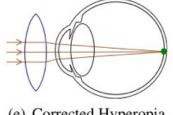
(Pamplona)











(b) Myopia

(c) Corrected Myopia

(d) Hyperopia

(e) Corrected Hyperopia

Combining two lenses

using focal lengths

$$\frac{1}{f_{tot}} = \frac{1}{f_1} + \frac{1}{f_2}$$

using diopters

$$P_{tot} = P_1 + P_2$$

◆ example

$$\frac{1}{200mm} + \frac{1}{500mm} = \frac{1}{143mm}$$
 -or-

The diopters version of this calculation was incorrect when shown in class. It has been corrected here. Thanks to Bryan Huh for finding the error.

-or-
$$5.0 + 2.0 = 7.0$$
 diopters





- screw on to end of lens
- power is designated in diopters (usually)

Panasonic 45-200





→ changes longest focal length from 200mm to 143mm

$$\frac{1}{200mm} + \frac{1}{500mm} = \frac{1}{143mm}$$

- ◆ for a fixed image distance, it reduces the object distance
 - at f=200mm, this len's minimum object distance $s_0 = 1000$ mm
 - at these settings, its effective image distance must be

$$s_i = \frac{1}{\frac{1}{f} - \frac{1}{s_o}} = \frac{1}{\frac{1}{200mm} - \frac{1}{1000mm}} = 250mm$$

• with the closeup filter and the same settings of focal length and image distance, the in-focus object distance becomes

$$s_o = \frac{1}{\frac{1}{f} - \frac{1}{s_i}} = \frac{1}{\frac{1}{143mm} - \frac{1}{250mm}} = 334mm$$

 $3\times$

closer!

Canon

CZ6-5602

Close-up Lens / Bonnette Macro / Nahlinse / Lente Addizionale Diametro / lente de acercamiento / Оптический конвертер для макросъёмки / 近摄鏡片 / 近攝鏡 / CLOSE-UP 렌즈 / クローズアップレンズ

58mm Close-up Lens 500D

CANON INC.

Made in Japan/Fabriqué au Japon/Hecho en Japón/ Сделано в Японии/日本制造/日本製造

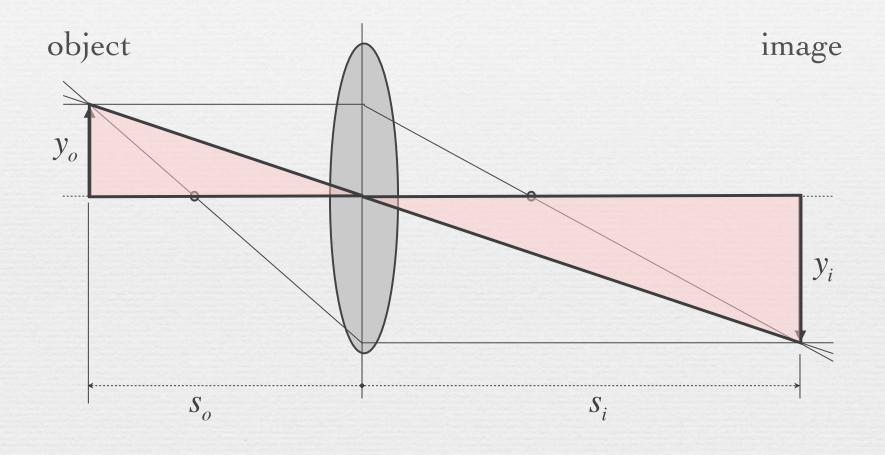
> 200mm lens no closeup filter $s_0 = 1000$ mm



200mm lens 500D closeup filter $s_0 = 334$ mm

poor man's macro lens

Magnification



$$M_T \triangleq \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

Canon

CZ6-5602

Close-up Lens / Bonnette Macro / Nahlinse / Lente Addizionale Diametro / lente de acercamiento / Оптический конвертер для макросъёмки / 近摄镜片 / 近攝鏡 / CLOSE-UP 렌즈 / クローズアップレンズ

58mm Close-up Lens 500D

CANON INC.

Made in Japan / Fabriqué au Japon / Hecho en Japón / Сделано в Японии / 日本制造 / 日本製造

> 200mm lens no closeup filter $S_0 = 1000 \text{mm}$

$$M_T = -\frac{s_i}{s_o} = \frac{250}{1000} = -1:4$$
 $M_T = -\frac{s_i}{s_o} = \frac{250}{334} = -3:4$

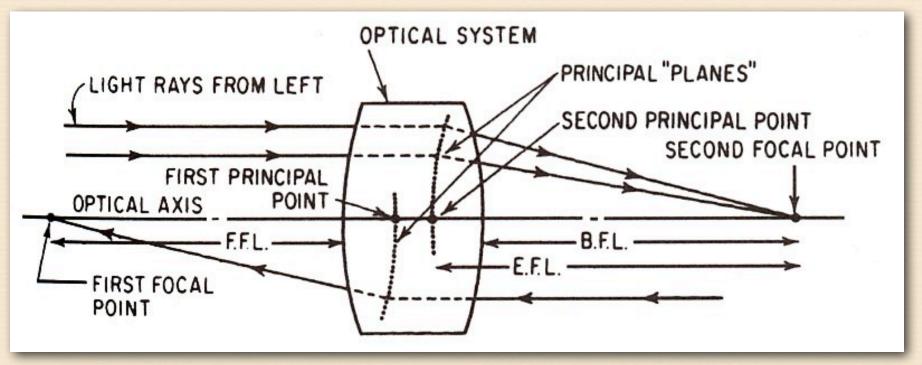


200mm lens 500D closeup filter $S_0 = 334 \text{mm}$

$$M_T = -\frac{s_i}{s_o} = \frac{250}{334} = -3:4$$

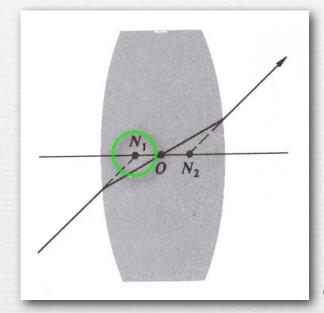
Thick lenses

an optical system may contain many lenses,
 but can be characterized by a few numbers



(Smith)

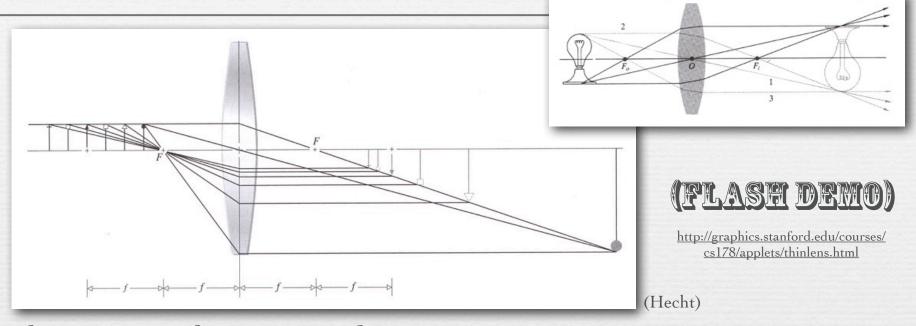
Center of perspective



(Hecht)

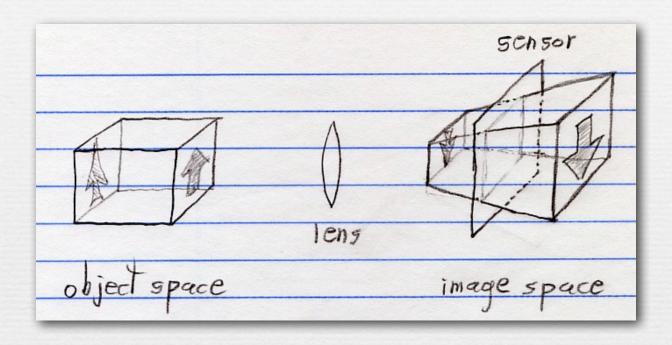
- in a thin lens, the *chief ray* from a point traverses the lens (through its optical center) without changing direction
- in a thick lens, the intersections of this ray with the optical axis are called the *nodal points*
- for a lens in air, these coincide with the principal points
- the first nodal point is the center of perspective

Lenses perform a 3D perspective transform



- → lenses transform a 3D object to a 3D image; the sensor extracts a 2D slice from that image
- ◆ as an object moves linearly (in Z),
 its image moves non-proportionately (in Z)
- ◆ as you move a lens linearly relative to the sensor, the in-focus object plane moves non-proportionately
- * as you refocus a camera, the image changes size!

Lenses perform a 3D perspective transform (contents of whiteboard)



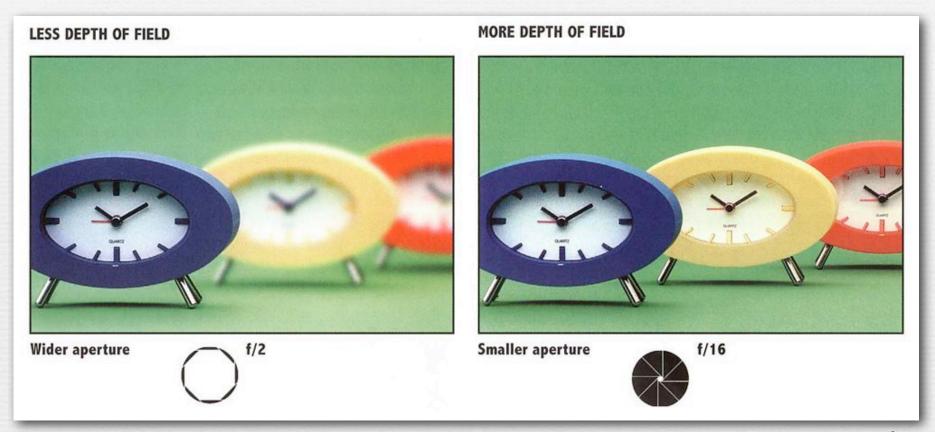
- → a cube in object space is transformed by a lens into a 3D frustum in image space, with the orientations shown by the arrows
- → in computer graphics this transformation is modeled as a 4 × 4 matrix multiplication of 3D points expressed in 4D homogenous coordinates
- in photography a sensor extracts a 2D slice from the 3D frustum; on this slice some objects will be sharply focused; others may be blurry

Recap

- → more implications of the Gaussian lens formula
 - convex lenses make real images; concave make virtual images
 - the power of a lens (in diopters) is 1 over its focal length
 - when combining two lenses, add their powers
 - adding a closeup filter allows a smaller object distance
 - changing object and image distances changes magnification
- → lenses perform a 3D perspective transform of object space
 - an object's apparent size is inversely proportional to its distance
 - linear lens motions move the in-focus plane non-linearly
 - focusing a lens changes the image size (slightly)

Questions?

Depth of field

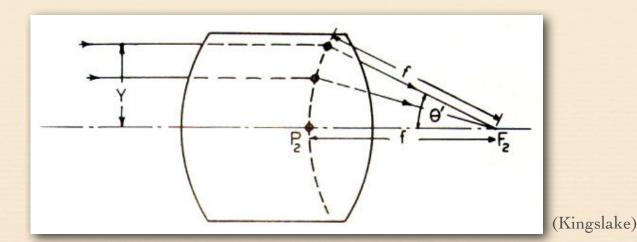


 $I = \frac{f}{\Lambda}$

(London)

→ lower N means a wider aperture and less depth of field

How low can N be?



 principal planes are the paraxial approximation of a spherical "equivalent refracting surface"

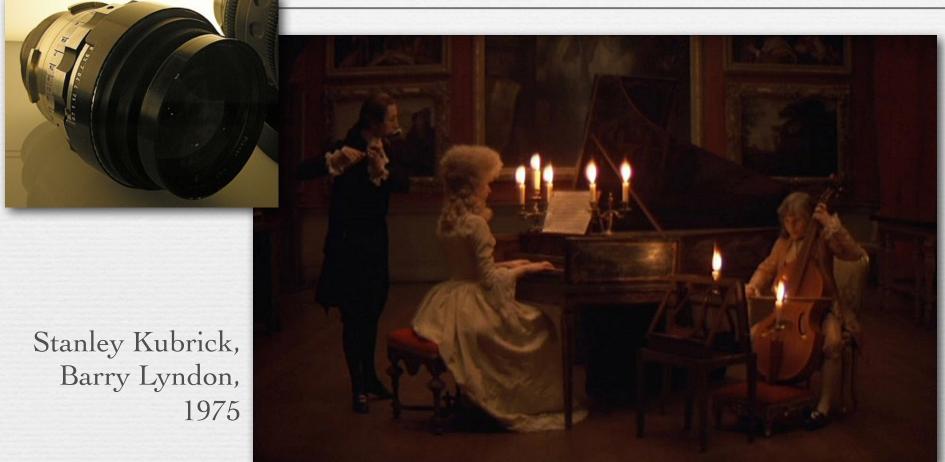
$$N = \frac{1}{2\sin\theta'}$$

- → lowest possible N in air is f/0.5
- → lowest N I've seen in an SLR is f/1.0



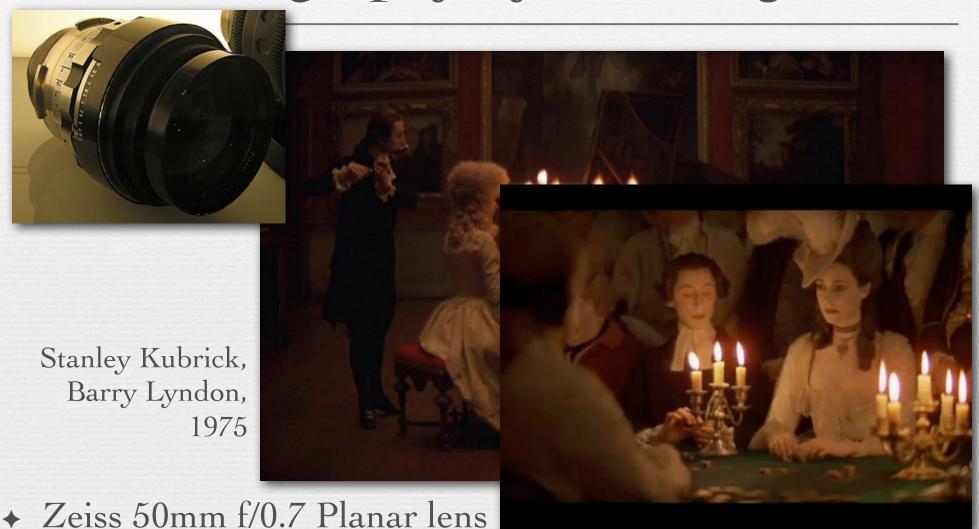
Canon EOS 50mm f/1.0 (discontinued)

Cinematography by candlelight



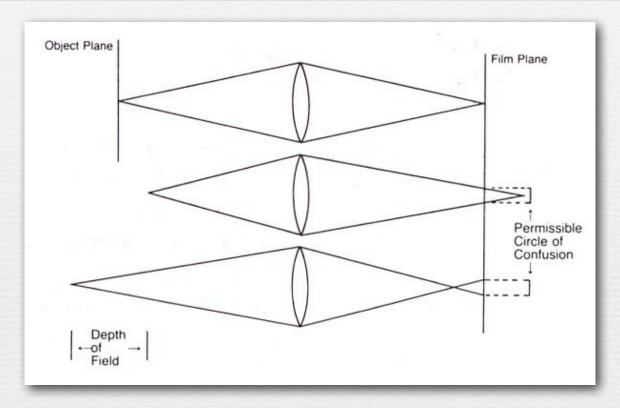
- → Zeiss 50mm f/0.7 Planar lens
 - originally developed for NASA's Apollo missions
 - very shallow depth of field in closeups (small object distance)

Cinematography by candlelight



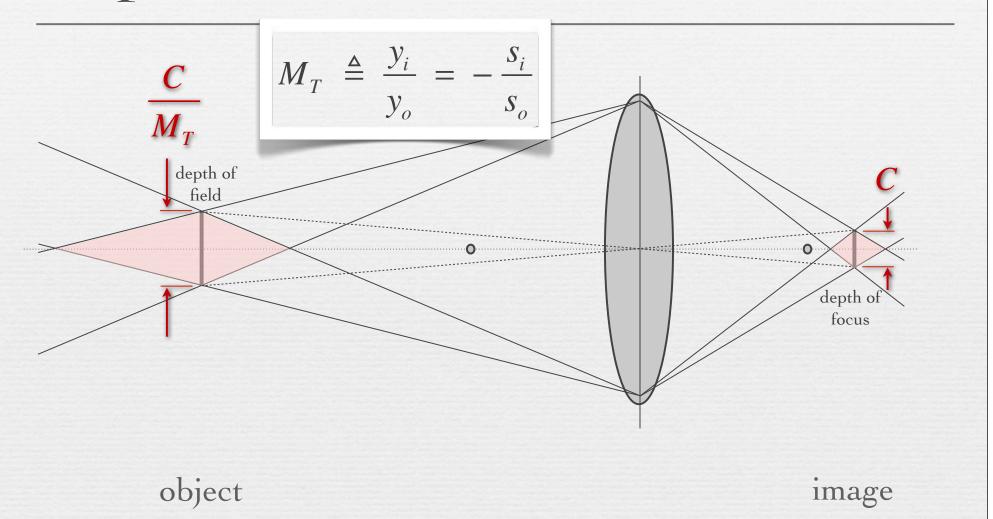
- - originally developed for NASA's Apollo missions
 - very shallow depth of field in closeups (small object distance)

Circle of confusion (C)

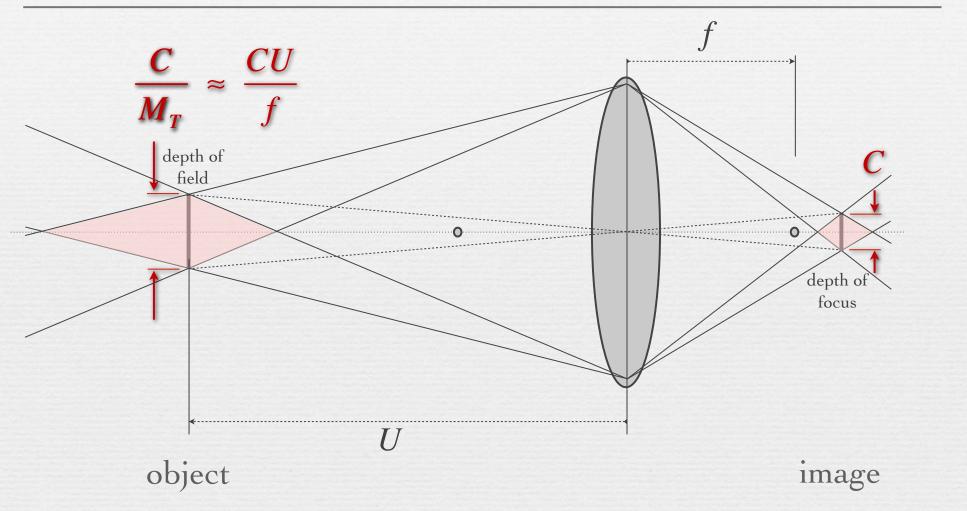


the right way to talk about C is in terms of angle subtended in the eye; we'll cover this later in the course

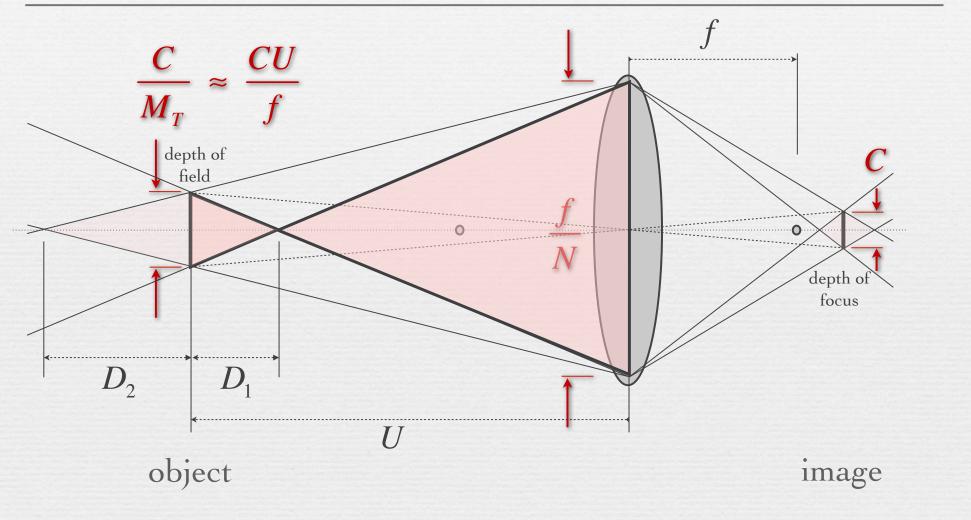
- ◆ C depends on sensing medium, reproduction medium, viewing distance, human vision,...
 - for print from 35mm film, 0.02mm (on negative) is typical
 - for high-end SLR, 6µ is typical (1 pixel)
 - larger if downsizing for web, or lens is poor



- ◆ DoF is asymmetrical around the in-focus object plane
- * conjugate in object space is typically bigger than C



- ◆ DoF is asymmetrical around the in-focus object plane
- → conjugate in object space is typically bigger than C



$$\frac{D_1}{CU/f} = \frac{U - D_1}{f/N} \dots D_1 = \frac{NCU^2}{f^2 + NCU}$$

$$D_2 = \frac{NCU^2}{f^2 - NCU}$$

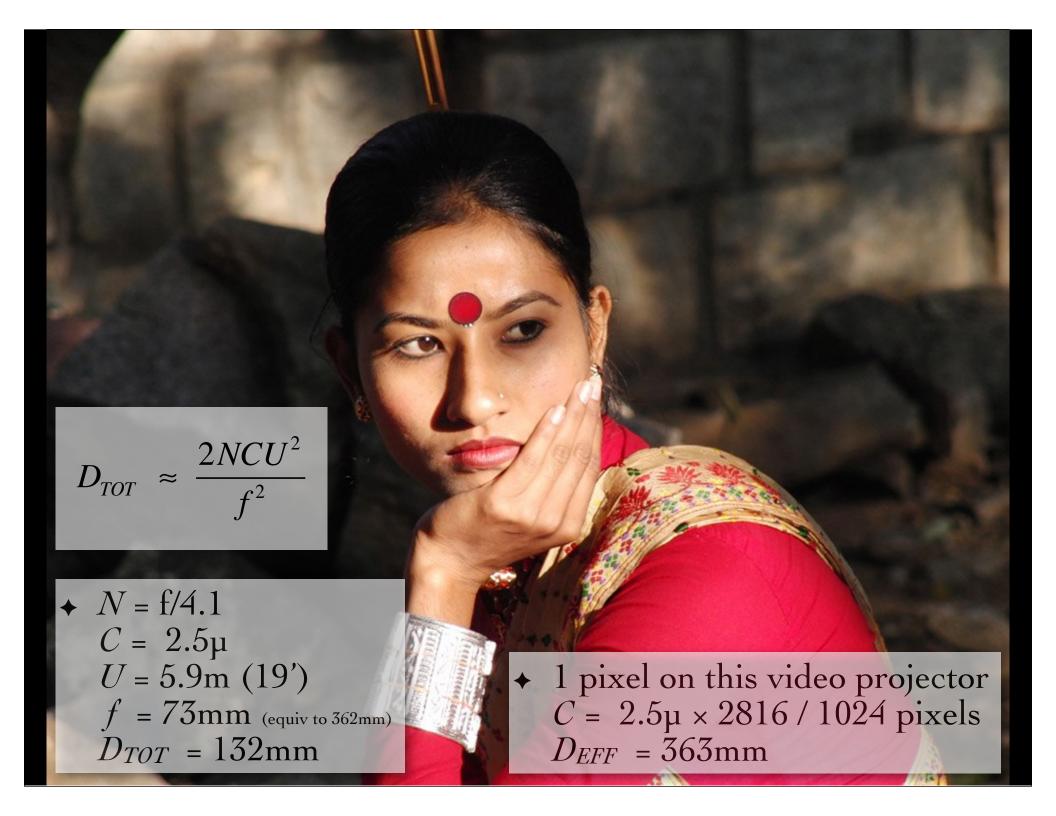
© Marc Levoy

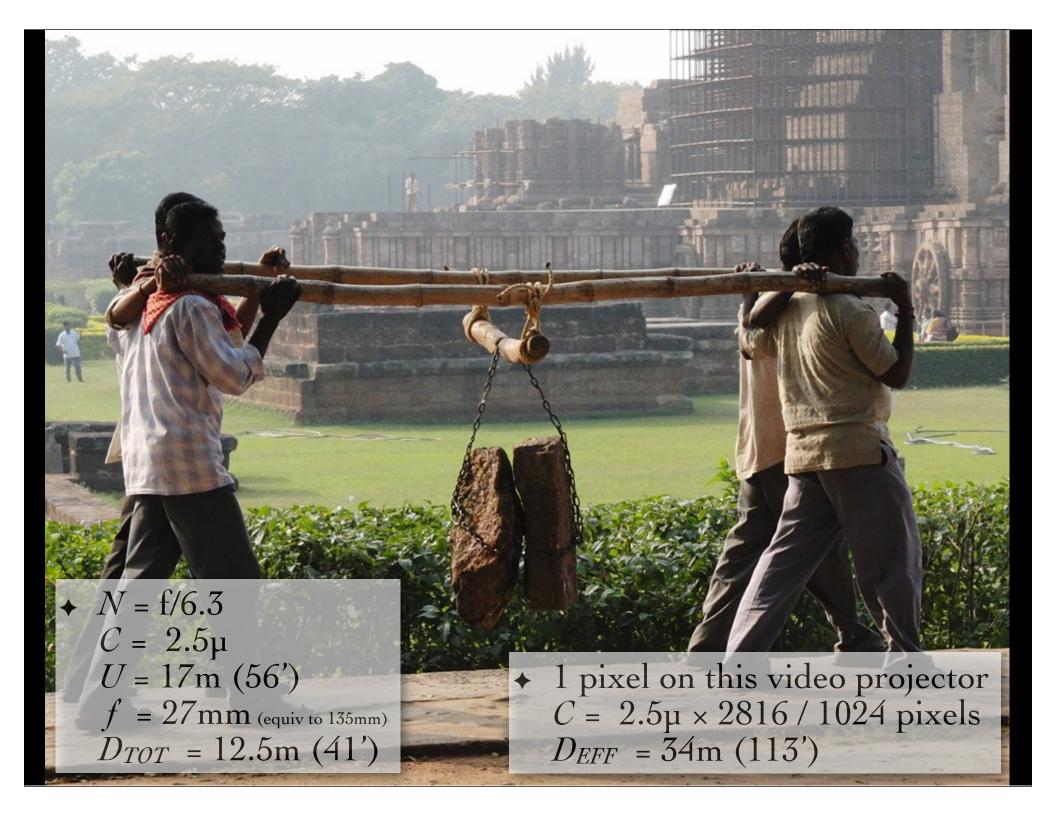
$$D_{TOT} = D_1 + D_2 = \frac{2NCU^2 f^2}{f^4 - N^2 C^2 U^2}$$

♦ $N^2C^2U^2$ can be ignored when conjugate of circle of confusion is small relative to the aperture

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- where
 - N is F-number of lens
 - C is circle of confusion (on image)
 - *U* is distance to in-focus plane (in object space)
 - f is focal length of lens









Canon MP-E 65mm 5:1 macro

N = f/2.8 $C = 6.4\mu$ U = 78mm f = 65mm



(use $N' = (1+M_T)N$ at short conjugates $(M_T=5 \text{ here})$) = f/16 $D_{TOT} = 0.29 \text{mm}!$

(Mikhail Shlemov)

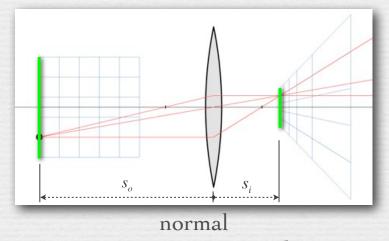
Sidelight: macro lenses

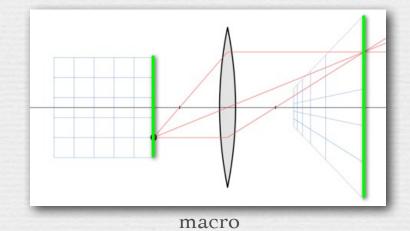
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$





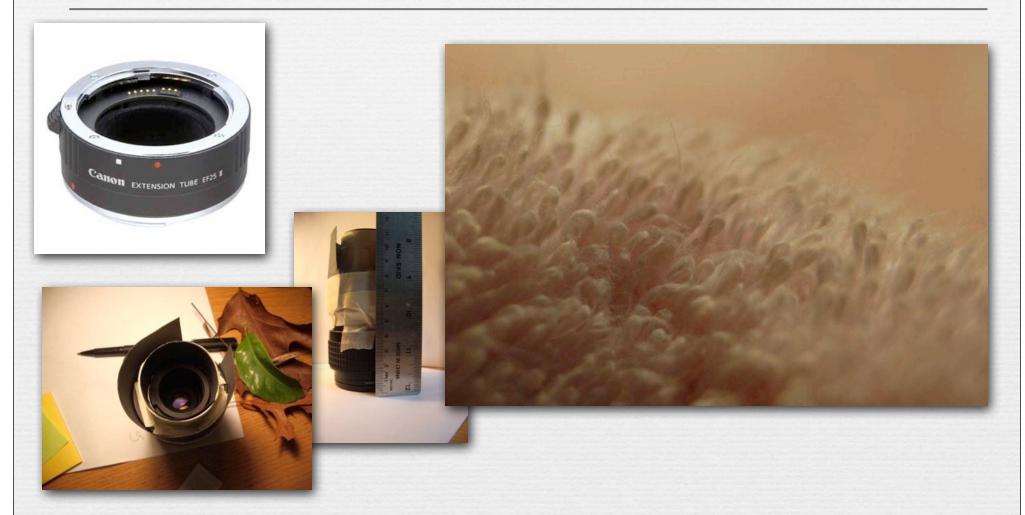
Q. How can the Casio EX-F1 at 73mm and the Canon MP-E 65mm macro, which have similar f's, have such different focusing distances?





- ♦ A. Because macro lenses are built to allow long si
 - this changes s_o , which changes magnification $M_T \triangleq -s_i/s_o$
 - macro lenses are also well corrected for aberrations at short so

Extension tube: fits between camera and lens, converts a normal lens to a macro lens



- → toilet paper tube, black construction paper, masking tape
- → camera hack by Katie Dektar (CS 178, 2009)

Extension tubes versus close-up filters



Canon 25mm



Canon f = 500 mm

- ♦ both allow closer focusing, hence greater magnification
- ◆ both degrade image quality relative to a macro lens
- extension tubes work best with wide-angle lenses;
 close-up filters work best with telephoto lenses
- extension tubes raise F-number, reducing light
- → need different close-up filter for each lens filter diameter

Extension tubes versus close-up filters versus teleconverters







Canon f = 500 mm



Nikon 1.4×

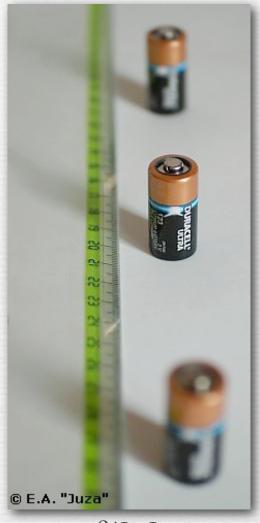
- → a teleconverter fits between the camera and lens, like an extension tube
- ◆ they increase f, narrowing FOV & increasing magnification, but they don't change the focusing range
- ♦ like extension tubes, they raise F-number, reducing light, and they are awkward to add or remove
- ◆ see http://www.cambridgeincolour.com/tutorials/macro-extension-tubes-closeup.htm

DoF is linear with F-number

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

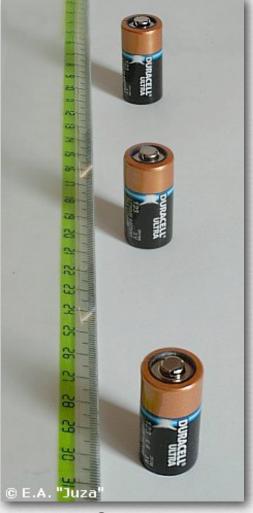


http://graphics.stanford.edu/courses/cs178/applets/dof.html



f/2.8

(juzaphoto.com)



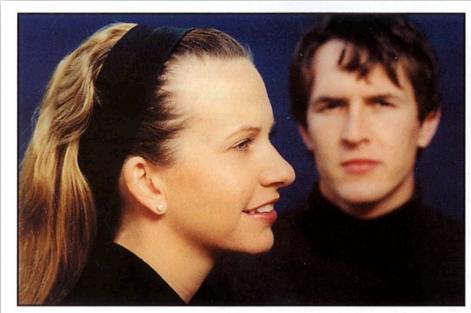
f/32

DoF is quadratic with subject distance

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$



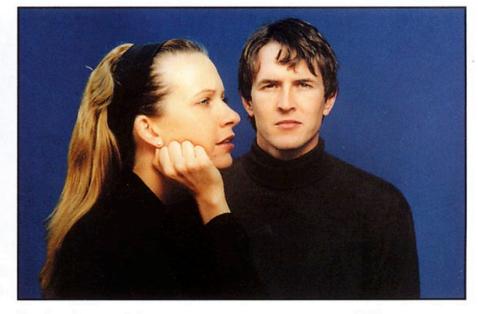
http://graphics.stanford.edu/courses/ cs178/applets/dof.html



Closer to subject



3 feet



Farther from subject



10 feet

(London)

Hyperfocal distance

the back depth of field

$$D_2 = \frac{NCU^2}{f^2 - NCU}$$

becomes infinite if

$$U \geq \frac{f^2}{NC} \triangleq H$$



♦
$$N = f/6.3$$

 $C = 2.5\mu \times 2816 / 1920 \text{ pixels}$
 $U = 17\text{m } (56')$
 $f = 27\text{mm } (\text{equiv to } 135\text{mm})$
 $D_{TOT} = 18.3\text{m } \text{on HD projector}$
 $H = 31.6\text{m } (104')$

♦ In that case, the front depth of field becomes

$$D_1 = \frac{NCU^2}{f^2 + NCU} = \frac{H}{2}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/ cs178/applets/dof.html

♦ so if I had focused at 32m, everything from 16m to infinity would be in focus on a video projector, including the men at 17m

DoF is inverse quadratic with focal length

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$



http://graphics.stanford.edu/courses/cs178/applets/dof.html



Longer focal length



180mm



Shorter focal length



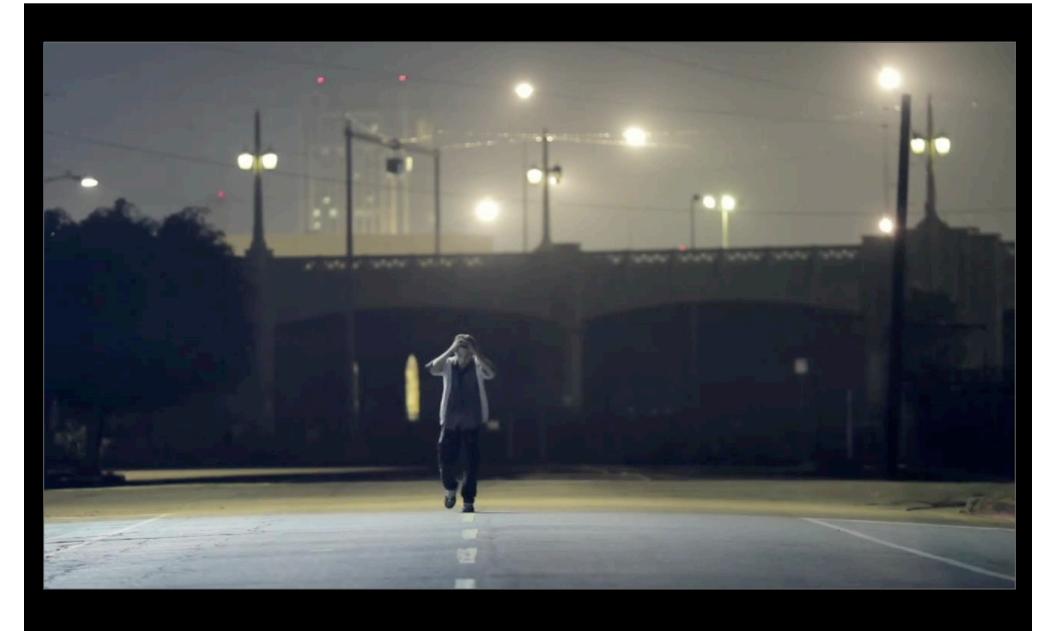
50mm

(London)

Q. Does sensor size affect DoF?

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- ◆ as sensor shrinks, lens focal length f typically shrinks to maintain a comparable field of view
- ◆ as sensor shrinks, pixel size C typically shrinks
 to maintain a comparable number of pixels in the image
- \star thus, depth of field D_{TOT} increases linearly with decreasing sensor size on consumer cameras
- this is why amateur cinematographers are drawn to SLRs
 - their chips are larger than even pro-level video camera chips
 - so they provide unprecedented control over depth of field



Vincent Laforet, Nocturne (2009) Canon 1D Mark IV

DoF and the dolly-zoom

 \star if we zoom in (increase f) and stand further back (decrease U) by the same factor

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

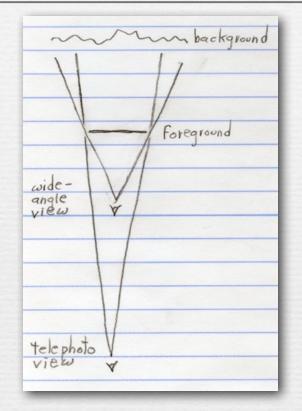
- depth of field stays the same, but background gets blurrier!
 - useful for macro when you can't get close enough



50mm f/4.8



Macro photography using a telephoto lens (contents of whiteboard)



- changing from a wide-angle lens to a telephoto lens and stepping back,
 you can make a foreground object appear the same size in both lenses
- * and both lenses will have the same depth of field on that object
- ♦ but the telephoto sees a smaller part of the background (which it blows up to fill the field of view), so the background will appear blurrier

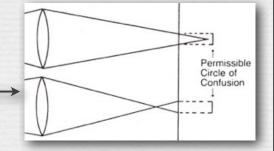
Parting thoughts on DoF: the zen of *bokeh*



Canon 85mm prime f/1.8 lens

(wikipedia.org)

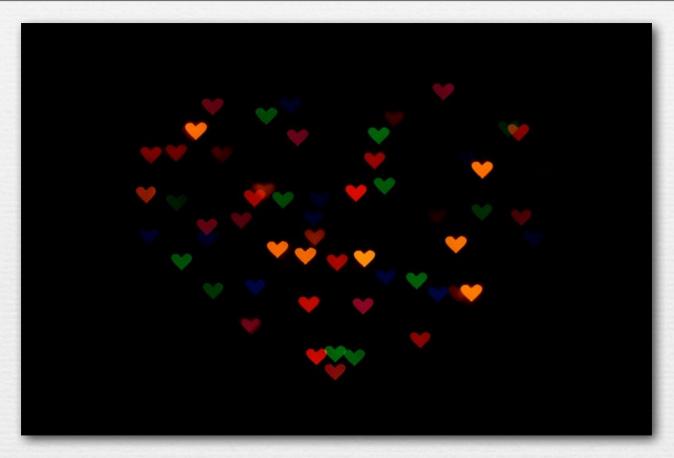
- the appearance of small out-of-focus features in a photograph with shallow depth of field
 - determined by the boundary of the aperture
 - people get religious about it
 - but not every picture with shallow DoF has evident bokeh...





Natasha Gelfand (Canon 100mm f/2.8 prime macro lens)

Games with bokeh



- → picture by Alice Che (CS 178, 2010)
 - heart-shaped mask in front of lens
 - subject was Christmas lights
 - photograph was misfocused and under-exposed

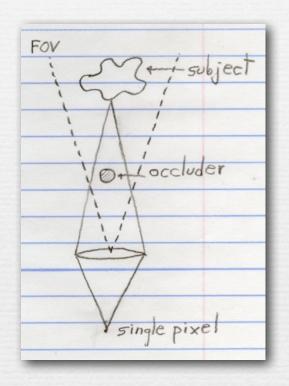
Parting thoughts on DoF: seeing through occlusions



(Fredo Durand)

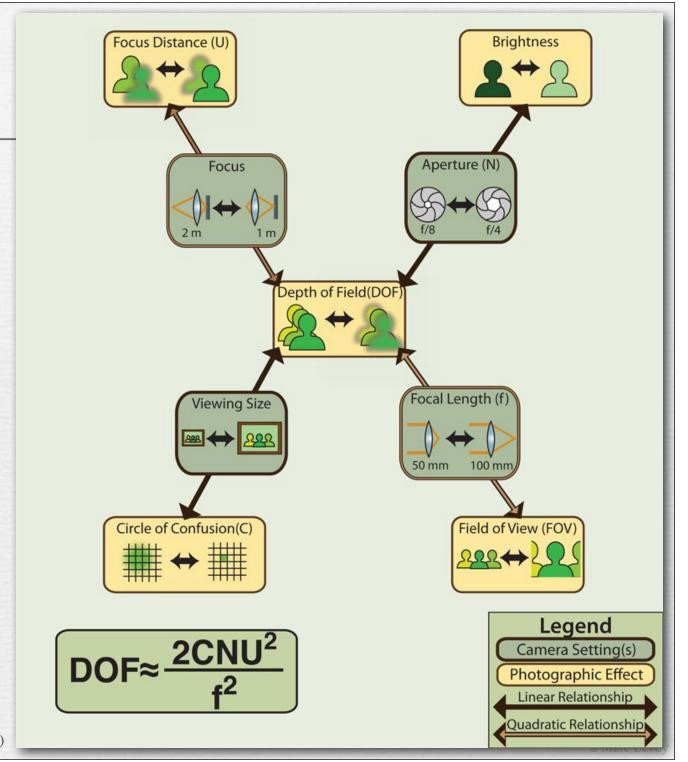
- depth of field is not a convolution of the image
 - i.e. not the same as blurring in Photoshop
 - DoF lets you eliminate occlusions, like a chain-link fence

Seeing through occlusions using a large aperture (contents of whiteboard)



- ♦ for a pixel focused on the subject, some of its rays will strike the occluder, but some will pass to the side of it, if the occluder is small enough
- the pixel will then be a mixture of the colors of the subject and occluder
- thus, the occluder reduces the contrast of your image of the subject, but it doesn't actually block your view of it

Tradeoffs affecting depth of field



(Eddy Talvala)

Recap

• depth of field (D_{TOT}) is governed by circle of confusion (C), aperture size (N), subject distance (U), and focal length (f)

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- · depth of field is linear in some terms and quadratic in others
- if you focus at the hyperfocal distance $H = f^2/NC$, everything from H/2 to infinity will be in focus
- depth of field increases linearly with decreasing sensor size
- → useful sidelights
 - bokeh refers to the appearance of small out-of-focus features
 - you can take macro photographs using a telephoto lens
 - · depth of field blur is not the same as blurring an image

